[I L / A S] IMAGE [I L / A S]

The Bulletin of the International Linear Algebra Society

Serving the International Linear Algebra Community

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ILAS President/Vice-President's Annual Report: October 1997

1. The following persons have been elected to ILAS offices with terms that began on March 1, 1997:

Board of Directors: VOLKER MEHRMANN and JANE DAY (three-year terms ending February 29, 2000).

The following persons continue in their offices to which they were previously elected:

President: RICHARD A. BRUALDI (term ends February 28, 1999).

Vice-President: DANIEL HERSHKOWITZ (term ends February 28, 1998).

Secretary/Treasurer: JAMES R. WEAVER (term ends February 29, 2000).

Board of Directors: JOSE DIAS DA SILVA and PETER LANCASTER (terms end February 28, 1998);

RAJENDRA BHATIA and LUDWIG ELSNER (terms end February 28, 1999).

Jose Dias da Silva was appointed to fill the vacancy created by the death of Robert C. Thompson.

2. The Nominating Committee for 1997 consists of: Chi-Kwong Li (chair, appointed by the president), Angelika Bunse-Gerstner and Michael Neumann (appointed by the Board of Directors), and Russell Merris and Graciano de Oliveira (appointed by the president's Advisory Committee).

3. This coming year there are elections for two positions on the Board that become vacant on March 1, 1998. The candidates chosen by the Nominating Committee are: Pauline van Den Driessche (Canada), Nicholas J. Higham (UK), Roger A. Horn (USA), and Jose Dias da Silva (Portugal). Following an election, Roger A. Horn and Jose Dias da Silva were declared as elected to the Board for the three-year term ending February 28, 2001. There is also an election for the position of Vice President that becomes vacant on March 1, 1998. Daniel Hershkowitz has held that position for one three-year term. According to ILAS by-laws, a Vice President may serve two consecutive terms and if the Vice President is renominated there may be only one nominee for that position. The Nominating Committee has chosen Daniel Hershkowitz as the nominee for the office of Vice President for the three-year term ending February 28, 2001.

4. ILAS owes a great debt of gratitude to Steven J. Leon for his important service in editing and producing *Image* for eight years. The first two issues of *Image*, published in January 1988 and January 1989, were edited by the late Robert C. Thompson. With Issue No. 3 (July 1989) and continuing through Issue No. 10 (January 1993), *Image* was jointly edited by Steven J. Leon and Robert C. Thompson. Steve was the sole Editor of *Image* for Issues No. 11 (July 1993) and No. 12 (January 1994). Issues No. 13 (July 1994) through No. 18 (Winter/Spring 1997) were jointly edited by Steven J. Leon and George P. H. Styan. Beginning with this Issue No. 19 (Summer/Fall 1997), George assumes the position of sole Editor of *Image*. His term of office ends with Issue No. 24 (Winter/Spring 2000). George is now ably assisted by S. W. Drury, Stephen J. Kirkland, Chi-Kwong Li, Simo Puntanen, Peter Šemrl, Hans Joachim Werner, and Fuzhen Zhang. On behalf of ILAS we thank Steve Leon for his work for our society, and we look forward to working with George in our capacity as President and Vice-President, respectively, of ILAS.

5. Thanks to the hard work of the Corporate Sponsors Committee (Carolyn Eschenbach, chair), ILAS currently has six sponsors: Addison-Wesley, Birkhäuser, International Publishers Distributor, MathWorks, SIAM, and Walter de Gruyter.

6. On August 14-17, 1996 the 6th ILAS Conference was held in Chemnitz, Germany at the Technical University Chemnitz-Zwickau with 189 participants from 32 countries. There were 18 invited plenary lectures and 14 presentations in 4 minisymposia. In addition, there were 101 contributed talks on a wide range of topics. Highlights of the meeting included: the presentation of the Hans Schneider prize lecture by Michael Boyle (who shared the prize with David Handelman), the Olga Taussky-Todd lecture given by Robert Guralnick on "Traces and generations of matrix algebras," and a banquet talk by Bertram Huppert. The organizing committee consisted of: Bart De Moor, Graciano De Oliveira, Ludwig Elsner, Thomas J. Laffey, Volker Mehrmann (chair), Gerhard Michler, Michael Neumann, and Frank Uhlig. The local arrangements committee consisted of: D. Happel, F. Lowke, C. Rost, and B. Silbermann.

7. The ILAS Symposium on Fast Algorithms for Control, Signals, and Image Processing was held on June 6-9, 1997 at University of Manitoba, Institute of Industrial Mathematical Sciences, Winnipeg, Canada. The symposium was held in conjunction with the summer meeting of the Canadian Mathematical Society (CMS). In addition to the symposium there were linear algebra sessions at the CMS meeting. The organizing committee consisted of Pauline van den Driessche, Thomas Kailath, Peter Lancaster, Dianne P. O'Leary, Robert J. Plemmons, Hans Schneider, P. N. Shivakumar (chair). The program committee consisted of: Moody Chu, Biswa Datta, Brent Ellerbroek, Georg Heinig, Franklin Luk, Dianne O'Leary (co-chair), Haesun Park, Robert J. Plemmons (co-chair), Ali Sayed, Hans Schneider, P. N. Shivakumar, and Paul Van Dooren.

8. The following ILAS conferences are planned for the near future:

a. The 7th ILAS Conference will be held at the University of Wisconsin, Madison, Wisconsin, USA, on June 3-6, 1998. The conference is being dedicated to Hans Schneider in recognition of his enormous contributions to linear algebra and the linear algebra community. A flyer, including a tentative program, a call for papers, and registration procedures will be mailed in October 1997 to all ILAS members. The organizing committee consists of Richard A. Brualdi (chair), Bryan Cain, Biswa Datta, Jose Dias da Silva, Shmuel Friedland, Moshe Goldberg, Uriel Rothblum, Jeffrey Stuart, Daniel Szyld, and Richard S. Varga.

b. The 8th ILAS Conference will be held at the Universitat Politecnica de Catalunya, Barcelona, Spain on July 19-22, 1999. The program committee consists of R. Bru, L. de Alba, G. de Oliveira, I. García-Planas (co-chair for local arrangements), J. M. Gracia, V. Hernandez, N. J. Higham, R. A. Horn, T. J. Laffey (co-chair), F. Mateos, F. Puerta (chair), and P. Van Dooren.

c. The 9th ILAS Conference, Technion, Haifa, Israel, summer 2001.

d. The 10th ILAS Conference, Auburn University, Auburn, USA, summer 2002.

9. ELA—*The Electronic Journal of Linear Algebra*: As of September 1997, the second volume (1997 volume) has published 2 papers. ELA's primary site is at the Technion, Haifa (Israel). Mirror sites are located in Temple University, Philadelphia, in the University of Chemnitz (Germany) and in the University of Lisbon (Portugal).

10. ILAS-NET: As of October 26, 1997, we have circulated 677 ILAS-NET announcements. ILAS-NET currently has 533 subscribers.

11. ILAS INFORMATION CENTER (IIC) has a daily average of 250 information requests (not counting FTP operations). IIC's primary site is at the Technion, Haifa (Israel). Mirror sites are located in Temple University, Philadelphia, in the University of Chemnitz (Germany) and in the University of Lisbon (Portugal).

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Technion–Haifa, Israel

Why do I want to be a Linear Algebraist?

In answer to this question, posed by Chi-Kwong Li in the last issue of *Image* [No. 18, Winter/Spring 1997, p. 23], we received the following answers:

(a) I like the linear algebra community very much; (b) Linear algebra touches a vast assortment of areas; (c) Linear algebra is very natural and intuitive; (d) One day I hope to have a licence plate with MATRIX on it.

HENRY WOLKOWICZ: henry@orion.math.uwaterloo.ca
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As a mathematician, I have always been intrigued by the contrast and interplay between the finite and discrete versus the infinite and continuous. Linear algebra provides me with the vehicle to crack open the former, which in turn sheds light on the mystery of problems in a typical setup of the latter, operator theory.

PEI YUAN WU: pywu@cc.nctu.edu.tw
National Chiao Tung University, Hsinchu, Taiwan

ILAS Treasurer's Report: March 1, 1996–February 28, 1997

by James R. Weaver, *University of West Florida, Pensacola*

Balance on hand March 1, 1996			
Certificate of Deposits (CD)	19,000.00		
Vanguard	7,962.20		
Checking Account	16,904.80		43,867.00
**			
Checking Account Balance on March 1, 1996			16,904.80
March 1996			
Income:			
Dues	180.00		
Interest (First Union)	22.05		
Interest (First Union)	13.38	215.43	
Expenses:			
Sec. of State	70.00	70.00	145.43
April 1996			
Income:			
Dues	60.00		
Interest on CD (FU)	23.38		
Interest (First Union)	15.00		
Contributions			
ILAS- 6th Conf.	240.00	338.38	
Expenses:			
Production and Mail "IMAGE"	1193.78	1,193.78	(855.40)
May 1996			
Income:			
Dues	100.00		
Interest on CD (HS)	75.38		
Interest on CD (OT/JT)	29.32		
Interest on CD (First Union)	68.42		
Interest (First Union)	14.31		
Contributions			
ILAS-6th Conf.	870.00		
Hans Schneider Prize	30.00	1,187.43	
Expenses:			
Postmaster (Stamps & Mailing)	54.66		
Benton's (3 HS Plaques)	128.40		
Lisa M. Weaver (Prep. of 95/96 Ann. Trea. Rep.)	48.60	231.66	955.77
June 1996			
Income:			
Dues	00.00		
Interest (First Union)	14.22		
Contributions			
ILAS-6th Conf.	1320.00		
OT/JT Fund	310.14	1,644.36	
Expences:	00.00	00.00	1,644.36
July 1996			
Income:			
Dues	180.00		
Interest (First Union)	17.49		
Interest on CD (FU)	23.62		
Interest on CD (GEN)	69.94		
Interest on CD (HS)	75.38		
Interest on CD (OT/JT)	29.32		
Contributions			
General	1.00		
ILAS-6th Conf.	1120.00	1,516.75	

Expenses:			
Postmaster (Stamps)	78.08		
Postmaster (Stamps-Dues Not.)	137.30		
James C Weaver (Supplies & Preparing Dues Not.)	89.97	305.35	1,211.40
August 1996			
Income:			
Dues	00.00		
Interest (First Union)	15.44	15.44	
Expenses:			
M. Michael Boyle- Hans Schneider Prize	1100.00		
Robert Guralnick- OT/JT Lecture Fund	1300.00		
ILAS-6th Conf.	3000.00		
Refund-6th Conf.	140.00	5,540.00	(5,524.56)
September 1996			
Income:			
Dues	3025.22		
Interest on CD (GF)	69.95		
Interest on CD (GF)	69.95		
Interest (First Union)	15.36		
Contributions			
General	81.00		
Conference	50.00		
H. Schneider Prize	256.00		
F. Uhlig Fund	91.00		
OT/JT Lec. Fund	246.00		
ILAS-6th Conf.	980.00	4,884.48	
Expenses:			
Lisa M. Weaver(Recording & Updating Files)	102.00		
Office Depot-Supplies	33.85		
Postmaster (Stamps & Mailing)	241.89		
Mail Boxes ETC (400 Ballots)	34.24		
James C. Weaver(ILAS Files & Preparing/Mailing Ballots)	102.00		
Service Charge	2.10	516.08	4,368.40
October 1996			
Income:			
Dues	00.00		
Interest (First Union)	15.93	15.93	
Expenses:			
Producing & Mailing "IMAGE" George P.H. Styan	1333.11	1,333.11	(1,317.18)
November 1996			
Income:			
Dues	1120.00		
Interest (First Union)	14.78		
Interest on CD (GF)	69.94		
Interest on CD (OT/JT)	29.64		
Interest on CD (HS)	76.23		
Interest on CD (FU)	23.63		
Contributions			
General	120.00		
H. Schneider Prize	30.00		
OT/JT Lec. Fund	210.00		
Conference	10.00		
ILAS-6th Conf.	545.00		
Addison Wesley Pub.	200.00		
SIAM	200.00	2,649.22	
Expenses:			
Office Depot (Supplies)	47.06		
Postmaster (Stamps)	141.87		

