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Egerváry-ABS International Prize

With reference to the intensive development of ABS methods since 1981 at the University of Bergamo and to the seminal, albeit little-known, works of the late Professor Egerváry, where the class of algorithms later called the unscaled ABS class was already developed, an international prize is now offered by the Department of Mathematics of the University of Bergamo for the best work in the ABS field written by a single author whose age on December 31, 1997, was no more than 39 years. The prize consists of two million Italian lira, plus travel expenses (economy class if by air) and living expenses for three days, to come to Bergamo to present in a seminar the content of the winning paper. Submitted papers can be either in preprint or already published form. They must be sent to: The Director, Department of Mathematics, University of Bergamo, p.za Rosate 2, I-24129 Bergamo, Italy. Age must be certified by a copy of the passport. Papers must be received by June 30, 1998. The winner will be decided by an international commission of experts in ABS methods. The winner will receive the prize in Bergamo, where the work will be presented.

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Book Reviews

R. B. Bapat & T. E. S. Raghavan: *Nonnegative Matrices and Applications*

Reviewed by Fuzhen Zhang, Nova Southeastern University, Fort Lauderdale

Nonnegative Matrices and Applications by R. B. Bapat and T. E. S. Raghavan [Encyclopedia of Mathematics and Its Applications, Vol. 64, Cambridge University Press, xiii + 336 pp., ISBN 0-521-57167-7, 1997] is a very good text on matrix theory for graduate students and a valuable reference book for linear algebraists and researchers. The book contains seven chapters, beginning with the Perron-Frobenius theory and matrix games, and then covering doubly stochastic matrices, inequalities for positive semidefinite matrices, topics in combinatorial theory, and applications to statistics and economics. The book presents the theory of nonnegative matrices in an elegant way and shows connections of the subject with game theory, combinatorics, optimization, and mathematical economics. The book is based on the authors' research of recent years in the field, though some topics are standard. The first five chapters contribute to a good text on basic nonnegative matrix theory for first-year graduate students, with an option of Chapter 6 or 7, if teaching time permits. Some exercises are given after each chapter but students may expect more. The prerequisite is a good elementary linear algebra course and some basic matrix skills.

Ali S. Hadi: *Matrix Algebra as a Tool*

Reviewed by I. S. Alalouf, Université du Québec à Montréal

In the author's words the main purpose of the *Matrix Algebra as a Tool* by Ali S. Hadi [Duxbury, xi + 212 pp., ISBN 0-534-23712-6, 1996] is to present matrix algebra as a tool for students and researchers in various fields who find it necessary for their own [statistical] work. This is no easy task, since practitioners need to be acquainted with sophisticated notions such as vector spaces, eigenvalues and eigenvectors, and matrix diagonalization, and they need to reach such levels from relatively modest mathematical beginnings. The book, however, serves exactly that function: starting from the definition of a matrix and matrix operations, it runs through the fundamental notions in barely more than 200 pages. For people with limited mathematical background, this is the most direct route to autonomy in linear algebra. Of course, there is no room for proofs. Theorems are stated, and they are made credible through numerical examples and counterexamples. These examples are surprisingly effective, and for those not mathematically inclined, they are more convincing, and they offer more insight than formal proofs would. (Proofs are not entirely absent, however: the occasional corollary is proven, usually to pedagogical advantage.) The subject matter is geared specifically to practitioners, which means that many of the things that are standard fare in introductory books are either glossed over or omitted: there are no minimal polynomials, no elementary divisors, no dual spaces, no Jordan forms. And the approach is coordinate bound: all vectors are expressed in terms of the standard coordinate system. On the other hand, the book covers such relatively esoteric things as L_p-norms, generalized inverses, and Kronecker products. The approach is mindful of the beginner. Whenever possible, a new concept is introduced through an application, or a numerical example, or a geometrical interpretation. Linear dependence is first defined for two vectors, the determinant for 2×2 matrices, and linear transformations in $\mathbf{R}^2$. This is especially true in the early chapters. Chapter 1 is a leisurely introduction to matrices, a matrix being defined merely as an array and motivated by examples such as graphs and incidence matrices, and connectivity matrices in the social sciences. Chapter 2 also starts in this vein, with matrix operations introduced slowly and concretely. The special concern for statistical applications is already apparent here with topics not usually mentioned so early: Hadamard products, idempotent matrices, or multiplication of partitioned matrices. The Kronecker product is discussed here for the first of many times, and the treatment of it is surprisingly detailed. Chapter 3, dealing with linear independence, marks a sudden jump in the level of abstraction, even though the rank of a matrix is defined rather concretely in terms of the appearance of its row-echelon form. The treatment deepens in Chapter 4, which covers vector geometry. This chapter is at once more conceptual and more concrete: it deals with abstract vector spaces, norms and scalar products, but these notions are given appealing geometric meaning. It is a dense and difficult chapter, though, with several notions cascading within a short space: spanning set, basis, subspace, orthogonal subspaces, orthogonal complements. Some of the definitions are necessarily a little vague, since the book resolutely avoids formal mathematics. A regrettable inaccuracy here is the way the sum of two vector spaces is treated: it is called a union and the operation is denoted by $\cup$. There is no indication that this is not a union in the set-theoretic sense. Chapter 5, reflecting a statisticians concerns, deals with what the author calls three matrix reductions: trace, determinant, and norm. In addition to the usual properties of traces, results are stated for idempotent matrices and for the trace of a Kronecker product.

The determinant is defined only for 2×2 matrices, and a result is given for diagonal and triangular matrices of any order. A nonsingular matrix is defined in terms of its determinant. L_p-norms are defined, and examples are given for $p = 1, 2$, and ∞. In Chapter 6, on matrix inversion, the main properties of inverses are stated, including rules for the inverse of a Kronecker product, of a partitioned matrix, and of special matrices. Generalized inverses are introduced, and although the author states that the discussion will be both brief and elementary, there is plenty there: a general form of a generalized inverse; an algorithm for computing one; several properties; and a discussion of special kinds of inverses (reflexive inverses and the Moore-Penrose inverse.) Chapter 7, on linear transformation, is a return to simple, concrete mathematics: small matrices, graphs, and geometric interpretations in $\mathbf{R}^2$. Useful notions such as orthogonal and oblique rotation and orthogonal projections are illustrated. This paves the way for linear models, discussed in Chapter 8 along with simultaneous equations. Eigenvalues and eigenvectors are introduced in Chapter 9, with their geometrical meaning amply demonstrated in $\mathbf{R}^2$. This is where, ostensibly, the determinant of an arbitrary square matrix is at last defined: it is the product of its eigenvalues. The reader may feel cheated; especially since the definition, promised in Chapter 5, is here presented as a result, not a definition. But it is in keeping with the approach of this book that details too technical are avoided. This chapter also deals with matrix diagonalization; the maximum and the minimum of quadratic forms; and the singular-value decomposition. Chapter 10 aims essentially at defining the Mahalanobis distance and the minimum volume ellipsoid. It builds up to it gradually and intuitively, so that the motivation for the Mahalanobis distance is nice and clear. The final chapter takes a cursory look at a variety of topics. Surprisingly, the important subject of positive definite and positive semidefinite matrices is relegated to this hodgepodge chapter.

All in all, the book is successful at what it attempts to do. It is elementary without being mechanical. It covers most of what a statistical practitioner needs to know, and makes some of the more sophisticated notions accessible to the mathematically untrained. I would also recommend the book to another class of readers, namely, statistics undergraduates whose knowledge of linear algebra is inadequate because the course or courses they followed did not cover the topics most useful to statisticians or were lacking in intuitive or geometrical content.

Russell Merris: *Multilinear Algebra*

Reviewed by S. W. Drury, McGill University, Montréal

Multilinear Algebra is a subject that has had an enormous impact over the last few decades, particularly in the domains of Differential Geometry and Differential Topology. The textbook long revered by the purists in the field is Marvin Marcus's monumental *Finite Dimensional Multilinear Algebra* [Part I, x + 292 pp., Dekker, 1973; Part II, xvi + 718 pp., Dekker, 1975]. Many developments have taken place since that book appeared and *Multilinear Algebra* by Russell Merris [Algebra, Logic and Applications Series, Gordon & Breach, viii + 332 pp., ISBN 90-5699-078-0, 1997] brings the field up to date. The book is especially suitable for use as a textbook in a graduate course in Multilinear Algebra. The first four chapters lay the groundwork. In the first two, Merris covers Partitions and Inner Product Spaces. He relates both of these topics to Graph Theory, the former by developing the degree sequence of the graph and the latter by studying the spectral theory of the Laplacian matrix of a graph. This is a feature of the book: wherever there are connections to Graph Theory, Merris develops them. This is refreshing because the emphasis coming from multilinear algebra is slightly different from that usually drawn between Graph Theory and Matrix Theory. Permutation groups and the Representation Theory of Finite Groups form the subject matter of the third and fourth chapters, and they are treated in a more-or-less traditional fashion. Then, at last, comes a very thorough chapter covering the basics of Tensor Spaces without being too dry. The next two chapters develop the theory of Symmetry Classes of Tensors and Generalized Matrix Functions. The treatment of these topics is the only available discussion in book form that is truly up to date. Nevertheless, many of the more recent results are left for the reader to track down on his own, the text not really contributing substantially to the reader's enlightenment. The Symmetry Classes chapter ends with a nice generalization of Pólya Enumeration together with a further application to NMR Spectroscopy. The Generalized Matrix Functions chapter describes the outstanding conjectures of the subject: the Permanent-on-Top Conjecture and the Soules Conjecture, and details recent progress. Finally there is a chapter on the Rational Representations of $GL(n, C)$, together with an application to the graph enumeration problem. This is a nice book, stimulating to read and excellent both as a reference book and as a textbook for a graduate course.

Fuzhen Zhang: *Linear Algebra: Challenging Problems for Students*

Reviewed by Wasin So, Sam Houston State University, Huntsville

Linear Algebra: Challenging Problems for Students by Fuzhen Zhang [Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, xii + 174 pp., ISBN 0-8018-5458-X (H), ISBN 0-8018-5459-8 (P), 1996] is a collection of 200 miscellaneous problems in linear algebra with hints and solutions. Since the book is not intended to be a text, but only a supplement, the reader is assumed to have at least a first course in linear algebra. Some problems, however, do demand more knowledge than a first course will provide, for instance the Jordan form and eigenvalue inequalities. The book is split into two parts: (1) Problems, (2) Hints and Solutions. For easy reference, the 200 problems are roughly divided into five chapters: 1. Vector Spaces, 2. Determinant, Inverses, Rank, and Linear Equations, 3. Matrices, Linear Transformations and Eigenvalues, 4. Special Matrices, and 5. Inner Product Spaces. The selection of problems is based purely on the author's taste. Paul Halmos once said that solving problems is fun provided that the problems are neither too hard nor too easy. Overall the reviewer believes that the author does provide a lot of fun for the reader. To achieve a certain elegancy, however, the hints and solutions tend to lack explanation. The reviewer has found the book to be very useful in writing exams for a linear algebra course. It is hard to write a perfect book, but the author has made a list of known errors and misprints available at the website `http://www.polaris.nova.edu/~zhang`.

New & Forthcoming Books on Linear Algebra & Related Topics: 1997–1998

Listed below are some new and forthcoming books on linear algebra and related topics that have been published in 1997 or in 1998, or are scheduled for publication in 1998; (P) denotes paperback and (H) hard cover. This list updates and augments our *Guide* [69]. The references to CMP = *Current Mathematical Publications* and MR = *Mathematical Reviews* were found using MathSciNet and to Zbl = *Zentralblatt für Mathematik/MATH Database* by visiting `http://www.emis.de/cgi-bin/MATH` (three free hits are allowed for non-subscribers). The following public websites were also helpful:

Amazon—"Earths biggest bookstore" : `http://www.amazon.com`

Barnes & Noble: `http://shop.barnesandnoble.com/BookSearch/search`

COPAC: Online Public Access Catalogue [to some of the largest university research libraries in the UK and Ireland]: `http://copac.ac.uk/copac/`

Interloc: "The worlds oldest and largest database of out-of-print, rare, and antiquarian books": `http://daniel.interloc.com/`

Library of Congress Catalogs: Browse Search: `http://lcweb.loc.gov/catalog/browse/`

Melvyl System: The University of California: `http://www.melvyl.ucop.edu/`

The New York Public Library—CATNYP, The Research Libraries On-Line Catalog: `http://149.123.101.18/search/`.

Many thanks go to Simo Puntanen and Götz Trenkler for their help. All additions and corrections are most welcome and should be sent by e-mail to George Styan at `styan@Math.McGill.CA`.

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by James R. Weaver, *University of West Florida, Pensacola*

Balance on hand March 1, 1997			
Certificate of Deposits (CD)	19,000.00		
Vanguard	8,342.84		
Checking Account	22,310.44		49,653.28
**			
Checking Account Balance on March 1, 1997			22,310.44
March 1997			
Income:			
Dues	780.00		
Interest (Gen)	155.84		
Contributions			
General Fund	52.00		
Hans Schneider Prize	42.00		
OT/JT Lecture Fund	2.00		
F. Uhlig Fund	12.00		
Conference Fund	2.00		
ILAS/LAA Fund	1000.00	2.045.84	
Expenses:			
Sec. of State	70.00	70.00	1,975.84
April 1997			
Income:			
Dues	220.00		
Interest on CD (HS)	74.56		
Interest on CD (OT/JT)	29.00		
Interest on CD (FU)	23.11		
Interest (Gen)	14.98		
Contributions			
General Fund	50.00		
OT/JT Lecture Fund	20.00	431.65	
Expenses:			
Postmaster (Stamps & Mailing)	40.60		
Lisa M. Weaver (Update Files)	49.50	90.10	341.55
May 1997			
Income:			
Interest (Gen)	14.66	14.66	
Expenses:			
Judy K. Weaver (Ann.Trea.Rpt)	108.50	108.50	(93.84)
June 1997			
Income:			
Dues	280.00		
Interest (Gen)	152.49	432.49	
Expences:	00.00	00.00	432.49
July 1997			
Income:			
Dues	320.00		
Interest (Gen)	14.64		
Interest on CD (HS)	75.38		
Interest on CD (OT/JT)	29.32		
Interest on CD (FU)	23.63	462.97	
Expenses:			
Producing & Mailing "IMAGE" George P.H.Styan	1328.73		
Office Depot (Supplies)	48.63		
Postmaster (Stamps & Ballot)	240.16		
Postmaster (Stamps-Dues Notice)	245.18		
Mail Boxes Etc.(400 Ballots)	25.68		
Judy K.Weaver (Early Dues Notice)	108.50	1,996.88	(1,533.91)

August 1997			
Income:			
Dues	1131.20		
Interest (Gen)	83.44		
Contributions			
General Fund	71.00		
Hans Schneider Prize	86.00		
OT/JT Lecture Fund	221.00		
F. Uhlig Fund	71.00		
Conference Fund	51.00	1,714.64	
Expenses:			
Judy K. Weaver (Ballots)	84.00		
Debit to accounting exchange rate	6.06	90.06	1,624.58
September 1997			
Income:			
Dues	1160.00		
Interest (Gen)	86.13		
Interest on CD (FU)	23.63		
Contributions			
General Fund	20.00		
Hans Schneider Prize	350.00		
OT/JT Lecture Fund	370.00		
F. Uhlig Fund	10.00	2,019.76	
Expenses:			
Office Depot (Supplies)	209.00		
Judy K. Weaver (Recording & Updating Files)	77.00	286.00	1,733.76
October 1997			
Income:			
Interest (Gen)	15.80	15.80	
Expenses:			
Mail Boxes Etc. (Ballots)	35.31		
Postmaster (Stamps)	771.24		
Judy K. Weaver (Ballots & Mailings)	217.00	1,023.55	(1,007.75)
November 1997			
Income:			
Dues	700.44		
Interest (Gen)	84.24		
Interest on CD (HS)	76.23		
Interest on CD (OT/JT)	29.64		
Contributions			
General Fund	1.00		
Hans Schneider Prize	25.00		
OT/JT Lecture Fund	10.00		
F. Uhlig Fund	5.00	931.55	
Expenses:			
Postmaster (2nd Dues-Stamps)	130.76		
Judy K. Weaver (Mailing)	77.00		
Commercial Service Charge	.75	208.51	723.04

December 1997			
Income:			
Dues	760.00		
Interest (Gen)	85.68		
Contributions			
General Fund	30.00		
Hans Schneider Prize	110.00		
Conference Fund	5.00	990.68	
Expenses:			
Producing & Mailing "IMAGE"			
George P.H. Styan	1307.00		
Judy K. Weaver (ILAS Files)	33.25		
Service Charge	3.67	1,343.92	(353.24)
January 1998			
Income:			
Dues	840.00		
Interest (Gen)	15.54		
Interest on CD (HS)	76.23		
Interest on CD (OT/JT)	29.64		
Interest on CD (FU)	23.37		
Contributions			
General Fund	40.00		
Hans Schneider Prize	110.00		
OT/JT Lecture Fund	310.00		
F. Uhlig Fund	115.00		
Conference Fund	10.00	1,569.78	
Expenses:			
Returned Item Charge	20.00		
Service Charge (Dec)	4.32	24.32	1,545.46
February 1998			
Income:			
Interest (Gen)	15.08	15.08	
Expenses			
Service Charge (Jan)	10.79	10.79	4.29

****

February 28, 1998		27,702.71
Account Balance		
Checking Account	27,702.71	
Certificate of Deposit (FU)	1,500.00	
Certificate of Deposit (Gen.)	10,000.00	
Certificate of Deposit (72% HS & 28% OT/JT)	7,500.00	
Vanguard (72% HS & 28% OT/JT)	8,882.15	55,584.86
General Fund	22,178.61	
Frank Uhlig Educational Fund	2,699.00	
Hans Schneider Prize	14,735.36	
Olga Taussky Todd/John Todd Fund	7,775.01	
Conference Fund	7,196.88	
ILAS/LAA Fund	1,000.00	55,584.86

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James R. Weaver
3/28/98

Thematic Term on Linear Algebra and Control Theory
Coimbra Astronomical Observatory, May 11–August 7, 1998

The Thematic Term on Linear Algebra an Control Theory will be held at the Coimbra Astronomical Observatory in the Spring of 1998: May 11–August 7, 1998. Some researchers will attend this event throughout the full three months, whilst others will participate for periods of one month or less. A weekly research seminar will be organized, led by either one of the permanent members or by guest speakers. There will be two summer schools aimed at graduate students, each consisting of two ten-hour courses:

1. "Structure and Design of Linear Systems" by Jean J. Loiseau and Petr Zagalak (May 21–29).
2. "Linear Algebra and Control Theory" by Paul Fuhrmann and Peter Lancaster (June 15–23).

To register, please, send an e-mail with: Your name, institution, e-mail and the indication that you wish to participate in this school to `lin98@hermite.cii.fc.ul.pt`. A registration fee of 15000 Portuguese Escudos is due and must be paid at the school. A Workshop will be held June 24–26, 1998, when most short-stay researchers will be present. A limited number of 15 minute contributed talks will be accepted. Please, submit your title and abstract to `lin98@hermite.cii.fc.ul.pt`.

Accommodation (Schools and Workshop): There are special prices for participants at several hotels. For information about accommodation/reservations, please contact: Mrs. Liliana Seabra, Centro Internacional de Matemática, Complexo do Observatório Astronómico, Almas de Freire, P-3040 Coimbra, Portugal; FAX: 351-39-445380; `lin98@hermite.cii.fc.ul.pt`.

Participants confirmed as of February 98 are (in alphabetical order): Itziar Baragaña (Univ. Pais Vasco, Spain)–May; Isabel Cabral (Univ. Nova de Lisboa, Portugal)–May/July; Joan J. Climent (Univ. Alicante, Spain)–June/2nd half; Jose A. Dias da Silva (Univ. Lisboa / CIM, Portugal)–May/July; Paul van Dooren (Catholic Univ. Louvain. Belgium)–June/last week; Paul Fuhrmann (Ben Gurion Univ., Israel)–May/July; Juan M. Gracia (Univ. Pas Vasco, Spain)–June/1st half; Diederich Hinrichsen (Univ. Bremen, Germany)–June/last week; Charles R. Johnson (College William and Mary, U.S.A.)–June/2nd half; Vladimir Kuera (Acad. Sciences, Czech Rep.)–May 11-17; Peter Lancaster (Univ. Calagary, Canada)–June; Jean J. Loiseau (CNRS, Nantes, France)–May; Michel Malabre (CNRS Nantes, France)–June/last week; Alexander Markus (Ben Gurion Univ., Israel)–June/2nd half; Nancy Nichols (Univ. Reading, UK)–June/last week; Fernando Puerta (Univ. Polit. Barcelona, Spain)–June/July; J. F. Queiró (Univ. Coimbra, Portugal)–May/July; Joachim Rosenthal (Univ. Notre Dame, USA)–June; E. Marques de S (Univ. Coimbra, Portugal)–May-July; Carsten Scherer (Tech. Univ. Delft, The Netherlands)–June/July; Hans Schumacher (CWI Amsterdam, The Netherlands)–June, last week; Fernando C. Silva (Univ. Lisboa, Portugal)–May/July; Ion Zaballa (Univ. Pas Vasco, Spain)–May/July; Petr Zagalak (Acad. Sciences, Czech Rep.)–May/June.

Organizing Committee: I. Zaballa (Univ. Pais Vasco, Spain), Coordinator: `mepzatej@lg.ehu.es`, Jose A. Dias da Silva (Univ. Lisboa / CIM, Portugal): `perdigao@hermite.cii.fc.ul.pt`, Fernando C. Silva (Univ. Lisboa, Portugal): `fcsilva@fc.ul.pt`. For further information please visit `http://hermite.cii.fc.ul.pt/lin98/`.

Matrix Theory Workshop in Honour of Professor Yik-Hoi Au-Yeung
The University of Hong Kong, May 15, 1998

A Matrix Theory Workshop, in honour of the retirement of Professor Yik-Hoi Au-Yeung, will be held in Room G5, James Hsioung Lee Science Building, University of Hong Kong, on Friday May 15, 1998. The programme of the workshop comprises of a number of talks on various aspects of matrix theory. Tentative speakers include: Che-Man CHENG (University of Macau), Siu-Por LAM (Chinese University of Hong Kong), Chi-Kwong LI (College of William and Mary, USA), Yiu-Tung POON (Iowa State University, USA), Lok-Shun SIU (University of Hong Kong), Bit-Shun TAM (Tam Kang University, Taiwan), Tin-Yau TAM (Auburn University, USA), Nam-Kiu TSING (University of Hong Kong).

Contributed talks are welcome, subject to availability of time slots. For further information, please contact: Dr. Nam-Kiu TSING, Department of Mathematics, The University of Hong Kong, Pokfulam Road, Hong Kong, Tel: (852) 2859-2251, Fax: (852) 2559-2225, `nktsing@hku.hk`.

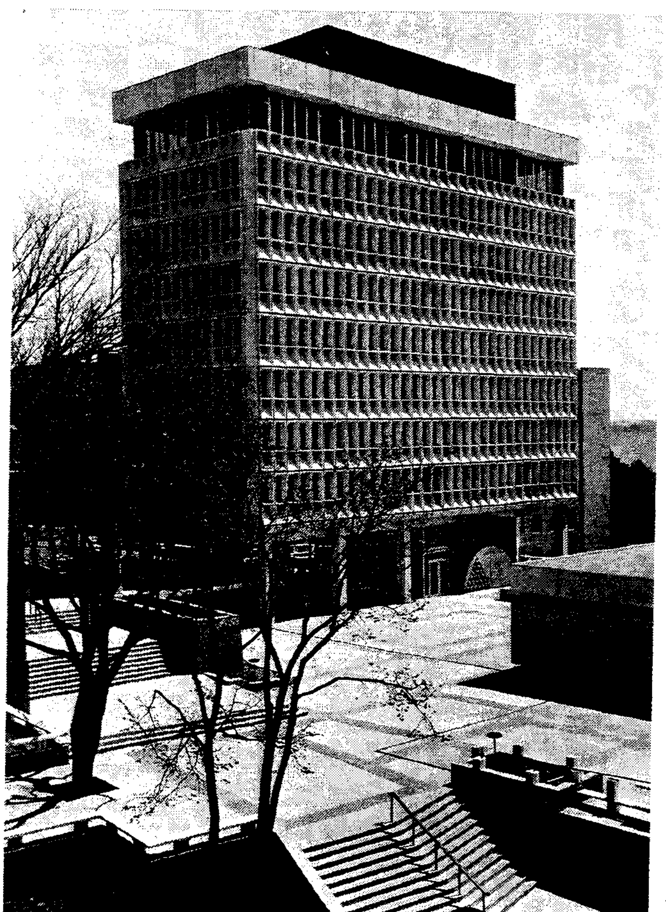
Seventh Conference of the International Linear Algebra Society
The "Hans Schneider Linear Algebra" Conference
Madison, Wisconsin, USA: June 3–6, 1998

The 7th Conference of the International Linear Algebra Society: The "Hans Schneider Linear Algebra" Conference will provide an opportunity to bring together researchers and educators in all aspects of pure and applied linear algebra and matrix theory in order to allow for a broad exchange of ideas and dissemination of recent developments and results. The conference will be dedicated to Hans Schneider in recognition of his enormous contributions to linear algebra and the linear algebra community. The program as of April 21, 1998, appears on pp. 16–21 below; the final program may be viewed as `http://math.wisc.edu/~brualdi/program.ps`.

THEMES: Algebraic, analytic and combinatorial matrix theory, numerical linear algebra, matrix perturbations, matrix stability and applications in engineering, linear algebra in control and systems theory, splines and linear algebra, applications of linear algebra to statistics, linear algebra education.