Refund-6th Conf.	60.00		
Lias M. Weaver (ILAS Files & Mailing)	79.50	328.43	2,320.79
December 1996			
Income:			
Dues	540.00		
Interest (First Union)	17.34		
Interest on CD (FU)	23.38		
Interest on CD (GF)	69.18		
Contributions			
OT/JT Lec. Fund	100.00		
H. Schneider Prize	120.00		
F. Uhlig Ed. Fund	100.00		
General	56.00		
Walter De Grayter, Inc.	200.00		
Inter.Pub.Dist.Inc.	200.00	1,425.90	
Expenses:	00.00	00.00	1,425.90
January 1997			
Income:			
Dues	820.00		
Interest (First Union)	18.29		
Interest on CD (HS)	76.23		
Interest on CD (OT/JT)	29.64		
Contributions			
H.Schneider Prize	85.00		
General	20.00		
OT/JT Lec. Fund	10.00	1,059.16	
Expenses:			
James C. Weaver (Update ILAS Records)	45.00	45.00	1,014.16
February 1997			
Income:			
Interest (First Union)	16.57		
Expenses:	00.00	00.00	16.57

****

February 28, 1997		22,310.44
Account Balance		
Checking Account	22,310.44	
Certificate of Deposit (FU)	1,500.00	
Certificate of Deposit (Gen.)	10,000.00	
Certificate of Deposit (72% HS & 28% OT/JT)	7,500.00	
Vanguard (72% HS & 28% OT/JT)	8,342.84	49,653.28
General Fund	20,237.08	
Frank Uhlig Educational Fund	2,392.26	
Hans Schneider Prize	13,321.65	
Olga Taussky Todd/John Todd Fund	6,573.41	
Conference Fund	7,128.88	49,653.28

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James R. Weaver
June 17, 1997

ILAS Treasurer's Report
Supplement from
Sparkasse Chemnitz
March 1, 1996 - February 28, 1997
Report by Volker Mehrmann and James R. Weaver

DM = Deutsche mark

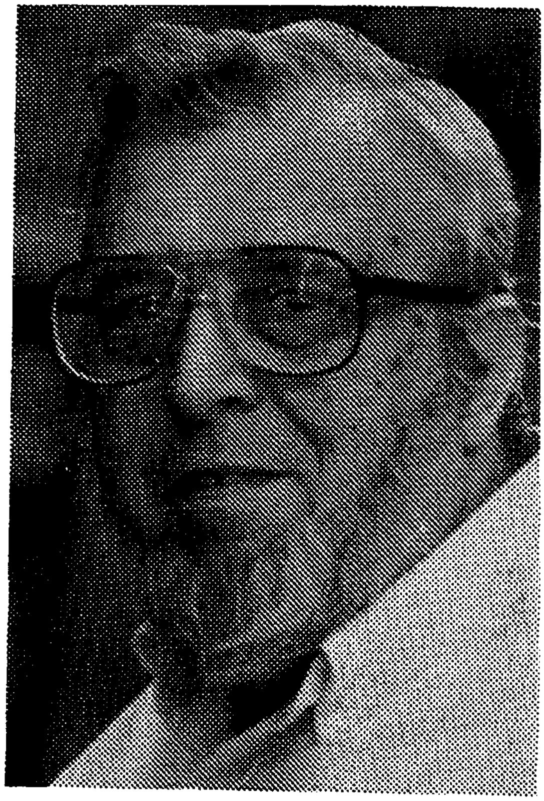
	DM	DM	DM
Balance on hand March 1, 1996			414.52
March 1996			
Income:			
Fees	45.00		
Fees	45.00	90.00	
Expenses:			
Fees	10.50	10.50	79.50
April 1996			
Income:			
Check	235.00		
Fees	130.50		
Fees	90.00		
Fees	45.00	500.50	
Expenses:	00.00	00.00	500.50
May 1996			
Income:			
Fees	90.00		
Check	60.00		
Check	165.00		
Fees	360.00		
Fees	180.00		
Fees	45.00		
Check	100.00		
Check	75.00		
Fees	57.40		
Check	390.00		
Fees	45.00		
Fees	90.00		
Fees	105.00		
Check	240.00		
Check	70.00	2.072.40	
Expenses:	00.00	00.00	2.072.40
June 1996			
Income:			
Check	165.00		
Check	55.00		
Fees	90.00		
Check	55.00		
Check	195.00		
Fees	180.00		
Fees	75.00		
Fees	45.00		
Fees	45.00		
Fees	75.00		
Fees	67.40		
Fees	105.00		
Check	120.00		
Fees	45.00		
Fees	45.00		
Check	100.00		
Fees	105.00		
Cash	15.00	1.582.40	
Expenses:			
Fees	20.10	20.10	1.562.30

July 1996			
Income:			
Check	430.00		
Fees	150.00		
Check	120.00		
Fees	67.40		
Check	150.00		
Check	130.00		
Fees	105.00		
Transfer	90.00	1.242.40	
Expenses:			
Transfer	336.64		
Transfer	313.38	650.02	592.38
August 1996			
Income:			
Check	130.00		
Check	70.00		
Check	105.52		
Check	4.420.60		
Check	144.00		
Cash	7.400.00		
Check	500.00		
Cash	1.171.00		
Check	500.00		
Check	500.00	14.941.12	
Expenses:			
Cash	514.50		
Cash	442.49		
Cash	900.00		
Transfer	165.00		
Transfer	126.50		
Transfer	1.190.00		
Transfer	604.38		
Transfer	6.051.30		
Transfer	3.436.44		
Transfer	693.62		
Transfer	150.00		
Fees	2.50		
Transfer	59.95		
Transfer	4.738.00	19.074.68	(4.133.56)
September 1996			
Income:	00.00	00.00	
Expenses:			
Fees	19.00		
Check	240.00		
Int.	4.72	263.72	(263.72)
October 1996			
Income:			
Interest	4.72	4.72	
Expenses:	00.00	00.00	4.72
November 1996			
Income:	00.00	00.00	
Expenses:			
Closing	727.44		
Fees	6.60		
Check	95.00	829.04	(829.04)

****
Account Balance (ILAS Conference 1996/97) February 28, 1997 00.00
****

James R. Weaver
June 17, 1997

John Stanley Maybee: 1928–1997



As a father of qualitative matrix theory, John Stanley Maybee was one of the earliest mathematicians to understand how deeply combinatorial thought can contribute insight into matrix structure.

After an undergraduate degree at the University of Maryland, John received his Ph.D. from the University of Minnesota in 1956 under Paul Rosenbloom. Always motivated by applications, he was a postdoc at the Courant Institute, and his thesis and initial work were in PDE's. His interests gradually drew him into matrix problems that lay at the heart of applications, and these matrix problems frequently involved a combinatorial/graph-theoretic aspect. In the mid-60's John was in the mathematics department at Purdue University; there in the economics department at the time was a remarkable collection of soon to be famous researchers, many of whom helped with the birth of modern mathematical economics. It was at this time that John developed an interest in qualitative matrix theory and, in papers with James Quirk and Lowell Bassett, developed the initial principles.

John moved to the University of Colorado in 1967, where he stayed until his retirement in 1993. Matrix and graph theory became the main emphasis of his research in Boulder, but he maintained his interest in modern applications as a consultant to the Department of Energy. John contributed to and in several cases began the study of several topics in matrix and graph theory, including sign stability, sign nonsingular and nearly-sign nonsingular matrices, tournament matrices, L-functions, competition graphs, bi-clique coverings of digraphs, inverse 0-patterns, and other topics in qualitative matrix theory and in graph theory. Before his retirement, he helped start the Applied Mathematics Department in the University of Colorado at Boulder and became an initial member. John's career was honored with a special meeting on combinatorial matrix analysis at his retirement (which Charlie helped to organize, and at which both Charlie and Tom gave a talk) and a subsequent special issue of *Linear Algebra and Its Applications* (LAA). But, like all good researchers, John had a deep curiosity about his mathematics and continued active research after retirement, including regular research discussions with both of us until his sudden death from a heart attack on May 2, 1997.

John blessed the lives of many, and all of us who had the pleasure of knowing him will greatly miss him. Charlie learned of John's work on qualitative matrix theory while in college and this was an early inspiration for his long time interest in matrices. Later on, interaction with John was one of the great pleasures of working in matrix theory, and John spent his last sabbatical at William and Mary with us. John was not only Tom's Ph.D. thesis advisor but a continuing collaborator and friend.

John is survived by nearly 100 papers (a few more to come), three books (including an upcoming volume on the applications of qualitative matrix theory), seven Ph.D. students, Carol his wife since 1955) and six children. Besides being good at and displaying an infectious enjoyment for mathematics, John had a keen enjoyment of life. His enthusiasm and vast store of anecdotes brightened many professional gatherings. Those who had the opportunity to attend one of the lovely dinners hosted by John and Carol (and, perhaps, some of the kids) were certain to remember the experience. John was nearly as good and inspired in the kitchen as in mathematics.

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International Linear Algebra Society (ILAS) Symposium on Fast Algorithms for Control, Signals and Image Processing

Winnipeg, Manitoba, Canada: June 6–8, 1997

Report by P. N. Shivakumar

This International Linear Algebra Society (ILAS) Symposium was organized by the Institute of Industrial Mathematical Sciences (IIMS) at the University of Manitoba, Canada. The Symposium was a Participating Institutions Conference of the Institute for Mathematics and its Applications (IMA) and was co-sponsored by the Fields Institute, the Centre de Recherches Mathématiques (Montréal) and the Manitoba HVDC Research Center. The Principal Symposium Organizers were: P. N. Shivakumar (IIMS, Univ. of Manitoba), Dianne P. O'Leary (Univ. of Maryland), Hans Schneider (Univ. of Wisconsin–Madison), and Robert J. Plemmons (Wake Forest University, Winston-Salem).