ORGANIZING COMMITTEE: R. A. Brualdi (chair), B. Cain, B. Datta, J. Dias da Silva, S. Friedland, M. Goldberg, U. Rothblum, J. Stuart, D. Szyld, and R. Varga.

CONFERENCE PROCEEDINGS: A special issue of the journal *Linear Algebra and Its Applications* (LAA), with special editors B. Cain, B. N. Datta, M. Goldberg, U. Rothblum, and D. Szyld. The issue will be dedicated to Hans Schneider. Speakers who wish to have their paper considered for publication in this special issue of LAA should submit their paper to one of the special editors. Only those papers that meet the standards of LAA and that are approved by the normal refereeing process will be published in the special issue. Deadline for submission is September 30, 1998.



Van Vleck Hall (photo courtesy: University of Wisconsin)

HOUSING: An entire (air-conditioned) dormitory, Chadbourne Hall, a two-minute walk to the mathematics building Van Vleck Hall, has been reserved for conference participants. A special floor will accommodate couples. Breakfast is included in the room rate of $40 single and $27 per person in double room. Conference participants are encouraged to stay at Chadbourne Hall. Rooms have also been set aside at a nearby Howard Johnson's Hotel. Those wishing to stay at the Howard Johnson's should make their reservations directly by calling: 608-251-5511. Mention the conference and the University to get the conference rate of $74 single and $84 double.

BANQUET: A banquet will be held on Friday, June 5, 1998, starting at 7 pm, in the Great Hall of the Memorial Union (the Union on the Terrace overlooking Lake Mendota) of UW–Madison. After dinner, several people will offer reminiscences and vignettes about Hans Schneider.

EXCURSION: An excursion is scheduled for Saturday afternoon, June 6, 1998, to the famous "House on the Rock" in nearby Spring Green. This is more than a fascinating house; it is a veritable large museum of remarkable mechanical and electrical contraptions with an incredible in-door carousel. Not to be missed!

WORKSHOP: A Numerical Range Workshop will precede the ILAS conference on June 1–2, 1998. For information please contact Chi-Kwong Li at `ckli@math.wm.edu` or visit `http://www.math.wm.edu/~ckli/wonra.html`.

Wednesday, June 3, 1998

9:00–9:45 am	B. ROSS BARMISH: *Risk-Adjusted Linear Algebra: A New Paradigm*
9:50–10:35 am	RAJENDRA BHATIA: *The Positive Definiteness of Some Matrices*
11:05–11:30 am	RAPHAEL LOEWY: *Class Function Inequalities for Positive Semidefinite Matrices*
11:35–12:00 noon	PETER LANCASTER: *The Inertia of Dissipative Quadratic Polynomials or Ostrowski-Schneider Revisited*
12:00–12:15 pm	*Conference Photo*
	MINISYMPOSIUM on Graph Theory and Linear Algebra I: *Organized by* R. Merris
2:00–2:25 pm	R. B. Bapat: *Linear estimation in models based on a graph*
2:30–2:55 pm	Pavel Yu. Chebotarev: *A graph interpretation of the Moore-Penrose inverse of the Laplacian matrix*
3:00–3:25 pm	Onn Chan: *Vertex orientations and immanantal inequalities*
3:30–3:55 pm	Douglas Klein: *Intrinsic metrics on graphs, distance matrices and graph invariants*
	MINISYMPOSIUM on Topological Methods in Linear Algebra I: *Organized by* M. Goldberg
2:00–2:25 pm	Shmuel Friedland: *The noncrossing rule revisited*
2:30–2:55 pm	Moshe Goldberg: *Not all polynomial mappings are continuous*
3:00–3:25 pm	Roy Meshulam: *Extremal problems for spaces of matrices*
3:30–3:55 pm	Joel Robbin: *Maslov index and matrix theory*
	MINISYMPOSIUM on Educational Issues in Linear Algebra I: *Organized by* D. Carlson and F. Uhlig
2:00–2:15 pm	Luz Maria DeAlba: *Teaching linear independence, span, and basis*
2:20–2:35 pm	David Carlson: *Teaching span and linear independence first*
2:40–2:55 pm	Jane M. Day: *Some ways to provide variety and insight*
3:00–3:20 pm	Richard O. Hill: *Still trying to keep the fog at bay—not completely succeeding*
3:25–3:45 pm	Thomas W. Polaski: *How foggy is it? Writing assignments and assessment in linear algebra*
	CORE LINEAR ALGEBRA I
2:00–2:15 pm	R. Cantó: *On the Segre characteristic of a block triangular matrix*
2:20–2:35 pm	Wasin So: *Linear operators preserving the number of nontrivial invariant polynomials*
2:40–2:55 pm	Enide Martins: *Eigenvalues of matrix commutators and Jordan products*
3:00–3:15 pm	J. D. Botha: *Products of diagonalizable matrices*
3:20–3:35 pm	Pei Yuan Wu: *Dilation to inflations of $S(\phi)$*
3:40–3:55 pm	Reinhard Nabben: *Tridiagonal matrices and their inverses*
	NUMERICAL AND COMPUTATIONAL LINEAR ALGEBRA I
2:00–2:15 pm	Daniel B. Szyld: *The mystery of asynchronous iterations convergence when the spectral radius is one*
2:20–2:35 pm	Ivo Marek: *An aggregation/disaggregation iteration scheme using general orderings*
2:40–2:55 pm	Ljiljana Cvetković: *Some convergence results about alternating iterations*
3:00–3:15 pm	Karen S. Braman: *Aspects of the multi-shift QR-Algorithm: Early deflation and maintaining well-focused shifts to achieve level 3 performance*
3:20–3:35 pm	Heike Faßbender: *Some connections between the SR/SZ and HR/HZ eigenvalue algorithms*
3:40–3:55 pm	Donald W. Robinson: *A useful quality indicator for a finite element calculation*
4:05–4:50 pm	CARL DE BOOR: *Linear Algebra Issues in Approximation Theory*
5:00–6:30 pm	*Reception: 9th Floor Lounge, Van Vleck Hall*

Thursday, June 4, 1998

8:30–9:15 am	GUERSHON HAREL: *A Developmental Model of Students' Understanding of Proofs: Historical, Philosopical, and Cognitive Considerations*
9:20–10:05 am	THOMAS J. LAFFEY: *Some New Inequalities in the Nonnegative Inverse Eigenvalue Problem*

Thursday, June 4, 1998 (cont'd)

MINISYMPOSIUM on Graph Theory and Linear Algebra II: *Organized by* R. Merris

10:35–11:00 am Miroslav Fiedler: *Additive compound matrices and graphs*

11:05–11:30 am Steve Kirkland: *Extremizing algebraic connectivity subject to graph theoretic constraints*

11:35–12:00 am Alex Pothen: *Distance matrices, their eigenvalues and vertex numbering of graphs*

MINISYMPOSIUM on Educational Issues in Linear Algebra II: *Organized by* D. Carlson and F. Uhlig

10:35–10:50 am R. Cantó: *On computer labs in linear algebra*

10:55–11:10 am Christopher Beattie: *An on-line course in linear algebra for first-year students*

11:15–11:35 am Jerry Uhl: *Matrices, Geometry and Mathematica*

11:40–11:55 am Steven J. Leon: *Using software to visualize eigenvalues and singular values*

12:00–12:20 pm A. Berman: *Linear transformations*

12:25–12:40 pm Biswa N. Datta: *Teaching of numerical linear algebra*

MINISYMPOSIUM on Structured Matrix Problems I: *Organized by* Lothar Reichel

10:35–11:00 am Daniela Calvetti: *On an inverse eigenvalue problem for Jacobi matrices and application to Gauss-Kronrod quadrature rules*

11:05–11:30 am Biswa N. Datta: *Recent results on partial pole and eigenstructure assignments for matrix second order control systems*

11:35–12:00 noon Greg Ammar: *The QR algorithm for orthogonal Hessenberg matrices*

12:05–12:30 pm William B. Gragg: *Stabilization of the UHQR algorithm*

CORE LINEAR ALGEBRA II

10:35–10:50 am Fuzhen Zhang: *Inequalities on Schur Complements*

10:55–11:10 am K. Okubo: *ρ-radius and some matrices*

11:15–11:30 am Tatjana Petek: *Spectrum and commutativity preserving mappings on Hermitian matrices*

11:35–11:50 am L. Elsner: *Orthogonal bases that lead to symmetric nonnegative matrices*

11:55–12:10 pm Richard Varga: *Hardy's inequality and ultrametric matrices*

12:15–12:30 pm Georg Heinig: *Minimal rank block Hankel matrix extensions and partial realization*

EIGENVALUES AND SINGULAR VALUES

10:35–10:50 am Hector Rojo: *On related bounds for the extreme eigenvalues*

10:55–11:10 am G. W. Stewart: *On the eigenstructure of graded matrices*

11:15–11:30 am Pál Rózsa: *Spectral properties of symmetrically reciprocal matrices*

11:35–11:50 am Oscar Rojo: *New lower (upper) bounds for the smallest (largest) singular value*

11:55–12:10 pm Xingzhi Zhan: *On the singular values of matrix sums*

12:15–12:30 pm Chi-Kwong Li: *Inequalities of the singular values of an off-diagonal block of a Hermitian matrix*

2:00–2:45 pm VOLKER MEHRMANN: *Jordan and Schur forms in Lie and Jordan Algebras*

INTERACTIVE-TEXT SESSION I

2:50–3:15 pm John R. Wicks: *Student-centered learning using a Mathematica Lab*

3:20–3:45 pm Eugene A. Herman: *The linear algebra modules project*

MINISYMPOSIUM on Linear Algebra Methods in Statistics I: *Organized by* H. J. Werner

2:50–3:15 pm Augustyn Markiewicz: *Matrix inequalities and their statistical applications*

3:20–3:45 pm George P. H. Styan: *Some comments on some selected matrix inequalities and equalities, with some statistical applications*

MINISYMPOSIUM on Matrix Equations, Inertia, Stability & Stabilization I: *Organized by* B. N. Datta

2:50–3:15 pm Peter Lancaster: *Optimal decay of energy for a system of oscillators*

3:20–3:45 pm T. Ando: *Simultaneous strong stability*

Thursday, June 4, 1998 (cont'd)

NUMERICAL AND COMPUTATIONAL LINEAR ALGEBRA II

2:50–3:05 pm Vlastimil Pták: *Krylov sequences of maximal length and convergence of GMRES*

3:10–3:25 pm Jinxi Zhao: *A quasi-minimal residual method based on incomplete orthogonalization for non-symmetric linear systems*

3:30–3:45 pm Zdeněk Strakoš: *Accuracy of three-term and two-term recurrences for Krylov space solvers*

MATRIX PROBLEMS FROM CONTROL AND SYSTEMS THEORY I

2:50–3:05 pm A. C. M. Ran: *Complete sets of solutions to algebraic Riccati equations*

3:10–3:25 pm Peter Benner: *The disk function and its relation to algebraic Riccati equations*

3:30–3:45 pm Mª I. García-Planas: *Structural invariants for linear differential-algebraic equations*

COMBINATORIAL MATRIX THEORY I

2:50–3:05 pm Peter M. Gibson: *Combinatorially orthogonal matrices and related graphs*

3:10–3:25 pm Bryan L. Shader: *Sparse orthogonal bases for hyperplanes*

3:30–3:45 pm Arnold R. Kräuter: *An upper bound for the permanent of a nonnegative matrix*

OPERATORS

2:50–3:05 pm Luis Verde-Star: *One-sided inverses of linear generalized differential operators*

3:10–3:25 pm Joan-Josep Climent: *General comparison theorems for splittings of nonsingular bounded operators*

3:30–3:45 pm Steve Galitsky: *On comparison of closeness measures of positively-defined operators numerical experiments*

INTERACTIVE-TEXT SESSION II

4:15–4:40 pm Elias Deeba: *Teaching linear algebra using an interactive text*

4:45–5:10 pm Gerald J. Porter: *Teaching linear algebra as a lab course*

MINISYMPOSIUM on Linear Algebra Methods in Statistics II: *Organized by* H. J. Werner

4:15–4:40 pm N. Rao Chaganty: *The analysis of longitudinal data using quasi-least squares*

4:45–5:10 pm H. J. Werner: *On some estimating techniques for the general Gauss-Markov model*

MINISYMPOSIUM on Matrix Equations, Inertia, Stability & Stabilization II: *Organized by* B. N. Datta

4:15–4:40 pm David Carlson: *Recent advances in a problem on invariants*

4:45–5:10 pm Leiba Rodman: *Linear matrix equations and rational matrix functions*

NUMERICAL AND COMPUTATIONAL LINEAR ALGEBRA III

4:15–4:30 pm Mei-Qin Chen: *The stability and convergence of a direction set based algorithm for adaptive least squares problems*

4:35–4:50 pm Tibor Boros: *Fast solution of Vandermonde-type least-squares problems*

4:55–5:10 pm Kuiyuan Li: *An algorithm for the generalized tridiagonal eigenvalue problem*