The three-day meeting brought together researchers from the areas of Control Theory, Signal and Image Processing, and Computational Linear Algebra to discuss recent advances, trends, and future directions for research on fast algorithms. This interdisciplinary gathering emphasized modern methods in scientific computing and relevant linear algebra. On the first day, three two-hour short courses were given. Stephen Boyd (Stanford) spoke on Convex Optimization in Control, Signals and Image Processing; Raymond Chan (Hong Kong) discussed Iterative Methods for Toeplitz Systems; Tom Kailath (Stanford) gave an introduction to Fast Algorithms for Structured Matrices. These talks are available on the authors' websites. Plenary talks were given by C. C. Paige (McGill), G. W. Stewart (Maryland), Haesun Park (Minnesota), L.Kaufman (Bell Labs), M. Hanke (Karlsruhe), E. Chu (Guelph), and S. Qiao (McMaster). In addition there were six minisymposia and three contributed paper sessions. In all, there were about 80 participants in the Symposium.

An important feature of the meeting was the overlap with the Canadian Mathematical Society (CMS) Summer meeting (June 6–9). A special Session on Linear Algebra at the CMS Meeting (June 9) was organized by IIMS with main speakers Roger Horn (Utah) and Paul van Dooren (Louvain). This was a unique experience with the sessions of each meeting open to the other and having common coffee breaks, banquet and excursions. Several members of the Symposium also took part in a Graduate Student Seminar of CMS, organized by IIMS. The speakers were Hans Schneider and Paul van Dooren. Social occasions included the banquet, a 'Dinner in the City', and a CMS/ILAS barbecue. After the meeting there were many letters of praise regarding the scientific content and the friendly atmosphere of the meetings. A special issue of *Linear Algebra and Its Applications* will be devoted to papers presented at this Symposium.

I$\begin{bmatrix}1 & 9\\ 9 & 7\end{bmatrix}_{\text{stanbul}}$

Sixth International Workshop on "Matrix Methods for Statistics"

İstanbul, Turkey: August 16–17, 1997

Report by Hans Joachim Werner

The Sixth International Workshop on "Matrix Methods for Statistics" was held on the top floor of The Marmara Hotel in İstanbul, Turkey, on Saturday–Sunday, August 16–17, 1997, the weekend immediately preceding the 51st Session of the *International Statistical Institute* (ISI). This Workshop was co-sponsored by the Turkish Scientific and Technical Research Council (TÜBİTAK), the Turkish Statistical Society, and the International Linear Algebra Society (ILAS). The Local Organizing Committee comprised Fikri Akdeniz (University of Çukurova, Turkey; chair), Ömer L. Gebizlioğlu (University of Ankara, Turkey), and Cemil Yapar (Karadeniz Technical University, Turkey). The International Programme Committee comprised R. William Farebrother (Victoria University of Manchester, England), Simo Puntanen (University of Tampere, Finland), George P. H. Styan (McGill University, Canada; vice-chair), and Hans Joachim Werner (University of Bonn, Germany; chair). This Workshop was the sixth in an ongoing series. The previous five Workshops were held as follows: (1) Tampere, Finland: August

1990, (2) Auckland, New Zealand: December 1992, (3) Tartu, Estonia: May 1994, (4) Montréal, Québec, Canada: July 1995, and (5) Shrewsbury, England: July 1996. The Seventh International Workshop on Matrices and Statistics, with Special Emphasis on Multivariate Analysis in Celebration of T. W. Anderson's 80th Birthday will be held in the Summer 1998; for further details contact George Styan at `styan@Math.McGill.CA`.

The purpose of this Workshop was to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between matrix theory and statistics. Participants came from Austria, Belgium, Bulgaria, Canada, Czech Republic, Estonia, Finland, Germany, Japan, The Netherlands, Portugal, Sweden, Switzerland, Turkey, United Kingdom, and the United States. Funding for some travel and local expenses was supported in part by the Turkish Scientific and Technical Research Council (TÜBİTAK).

The Workshop began with a talk by the President of the Turkish Statistical Association: Ömer L. Gebizlioğlu, *Ankara University*, on: "The Turkish Statistical Association, Statistics in Turkey, and Statisticians in Turkey". This was followed by 35 papers presented in person in 8 plenary sessions; there were also 3 papers presented "by title". These 38 papers were

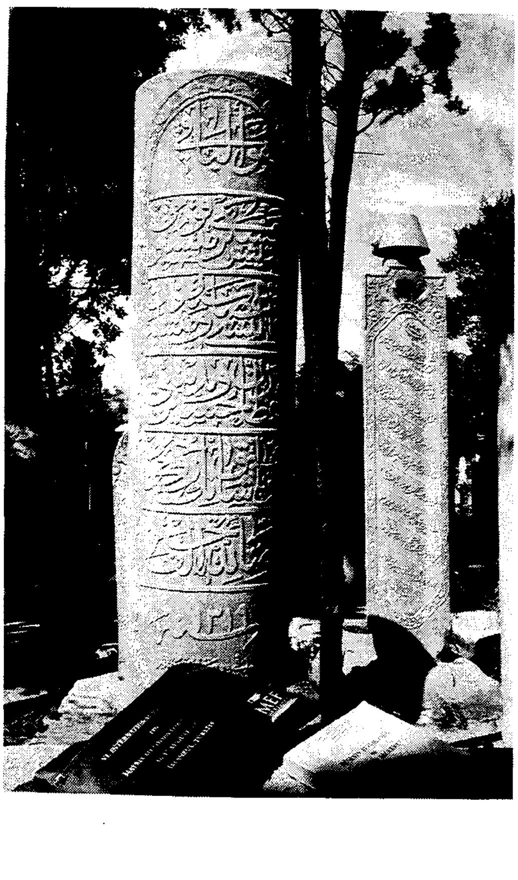
1. Abdul Sattar Rashid Salim AL-KHALİDİ *(Dokuz Eylul University, Izmir)*: Global Minimum Solutions of Nonparametric Estimation Problems for Truncated Distributions
2. T. W. ANDERSON *(Stanford University)*: Asymptotic Distributions of Characteristic Roots and Vectors with Different Rates of Convergence
3. Tsuyoshi ANDO *(Hokusei Gakuen University, Sapporo)*: Eigenvalue Inequalities for Hadamard Products
4. Olcay ARSLAN *(Çukurova University, Adana)* and Nedret BİLLOR* *(Çukurova University, Adana)*: Robust Liu Estimator for Regression Based on M-Estimator
5. Georgi N. BOSHNAKOV *(University of London and Bulgarian Academy of Sciences, Sofia)*: Multi-companion matrices
6. N. Rao CHAGANTY* *(Old Dominion University, Norfolk)*, John J. SWETITS *(Old Dominion University, Norfolk)* and Akhil K. VAISH *(University of North Carolina, Charlotte)*: Spectral Value Type Decomposition of a Positive Definite Matrix
7. Miroslav FIEDLER *(Academy of Sciences of the Czech Republic, Prague)*: Ultrametric Matrices and Cluster Analysis
8. Ömer L. GEBİZLİOĞLU* *(Ankara University)* and Fazıl A. ALİEV *(Ankara University)*: On the Robust Estimation for Discrete Space Spatial Processes
9. Gene H. GOLUB *(Stanford University)*: Efficient Algorithms for Least-squares Type Problems
10. Ulrike GRÖMPING *(University of Dortmund)*: One-sided Likelihood Ratio Tests for Linear Inequalities on the Parameters of Normal Linear Models
11. Patrick J. F. GROENEN* *(Leiden University)*, Willem J. HEISER *(Leiden University)* and Jacqueline J. MEULMAN *(Leiden University)*: Iterative Majorization in Distance Smoothing for Multidimensional Scaling to Avoid Local Minima
12. Ali S. HADI* *(Cornell University)* and Hans NYQUIST *(University of Umeå)*: Fréchet Distance as a Diagnostic Tool for Diagnosing Elliptically Symmetric Multivariate Data
13. Selahattin KAÇIRANLAR* *(Çukurova University, Adana)*, Sadullah SAKALLIOĞLU *(Çukurova University, Adana)*, Fikri AKDENİZ *(Çukurova University, Adana)* and George P. H. STYAN *(McGill University, Montréal)*: A New Biased Estimator in Linear Regression and Comparisons with Some Other Estimators
14. André KLEIN* *(University of Amsterdam)*, Guy MÉLARD *(Université Libre de Bruxelles)* and Toufik ZAHAF *(Université Libre de Bruxelles)*: Construction of the Exact Fisher Information Matrix of Time Series Models by means of Matrix Differentiation Rules
15. Tõnu KOLLO* *(University of Tartu)* and Maria ZELTSER *(University of Tartu)*: Pattern Matrix and Its Generalizations
16. Alexander KOVAČEC *(University of Coimbra)*: On some Occurrences and Characterizations of the Bruhat Order of the Symmetric Group
17. Wolfgang POLASEK *(University of Basel)* and Shuangzhe LIU* *(University of Basel)*: MANOVA Models: A Bayesian Analysis
18. Song-Gui WANG *(Chinese Academy of Sciences, Beijing)*, Erkki P. LISKI *(University of Tampere)* and Tapio NUMMI* *(University of Tampere)*: TWO-WAY Selection of Covariables in Multivariate Growth Curve Models
19. Fikri ÖZTÜRK* *(University of Ankara)* and Fikri AKDENİZ *(Çukurova University, Adana)*: Ill-Conditioning and Multicollinearity
20. Albert W. MARSHALL *(The University of British Columbia, Vancouver)* and Ingram OLKIN* *(Stanford University)*: Functional Equation Characterizations of Non-normal Distributions
21. Mustafa Ç. PINAR *(Bilkent University, Ankara)*: On the Finite Computation of the ℓ_1 Estimator from Huber's M-Estimator in Linear Regression
22. Vlastimil PTÁK *(Academy of Sciences of the Czech Republic, Prague)*: Geometric Means of Operators
23. Simo PUNTANEN *(University of Tampere)*: Comparing Estimators in Reduced Linear Models
24. Simo PUNTANEN* *(University of Tampere)* and George P. H. STYAN *(McGill University, Montréal)*: A Second Guide to Books on Matrices and Books on Inequalities, with Statistical and Other Applications
25. C. Radhakrishna RAO *(The Pennsylvania State University, University Park)*: Statistical Solutions to Some Matrix Problems
26. Burkhard SCHAFFRIN *(The Ohio State University)*: Softly Unbiased Estimation — Part I: The Gauss-Markov Model
27. Gülhan ALPARGU *(McGill University, Montréal)* and George P. H. STYAN* *(McGill University, Montréal)*: Some Comments on the Wielandt Inequality and Its Connection to the Frucht-Kantorovich Inequality