MATRIX PROBLEMS FROM CONTROL AND SYSTEMS THEORY II

4:15–4:30 pm F. Puerta: *Topological structure of the set of $(A, B)^t$-invariant subspaces*

4:35–4:50 pm J. Clotet: *Bounding the distant of a controllable system to an uncontrollable one*

4:55–5:10 pm Cristina Jordán: *Controllability completions problems of matrices in upper canonical form*

COMBINATORIAL MATRIX THEORY II

4:15–4:30 pm J. A. Dias da Silva: *Rank partition of a matroid and the rank partition of its dual*

4:35–4:50 pm James R. Weaver: *Tournament matrices and the Brualdi-Li Conjecture*

4:55–5:10 pm Olga Azenhas: *Combinatorics of Littlewood-Richardson fillings and matrix spectral problems*

MATRIX PERTUBATIONS

4:15–4:30 pm Xiao-Wen Chang: *Componentwise perturbation analyses for the QR factorization*

4:35–4:50 pm Erxiong Jiang: *Perturbations in the eigenvalues of a symmetric tridiagonal matrix*

4:55–5:10 pm Yimin Wei: *Perturbation bound of a singular linear system*

5:15–6:15 pm ILAS Business Meeting

Friday, June 5, 1998

8:30–9:15 am	DANIEL HERSHKOWITZ: *The Combinatorial Structure of Generalized Eigenspaces from Nonnegative Matrices to General Matrices*
9:20–10:05 am	NICHOLAS J. HIGHAM: *QR Factorization with Complete Pivoting and Accurate Computations of the SVI*
	INTERACTIVE LAB SESSION I:
10:35–11:30 am	Gerald J. Porter: *Lab Session*
11:35 am–12:30 pm	Elias Deeba: *Lab Session*
	MINISYMPOSIUM on Linear Algebra Methods in Statistics III: *Organized by* H. J. Werner
10:35–11:00 am	Carl D. Meyer: *Sensitivity of Markov chains*
11:05–11:30 am	Adi Ben-Israel: *Integration using matrix volume: Applications in probability and statistics*
11:35–12:00 noon	Peter McCullagh: *Factorial models and invariant subspaces*
12:05–12:30 pm	John S. Chipman: *Linear restrictions, rank reduction, and biased estimation in linear regression*
	MINISYMPOSIUM on Matrix Equations, Inertia, Stability & Stabilization III: *Organized by* B. N. Datta
10:35–11:00 am	Bryan E. Cain: *Generalizations of Sylvester's theorem and their associated canonical forms*
10:05–11:30 am	Biswa N. Datta: *Inertia, stability, and stabilization*
11:35–12:00 noon	P. A. Fuhrmann: *On the Lyapunov equation and coinvariant subspaces of Hardy spaces*
12:05–12:30 pm	S. P. Bhattacharyya: *A generalization of the classical Hermit-Bieler theorem with applications to control*
	MINISYMPOSIUM on Numerical Linear Algebra I: *Organized by* M. Overton
10:35–11:00 am	James W. Demmel: *When can the SVD be computed accurately?*
11:05–11:30 am	Danny Sorenson: *Deflation schemes of implicitly restarted Arnoldi methods*
11:35–12:00 noon	Ming Gu: *On rank-revealing factorizations*
12:05–12:30 pm	Alan Edelman: *Algorithms for nearest canonical forms*
	STRUCTURED MATRICES
10:35–10:50 am	Georg Heinig: *Stabilized superfast Toeplitz solvers based on interpolation*
10:55–11:10 am	Steven J. Kifowit: *A superfast version of the split Schur algorithm*
11:15–11:30 am	David T. Clydesdale: *A superfast algorithm for solving positive definite block Toeplitz systems*
11:35–11:50 am	William F. Trench: *Asymptotic distribution of the even and odd spectra of real symmetric Toeplitz matrices*
11:55–12:10 pm	Niloufer Mackey: *Structure-preserving Jacobi methods for doubly-structured matrices*
12:15–12:30 pm	Vadim Olshevsky: *Eigenvector computations for almost-unitary-Hessenberg matrices via discrete transmission lines*
	NONNEGATIVE MATRICES
10:35–10:50 am	Jane M. Day: *On a theorem and conjecture of Garloff*
10:55–11:10 am	Shaun M. Fallat: *On the spectrum of totally nonnegative matrices*
11:15–11:30 am	Roy A. Mathias: *A new approach to totally positive matrices*
11:35–11:50 am	Naomi Shaked-Monderer: *Support-restricted rank 1 representations of completely positive matrices*
11:55–12:10 pm	D. Dale Olesky: *Relations between Perron-Frobenius results for matrix pencils*
12:15–12:30 pm	Siegfried M. Rump: *Generalized Perron-Frobenius theory*
2:00–2:25 pm	JUDITH J. McDONALD: *Splittings of Singular M-matrices*
2:30–2:55 pm	FERNANDO C. SILVA: *On the Eigenvalues of Sums and Products of Matrices*
	INTERACTIVE LAB SESSION II
3:35–4:30 pm	Eugene A. Herman: *Lab Session*
4:35–5:30 pm	John R. Wicks: *Lab Session*

Friday, June 5, 1998 (cont'd)

	MINISYMPOSIUM on Structured Matrix Problems II: *Organized by* Lothar Reichel
3:25–3:50 pm	Vadim Olshevsky: *Pivoting for structured matrices with applications*
3:55–4:20 pm	Thilo Penzl: *A cyclic low rank Smith method for large, sparse Lyapunov equations*
4:25–4:50 pm	Misha Kilmer: *Preconditioners for structured matrices arising in mine detection*
4:55–5:20 pm	James G. Nagy: *Kronecker product decomposition of block Toeplitz matrices*
	MINISYMPOSIUM on Matrix Equations, Inertia, Stability & Stabilization IV: *Organized by* B. N. Datta
3:25–3:50 pm	Shaul Gutman: *Worst case design for robust compensation*
3:55–4:20 pm	Georg Heinig: *Inertia of matrix polynomials and Block Hankel matrices*
4:25–4:50 pm	Sergio Bittanti: *Invariant reformulations of periodic systems*
	CORE LINEAR ALGEBRA III
3:25–3:40 pm	Michael Neumann: *Partial norms and the convergence of general products of matrices*
3:45–4:00 pm	Roger A. Horn: *The moment matrix and some applications*
4:05–4:20 pm	Vladimir A. Strauss: *The matrix moment problem: a definitizable version*
4:25–4:40 pm	Pedro Freitas: *On an action of the symplectic group*
4:45–5:00 pm	Kenneth R. Driessel: *On the geometry of some isospectral surfaces*
5:05–5:20 pm	S. P. Tan: *Principal congruence subgroups of the Hecke group*
5:25–5:40 pm	Natália Bebiano: *Some matrix inequalities in physics*
5:45–6:00 pm	John Maroulas: *On the numerical range of rational matrix functions*
	NUMERICAL AND COMPUTATIONAL LINEAR ALGEBRA IV
3:25–3:40 pm	David Saunders: *Solving parametric linear systems*
3:45–4:00 pm	Barry Nelson: *How solving linear equations can benefit from differential equation*
4:05–4:20 pm	Yoopyo Hong: *A group homology method for symmetric eigen problems*
4:25–4:40 pm	Eugene C. Boman: *Computing preconditioners via subspace projection*
4:45–5:00 pm	Françoise Tisseur: *Structured backward error and condition number of quadratic eigenvalue problems*
5:05–5:20 pm	Xiao-Wen Chang: *Componentwise pertubation analyses for the QR factorization*
5:25–5:40 pm	Thomas Szirtes: *A matrix method to generate dimensionless variables*
5:45–6:00 pm	Frank Uhlig: *General polynomial roots and their multiplicities on $O(n)$ memory and $O(n^2)$ time*
	COMBINATORIAL MATRIX THEORY III
3:25–3:40 pm	Elena Shamis: *On a duality between matrices and Σ-proximities*
3:45–4:00 pm	P. van den Driessche: *The power method in Max Algebra distance four*
4:05–4:20 pm	Michael Tsatsomeros: *Extremal properties of ray-nonsingular matrices*
4:25–4:40 pm	Carolyn Eschenbach: *Eigenvalue distribution of certain ray patterns*
4:45–5:00 pm	A. Torres-Cházaro: *Construction of binary constant weight codes with minimum distance 4*
5:05–5:20 pm	G. Sampath: *Ordered adjacency matrices*
5:25–5:40 pm	M. Antonia Duffner: *Linear transformations converting immanants*
5:45–6:00 pm	António Leal Duarte: *Eigenvalues of principal submatrices of acyclic matrices*
6:30–10:00 pm	BANQUET: Great Hall, Memorial Union

Saturday, June 6, 1998

	MINISYMPOSIUM on Numerical Linear Algebra II: *Organized by* M. Overton
8:30–8:55 am	Roy Mathias: *Careful analysis can increase speed of eigenvalue algorithms*
9:00–9:25 am	Julio Moro: *Weyl-type relative pertubation bounds for eigenvalues of hermitian matrices*
9:30–9:55 am	Nicholas J. Higham: *The nearest definite pair and the inner numerical radius*
10:00–10:25 am	Ilse C. F. Ipsen: *If a matrix has only a single eigenvalue, how sensitive is this eigenvalue?*

Saturday, June 6, 1998 (cont'd)

CORE LINEAR ALGEBRA IV

8:30–8:45 am	André Klein: *On the solution of Stein's equation, the Bezoutian, and statistical information*
8:50–9:05 am	Tian-Gang Lei: *Positive semi-definiteness in a generalized group algebra*
9:10–9:25 am	Rafael Bru: *Group involutory matrices*
9:30–9:45 am	M. C. Gouveia: *On the Stein and Lyapunov equations over rings*
9:50–10:05 am	Ralph DeMarr: *Groups of matrices*
10:10–10:25 am	Ilya M. Spitkovsky: *An update on the factorization problem for block triangular matrix functions*

MATRIX PROBLEMS FROM CONTROL AND SYSTEMS THEORY III

8:30–8:45 am	A. Compta: *Observable pairs having prescribed observable and nilpotent supplementary blocks*
8:50–9:05 am	K.-H. Förster: *On the nonnegative realization problem for rational functions*
9:10–9:25 am	Fernando Rincon: *Towards a design of a robust eigenvalue assignment method*
9:30–9:45 am	Mª D. Magret: *Linear differential-algebraic equations. A geometric approach*
9:50–10:05 am	Néstor Thome: *Balanced model reduction of discrete-time systems by state-feedbacks*
10:10–10:25 am	Susan Furtado: *Completion problems and some applications to linear systems*

COMBINATORIAL MATRIX THEORY IV

8:30–8:45 am	Sang-Gu Lee: *On a sign-pattern matrix and its related algorithms for L-matrices*
8:50–9:05 am	Frank Hall: *Sign patterns of idempotent matrices*
9:10–9:25 am	Zhongshan Li: *Potentially nilpotent sign pattern matrices*
9:30–9:45 am	David P. Stanford: *Patterns that allow given row and column sums*
9:50–10:05 am	Jeffrey Stuart: *Sign k-potent sign pattern matrices*
10:10–10:25 am	Juan Torregrosa: *The sign symmetric P-matrix completion problem*

MATRIX CANONICAL FORMS

8:30–8:45 am	Dennis I. Merino: *A note on quaternion matrices and complex partitioned matrices*
8:50–9:05 am	D. Steven Mackey: *Hamiltonian square roots of skew-Hamiltonian matrices*
9:10–9:25 am	Mª I. García-Planas: *A normal form for contragradient pencils depending on parameters*
9:30–9:45 am	Bit-Shun Tam: *On the Jordan canonical forms of an eventually nonnegative matrix and related topics*
9:50–10:05 am	Dennis I. Merino: *The Jordan canonical forms of orthogonal and skew-symmetric matrices*
10:10–10:25 am	Mark A. Mills: *Layers of matrices: The conjunctivity-similarity equivalence class of matrices*
11:00–11:45 am	URIEL G. ROTHBLUM: *Matrix Scalings: Existence, Characterization, and Computation*

Numerical Analysis and Computers: 50 Years of Progress
University of Manchester, Manchester, England, June 16–17, 1998

This conference, organized by Nick Higham and David Silvester of the Manchester Centre for Computational Mathematics, ties in with the celebrations in Manchester to mark the 50th Anniversary of the birth of "the Baby", the first stored-program electronic digital computer—born at the University of Manchester on June 21, 1948. The meeting runs from 9 am on Tuesday, June 16 until 2 pm on Wednesday, June 17. The aim is to describe how numerical analysis has been influenced by the development of computers over the last 50 years. The meeting comprises invited talks covering key areas including numerical linear algebra, optimisation, ODEs, PDEs and computational fluid dynamics, and will contain historical remarks, perspectives and anecdotes. The influence of high-performance computing on the future of numerical analysis will also be assessed. A dinner will be held on the Tuesday evening. The speakers include: Jack Dongarra (Knoxville), Brian Ford (NAG), Ian Gladwell (SMU, Dallas), Gene Golub (Stanford), Cleve Moler (The MathWorks), Bill Morton (Bath and Oxford), Mike Powell (Cambridge), Mark Sofroniou (Wolfram Research), Andrew Stuart (Stanford), Nick Trefethen (Oxford), and Joan Walsh (Manchester). For further details and registration forms please visit `http://www.ma.man.ac.uk/NAC98`. There is no registration fee for graduate students. The meeting is sponsored by the London Mathematical Society, NAG Ltd., and Wolfram Research.

Western Canada Linear Algebra Meeting Victoria, British Columbia, July 30–31, 1998

The Western Canada Linear Algebra Meeting (W-CLAM) provides an opportunity for mathematicians in western Canada working in linear algebra and related fields to meet, present accounts of their recent research, and to have informal discussions. While the meeting has a regional base, it also attracts people from outside the geographical area. Participation is open to anyone who is interested in attending or speaking at the meeting. Previous W-CLAMs were held in Regina (1993), Lethbridge (1995) and Kananaskis (1996).

The fourth W-CLAM will be held in Victoria, B.C., July 30–31, 1998. Participants are invited to contribute a talk on linear algebra or its applications (30–45 minutes, depending on the number of participants). Abstracts should be sent to Steve Kirkland `kirkland@math.uregina.ca` or Michael Tsatsomeros `tsat@math.uregina.ca` by June 15, 1998. The participation fee for the meeting is $20 (Canadian), to be collected at the meeting. This fee will be waived for participating students. Some support towards expenses will be available to a maximum of 6 students or post-doctoral fellows who wish to give a talk, on a first-come first-served basis. For up-to-date information on W-CLAM '98 please visit: `http://www.math.uregina.ca/~tsat/wclam.html/`. The W-CLAM Organizing Committee comprises: Hadi Kharaghani: `hadi@cs.uleth.ca`; Stephen Kirkland: `kirkland@math.uregina.ca`; Peter Lancaster: `lancaste@acs.ucalgary.ca`; Dale Olesky: `dolesky@csr.csc.uvic.ca`; Michael Tsatsomeros: `tsat@math.uregina.ca`; and Pauline van den Driessche: `pvdd@math.uvic.ca`.

Following W-CLAM '98, the University of Lethbridge will be offering a mini-course entitled "Coding, Cryptography and Computer Security" on August 3–7, 1998. For further information on the mini-course, please contact Hadi Kharaghani or for up-to-date information visit `http://www.cs.uleth.ca/workshop/`.

Seventh International Workshop on Matrices and Statistics, in Celebration of T. W. Anderson's 80th Birthday Fort Lauderdale, Florida: December 11–14, 1998

The Seventh International Workshop on Matrices and Statistics, in celebration of T. W. Anderson's 80th birthday, will be held at Nova Southeastern University, Fort Lauderdale, Florida, on the weekend from Friday, December 11 through Monday, December 14, 1998. For their support of this Workshop we are very grateful to the International Linear Algebra Society (ILAS), Nova Southeastern University, and the Statistical Society of Canada (SSC). The International Organizing Committee for this Workshop comprises R. William Farebrother (Victoria University of Manchester, England), Simo Puntanen (University of Tampere, Tampere, Finland), George P. H. Styan (McGill University, Montréal, Québec, Canada; chair), and Hans Joachim Werner (University of Bonn, Bonn, Germany; vice-chair). The Local Organizing Committee at Nova Southeastern University comprises Naomi D'Alessio, William D. Hammack, David Simon and Fuzhen Zhang (chair). This Workshop is the seventh in a series. The previous six Workshops were held as follows: (1) Tampere, Finland: August 1990, (2) Auckland, New Zealand: December 1992, (3) Tartu, Estonia: May 1994, (4) Montréal, Québec, Canada: July 1995, (5) Shrewsbury, England: July 1996, and (6) İstanbul, Turkey: August 1997. The 8th Workshop will be held in Tampere, Finland, August 6–9, 1999.

The purpose of this Seventh Workshop, the first to be held in the United States, is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between matrix theory and statistics. This Workshop will provide a forum through which scientists working in the areas of linear algebra/matrix theory and/or statistics may be better informed of the latest developments and newest techniques and may exchange ideas with researchers from several different countries. Selected fully-refereed papers from this Workshop will be published in a Special Issue on Linear Algebra and Statistics of *Linear Algebra and Its Applications*. The Workshop will include invited and contributed talks. These talks and the informal workshop atmosphere will guarantee an intensive exchange of ideas. We expect special coverage of some of the topics that have highlighted Ted Anderson's research: multivariate statistical analysis and econometrics, as well as matrix inequalities. Accommodation, at very favorable rates, has been arranged at the Rolling Hills Hotel & Golf Resort (Official Hotel & Resort of the Miami Dolphins, film location for the hit comedy "Caddyshack"); this hotel is about 10 minutes walk from Nova Southeastern University. Please visit `http://www.polaris.nova.edu/MST/conf/FMW/`, where all available information about this Workshop will be updated frequently.

Image Problem Corner

We present solutions to Problems 18-4, 19-1, 19-2 and 19-5 and we introduce six new problems (page 32). We invite readers to submit solutions, as well as new problems, for publication in *Image*. We look forward particularly to receiving solutions to Problems 18-1, 19-3 and 19-4! Please send all material to George Styan BOTH in LaTeX code embedded as text only by e-mail to `styan@Math.McGill.CA` AND by regular air mail (nicely printed copy, please!) to Professor George P. H. Styan, Editor-in-Chief: *Image*, Dept. of Mathematics & Statistics, McGill University, 805 ouest, rue Sherbrooke Street West, Montréal, Québec, Canada H3A 2K6. Many thanks go to *Image* Senior Associate Editor Hans Joachim Werner for his tremendous help with this *Image* Problem Corner. –Ed.

Problem 18-1: 5×5 **Complex Hadamard Matrices**
Proposed by S. W. DRURY, *McGill University, Montréal, Québec, Canada.*

The Editor has not yet received a solution to this problem–indeed even the Proposer has not yet found a solution!

Show that every 5×5 matrix U with complex entries $u_{j,k}$ of constant absolute value one that satisfies $U^*U = 5I$ can be realized as the matrix $(\omega^{jk})_{j,k}$ where ω is a complex primitive fifth root of unity by applying some sequence of the following: (1) A rearrangement of the rows, (2) A rearrangement of the columns, (3) Multiplication of a row by a complex number of absolute value one, (4) Multiplication of a column by a complex number of absolute value one.

Problem 18-4: Bounds for a Ratio of Matrix Traces
Proposed by SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

Let $\Sigma > 0$ be an $n \times n$ matrix with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$, X be an $n \times k$ matrix such that $X'X = I$ and $n \geq 2k$. Show that

$$1 \leq \frac{\mathrm{tr} X'\Sigma^2 X}{\mathrm{tr}(X'\Sigma X)^2} \leq \frac{\Sigma_{j=1}^k(\lambda_j + \lambda_{n-j+1})^2}{4\Sigma_{j=1}^k \lambda_j\lambda_{n-j+1}}.$$

Solution 18-4.1 by NATÁLIA BEBIANO and JOÃO DA PROVIDENCIA, *University of Coimbra, Coimbra, Portugal* & CHI-KWONG LI, *College of William and Mary, Williamsburg, Virginia, USA.*

We prove the inequalities for complex Hermitian matrices. Let X be $n \times k$ such that $X^*X = I_k$. Then there is a unitary matrix U with X in the first k columns. Let $M := U^*\Sigma U = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix}$ with $A = X^*\Sigma X$. Then the inequalities become:

$$1 \leq \frac{\mathrm{tr}\,(A^2 + BB^*)}{\mathrm{tr}\,A^2} \leq \frac{\sum_{j=1}^k(\lambda_j + \lambda_{n-j+1})^2}{4\sum_{j=1}^k \lambda_j\lambda_{n-j+1}}.$$

The first inequality is clear. To prove the second inequality, we focus on the optimal matrix M that yields the maximum ratio $\mathrm{tr}\,(A^2 + BB^*)/\mathrm{tr}\,A^2$.

We first consider the case when $n = 2k$. Suppose B has singular value decomposition $B = Y\tilde{B}Z$, where Y and Z are $k \times k$ unitary matrices, and $\tilde{B} = \mathrm{diag}(b_1, \ldots, b_k)$ with $b_1 \geq \cdots \geq b_k \geq 0$. We may actually assume that $B = \tilde{B}$, otherwise, replace M by $\begin{pmatrix} YAY^* & YBZ \\ Z^*B^*Y^* & Z^*CZ \end{pmatrix}$. Now, for every Hermitan S, and $t \in \mathbf{R}$ in a neighborhood of 0, we can construct a matrix

$$\begin{pmatrix} A(t) & B(t) \\ B(t)^* & C(t) \end{pmatrix} = e^{-itS}Me^{itS},$$

where $A(t)$ is $k \times k$. Moreover, the differentiable function $f(t) = \mathrm{tr}\,(A(t)^2 + B(t)B(t)^*)/\mathrm{tr}\,A(t)^2$ attains a maximum at $t = 0$. Let $Q = I_k \oplus 0_{n-k}$. Then, $\mathrm{tr}\,(A^2 + BB^*) = \mathrm{tr}\,(M^2Q)$ and $\mathrm{tr}\,(A^2) = \mathrm{tr}\,(MQMQ)$. A simple computation shows that

$$\frac{d}{dt}\log f(t)\Big|_{t=0} = -i\frac{\operatorname{tr}(S[M^2,Q])}{\operatorname{tr}(M^2Q)} + 2i\frac{\operatorname{tr}(S[M,QMQ])}{\operatorname{tr}(MQMQ)} = 0.$$

Since S is arbitrary, we have

$$\frac{[M^2,Q]}{\operatorname{tr}(A^2+BB^*)} - 2\frac{[M,QMQ]}{\operatorname{tr}(A^2)} = \frac{\begin{pmatrix} 0 & -B^*A-CB^* \\ AB+BC & 0\end{pmatrix}}{\operatorname{tr}(A^2+BB^*)} - 2\frac{\begin{pmatrix} 0 & -B^*A \\ AB & 0\end{pmatrix}}{\operatorname{tr}(A^2)} = 0.$$

One can then deduce that $AB = \gamma BC$ for $\gamma = \operatorname{tr}(\frac{1}{2}A^2)/\operatorname{tr}(\frac{1}{2}A^2+BB^*) > 0$. Thus A is in block form $A_1\oplus\cdots\oplus A_m$ according to the multiplicities of the distinct singular values of B. Now, we can use a unitary matrix W of the form $W = W_1\oplus\cdots\oplus W_m$ conformed with that of A so that $W^*AW = \operatorname{diag}(a_1,\dots,a_k)$. Clearly, we have $W^*BW = B$. Replace M by $(W^*\oplus W^*)M(W\oplus W)$, we may assume that $A = \operatorname{diag}(a_1,\dots,a_k)$ and $B = \operatorname{diag}(b_1,\dots,b_k)$ are both in diagonal form, and we still have $AB = \gamma BC$. Hence $C = \operatorname{diag}(c_1,\dots,c_m)\oplus\tilde{C}$ if B has m positive singular values. In particular, M is a direct sum of $\operatorname{diag}(a_{m+1},\dots,a_k)\oplus\tilde{C}$ and 2×2 principal submatrices $M_j = \begin{pmatrix} a_j & b_j \\ b_j & c_j\end{pmatrix}$ for $j=1,\dots,m$. Moreover,

$$\operatorname{tr}(A^2+BB^*)/\operatorname{tr}A^2 = \sum_{i=1}^k (a_i^2+b_i^2)/\sum_{i=1}^k a_i^2.$$