28. Gülhan ALPARGU *(McGill University, Montréal)* and George P. H. STYAN* *(McGill University, Montréal)*: A Further Bibliography on the Frucht-Kantorovich Inequality and on Some Related Inequalities
29. S. W. DRURY *(McGill University, Montréal)* and George P. H. STYAN* *(McGill University, Montréal)*: An Extension of an Inequality of Bloomfield-Watson and Knott
30. Yoshio TAKANE* *(McGill University, Montréal)* and Haruo YANAI *(National Center for University Entrance Examination, Tokyo)*: On Oblique Projectors
31. Ene-Margit TIIT *(University of Tartu)*: Boundary Correlation Matrices
32. Imbi TRAAT *(University of Tartu)*: A Note on Matrix Moments and Cumulants
33. Yimin WEI *(Fudan University, Shanghai)*: The Weighted Moore-Penrose Inverse of Modified Matrices
34. Hans Joachim WERNER *(University of Bonn)*: Some Matrix Methods for the General Gauss-Markov Model
35. Simo PUNTANEN *(University of Tampere)*, George P. H. STYAN *(McGill University, Montréal)* and Hans Joachim WERNER* *(University of Bonn)*: Two New Accessible Proofs that the Linear Estimator Gy is the Best Linear Unbiased Estimator
36. Yimin WEI *(Fudan University, Shanghai)* and Hans Joachim WERNER* *(University of Bonn)*: Some Further Results on $\{2\}$-Inverses
37. Kenichi KIKUCHI *(National Center for University Entrance Examination, Tokyo)* and Haruo YANAI* *(National Center for University Entrance Examination, Tokyo)*: Orthogonal Projectors onto the Intersection of Subspaces and Their Applications to Multivariate Linear Models
38. Cemil YAPAR* *(Karadeniz Technical University, Trabzon)*, İhsan ÜNVER *(Karadeniz Technical University, Trabzon)* and Selahattin MADEN *(Karadeniz Technical University, Trabzon)*: On Using Inequality Constrained Least Squares to Delineate the Effects of Misspecification in Linear Models

Please visit the website `http://eos.ect.uni-bonn.de/werner.html` for the complete Workshop Programme booklet, with abstracts (in English) of all 38 papers. Nicely printed (hard) copies of the two bibliographies [24] and [28], bound together (80 pp.), and/or soft copies in LaTeX, are available from George Styan: `styan@Math.McGill.CA`. It is expected that selected fully-refereed papers from this Sixth International Workshop will be published in the Seventh Special Issue on Linear Algebra and Statistics of *Linear Algebra and Its Applications*, edited by Simo Puntanen, George P. H. Styan and Hans Joachim Werner—the Sixth Special Issue has just been published: vol. 264 (October 1997).

A delicious Workshop Dinner (featuring çipura = gilt-head bream) was served at the Süper Köşem Restaurant by the edge of the Bosphorus, followed by an enjoyable Belly Dancer Show. Since many participants were accompanied by their spouses, Süheyla Akdeniz (Adana, Turkey) and Magdala Werner (Meckenheim, Germany) organized spontaneously two tours: a visit of the Topkapı Palace on the Saturday, and a Bosphorus Cruise on the Sunday. Many thanks go to both of them! Without their enormous help these tours would not have been possible.

In *Image*, No. 15, Summer 1995, p. 24, we challenged readers to identify a certain gentleman and asked "Is his book on Linear Algebra, published in 1882, the first book ever published on Linear Algebra?" In *Image*, No. 16, Winter 1996, p. 29, we identified the gentleman as Hüseyin Tevfik Paşa (1832–1901) and the book as *Linear Algebra*, published (in English) by A. H. Boyajian in Constantinople in 1882. Guided by Fikri Akdeniz, we visited the tomb of Hüseyin Tevfik Paşa in the Eyüp cemetery in İstanbul on Tuesday, August 19, 1997. The tomb (photo by Simo Puntanen) is located on the right-hand side of Beybaba Street on the way from the tomb of Ferhat Paşa to the tomb of Feridun Bey. The inscription on the tomb, translated into English, is "He is eternal. Pray for the soul of Hüseyin Tevfik Paşa from Vidin, member of the High Military Inspection Committee and one of the High Marshals."



"In the unpublished Memoirs of the noted Ottoman mathematician Salih Zeki, Hüseyin Tevfik Paşa appears as a man of exceptional ability and honest character who everywhere encouraged scientific research. He reached as high a point in his scientific endeavours as permitted by his sterile environment. This remarkable man in the history of science in Turkey was later almost entirely forgotten and all his works written in Turkish were regrettably lost! Of his venerable memory, only a street bearing his name in Vezneciler (a quarter of İstanbul south of the Golden Horn and west of the Sultan Ahmet Square or Hippodrome) and a mute tombstone in the Eyüp cemetery remain." —Kâzım Çeçen (1988), *Hüseyin Tevfik Paşa ve "Linear Algebra"*, İstanbul Teknik Üniversitesi Bilim ve Teknoloji Tarihi Araştırma Merkesi Yayın, no: 5, 1988, p. 15.

ods for Statistics" — İstanbul, Turkey: August 16–17, 1997.
imo Puntanen.

International Calendar of Events in Linear Algebra & Related Topics

1998

January 5–9: Haifa, Israel. 10th Haifa Matrix Theory Conference. Dept of Mathematics & Institute for Advanced Studies in Mathematics, Technion. [Dept of Mathematics, Technion–Israel Institute of Technology, Haifa 32000; `hershkow@tx.technion.ac.il`, `http://math.technion.ac.il/iic`]

June 1–2: Madison, Wisconsin. Fourth Workshop on Numerical Ranges and Numerical Radii. Dept of Mathematics, Univ of Wisconsin. [Chi-Kwong Li, Dept of Mathematics, College of William and Mary, Williamsburg, VA 23187; `http://www.math.wm.edu/~ckli/wonra.html`]

June 3–6: Madison, Wisconsin. 7th International Linear Algebra Society (ILAS) Conference: The "Hans Schneider Linear Algebra" Conference. Univ of Wisconsin–Madison. [Richard Brualdi, Dept of Mathematics, Univ of Wisconsin, Van Vleck Hall, 480 Lincoln Drive, Madison, WI 53706-1388; `brualdi@math.wisconsin.edu`]

Dates and Location TBA: 7th International Workshop on Matrices and Statistics, with Special Emphasis on Multivariate Analysis in Celebration of T. W. Anderson's 80th Birthday (Summer 1998). [George Styan: `styan@Math.McGill.CA`]

July 24–25: Winnipeg, Manitoba. 2nd International Conference on Matrix-Analytic Methods in Stochastic Models. [A. S. Alfa, Dept Mechanical & Industrial Engrg, Univ of Manitoba, Winnipeg, Manitoba R3T 5V6; `alfa@cc.umanitoba.ca`]

July 30–31: Victoria, British Columbia. 4th Western Canada Linear Algebra Meeting. [Steve Kirkland, Dept of Mathematics and Statistics, Univ of Regina, Regina, Saskatchewan S4S 0A2; `kirkland@math.uregina.ca`]

August 3–7: Lethbridge, Alberta. Mini-Course on Coding, Cryptography and Computer Security. Univ of Lethbridge. [Hadi Kharaghani; `http://www.cs.uleth.ca/workshop/`]

August 8–13: Zhangjiagye, Hunan Province, China. 3rd China Matrix Theory Conference. Chinese Mathematical Society & Xiangtan Polytechnic Univ. [Bit-Shun Tam: `bsm01@mail.tku.edu.tw`, Dept of Mathematics, Tamkang Univ, Tamsui, Taiwan 25137]

1999

July 19–22: Barcelona, Spain. 8th International Linear Algebra Society (ILAS) Conference. Universitat Politecnica de Catalunya. [Isabel García-Planas, Marqués de Sentmenat 63-4-3, E-08029 Barcelona; `igarcia@mal.upc.es`]

August 7–8: Tampere, Finland. 8th International Workshop on Matrices and Statistics. [Simo Puntanen, Dept of Mathematical Sciences, Univ of Tampere, P.O. Box 607, FIN-33101 Tampere, Finland; `sjp@uta.fi`]

August 10–18: Helsinki, Finland. 52nd Biennial Session of the International Statistical Institute (ISI). [Ilkka Mellin, Statistics Finland, FIN-00022 Statistics Finland; `isi99@stat.fi`, `http://www.stat.fi/isi99/`]

August 19–23: Tartu, Estonia. 6th Tartu Conference on Multivariate Statistics. Univ of Tartu. [Ene-Margit Tiit, Inst. Math. Statistics, Univ of Tartu, Liivi 2, EE-2400 Tartu; `etiit@ut.ee`]

2000

May 15–20: Guanajuato, México. 5th World Congress 2000. Centro de Investigación en Matematicas. [V Pérez-Abreu, CIMAT, AP 402, Guanajuato, Gto. 36000; `pabreu@buzon.main.conacyt.mx`]

2001

Dates TBA: Haifa, Israel. 9th International Linear Algebra Society (ILAS) Conference: Challenges. [Richard Brualdi, Dept of Mathematics, Univ of Wisconsin, Madison, WI 53706-1388; `brualdi@math.wisconsin.edu`]

Dates TBA: Seoul, Korea. 53rd Biennial Session of the International Statistical Institute (ISI). [ISI Permanent Office, 428 Prinses Beatrixlaan, Postbus 950, NL-2270 AZ Voorburg; `isi@cs.vu.nl`]

2002

Dates TBA: Auburn, Alabama. 10th International Linear Algebra Society (ILAS) Conference: Challenges. [Frank Uhlig, Dept of Mathematics, Auburn Univ, Alabama, AL 36849-5310; `uhligfd@mail.auburn.edu`] *See Image 16:28.*

See also: `http://www.math.technion.ac.il/iic/conferences.html`; `http://www.netlib.org/confdb/conf_list.html`, `http://www.ams.org/mathcal/`, *and* `http://wwwmath.uni-muenster.de/math/inst/statistik/publ/icse/index.html`.

Seventh Conference of the International Linear Algebra Society
The "Hans Schneider Linear Algebra" Conference
Madison, Wisconsin, USA: June 3–6, 1998

The 7th Conference of the International Linear Algebra Society: The "Hans Schneider Linear Algebra" Conference will provide an opportunity to bring together researchers and educators in all aspects of pure and applied linear algebra and matrix theory in order to allow for a broad exchange of ideas and dissemination of recent developments and results. The conference will be dedicated to Hans Schneider in recognition of his enormous contributions to linear algebra and the linear algebra community.

THEMES: Algebraic, analytic and combinatorial matrix theory, numerical linear algebra, matrix perturbations, matrix stability and applications in engineering, linear algebra in control and systems theory, splines and linear algebra, applications of linear algebra to statistics, linear algebra education.

ORGANIZING COMMITTEE: R. A. Brualdi (chair), B. Cain, B. Datta, J. Dias da Silva, S. Friedland, M. Goldberg, U. Rothblum, J. Stuart, D. Szyld, and R. Varga.

INVITED SPEAKERS: R. Barmish, Madison (USA), R. Bhatia, New Delhi (India), G. Harel, West Lafayette (USA), D. Hershkowitz, Haifa (Israel), N. Higham, Manchester (England, UK), T. Laffey, Dublin (Ireland), P. Lancaster, Calgary (Canada), R. Loewy, Haifa (Israel), J. McDonald, Regina (Canada), V. Mehrmann, Chemnitz (Germany), U. Rothblum, Haifa (Israel), and F. Silva, Lisbon (Portugal). The ILAS-LAA Lecturer will be C. de Boor, Madison (USA).

INVITED MINISYMPOSIA: Topological methods in linear algebra (M. Goldberg), Linear algebra methods in statistics (H. J. Werner), Graph theory and linear algebra (R. Merris), Numerical linear algebra (M. Overton), Matrix inertia and stability (B. N. Datta), and Educational issues in linear algebra (D. Carlson & F. Uhlig).