For $j = 1,\dots,m$, using the fact that $\begin{pmatrix} a_j & b_j \\ b_j & c_j\end{pmatrix}$ is unitarily similar to $\operatorname{diag}(\mu_j,\nu_j)$ for two eigenvalues μ_j and ν_j of M, we may write $a_j^2+b_j^2 = a_j(\mu_j+\nu_j)-\mu_j\nu_j$. For those $j > m$ (if exist), one should clearly choose a_j to be the smallest eigenvalues of M available after removing $\mu_1,\dots,\mu_m$, and $\nu_1,\dots,\nu_m$. Consequently, to determine the maximum of $\operatorname{tr}(A^2+BB^*)/\operatorname{tr}A^2$, one may consider the maxium of

$$g = \frac{\sum_{i=1}^m (a_i(\mu_i+\nu_i)-\mu_i\nu_i) + \sum_{i=m+1}^k \eta_i}{\sum_{i=1}^m a_i^2 + \sum_{i=m+1}^k \eta_i}$$

among all possible choices of $m+k$ eigenvalues $\mu_1,\dots,\mu_m,\nu_1,\dots,\nu_m,\eta_{m+1},\dots,\eta_k$ of Σ, where $0\le m\le k$, subject to the constraints that

$$a_i\in[\nu_i,\mu_i],\quad i=1,\dots,m.$$

Now, maximizing g under the above constraints is not easy. We derive an upper bound for the optimal g by relaxing the conditions on a_j for $j = 1,\dots,m$, namely, we only assume that $a_j^2+b_j^2 = a_j(\mu_j+\nu_j)-\mu_j\nu_j$ for some eigenvalues $\mu_1,\dots,\mu_k,\nu_1,\dots,\nu_k$ of Σ without requiring $a_j\in[\nu_j,\mu_j]$ for $j\le m$, and a_j to be an eigenvalue for $j>m$. Thus, we maximize

$$\frac{\sum_{i=1}^k (a_i(\mu_i+\nu_i)-\mu_i\nu_i)}{\sum_{i=1}^k a_i^2}.$$

For a fixed choice of μ_i and ν_i, we choose $(a_1,\dots,a_k)$ to maximize the expression and obtain the equations

$$\frac{(\mu_i+\nu_i)}{\sum_{j=1}^k (a_j(\mu_j+\nu_j)-\mu_j\nu_j)} = 2\frac{a_i}{\sum_{j=1}^k a_j^2},\quad j=1,\dots,k,$$

at a critical point. This leads to

$$\frac{a_i}{(\mu_i+\nu_i)} = \frac{\sum_{j=1}^k a_j^2}{2\sum_{j=1}^k (a_j(\mu_j+\nu_j)-\mu_j\nu_j)}.$$

Let $\alpha = a_i/(\mu_i+\nu_i)$. Then $2\alpha = \frac{\alpha^2 S_1}{\alpha S_1 - S_2}$ where $S_1 = \sum_{j=1}^k (\mu_j+\nu_j)^2$ and $S_2 = \sum_{j=1}^k \mu_j\nu_j$. Thus $\alpha = 2S_2/S_1$. In other words, the maximum occurs at

$$\frac{1}{2\alpha} = \frac{\sum_{j=1}^k(\mu_j+\nu_j)^2}{4\sum_{j=1}^k \mu_j\nu_j} = 1 + \frac{\sum_{j=1}^k(\mu_j-\nu_j)^2}{4\sum_{j=1}^k \mu_j\nu_j},$$

Next, one can prove that this is no larger than

$$\frac{\sum_{j=1}^{k}(\lambda_j+\lambda_{n-j+1})^2}{4\sum_{j=1}^{k}\lambda_j\lambda_{n-j+1}} = 1+\frac{\sum_{j=1}^{k}(\lambda_j-\lambda_{n-j+1})^2}{4\sum_{j=1}^{k}\lambda_j\lambda_{n-j+1}}.$$

In fact, if we arrange μ_i ad ν_i in descending order $x_1 \geq \cdots \geq x_{2k}$, we see that $\sum_{i=1}^{2k} x_j x_{2k-j+1} \leq \sum_{i=1}^{2k} x_j x_{i_j}$ and $\sum_{i=1}^{2k}(x_j - x_{2k-j+1})^2 \geq \sum_{i=1}^{2k}(x_j - x_{i_j})^2$ for any permutation $(i_1, \ldots, i_{2k})$ of $(1, \ldots, 2k)$. Hence, we may assume that $\mu_1 \geq \cdots \geq \mu_k \geq \nu_k \geq \cdots \geq \nu_1$ when maximum occurs. Moreover, if there exists j with $1 \leq j \leq k$ such that $(\mu_j, \nu_j) \neq (\lambda_j, \lambda_{n-j+1})$, one can improve the maxmum by replacing (μ_j, ν_j) by $(\lambda_j, \lambda_{n-j+1})$ at the first occurence of this inequality.

For the case when $n > 2k$, we can apply the arguments to the leading $2k \times 2k$ matrix of the optimal matrix M and get the conclusion. □

Solution 18-4.2 by SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

It's easy to see the first relationship. Define

$$\phi = \frac{1}{2}\log g - \frac{1}{2}\log h - \frac{1}{2}\mathrm{tr}\, L(X'X - I)$$

where $g = \mathrm{tr}\, X'\Sigma^2 X$, $h = \mathrm{tr}\,(X'\Sigma X)^2$ and $L' = L$. Clearly,

$$d\phi = \frac{1}{g}\mathrm{tr}\, X'\Sigma^2 dX - \frac{2}{h}\mathrm{tr}\, X'\Sigma X X'\Sigma dX - \mathrm{tr}\, LX'dX.$$

By letting $d\phi = 0$ and solving L we get

$$\frac{1}{g}X'\Sigma^2 - \frac{2}{h}X'\Sigma X X'\Sigma = \frac{1}{g}X'\Sigma^2 X X' - \frac{2}{h}(X'\Sigma X)^2 X'. \tag{1}$$

Postmultiplying it by ΣX gives

$$\frac{1}{g}X'\Sigma^3 X - \frac{2}{h}X'\Sigma X X'\Sigma^2 X = \frac{1}{g}X'\Sigma^2 X X'\Sigma X - \frac{2}{h}(X'\Sigma X)^3. \tag{2}$$

Taking transpose results in

$$\frac{1}{g}X'\Sigma^3 X - \frac{2}{h}X'\Sigma^2 X X'\Sigma X = \frac{1}{g}X'\Sigma X X'\Sigma^2 X - \frac{2}{h}(X'\Sigma X)^3. \tag{3}$$

From (2) and (3) we see that $(\frac{1}{g} - \frac{2}{h})(MN - NM) = 0$, and hence $MN = NM$, where $M = X'\Sigma^2 X > 0, N = X'\Sigma X > 0$, and $\frac{1}{g} - \frac{2}{h} < 0$ as $2g \geq 2h > h > 0$. Without loss of generality, assume $\Sigma = \mathrm{diag}(\lambda_1, ..., \lambda_n)$. Write $M = TUT'$, $N = TVT'$, $T'X' = W$, where $U = \mathrm{diag}(u_1, ..., u_k) > 0$, $V = \mathrm{diag}(v_1, ..., v_k) > 0$, $T'T = TT' = I$. Then (1) leads to

$$\frac{1}{g}W\Sigma^2 - \frac{2}{h}VW\Sigma = \frac{1}{g}UW - \frac{2}{h}V^2 W \tag{4}$$

Examine $(w_{i1}, ..., w_{in})$, the i-th row of W $(i = 1, ..., k)$ and rewrite (4) as

$$w_{ij}F(j) = 0, \tag{5}$$

where $F(j) = \frac{1}{g}\lambda_j^2 - \frac{2}{h}v_i\lambda_j - \frac{1}{g}u_i + \frac{2}{h}v_i^2$, $j = 1, ..., n$. It's not possible that $w_{i1} = ... = w_{in} = 0$, as $WW' = I$. Due to (5) we must have either (a) $w_{ir} \neq 0$, $F(r) = 0$; or (b) $0 \neq w_{ir} \neq w_{is} \neq 0$, $F(r) = F(s) = 0$, $r \neq s$. In (a) we have

$$\left(\frac{2v_i}{h}\right)^2 - \frac{4}{g}\left(\frac{2v_i^2}{h} - \frac{u_i}{g}\right) = 0,$$

i.e., $h^2 u_i = (2gh - g^2)v_i^2$, then $g = h$ (as $u_i \geq v_i^2$), which leads to the lower bound (first relationship), not an upper bound. In (b) we have

$$\lambda_r + \lambda_s = \frac{2g}{h} v_i, \; \lambda_r \lambda_s = \frac{2g}{h} v_i^2 - u_i.$$

Solving u_i and v_i we obtain

$$\frac{g}{h} = \frac{\Sigma_{i=1}^k u_i}{\Sigma_{i=1}^k v_i} = \frac{(\frac{h}{2g})\Sigma_{i=1}^k (\lambda_r + \lambda_s)^2 - \Sigma_{i=1}^k \lambda_r \lambda_s}{(\frac{h}{2g})^2 \Sigma_{i=1}^k (\lambda_r + \lambda_s)^2}$$

and hence

$$\frac{g}{h} = \frac{\Sigma_{i=1}^k (\lambda_r + \lambda_s)^2}{4\Sigma_{i=1}^k \lambda_r \lambda_s}. \tag{6}$$

Note that r and s $(1 \leq r \neq s \leq n)$ relate to i $(i = 1, ..., k)$. Maximizing (6) we obtain the result. □

COMMENT. Note that the result has applications on efficiency comparisons in statistics (partly presented at Tinbergen Institute, Amsterdam in 1993 and Adam Mickiewicz University, Poznan in 1995).

Problem 19-1: Eigenvalues

Proposed by JÜRGEN GROß & GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let $\mathbf{a}$ and $\mathbf{b}$ be two nonzero $n \times 1$ real vectors and consider the matrix

$$\mathbf{A} = \alpha\,\mathbf{aa}' + \beta\,\mathbf{ab}' + \gamma\,\mathbf{ba}' + \delta\,\mathbf{bb}',$$

where $\alpha, \beta, \gamma, \delta$ are real scalars such that $\beta = -\gamma$ and $\gamma^2 = -\alpha\delta$. Find the (nonzero) eigenvalues of $\mathbf{A}$.

Solution 19-1.1 by DAVID A. HARVILLE, *IBM Thomas J. Watson Research Center, Yorktown Heights, New York, USA.*

Let x, y, w, and z represent real numbers such that $xw = \alpha$, $xz = -\gamma$, $yw = \gamma$, $yz = \delta$, so that

$$\mathbf{A} = \alpha\mathbf{aa}' - \gamma\mathbf{ab}' + \gamma\mathbf{ba}' + \delta\mathbf{bb}' = (x\mathbf{a} + y\mathbf{b})(w\mathbf{a} + z\mathbf{b})'.$$

That such numbers exist is evident upon observing (in light of the condition $\gamma^2 = -\alpha\delta$, which implies either that α and δ are opposite in sign or that one or both of them equal 0) that (1) if $\alpha > 0$ (in which case $\delta \leq 0$), we can take $w = x = \sqrt{\alpha}$, $z = -y$, and either $y = \sqrt{-\delta}$ or $y = -\sqrt{-\delta}$ (depending on whether $\gamma \geq 0$ or $\gamma < 0$); (2) if $\alpha = 0$ (in which case $\gamma = 0$), we can take $w = x = 0$ and either $z = y = \sqrt{\delta}$ or $z = -y$ and $y = \sqrt{-\delta}$ (depending on whether $\delta \geq 0$ or $\delta < 0$); and (3) if $\alpha < 0$ (in which case $\delta \geq 0$), we can take $w = -x$, $x = \sqrt{-\alpha}$, and either $z = y = -\sqrt{\delta}$ or $z = y = \sqrt{\delta}$ (depending on whether $\gamma \geq 0$ or $\gamma < 0$). Then, it follows from a well-known result, which is given by Theorem 21.10.1 of Harville (1997) [*"Matrix Algebra From a Statistician's Perspective"*, Springer-Verlag, New York], that $n - 1$ of the eigenvalues of $\mathbf{A}$ are 0, and the remaining eigenvalue is $(w\mathbf{a} + z\mathbf{b})'(x\mathbf{a} + y\mathbf{b}) = \alpha\mathbf{a}'\mathbf{a} + \gamma\mathbf{a}'\mathbf{b} - \gamma\mathbf{b}'\mathbf{a} + \delta\mathbf{b}'\mathbf{b} = \alpha\mathbf{a}'\mathbf{a} + \delta\mathbf{b}'\mathbf{b}$. □

Solution 19-1.2 by CHI-KWONG LI and ROY MATHIAS, *The College of William & Mary, Williamsburg, Virginia, USA.*

Note that $A = (\,a \quad b\,)\,B\,(\,a \quad b\,)'$, where

$$B = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} = \begin{pmatrix} \alpha & -\gamma \\ \gamma & \delta \end{pmatrix}$$

has determinant zero by the given condition. Thus A has at most rank one and the nonzero eigenvalue (if exists) equals $\operatorname{tr} A = \operatorname{tr}\left((\,a \quad b\,)\,B\,(\,a \quad b\,)'\right) = \operatorname{tr}\left(B\,(\,a \quad b\,)'\,(\,a \quad b\,)\right) = \alpha(a'a) + \delta(b'b)$. □

Solution 19-1.3 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

It suffices to consider the following two exhaustive cases.

Case 1. Let $\alpha = 0$. Then $\gamma = 0$ and $\beta = 0$ so that A becomes $A = \delta bb'$ which is a matrix with rank ≤ 1. This matrix has rank 1 if and only if $\delta b \neq 0$. If it has rank 1, then its only nonzero eigenvalue is given by $\operatorname{tr}(A) = \delta b'b$.