CALL FOR PAPERS: Contributed papers from all areas of linear algebra and its applications are solicited. Papers that fall within the scope of the conference will be scheduled for 15 minute presentations (with 5 minute intervals between talks) in parallel sessions. In order to be assured of getting on the program, an abstract (at most one page) must be submitted by March

Miriam and Hans Schneider: Photo by Harold V. Henderson in Dunedin, New Zealand, August 1995.

1, 1998: By e-mail (preferred) to: `ilas98@math.wisc.edu` or to: ILAS98-Madison, Dept. of Mathematics, University of Wisconsin, Madison, Van Vleck Hall, 480 Lincoln Drive, Madison, WI 53706-1388, USA. Abstracts should mention all authors and their affiliations, specify key words, and identify the speaker and give her or his e-mail address. A LaTeX-template for abstracts is available on the website: `http://www.math.wisc.edu/~brualdi/template`. It can also be obtained by anonymous ftp to `brualdi.math.wisc.edu`. The LaTeX-template is also available from the ILAS Information Center. Speakers are urged to use the abstract template if at all possible.

CONFERENCE PROCEEDINGS: A special issue of the journal *Linear Algebra and Its Applications* (LAA), with special editors B. Cain, B. N. Datta, M. Goldberg, U. Rothblum, and D. Szyld. The issue will be dedicated to Hans Schneider. Speakers who wish to have their paper considered for publication in this special issue of LAA should submit their paper to one of the special editors. Only those papers that meet the standards of LAA and that are approved by the normal refereeing process will be published in the special issue. Deadline for submission is September 30, 1998.

HOUSING: An entire (air-conditioned) dormitory, Chadbourne Hall, a two-minute walk to the mathematics building Van Vleck Hall, has been reserved for conference participants. A special floor will accommodate couples. Breakfast is included in the room rate of $40 single and $27 per person in a double room. Conference participants are encouraged to stay at Chadbourne Hall. To reserve a room in Chadbourne Hall complete and then return the enclosed form to the address indicated on the form. Full payment will be due at check-in. Rooms have also been set aside at a nearby Howard Johnson's Hotel. Those wishing to stay at the Howard Johnson's Hotel should make their reservations directly by calling: 608-251-5511. Mention the conference and UW to get the conference rate of $74 single and $84 per person in a double room.

BANQUET: A banquet will be held on Friday, June 5, 1998, starting at 7 p.m., in the Great Hall of the Memorial Union (the Union on the Terrace overlooking Lake Mendota) of UW-Madison. Dinner choices will be Boneless Cornish Game Hen, Stuffed Rainbow Trout, and Eggplant & Pasta (vegetarian). Dessert will be the world famous UW fudge-bottom pie. After dinner, several people will offer reminiscences and vignettes about Hans Schneider.

EXCURSION: An excursion is scheduled for Saturday afternoon, June 6, 1998, to the famous "House on the Rock" in nearby Spring Green. This is more than a fascinating house; it is a veritable large museum of remarkable mechanical and electrical contraptions with an incredible in-door carousel. Not to be missed! The cost of the excursion is $20.

FINANCIAL SUPPORT: The conference has been approved and partially funded by the Participating Institutions of the Institute for Mathematics and its Applications (IMA) in Minneapolis. Approval implies that a member of a university which is a IMA Participating Institution can request from her or his Dept. Chair to use PI funds held at the IMA to attend the conference. Other sources of support are currently being investigated.

NUMERICAL RANGE WORKSHOP: A workshop on the numerical range will precede the conference on June 1–2, 1998. For information contact Chi-Kwong Li, Dept. of Mathematics, The College of William and Mary, Williamsburg, VA 23187, USA; `ckli@math.wm.edu`, or visit the website: `http://www.math.wm.edu/~ckli/wonra.html`.

REGISTRATION FEES: $60: Early registration for ILAS members (by April 1, 1998); $70: Early registration for non-ILAS members (by April 1, 1998); $70: Late registration for ILAS members (after April 1, 1998); $80: Late registration for non-ILAS members (after April 1, 1998); Student (registration fee waived).

WEATHER: **Theorem.** *The sun will be shining, a cool breeze will be blowing over Lake Mendota, and the temperature will be in the 70s (Fahrenheit).* **Proof:** To be given during the conference.

Fourth Western Canada Linear Algebra Meeting
Victoria, British Columbia, Canada: July 30–31, 1998

The Western Canada Linear Algebra Meeting (W-CLAM) provides an opportunity for mathematicians in western Canada working in linear algebra and related fields to meet, present accounts of their recent research, and to have informal discussions. While the meeting has a regional base, it also attracts people from outside the geographical area. Participation is open to anyone who is interested in attending or speaking at the meeting. Previous W-CLAMs were held in Regina (1993), Lethbridge (1995) and Kananaskis (1996).

The fourth W-CLAM will be held in Victoria, B.C., July 30–31, 1998. Participants are invited to contribute a talk on linear algebra or its applications (30–45 minutes, depending on the number of participants). Abstracts should be sent to Steve Kirkland or Michael Tsatsomeros; the deadline for abstracts is June 15, 1998. A second announcement will contain details concerning accommodations. Up-to-date information on this 1998 Western Canada Linear Algebra Meeting (W-CLAM '98) can be found at: `http://www.math.uregina.ca/~tsat/wclam.html/`. The W-CLAM Organizing Committee comprises: Hadi Kharaghani: `hadi@cs.uleth.ca`; Stephen J. Kirkland: `kirkland@math.uregina.ca`; Peter Lancaster: `lancaste@acs.ucalgary.ca`; Dale Olesky: `dolesky@csr.csc.uvic.ca`; Michael Tsatsomeros: `tsat@math.uregina.ca`; and Pauline van den Driessche: `pvdd@math.uvic.ca`.

Following W-CLAM '98, the University of Lethbridge will be offering a mini-course entitled "Coding, Cryptography and Computer Security" on August 3–7, 1998. For further information on the mini-course, please contact Hadi Kharaghani or for up-to-date information visit `http://www.cs.uleth.ca/workshop/`.

Third China Matrix Theory Conference
Zhangjiagye, Hunan Province, China: August 8–13, 1998

The 3rd China Matrix Theory Conference will be held in Zhangjiagye, Hunan Province, China, from August 8–13, 1998. Supported by the Chinese Mathematical Society and with the collaboration of a national committee, this conference is being organized by Xiangtan Polytechnic University. There is no funding to support participants. Zhangjiagye is near the north-north-west border of Hunan Province, northwest of its capital city of Changsha. A conference proceedings (in English) will be published. Deadline for submission of titles and abstracts is February 28, 1998. For further information please contact: Bit-Shun Tam: `bsm01@mail.tku.edu.tw`, or write to: Prof. Huang Liping, Institute of Mathematics, Xiangtan Polytechnic University, Xiangtan 411201, China.

SIAM Linear Algebra Prize: 1997

The 1997 SIAM Activity Group on Linear Algebra Prize will be awarded jointly to Ming Gu (UCLA) and Stanley Eisenstat (Yale University) for their paper, "A Divide-and-Conquer Algorithm for the Symmetric Tridiagonal Eigenproblem," and to Gerard Sleijpen and Henk Van Der Vorst (both of the Mathematical Institute at University of Utrecht, The Netherlands) for their paper, "A Jacobi-Davidson Iteration Method for Linear Eigenvalue Problems." The award ceremony will take place on November 1, 1997, during the Sixth SIAM Conference on Applied Linear Algebra at Snowbird, Utah. Cited for honorable mention was the paper, "A Formula for Computation of the Real Stability Radius," co-authored by L. Qui (Hong Kong University of Science), B. Bernhardsson and A. Rantzer (Lund Institute of Technology, Sweden), E. J. Davison (University of Toronto), P. M. Young (MIT), and J. C. Doyle (CalTech). The selection committee was chaired by Biswa Nath Datta (Northern Illinois University). Other members of the committee: Tony Chan (UCLA), Ludwig Elsner (Universität Bielefeld, Germany), Anne Greenbaum (New York University), and Jim Varah (University of British Columbia).

A Quotation About Linear Algebra

We looked up "Linear Algebra" at `http://math.furman.edu/~mwoodard/mquot.html`, the Mathematical Quotations website maintained by Mark Woodard in the Dept. of Mathematics at Furman University (Greenville, South Carolina) and found this entry: "We [he and Halmos] share a philosophy about linear algebra: we think basis-free, we write basis-free, but when the chips are down we close the office door and compute with matrices like fury.— *Paul Halmos Celebrating 50 Years of Mathematics* [John H. Ewing & F. W. Gehring, eds., Springer-Verlag, New York, viii + 320 pp., 1991]."

Image Publication Schedule: 1998

We expect that the two issues of *Image* for 1998 will be published in April 1998 (No. 20) and October 1998 (No. 21). Please send all material for inclusion in these issues by e-mail in LaTeX (or plain ASCII) to George Styan: `styan@Math.McGill.CA` to arrive, respectively, by February 1, 1998, and August 1, 1998.

Image Problem Corner

We present solutions to four of the six problems appearing in our previous Problem Corner: *Image*, no. 18 (Winter/Spring 1997), p. 32, and we introduce five new problems. We invite readers to submit solutions, as well as new problems, for publication in *Image*. We look forward particularly to receiving solutions to Problems 18-1 and 18-4! Please send all material to George Styan in LaTeX and by e-mail to `styan@Math.McGill.CA`.

Problem 18-1: 5×5 **Complex Hadamard Matrices**
Proposed by S. W. DRURY, *McGill University, Montréal, Québec, Canada.*

The Editor has not yet received a solution to this problem–indeed even the Proposer has not yet found a solution!

Show that every 5×5 matrix U with complex entries $u_{j,k}$ of constant absolute value one that satisfies $U^*U = 5I$ can be realized as the matrix $(\omega^{jk})_{j,k}$ where ω is a complex primitive fifth root of unity by applying some sequence of the following: (1) A rearrangement of the rows, (2) A rearrangement of the columns, (3) Multiplication of a row by a complex number of absolute value one, (4) Multiplication of a column by a complex number of absolute value one.

Problem 18-2: 7×7 **Complex Hadamard Matrices**
Proposed by S. W. DRURY, *McGill University, Montréal, Québec, Canada.*

Show that there exist two 7×7 nonzero complex matrices U and V with the following bizarre properties.

1. For all j, k with $1 \leq j \leq 7$ and $1 \leq k \leq 7$, either $|u_{j,k}| = 1$ and $v_{j,k} = 0$ or $|v_{j,k}| = 1$ and $u_{j,k} = 0$.
2. For every complex number z of absolute value 1, the matrix $W = U + zV$ satisfies $W^*W = 7I$.