Case 2. Let $\alpha \neq 0$. Then it is easy to see that A can be rewritten as $A = (\alpha a - \beta b)\left(a + \frac{\beta}{\alpha} b\right)'$ which again is obviously a matrix of rank ≤ 1. This matrix has rank 1 if and only if $\alpha a \neq \pm\beta b$. If it has rank 1, then the only nonzero eigenvalue is given by $\mathrm{tr}(A) = \left(a + \frac{\beta}{\alpha} b\right)'(\alpha a - \beta b) = \alpha a'a - \frac{\beta^2}{\alpha} b'b$. □

Solutions were also received from DAVID CALLAN, *University of Wisconsin, Madison, Wisconsin, USA*; KRZYSZTOF KRASUSKI & MAREK BIAŁKOWSKI, *Warsaw University of Technology, Warsaw, Poland*; STEVEN J. LEON, *University of Massachusetts–Dartmouth, North Dartmouth, Massachusetts, USA*; DENNIS I. MERINO, *Southeastern Louisiana University, Hammond, Louisiana, USA*; INGRAM OLKIN, *Stanford University, Stanford, CA, USA*; SHAYLE R. SEARLE, *Cornell University, Ithaca, New York, USA*; DAFNA SHASHA, *Tel-Hai Rodman College, Upper Galilee, Israel*; YONGGE TIAN, *Concordia University, Montréal, Québec, Canada*; WILLIAM F. TRENCH, *Divide, Colorado, USA*; HENRY WOLKOWICZ, *University of Waterloo, Ontario, Canada*; and from the proposers JÜRGEN GROß & GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany*.

Problem 19-2: Non-negative Definite Matrices
Proposed by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany*.

A matrix $A \in \mathbb{C}^{n\times n}$ is said to be non-negative definite if $\mathrm{Re}(x^\star Ax) \geq 0$ for all vectors $x \in \mathbb{C}^n$. Prove that A is non-negative definite if and only if its Moore-Penrose inverse $A^\dagger$ is non-negative definite.

Solution 19-2.1 by R. B. BAPAT, *Indian Statistical Instiute, New Delhi, India.*

Since $Re(x^*Ax) = \frac{1}{2}x^*(A + A^*)x$, $A \in C^{n\times n}$ is non-negative definite if and only if $A + A^*$ is positive semidefinite (i.e., $x^*(A + A^*)x \geq 0$ for all $x \in C^{n\times n}$.) Suppose A is non-negative definite and let $x \in \mathcal{N}(A)$, the null space of A. Then $x^*(A + A^*)x = 0$ and since $A + A^*$ is positive semidefinite, $(A + A^*)x = 0$. Thus $A^*x = 0$ and $x \in \mathcal{N}(A^*)$. Similairly we can show that $x \in \mathcal{N}(A^*)$ implies $x \in \mathcal{N}(A)$ and hence $\mathcal{N}(A) = \mathcal{N}(A^*)$. It follows that $\mathcal{C}(A) = \mathcal{C}(A^*)$, where $\mathcal{C}(\cdot)$ denotes column space. This, together with the well-known fact that $\mathcal{C}(A^\dagger) = \mathcal{C}(A^*)$ leads to $\mathcal{C}(A^{\dagger*}) = \mathcal{C}(A^\dagger)$. Thus $A^{\dagger*} = A^\dagger X$ for some matrix X. Then

$$A^\dagger AA^{\dagger*} = A^\dagger AA^\dagger X = A^\dagger X = A^{\dagger*}$$

and hence $A^\dagger = A^\dagger A^* A^{\dagger*}$. Using these equations we get $A^\dagger + A^{\dagger*} = A^\dagger(A + A^*)A^{\dagger*}$, which is positive semidefinite. Therefore $A^\dagger$ is non-negative definite. This proves the "only if" part. The "if" part follows at once since $A^{\dagger\dagger} = A$. □

Solution 19-2.2 by CHI-KWONG LI and ROY MATHIAS, *The College of William & Mary, Williamsburg, Virginia, USA.*

Let S be invertible, and let k be a positive integer. Then a given A is non-negative definite if and only if one of the following holds: (a) S^*AS is non-negative definite, (b) $A^* + A$ is non-negative definite, (c) $A \oplus 0_k$ is non-negative definite. Also, (d) if A is invertible and non-negative definite then $A^\dagger = A^{-1} = A^{-1}A^*(A^{-1})^*$ is also non-negative definite. Finally, since $A = (A^\dagger)^\dagger$ it is sufficient to show that A is non-negative definite implies that $A^\dagger$ is non-negative definite.

Solution I (via the Schur Decomposition): Let $A = UTU^*$ where U is unitary and T upper triangular with the eigenvalues of A on the diagonal. Let us assume that A has k non-zero eigenvalues and that $t_{ii} \neq 0$, for $i \leq k$ and $t_{ii} = 0$ for $i \geq k+1$. By (a) above T is non-negative definite, and so by (b) so is $T + T^*$. Since $(T + T^*)_{ii} = 0$ for $i > k$ it follows that the whole of the ith row and the ith column of $(T + T^*)$ is 0 for $i > k$. Since T is upper triangular it follows that $T = T_1 \oplus 0_{n-k}$ where T_1 is invertible. Hence $T^\dagger = T_1^{-1} \oplus 0_{n-k}$ is non-negative definite by (d) and (c), and so is $A^\dagger = UT^\dagger U^*$.

Solution II (via SVD): Suppose A is non-negative definite and has singular value decomposition UDV. Then $DVU = U^*AU$ and $DVU + U^*V^*D$ is non-negative definite. If A has rank k, then the last $(n-k)$ rows of DVU are zero, and thus the lower $(n-k) \times (n-k)$ block of $DVU + U^*V^*D$ is zero. Since $DVU + U^*V^*D$ is hermitian non-negative definite, its $(1,2)$ and $(2,1)$ block must be zero as well. It follows that $DVU = B \oplus 0_{n-k}$, where B is $k \times k$ invertible non-negative definite. Hence $U^*V^*D^\dagger = B^{-1} \oplus 0_{n-k}$ is non-negative definite by (d) and (c), and so is $A^\dagger = V^*D^\dagger U^*$. □

Solution 19-2.3 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

Since $\mathrm{Re}(x^*Ax) = x^*(A + A^*)x/2$, it is evident that A is non-negative definite if and only if the Hermitian matrix $A + A^*$ is non-negative definite. Clearly, if B is Hermitian non-negative definite, then $x^*Bx = 0 \iff Bx = 0$. We recall that a matrix C is an EP-matrix if and only if $\mathcal{N}(C) = \mathcal{N}(C^*)$ or, equivalently, $\mathcal{R}(C) = \mathcal{R}(C^*)$; here $\mathcal{N}(\cdot)$ denotes the null space and $\mathcal{R}(\cdot)$ the range (column space). We proceed as follows. First, we show that each non-negative definite matrix A is necessarily an EP-matrix. With this in mind, we then prove that $A^\dagger$ is non-negative definite whenever A is so. For that purpose, let A be non-negative definite. Moreover, let x be an arbitrary vector from $\mathcal{N}(A)$. Then $Ax = 0$ and it follows from $0 = \mathrm{Re}(x^*Ax) = x^*(A + A^*)x/2$ that $(A + A^*)x = 0$ and so $A^*x = 0$. Thus $\mathcal{N}(A) \subseteq \mathcal{N}(A^*)$. In virtue of $\mathrm{rank}(A) = \mathrm{rank}(A^*)$, therefore $\mathcal{N}(A) = \mathcal{N}(A^*)$, i. e. A is EP, as claimed. Consequently $AA^\dagger = A^\dagger A$; for notice that $AA^\dagger$ represents the orthogonal projector onto $\mathcal{R}(A)$ whereas $A^\dagger A$ represents the orthogonal projector onto $\mathcal{R}(A^*)$. The defining equations of $A^\dagger$—the so-called Penrose equations—further tell us that $A^\dagger = A^\dagger AA^\dagger$ and $AA^\dagger = (AA^\dagger)^*$. Therefore $A^\dagger = A^\dagger AA^\dagger = AA^\dagger A^\dagger = (AA^\dagger)^*A^\dagger = A^{\dagger *}A^*A^\dagger$. For each $x \in \mathbb{C}^n$, hence $0 \geq \mathrm{Re}(x^*A^\dagger x) = \mathrm{Re}(x^*A^{\dagger *}A^*A^\dagger x)$, thus completing the proof of $A^\dagger$ being non-negative definite whenever A is so. That the reverse implication is also true follows from the fact that $(A^\dagger)^\dagger = A$. □

Problem 19-3: Characterizations Associated with a Sum and Product

Proposed by ROBERT E. HARTWIG, *North Carolina State University, Raleigh, North Carolina, USA*, PETER ŠEMRL, *University of Maribor, Maribor, Slovenia* & HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

The Editor has so far received only one solution (in addition to a solution by the Proposers) to this problem–we look forward to receiving more solutions!

1. Characterize square matrices A and B satisfying $AB = pA + qB$, where p and q are given scalars.
2. More generally, characterize linear operators A and B acting on a vector space $\mathcal{X}$ satisfying $ABx \in \mathrm{Span}(Ax, Bx)$ for every $x \in \mathcal{X}$.

Problem 19-4: Eigenvalues of Positive Semidefinite Matrices

Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

The Editor has so far received only one solution (from the Proposer) to this problem–we look forward to receiving more solutions!

Show that there are constants $\gamma \in (0, \frac{1}{2}]$ and $\varepsilon \in (0, \frac{1}{2})$ such that, if $A_1, \ldots, A_n$ are non-negative definite $r \times r$ matrices of rank one satisfying

1. $A_1 + \cdots + A_n = I_r$ and
2. $\mathrm{trace}(A_i) < \gamma$ for each $i = 1, \ldots, n$,

then there is a subset σ of $\{1, 2, \ldots, n\}$ such that the eigenvalues of $A_\sigma = \sum_{i\in\sigma} A_i$ all lie in the interval $(\varepsilon, 1 - \varepsilon)$.

Problem 19-5: Symmetrized Product Definiteness?

Proposed by INGRAM OLKIN, *Stanford University, Stanford, California, USA.*

Let A and B be real symmetric matrices. Then prove or disprove:

1. If A and B are both positive definite (non-negative definite) then $AB + BA$ is positive definite (non-negative definite).
2. If A and $AB + BA$ are both positive definite (non-negative definite) then B is positive definite (non-negative definite).

COMMENT: This problem is discussed by Peter Lax on pp. 120–121 of his new book *Linear Algebra* [Wiley, 1997, ISBN 0-471-11111-2] recently reviewed in *Image*, 18, pp. 4–5. —Ed.

Solution 19-5.1 by DAVID CALLAN, *University of Wisconsin, Madison, Wisconsin, USA.*

1. The first assertion is false, even for positive entry matrices. For example, take $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}$. Then $\det(AB + BA) = -1$ and $AB + BA$ fails to be non-negative definite.

2. The second assertion is true if A is positive definite. Since $\mathbf{x}'(AB + BA)\mathbf{x} = 2\mathbf{x}'AB\mathbf{x}$, the other hypothesis implies $\mathbf{x}'AB\mathbf{x} > 0\ (\geq 0)$ for all $\mathbf{x} \neq \mathbf{0}$. If the conclusion fails, then B has an eigenvalue $\lambda \leq 0\ (< 0)$ with eigenvector $\mathbf{x}$, say. But then $\mathbf{x}'AB\mathbf{x} = \lambda\mathbf{x}'A\mathbf{x} \leq 0\ (< 0)$, a contradiction. □

Solution 19-5.2 by SHUANGZHE LIU, *Universität Basel, Basel, Switzerland.*

1. For positive definite (non-negative definite) matrices A and B, $AB+BA$ is not positive definite (non-negative definite) in general. Here $x'(AB + BA)x = 2x'ABx$, for any real vector x. An example:

$$A = \begin{pmatrix} 3 & -2 \\ -2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, \quad AB = \begin{pmatrix} 12 & -2 \\ -8 & 2 \end{pmatrix}, \quad AB + BA = \begin{pmatrix} 24 & -10 \\ -10 & 4 \end{pmatrix}$$

.

2. Let μ be an eigenvalue of B, y be its associated eigenvector, then $By = \mu y$ and $y'(AB+BA)y = 2y'ABy = 2\mu y'Ay$. Hence (1) IF A is positive definite, then B is positive definite (non-negative definite) whenever $AB + BA$ is positive definite (non-negative definite). (2) When A and $AB + BA$ are both non-negative definite, B need not be non-negative definite. An example:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad AB + BA = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix}$$

.

Solution 19-5.3 by DAVID LONDON, *Technion, Haifa, Israel.*

1. Both parts of assertion 1 are false. Let $A = \begin{pmatrix} 101 & 100 \\ 100 & 101 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. A and B are positive definite while $AB + BA = \begin{pmatrix} 202 & 300 \\ 300 & 404 \end{pmatrix}$ is not non-negative definite.

2. The positive definite version of assertion 2 is true. Without loss of generality, we may assume that $B = \operatorname{diag}(b_1, b_2, \cdots, b_n)$ is diagonal. Let $A = (a_{ij})$ and $AB + BA = (a_{ij}(b_i + b_j))$ be positive definite. It follows that $a_{ii} > 0$ and $2a_{ii}b_i > 0$ for $i = 1, \ldots, n$. Hence, $b_i > 0, i = 1, \ldots, n$, i.e., B is positive definite.