Solution 18-2.1 by S. W. DRURY, *McGill University, Montréal, Québec, Canada.*

Let the matrices

$$U = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega & \omega^4 & \omega^2 & 1 & \omega^4 & \omega^3 \\ 1 & \omega^3 & 0 & \omega^2 & \omega^4 & 0 & 1 \\ 1 & 0 & 0 & 0 & \omega^3 & 0 & 0 \\ 1 & \omega^3 & \omega^2 & \omega^5 & 1 & \omega^2 & \omega^4 \\ 1 & 0 & 0 & 0 & \omega^3 & 0 & 0 \\ 1 & \omega^5 & 0 & \omega^4 & \omega^2 & 0 & \omega^2 \end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \omega^3 & 0 \\ 0 & \omega^5 & \omega^2 & \omega^2 & 0 & 1 & \omega^4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \omega^2 & \omega^3 & \omega^5 & 0 & \omega^5 & \omega \\ 0 & 0 & \omega^5 & 0 & 0 & \omega^2 & 0 \end{pmatrix},$$

where ω is a primitive sixth root of unity (e.g. $\omega = e^{\pi i/3}$). Then, one easily checks that $U^\star V = V^\star U = 0$ and $U^\star U + V^\star V = 7I$. It follows that if z is a complex number of absolute value 1 then $W = U + zV$ is a 7×7 matrix with entries of absolute value 1 and yet $W^\star W = 7I$. □

Problem 18-3: A Further Matrix Version of the Cauchy-Schwarz Inequality
Proposed by SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

Prove that $(X'AX)^+ \leq X^+A^+X'^+$, where $A \geq 0$, $\mathcal{R}(XX'A) \subseteq \mathcal{R}(A)$; here $\mathcal{R}(\cdot)$ denotes range (column space) and $(\cdot)^+$ the Moore-Penrose inverse.

Solution 18-3.1 by SIMO PUNTANEN, *University of Tampere, Tampere, Finland.*

According to Theorem 1 in Baksalary & Puntanen (1991) [*Aequationes Mathematicae* 41:103–110], if $A \geq 0$ then

$$X'P_AX(X'AX)^-X'P_AX \leq X'A^+X \tag{1}$$

with equality if and only if $\mathcal{R}(AX) = \mathcal{R}(P_AX)$; here P_A is the orthogonal projector onto the column space of A and $(\cdot)^-$ refers to an arbitrary generalized inverse. The matrix inequality (1) is a special case of Theorem 2.1 in Pečarić, Puntanen & Styan (1996) [*Linear Algebra Appl.* 237/238:455–477]. To prove $(X'AX)^+ \leq X^+A^+X'^+$, we first note that assumption $\mathcal{R}(XX'A) \subseteq \mathcal{R}(A)$ is equivalent to $P_AXX'A = XX'A$. Hence, in view of $\mathcal{R}\left(X(X'AX)^+X'\right) = \mathcal{R}\left(X(X'AX)\right) = \mathcal{R}\left(XX'A\right)$, we can delete the orthogonal projectors P_A from (1) and thus obtain

$$X'X(X'AX)^+X'X \leq X'A^+X. \tag{2}$$

Pre- and post-multiplying (2) with $(X'X)^+$ yields

$$P_{X'}(X'AX)^+P_{X'} \leq (X'X)^+X'A^+X(X'X)^+ = X^+A^+X'^+. \tag{3}$$

In (3), the orthogonal projectors $P_{X'}$ can be deleted since $\mathcal{R}\left((X'AX)^+\right) \subseteq \mathcal{R}\left(X'\right)$ and thus the proof is completed. □

Solution 18-3.2 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

In this solution we wish to demonstrate how the claimed matrix inequality can be established by means of a powerful result from statistics. For the sake of clarity, we start with stating the Gauss-Markov model. Then we review briefly the famous BLUE approach and show how the well-known BLUE-Conditions generate a myriad of matrix inequalities between certain positive semidefinite matrices. It is interesting that our approach reveals, at the same time, also a technique which enables us to verify many conjectured inequalities such as the claimed one.

The general Gauss-Markov model

$$\mathcal{L} = (y,\ X\beta,\ V)$$

characterizes an observable random vector y by requiring $\mathcal{E}(y) = X\beta$ and $\mathcal{D}(y) = V$ ($\mathcal{E}$ and $\mathcal{D}$ denote expectation and dispersion, respectively), where the model matrices $X \in \mathbb{R}^{n\times m}$ and $V \in \mathbb{R}^{n\times n}$ are fixed and known and where V is assumed to be positive semidefinite. The linear transformation $\gamma := L\beta$, with $L \in \mathbb{R}^{p\times m}$ fixed and known, is linearly unbiasedly estimable if and only if $\mathcal{R}(L') \subseteq \mathcal{R}(X')$. In what follows, let $\gamma = L\beta$ be linearly unbiasedly estimable. The linear estimator Cy is then defined as the *traditional* BLUE (best linear unbiased estimator) of γ under the model $\mathcal{L}$ whenever

$$Cy \in \operatorname{argmin}_{Dy\in\mathcal{C}_\mathcal{L}} \mathcal{D}(Dy);$$

here $\mathcal{C}_\mathcal{L} := \{Dy \mid DX = L\}$ and minimization is with respect to the Löwner ordering. Observe that all the statistics Dy with $DX = L$ are linear unbiased estimators for $\gamma = L\beta$ under the model $\mathcal{L}$. We are now in the position to quote the following powerful result from statistics; see, e. g. (18) in P. Schönfeld and H. J. Werner (1987) ["A note on C. R. Rao's wider definition BLUE in the general Gauss-Markov model", *Sankhyā Ser. B* 49:1–8]; compare also Theorem 3.1 in C. R. Rao (1973) ["Representations of the best linear unbiased estimators in the Gauss-Markoff model with a singular dispersion matrix", *J. Multivariate Anal.* 3:276–292].

Theorem 1: *Let Q be any matrix such that $\mathcal{R}(Q) = \mathcal{N}(X')$; $\mathcal{N}(\cdot)$ indicates the null space. Then $\hat{\gamma} := Cy$ is a representation for the traditional BLUE of $\gamma = L\beta$ in the model $\mathcal{L}$ if and only if*

$$C(X,\ VQ) = (L,\ 0). \tag{4}$$

The conditions (4) are the so-called BLUE-Conditions. Recall that the dispersion of any statistic By is always given by $\mathcal{D}(By) = BVB'$. From Theorem 1 and the definition of the traditional BLUE we therefore directly get the following result between positive semidefinite matrices.

Corollary 1: *For given matrices $X \in \mathbb{R}^{n\times m}$ and $V \in \mathbb{R}^{n\times n}$, with V positive semidefinite, let Q be any matrix such that $\mathcal{R}(Q) = \mathcal{N}(X')$. Moreover, let C be an arbitrary but fixed matrix with $CVQ = 0$. For each matrix D satisfying $DX = CX$ we then have the following matrix inequality*

$$CVC' \leq DVD', \tag{5}$$

with equality in (5) *if and only if $DVQ = 0$.*

We can obviously generate a lot of matrix inequalities by means of this result. Conversely, however, this result also provides us with a simple technique which, in very many cases, enables us to verify that a conjectured (or claimed) matrix inequality is indeed correct. The claim that under the assumption

$$XX'\mathcal{R}(A) \subseteq \mathcal{R}(A) \tag{6}$$

the matrix inequality

$$(X'AX)^+ \leq X^+A^+X'^+ \tag{7}$$

holds can serve as a simple example.

For that purpose, observe first that the inclusion (6) implies

$$\mathcal{N}(X') = [\mathcal{N}(X') \cap \mathcal{R}(A)] \oplus [\mathcal{N}(X') \cap \mathcal{N}(A)], \tag{8}$$

where $\oplus$ indicates a direct sum; see, for instance, Theorem 5.5 along with Theorem 5.3 in H. J. Werner (1992) ["G-inverses of matrix products", in Data Analysis and Statistical Inference (S. Schach and G. Trenkler, eds.), pp. 531–546, *Verlag Josef Eul*, Bergisch Gladbach]. Now put $V := A^+$ and $C := (X'AX)^+X'A$. By assumption, $A \geq 0$. Note that $AA^+ = A^+A = P_{\mathcal{R}(A)}$; here $P_{\mathcal{R}(A)}$ denotes the orthogonal projector onto $\mathcal{R}(A)$ along $\mathcal{N}(A)$. In view of (8), therefore $X'AA^+Q = 0$, where Q is defined as in Corollary 1. Consequently, $CVQ = 0$. Check that $CVC' = (X'AX)^+$. Put $D := X^+A^+A$. Clearly, $DVD' = X^+A^+X'^+$. According to Corollary 1, it is now evident that (7) holds whenever $DX = CX$. Since $CX = P_{\mathcal{R}(X'AX)}$, we therefore try to prove $DX = P_{\mathcal{R}(X'AX)}$. Clearly, $DX = X^+A^+AX$. In view of (6) and $A^+A = P_{\mathcal{R}(A)}$, $A^+AXX'A = XX'A$. Since $X^+XX' = X'$, therefore $X^+A^+AXX'AX = X'AX$. Moreover, $\mathcal{N}(X^+A^+AX) = \mathcal{N}(A^+AX) = \mathcal{N}(AX) = \mathcal{N}(X'AX)$. As $\mathcal{N}(X'AX)$ is the orthogonal complement of $\mathcal{R}(X'AX)$, it now follows that DX coincides with the orthogonal projector $P_{\mathcal{R}(X'AX)}$. Hence, as desired, $DX = CX$.

COMMENT. We conclude this small note with mentioning that, according to Corollary 1, we have equality in (7) if and only if $X^+A^+Q = 0$ or, equivalently, $A^+\mathcal{N}(X') \subseteq \mathcal{N}(X')$. It is left to the reader to find other conditions, such as $A^+\mathcal{R}(X) \subseteq \mathcal{R}(X)$ or $A\mathcal{R}(X) \subseteq \mathcal{R}(X)$, which are equivalent to $X^+A^+Q = 0$. □

A solution was also received from the Proposer: SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

Problem 18-4: Bounds for a Ratio of Matrix Traces

Proposed by SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

Let $\Sigma > 0$ be an $n \times n$ matrix with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$, X be an $n \times k$ matrix such that $X'X = I$ and $n \geq 2k$. Show that

$$1 \leq \frac{\mathrm{tr} X'\Sigma^2 X}{\mathrm{tr}(X'\Sigma X)^2} \leq \frac{\Sigma_{j=1}^k(\lambda_j + \lambda_{n-j+1})^2}{4\Sigma_{j=1}^k \lambda_j \lambda_{n-j+1}}.$$

The only solution that the Editor has received so far is from the Proposer!

Problem 18-5: Eigenvalues and Eigenvectors of a Patterned Matrix
Proposed by I. S. ALALOUF & K. BRENDA MACGIBBON, *Université du Québec à Montréal, Montréal, Québec, Canada.*

Find the eigenvalues and eigenvectors of the $n \times n$ patterned matrix $A = \{a_{ij}\} = \{i - j\} \quad (i, j = 1, 2, \ldots, n)$.

Solution 18-5.1 by JÜRGEN GROß & GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

The patterned matrix $\mathbf{A} = \{i - j\}$ for $i, j = 1, \ldots, n$ can be written as

$$\mathbf{A} = \mathbf{ab}' - \mathbf{ba}'$$

where $\mathbf{a} = (1, \ldots, n)'$ and $\mathbf{b} = (1, \ldots, 1)'$. We derive the eigenvalues and eigenvectors of $\mathbf{A}$ when $\mathbf{a}$ and $\mathbf{b}$ are arbitrary linearly independent vectors:

The matrix $\mathbf{A}$ is skew-symmetric and therefore has purely imaginary eigenvalues. Moreover, the algebraic multiplicity of 0 as an eigenvalue of $\mathbf{A}$ is $n - \text{rank}(\mathbf{A})$ due to normality of $\mathbf{A}$. But since $\mathbf{a}$ and $\mathbf{b}$ are linearly independent, the rank of the matrices $\mathbf{ab}'$ and $-\mathbf{ba}'$ is additive, cf. Theorem 3.6.1 in Rao & Bhimasankaram (1992) [*Linear Algebra*, Tata McGraw-Hill, New Delhi], i.e., $\text{rank}(\mathbf{A}) = \text{rank}(\mathbf{ab}' - \mathbf{ba}') = \text{rank}(\mathbf{ab}') + \text{rank}(-\mathbf{ba}') = 2$.

OBSERVATION 1. The $n \times n$ real matrix $\mathbf{A}$ has $n-2$ zeros and one conjugate pair of nonzero imaginary numbers as eigenvalues.

Straightforward computations show that $\mathbf{A}^3 = -\varrho\mathbf{A}$, where $\varrho = \mathbf{a}'\mathbf{ab}'\mathbf{b} - (\mathbf{a}'\mathbf{b})^2$ is strictly positive due to the Cauchy-Schwarz inequality, cf. Theorem 5.1.4 in Horn & Johnson (1985) [*Matrix Analysis*, Cambridge Univ. Press]. The identity $\mathbf{A}^3 = -\varrho\mathbf{A}$ may equivalently be written as $(\mathbf{A} - i\sqrt{\varrho}\mathbf{I})(\mathbf{A} + i\sqrt{\varrho}\mathbf{I})\mathbf{A} = \mathbf{0}$ and thus the polynomial $q(t) = (t - i\sqrt{\varrho})(t + i\sqrt{\varrho})t$ annihilates $\mathbf{A}$. In view of Observation 1 we have

OBSERVATION 2. The polynomials $q(t)$ and $q(t)/t$ are the minimal polynomials of $\mathbf{A}$ in case $n \geq 3$ and $n = 2$, respectively.

Now, from Observation 2 the eigenvalues of the matrix $\mathbf{A}$ are

$$\{0, +i\sqrt{\varrho}, -i\sqrt{\varrho}\}, \quad i = \sqrt{-1}, \quad \varrho = \mathbf{a}'\mathbf{ab}'\mathbf{b} - (\mathbf{a}'\mathbf{b})^2.$$

Of course 0 is not an eigenvalue of $\mathbf{A}$ when $n = 2$. For the special case which is addressed in the problem we obtain

$$\varrho = n \sum_{k=1}^{n} k^2 - \left(\sum_{k=1}^{n} k \right)^2 = \frac{n^2(n^2 - 1)}{12}.$$

From the identity $q(\mathbf{A}) = \mathbf{0}$ and the commutativity of $\mathbf{A} - i\sqrt{\varrho}\,\mathbf{I}$ and $\mathbf{A} + i\sqrt{\varrho}\,\mathbf{I}$, we easily see that any vector

$$\mathbf{x} = (\mathbf{A} \pm i\sqrt{\varrho}\,\mathbf{I})\mathbf{Az}, \qquad \mathbf{z} \text{ arbitrary } n \times 1 \text{ such that } \mathbf{x} \neq \mathbf{0},$$

is an eigenvector corresponding to the eigenvalue $\pm i\sqrt{\varrho}$ respectively. Moreover, the identity $\mathbf{A}^3 = -\varrho\mathbf{A}$ shows that the matrix $\mathbf{A}^- = (-1/\varrho)\mathbf{A}$ is a generalized inverse of $\mathbf{A}$ (in fact it is the unique Moore-Penrose inverse). Since the null space of $\mathbf{A}$ equals the range of $\mathbf{I} - \mathbf{A}^-\mathbf{A}$, any vector

$$\mathbf{x} = \mathbf{z} + \frac{1}{\varrho}\mathbf{AAz}, \qquad \mathbf{z} \text{ arbitrary } n \times 1 \text{ such that } \mathbf{x} \neq \mathbf{0},$$

is an eigenvector corresponding to the eigenvalue 0. □

Solution 18-5.2 by EUGENE A. HERMAN, *Grinell College, Grinell, Iowa, USA.*

Zero is an eigenvalue of multiplicity $n - 2$ for the $n \times n$ matrix $A = \{a_{ij}\} = \{i - j\}$, as can be seen from the fact that the difference between any two consecutive rows of A is the vector of all 1's. To find the two complex conjugate eigenvalues, we first note that every row of A is an eigenvector of A^2 with eigenvalue $-c_n = -n^2(n^2 - 1)/12$, as may be verified by a direct calculation. Therefore, selecting any row R of A, we check that A has the eigenvectors $AR \pm i\sqrt{c_n}R$ with eigenvalues $\pm i\sqrt{c_n}$: $A(AR + i\sqrt{c_n}R) = A^2R + i\sqrt{c_n}AR = -c_nR + i\sqrt{c_n}AR = i\sqrt{c_n}(i\sqrt{c_n}R + AR)$ □

Solution 18-5.3 by PRZEMYSŁAW KOSOWSKI, *Polish Academy of Sciences, Warsaw, Poland.*

We show that the eigenvalues of the $n \times n$ patterned matrix $A = \{a_{ij}\} = \{i - j\}$ $(i, j = 1, 2, \ldots, n)$ are equal to $\lambda_{1,2} = \mp i\sqrt{\frac{n^2(n^2-1)}{12}}$ and $\lambda_3 = \ldots = \lambda_n = 0$. To prove this we consider the more general case $A = xy' - yx' = \{x_iy_j - y_ix_j\}$, where $x, y \in \mathbb{R}^n$ are assumed to be linearly independent. First we orthogonalize the vectors x, y. We find $u, w \in \operatorname{span}\{x, y\}$ such that $\|u\| = \|w\| = 1$ and $u'w = 0$; here $\|u\|^2 = u'u$. Let $u = \frac{y}{\|y\|}$ and $w = \frac{q}{\|q\|}$, where $q = x - (x'u)\,u$. Then A can be rewritten in the form $A = -r\,(uw' - wu')$, where $r = \|y\| \cdot \|q\| = \sqrt{(x'x)\,(y'y) - (x'y)^2}$. Denote by $q_1, q_2, \ldots, q_n$ the set of the eigenvectors of A corresponding to the eigenvalues $\lambda_1, \lambda_2, \ldots \lambda_n$. We see that $Aw = -ru$ and $Au = rw$, so $A\,(w \pm iu) = \pm ir\,(w \pm iu)$. We can take $q_{1,2} = w \pm iu$ as the first two eigenvectors and $q_3, \ldots, q_n$ may be chosen as an arbitrary set of linearly independent vectors such that $q_j'u = q_j'w = 0$ for $j = 3, \ldots, n$. Then $Aq_j = \lambda_j q_j$, $j = 1, 2, \ldots, n$, where $\lambda_{1,2} = \pm ir$, $\lambda_3 = \ldots = \lambda_n = 0$. Now we consider the case of this problem: $x = (1, 2, \ldots, n)'$ and $y = (1, 1, \ldots, 1)'$. Then $A = xy' - yx' = \{i - j\}$. Thus we obtain $x'x = \frac{n(n+1)(2n+1)}{6}$, $y'y = n$, $x'y = \frac{n(n+1)}{2}$, so $r = \sqrt{\frac{n^2(n^2-1)}{12}}$. Moreover, for the vectors $u = (u_1, \ldots, u_n)'$, $w = (w_1, \ldots, w_n)'$ we have $u_j = \frac{\sqrt{n}}{n}$, $w_j = \sqrt{n}\,(j - \frac{n+1}{2})/r$ for $j = 1, \ldots, n$; so we can set $q_{1,2} = w \pm iu$ and $q_j = e_j - u_ju - w_jw$ for $j = 3, \ldots, n$, where e_j denotes jth unit vector. Then $\{q_1, \ldots, q_n\}$ forms the set of eigenvectors of A. □

Solution 18-5.4 by CHI-KWONG LI, *The College of William and Mary, Williamsburg, Virginia, USA.*

Let $x = (1, 2, \ldots, n)'$ and $y = (1, \ldots, 1)'$. Then $A = xy' - yx' = BC'$ with $B = (x, -y)$ and $C = (y, x)$. It is clear that the eigenspace of the zero eigenvalue equals $\{x, y\}^\perp$ and has dimension $n - 2$. The nonzero eigenvalues of $A = BC'$ are the same as those of the nonsingular matrix

$$C'B = \begin{pmatrix} n(n+1)/2 & -n \\ n(n+1)(2n+1)/6 & -n(n+1)/2 \end{pmatrix},$$

which can be easily checked to be $\pm i\{(n-1)n^2(n+1)/12\}^{1/2}$. Moreover, if $u \in \mathbb{C}^2$ is an eigenvector of $C'B$ then Bu is an eigenvector of BC'. Hence one can use the eigenvectors of $C'B$ to get the eigenvectors of BC' of the 2 nonzero eigenvalues. For example, one can use $u = \left(2, (n+1) - i\sqrt{(n^2-1)/3}\right)'$ for the eigenvalue $i\{(n-1)n^2(n+1)/12\}^{1/2}$, and use $u = \left(2, (n+1) + i\sqrt{(n^2-1)/3}\right)'$ for the eigenvalue $-i\{(n-1)n^2(n+1)/12\}^{1/2}$. We note that this proof works for any rank 2 skew-symmetric matrix. □

Solution 18-5.5 by CARL D. MEYER, JR. *North Carolina State University, Raleigh, North Carolina, USA.*

The eigenvalues are 0 (repeated $n - 2$ times) and $\pm\beta i$ where $\beta = \frac{n}{2}\sqrt{\frac{n^2-1}{3}}$. Any subset of $n - 2$ columns of $A^2/\beta^2 + I$ is an eigenbasis corresponding to 0, and eigenvectors associated with $\pm\beta i$ are $\left[I \mp i(A/\beta)\right]e$ where $e = (1, 1, \ldots, 1)'$.