3. The non-negative definite version of assertion 2 is false. Let $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Then $AB + BA = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}$ is non-negative definite. □

Solution 19-5.4 by HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

The following four observations give the complete solution to this problem.

Observation 1: *Consider the matrices* $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. *Although A and B are both (real symmetric) non-negative definite, the matrix* $AB + BA = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$ *fails to be non-negative definite.*

Observation 2: *Consider the matrices* $A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix}$. *Although A and B are both (real symmetric) positive definite, the matrix* $AB + BA = \begin{pmatrix} 4 & 11 \\ 11 & 20 \end{pmatrix}$ *fails to be positive definite.*

Observation 3: *Consider the matrices* $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ *and* $B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. *Although the matrices* A *and* $AB + BA = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ *are both (real symmetric) non-negative definite, the matrix* B *is not so.*

Observation 4: *Let* A *and* B *be real symmetric matrices. If* A *is positive definite and* $AB + BA$ *is non-negative definite (positive definite), then* B *is non-negative definite (positive definite).*

Proof of Observation 4: Recall that for any square (possibly complex) $n \times n$ matrix C its field of values (often also called its numerical range) is $\mathcal{F}(C) := \{x^*Ax \,:\, x \in \mathbb{C}^n,\ x^*x = 1\}$. Since by assumption $AB + BA$ is non-negative definite (positive definite), the Properties 1.2.5 and 1.2.6 in Roger Horn & Charles R. Johnson [*Topics in Matrix Analysis*, Cambridge University Press, 1991] tell us that $\sigma(AB) \subseteq \mathcal{F}(AB) \subseteq \mathrm{RHP}_0 := \{z \in \mathbb{C} : \mathrm{Re}(z) \geq 0\}$ (respectively $\sigma(AB) \subseteq \mathcal{F}(AB) \subseteq \mathrm{RHP} := \{z \in \mathbb{C} : \mathrm{Re}(z) > 0\}$); here $\sigma(AB)$ denotes the spectrum of AB, that is, the point set of eigenvalues of the matrix AB. From matrix theory it is well known (see, for example, Exercise 9.12 in R. B. Bapat [*Linear Algebra and Linear Models*, Hindustan Book Agency, Delhi 1993]) that if C and D are $n \times n$ matrices then CD and DC have the same eigenvalues. Hence, in particular, $\sigma(AB) = \sigma(A^{\frac{1}{2}}BA^{\frac{1}{2}})$ where $A^{\frac{1}{2}}$ is the unique square root of A. The matrix $A^{\frac{1}{2}}BA^{\frac{1}{2}}$ is symmetric and so all its eigenvalues are real. In view of $\sigma(A^{\frac{1}{2}}BA^{\frac{1}{2}}) = \sigma(AB) \subseteq \mathrm{RHP}_0$ (respectively $\sigma(A^{\frac{1}{2}}BA^{\frac{1}{2}}) = \sigma(AB) \subseteq \mathrm{RHP}$) it now follows that $A^{\frac{1}{2}}BA^{\frac{1}{2}}$ is non-negative definite (positive definite). Since $A^{\frac{1}{2}}$ is positive definite, $A^{\frac{1}{2}}BA^{\frac{1}{2}}$ is non-negative definite (positive definite) if and only if B is non-negative definite (positive definite). This completes the proof of Observation 4. □

Solution 19-5.5 by HENRY WOLKOWICZ, *University of Waterloo, Waterloo, Ontario, Canada.*

Let A and B be real symmetric matrices.

1. If A and B are positive definite (non-negative definite), then it is not true that $AB+BA$ is positive definite (non-negative definite).

   Proof. The proof follows (quickly) from using the following MATLAB program (courtesy of Mike Overton, Courant Institute, NY).

```
for try = 1:10,
    X = diag(rand(10,1));
    Z = rand(10,10); Z = Z'*Z;
    XZZX = X*Z + Z*X;
    [eig(X) eig(Z) eig(XZZX)]
    pause
end;
```

   A specific example given by R. D. C. Monteiro and Y. Zanjacomo (1997) ["A note on the existence of the Alizadeh-Haeberly-Overton direction for semidefinite programming", *Mathematical Programming*, 78:393–401; `http://www.isye.gatech.edu/]~monteiro/tech_reports/note.ps`] is

$$A = \begin{bmatrix} 8 & 1 \\ 1 & 0.5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 0 & 16 \end{bmatrix}.$$

   The eigenvalues of A are: 8.1310 and 0.3690. The eigenvalues of $AB + BA$ are: -1 and 33. □

2. If A and $AB+BA$ are both positive semidefinite, then it is not true that B is positive semidefinite: consider the diagonal matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}.$$

   But for A and $P = AB + BA$ both positive definite, then it is true that B is positive definite.

   Proof. Let $B = UDU^t$ be the orthogonal diagonalization of B. Then A and $U^tPU = (U^tAU)D + D(U^tAU)$ are both positive definite, i.e. we could have assumed that $B = D$ was diagonal and so we write $P = AD + DA$. But, then we can solve the consistent linear equation in the diagonal matrix D to get $D_{ii} = P_{ii}/(2A_{ii}) > 0,\ \forall i.$ □

We add some observations. An immediate corollary to the proof of part 2 is

Corollary 1: *If A and $AB+BA$ are nonnegative definite and the diagonal of A is positive, then B is nonnegative definite.*

In fact, these types of questions arise indirectly in certain algorithms for solving semidefinite programming (SDP) problems such as

$$\min\{\operatorname{trace} CX : \mathcal{A}X = b,\ X \succeq 0\},$$

where $X \succeq 0$ denotes positive semidefiniteness, C is a given symmetric matrix and $\mathcal{A}$ is a given linear operator. These algorithms start with two given $n \times n$ symmetric positive definite matrices $\bar{Z}, \bar{X} \succ 0$ and the scalar $\mu = \operatorname{trace} \bar{Z}\bar{X}/n$. They then take a Newton step for solving the nonlinear system of equations:

$$\begin{pmatrix} Z + C - \mathcal{A}^* y \\ b - \mathcal{A}(X) \\ ZX - \mu I \end{pmatrix} =: \begin{pmatrix} F_d \\ F_p \\ F_c \end{pmatrix} = 0. \tag{1}$$

The first two equations are linear while the nonlinearity arises from the third equation. This equation also results in the system being overdetermined, since it maps symmetric matrices to possibly nonsymmetric matrices. One approach is to symmetrize this equation, resulting in $ZX + XZ - 2\mu I = 0$. (More general symmetrizations are discussed in R.D.C. Monteiro and Y. Zhang (1996) ["A unified analysis for a class of path-following primal-dual interior-point algorithms for semidefinite programming", Technical report, Georgia Tech, Atlanta, GA, 1996, `ftp://pc5.math.umbc.edu/pub/mz_sdp.ps.gz` or `ftp://pc5.math.umbc.edu/pub/mz_sdp.dvi.gz`].) The linearization of the resulting square system has a unique solution (called the AHO direction; see F. Alizadeh, J.-P. A. Haeberly, and M. L. Overton (1996) ["Primal-dual interior-point methods for semidefinite programming: convergence rates, stability and numerical results", NYU Computer Science Dept Technical Report 721, Courant Institute of Mathematical Sciences, to appear in SIOPT]) if $ZX + XZ$ is positive semidefinite. (This is shown in M. Todd, K. C. Toh, and R. H. Tutuncu (1996) ["On the Nesterov-Todd direction in semidefinite programming", Technical Report TR1154, School of OR and IE, Cornell University, Ithaca, NY] and M. Shida, S. Shindoh and M. Kojima (1996) ["Existence of search directions in interior-point algorithms for the SDP and monotone SDLCP", Technical report, Dept. of Information Sciences, Tokyo Institute of Technology, Tokyo, Japan].) However, as the above example shows, this may not hold.

However, the following is clear: if $ZX - \mu I = 0$ (i.e. Z, X lie on the so-called central path), then $ZX + XZ \succ 0$, since commutativity means common orthogonal diagonalization. Therefore, a natural question to ask is:

Problem 1: *Given $Z, X \succ 0,\ \mu > 0$, define a neighbourhood of the central path*

$$||ZX - \mu I|| < \epsilon \text{ or } ||Z^{\frac{1}{2}} X Z^{\frac{1}{2}} - \mu I|| < \epsilon$$

which guarantees that $ZX + XZ \succ 0$.

Such a result is discussed by Monteiro & Zanjacomo (1997) *op. cit.*, who show that

$$||Z^{\frac{1}{2}} X Z^{\frac{1}{2}} - \mu I|| < \mu/2$$

implies that the linearization has a unique solution. In fact, this does not imply $XZ + ZX \succ 0$ as the above example illustrates with $\nu = 8$. However, the triangle inequality (observation due to Mike Todd) does show that:

$$||ZX - \mu I|| < \mu \text{ implies } ZX + XZ \succ 0,$$

thus providing a neighbourhood of the central path. □

Solutions were also received from ROGER A. HORN, *University of Utah, Salt Lake City, Utah, USA*; CHI-KWONG LI & ROY MATHIAS, *The College of William and Mary, Williamsburg, Virginia, USA*; JORMA KAARLO MERIKOSKI & ARI VIRTANEN, *University of Tampere, Tampere, Finland*; FERNANDO C. SILVA, *University of Lisbon, Lisbon, Portugal*; DAFNA SHASHA, *Tel-Hai Rodman College, Upper Galilee, Israel*; YONGGE TIAN, *Concordia University, Montréal, Québec, Canada*; and from the proposer INGRAM OLKIN, *Stanford University, Stanford, CA, USA.*

New Problems

Problem 20-1: Eigenvalues

Proposed by BENITO HERNÁNDEZ–BERMEJO, *Universidad Nacional de Educación a Distancia. Madrid, Spain.*

Let R, P and K be $n \times n$ real matrices, where P is diagonal and invertible, K is skew-symmetric and $R = PK$. Prove that:

1. The eigenvalues of R appear in pairs of opposite sign of the form $\pm\sqrt{\xi}$, where ξ is a real number which may be positive, zero or negative.

2. If the diagonal entries of P are all positive or all negative, then the eigenvalues of R appear in pairs of opposite sign of the form $\pm\sqrt{\xi}$, where ξ is nonpositive, i.e., all eigenvalues are zero or imaginary.

Problem 20-2: Eigenvalues and eigenvectors of a patterned matrix

Proposed by EMILIO SPEDICATO, *University of Bergamo, Bergamo, Italy.*

Find the eigenvalues and eigenvectors of the $n \times n$ patterned matrix $A = \{a_{ij}\} = \{|i-j|\} \quad (i,j = 1,2,\ldots,n)$. [For the eigenvalues and eigenvectors of $A = \{a_{ij}\} = \{i-j\} \quad (i,j = 1,2,\ldots,n)$, see Problem 18-5 in *Image*, 19, 28–30.]

Problem 20-3: When is $\{X|X^n \leq A\}$ Convex ?

Proposed by G. E. TRAPP & BOHE WANG, *West Virginia University, Morgantown, West Virginia, USA.*

For a Hermitian non-negative definite matrix A and positive integer n, let $\mathcal{C}(n,A)$ be the following set of Hermitian matrices:

$$\mathcal{C}(n,A) = \{X|X = X^*, \quad X^n \leq A\}.$$

For what values of $n = 1,2,\ldots$ is the set $\mathcal{C}(n,A)$ convex? Here $\leq$ denotes the Löwner matrix partial ordering.

Problem 20-4: Continuity of Transposition?

Proposed by GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let A be a real $m \times n$ matrix. Find matrices B_{ij} depending only on the type of A such that $A^T = \sum_{j=1}^m \sum_{i=1}^n B_{ij}AB_{ij}$, where A^T is the transpose of A. [If such matrices existed, by the continuity of addition and multiplication of matrices, it would prove also the continuity of transposing.]

Problem 20-5: Matrix Similarity

Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

As is well known, if two real matrices are similar over the complex numbers, then they are similar over the real numbers. Show that this assertion holds for unitary similarity. Precisely, if two real matrices are unitarily similar over the complex numbers, then show that they are orthogonally similar over the real numbers.

Problem 20-6: Non-negative Definiteness and Square Roots

Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

Let $A \geq 0$, $B \geq C \geq 0$. Prove or disprove

$$B^{1/2}AB^{1/2} \geq C^{1/2}AC^{1/2};$$

here $\geq$ is the Löwner matrix partial ordering.

Please submit solutions, as well as new problems, to George Styan BOTH in LaTeX code embedded as text only by e-mail to `styan@Math.McGill.CA` AND by regular air mail (nicely printed copy, please!) to Professor George P. H. Styan, Editor-in-Chief: *Image*, Dept. of Mathematics & Statistics, McGill University, 805 ouest, rue Sherbrooke Street West, Montréal, Québec, Canada H3A 2K6. We look forward particularly to receiving solutions to Problems 18-1, 19-3 and 19-4!