DERIVATION. Since $A = ce' - ec'$ with $c = (1, 2, \ldots, n)'$, standard rank results insure $\operatorname{rank}(A) = 2$. Furthermore, A is skew-symmetric (and hence normal), so there are only two nonzero eigenvalues, and they must be of the form $\pm\beta i$. To evaluate β, set $\lambda_0 = 0$, $\lambda_1 = \beta i$, and $\lambda_2 = -\beta i$, and use the spectral theorem $f(A) = f(\lambda_0)P_0 + f(\lambda_1)P_1 + f(\lambda_2)P_2$ where P_k is the orthogonal projector onto $\mathcal{N}(A - \lambda_kI)$. The Lagrange formula $P_k = \prod_{j\neq k}(A - \lambda_jI)/\prod_{j\neq k}(\lambda_k - \lambda_j)$ yields

$$P_0 = \frac{A^2}{\beta^2} + I, \quad P_1 = -\frac{A^2}{2\beta^2} - i\frac{A}{2\beta}, \quad \text{and} \quad P_2 = \overline{P_1} = -\frac{A^2}{2\beta^2} + i\frac{A}{2\beta},$$

so, by using $f(z) = z^2$ along with $c'e = n(n+1)/2$ and $c'c = n(n+1)(2n+1)/6$, we have

$$-2\beta^2 = \text{trace}(A^2) = \text{trace}(ce' - ec')^2 = 2(c'e)^2 - 2nc'c = -\frac{n^2(n^2-1)}{6}, \quad \text{and thus} \quad \beta = \frac{n}{2}\sqrt{\frac{n^2-1}{3}}.$$

All eigenvectors corresponding to $\pm\beta i$ must be of the form $P_1 y$ and $\overline{P_1 y}$, so you can judiciously vary y to generate patterned eigenvectors to suit your tastes. For example, setting $y = 2e$ yields eigenvectors $[I \mp i(A/\beta)]e$ by noting that for $z = (1, -1, 0, \ldots, 0)'$,

$$e = Az \Longrightarrow e \in \mathcal{R}(A) = \mathcal{N}(A)^{\perp} = \mathcal{N}(P_0) \Longrightarrow A^2 e = -\beta^2 e \Longrightarrow P_1(2e) = e - i(Ae/\beta).$$

With $z = (2, -1, 0, \ldots, 0)'$, vector e can be replaced by c in the previous sentence. Finally, all vectors in $\mathcal{N}(A)$ are of the form $P_0 y$, and a basis for $\mathcal{N}(A)$ is obtained by extracting any subset of $n-2$ columns from P_0 (independence is guaranteed by the fact that $P_0 e = 0 = P_0 c$). Furthermore, by verifying that $AB = 0$, one sees that another simple basis for $\mathcal{N}(A)$ is given by the set of columns in

$$B = \begin{pmatrix} 1 & 2 & \cdots & (n-2) \\ -2 & -3 & \cdots & -(n-1) \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}.$$

□

Solution 18-5.6 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

The case $n = 1$ is trivial: since now $A = 0$, it is clear that 0 is the only eigenvalue of A. An associated eigenvector is 1. Next, let $n \geq 2$. Put

$$\alpha_n := \frac{n(n-1)}{2}, \quad \gamma_n := \frac{n}{2}\sqrt{\frac{n^2-1}{3}}\, i, \quad \tau_n := \frac{6}{(n-1)n(2n-1)},$$

where i is the purely imaginary number such that $i^2 = -1$. Define the $n \times 1$ vectors

$$c_1 := \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} + \tau_n(\gamma_n - \alpha_n)\begin{pmatrix} n-1 \\ n-2 \\ \vdots \\ 1 \\ 0 \end{pmatrix}, \quad c_2 := \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1 \end{pmatrix} - \tau_n(\gamma_n + \alpha_n)\begin{pmatrix} n-1 \\ n-2 \\ \vdots \\ 1 \\ 0 \end{pmatrix},$$

and, provided $n \geq 3$, the $n-2$ additional $n \times 1$ vectors

$$d_1 := \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad d_2 := \begin{pmatrix} 0 \\ 1 \\ -2 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \ldots, \quad d_{n-2} := \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ -2 \\ 1 \end{pmatrix}.$$

In what follows, we interpret all statements concerning d_j $(j = 1, 2, \ldots, n-2)$ as absent whenever $n = 2$. Since $Ad_j = 0$ $(j = 1, 2, \ldots, n-2)$, $Ac_1 = \gamma_n c_1$ and $Ac_2 = -\gamma_n c_2$, it follows that A has the two non-null eigenvalues γ_n and $-\gamma_n$ and, provided $n \geq 3$, a null eigenvalue of multiplicity $n-2$. It is further clear that $c_1,\ c_2,\ d_1,\ d_2,\ \ldots,\ d_{n-2}$ are linear independent right eigenvectors associated with the eigenvalues $\gamma_n,\ -\gamma_n,\ 0,\ 0,\ \ldots,\ 0$, respectively. □

Solutions were also received from CHRISTOPHER W. NORMAN, *Royal Holloway and Bedford New College, Egham, Surrey, England, UK* & WILLIAM F. TRENCH, *Divide, Colorado, USA.*

Problem 18-6: Two Imaginary Sisters of the Vector Cross-product
Proposed by GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let $\mathbf{a}$ be a non-zero vector from $\mathbb{R}3$. Show that there are just two square matrices $\mathbf{A}$ with purely imaginary entries which satisfy

$$\mathbf{A}^2\mathbf{x} = \mathbf{a} \times (\mathbf{a} \times \mathbf{x}) \quad \text{for all } \mathbf{x} \in \mathbb{R}3 \tag{9}$$

and identify them!

Note. As pointed out (subsequently) by GÖTZ TRENKLER, the problem statement should have read "Find two square matrices A with purely imaginary entries which satisfy ..." *We apologize for the incorrect (original) wording. –Ed.*

Solution 18-6.1 by CHI-KWONG LI, *The College of William and Mary, Williamsburg, Virginia, USA.*

It is clear that the mapping $x \mapsto a \times (a \times x)$ is linear. Consider $A^2x = a \times (a \times x)$ for the three standard basic vectors in $\mathbb{R}^3$. One sees that $A^2 = aa' - (a'a)I_3$. Alternatively, one can get the conclusion by considering A^2x for $x = a, b, c$, where $\{b, c\}$ is an orthonormal basis for $a^\perp$. Let $A = iB$ for a real matrix B. Then $B^2 = (a'a)P$ with $P = I_3 - (a'a)^{-1}aa' = VV'$, where $V = (b, c)$. As a result, B can be chosen to be $\pm\sqrt{a'a}P$, or a matrix of the form $\sqrt{a'a}PVS^{-1}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} SV'$ for some 2×2 real invertible matrix S. □

Solution 18-6.2 by WILLIAM F. TRENCH, *Divide, Colorado, USA.*

We will show that there are infinitely many matrices C in $\mathbb{R}^{3\times3}$ such that $-C^2x = (a \times x) \times a \quad$ for all $x \in \mathbb{R}^3$. If C is such a matrix then $A = iC$ satisfies the requirements of the problem. If $a = (a_1, a_2, a_3)'$ and $x = (x_1, x_2, x_3)'$ then

$$(a \times x) \times a = -Bx, \quad \text{where} \quad B = \begin{pmatrix} a_2^2 + a_3^2 & -a_1a_2 & -a_1a_3 \\ -a_2a_1 & a_1^2 + a_3^2 & -a_2a_3 \\ -a_3a_1 & -a_3a_2 & a_1^2 + a_2^2 \end{pmatrix}.$$

We want matrices C in $\mathbb{R}^{3\times3}$ such that $C^2 = B$. Since

$$-\frac{(a \times x) \times a}{||a||^2} = x - \frac{(a \cdot x)}{||a||^2}a \tag{10}$$

is the projection of x on the subspace of $\mathbb{R}^3$ orthogonal to a and $||a||^{-2}B$ is the matrix of this projection, it follows that $(||a||^{-2}B)^2 = ||a||^{-2}B$, which implies that if $C = \pm||a||^{-1}B$ then $C^2 = B$. To see where infinitely many other choices of C come from, note from (10) that $\lambda = 0$ is an eigenvalue of B and a is an associated eigenvector, while $||a||^2$ is an eigenvalue of multiplicity 2 and any nonzero vector orthogonal to a is an associated eigenvector. The matrices $\pm||a||^{-1}B$ have the same eigenspaces as B. However, there are infinitely many other solutions of $C^2 = B$ that do not have this property. For example, let $C = (a, p, q)\mathrm{diag}(0, ||a||, -||a||)(a, p, q)^{-1}$, where p and q are linearly independent and orthogonal to a. If $x = \alpha a + \beta p + \gamma q$ is an arbitrary vector in $\mathbb{R}^3$ then $C^2x = ||a||^2(\beta p + \gamma q) = -(a \times x) \times a$, and so $C^2 = B$. □

Solution 18-6.3 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

Let a be a nonzero vector from $\mathbb{R}3$. Since $a \times (a \times x) = -(a'aI_3 - aa')x$, clearly $A^2x = a \times (a \times x) \quad$ for all $x \in \mathbb{R}3$ iff $A^2 = -(a'aI_3 - aa')$. Observing $\left(I - \frac{aa'}{a'a}\right)^2 = I - \frac{aa'}{a'a}$ shows that A can be chosen as $\pm i\sqrt{a'a}\left(I - \frac{aa'}{a'a}\right)$. Notice, however, that there exist infinitely many further pairs of matrices with the desired property. For, if P is any column-orthonormal matrix with $PP' = I - \frac{aa'}{a'a}$, then the matrices $\pm i\sqrt{a'a}PFP'$ also satisfy (9) whenever F is such that $F^2 = I_2$. □

New Problems

Problem 19-1: Eigenvalues
Proposed by JÜRGEN GROß & GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let a and b be two nonzero $n \times 1$ real vectors and consider the matrix

$$\mathbf{A} = \alpha\,\mathbf{a}\mathbf{a}' + \beta\,\mathbf{a}\mathbf{b}' + \gamma\,\mathbf{b}\mathbf{a}' + \delta\,\mathbf{b}\mathbf{b}',$$

where $\alpha, \beta, \gamma, \delta$ are real scalars such that $\beta = -\gamma$ and $\gamma^2 = -\alpha\delta$. Find the (nonzero) eigenvalues of $\mathbf{A}$.

Problem 19-2: Non-negative Definite Matrices
Proposed by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

A matrix $A \in \mathbb{C}^{n\times n}$ is said to be non-negative definite if $\mathrm{Re}(x^*Ax) \geq 0$ for all vectors $x \in \mathbb{C}^n$. Prove that A is non-negative definite if and only if its Moore-Penrose inverse $A^\dagger$ is non-negative definite.

Problem 19-3: Characterizations Associated with a Sum and Product

Proposed by ROBERT E. HARTWIG, *North Carolina State University, Raleigh, North Carolina, USA,*
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1. Characterize square matrices A and B satisfying $AB = pA + qB$, where p and q are given scalars.
2. More generally, characterize linear operators A and B acting on a vector space $\mathcal{X}$ satisfying $ABx \in \mathrm{Span}(Ax, Bx)$ for every $x \in \mathcal{X}$.

Problem 19-4: Eigenvalues of Non-negative Definite Matrices
Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

Show that there are constants $\gamma \in (0, \frac{1}{2}]$ and $\varepsilon \in (0, \frac{1}{2})$ such that, if $A_1, \ldots, A_n$ are non-negative definite $r \times r$ matrices of rank one satisfying

1. $A_1 + \cdots + A_n = I_r$ and
2. $\mathrm{trace}(A_i) < \gamma$ for each $i = 1, \ldots, n$,

then there is a subset σ of $\{1, 2, \ldots, n\}$ such that the eigenvalues of $A_\sigma = \sum_{i\in\sigma} A_i$ all lie in the interval $(\varepsilon, 1-\varepsilon)$.

Problem 19-5: Symmetrized Product Definiteness?
Proposed by INGRAM OLKIN, *Stanford University, Stanford, California, USA.*

Let A and B be real symmetric matrices. Then prove or disprove:

1. If A and B are both positive definite (non-negative definite) then $AB + BA$ is positive definite (non-negative definite).
2. If A and $AB + BA$ are both positive definite (non-negative definite) then B is positive definite (non-negative definite).

Please submit solutions, as well as new problems, to George Styan in LaTeX and by e-mail to `styan@Math.McGill.CA`. We look forward particularly to receiving solutions to Problems 18-1 and 18-4!