RECEIVED
NOV 1 6 1998

The Bulletin of the International Linear Algebra Society

ILAS

IMAGE

ILAS

Serving the International Linear Algebra Community

Issue Number 21, pp. 1–28, October 1998

Editor-in-Chief: George P. H. STYAN

Editorial Assistants: Shane T. Jensen & Evelyn M. Styan

Department of Mathematics and Statistics
McGill University, Burnside Hall, Room 1005
805 ouest, rue Sherbrooke Street West
Montréal, Québec, Canada H3A 2K6

tel. (1-514) 398-3845
FAX (1-514) 398-3899
styan@math.mcgill.ca

Edited by: Robert C. THOMPSON (1988–1989); Steven J. LEON & Robert C. THOMPSON (1989–1993); Steven J. LEON (1993–1994); Steven J. LEON & George P. H. STYAN (1994–1997).

Senior Associate Editors: Steven J. LEON, Chi-Kwong LI, Simo PUNTANEN, Hans Joachim WERNER.
Associate Editors: S. W. DRURY, Stephen J. KIRKLAND, Peter ŠEMRL, Fuzhen ZHANG.

The Development of Linear Algebra in Portugal (Natália Bebiano) 3
New & Forthcoming Books on Linear Algebra & Related Topics: 1997–1999 8
Patricia James Eberlein: 1923–1998 17

Book Reviews

Badi H. Baltagi: *Econometrics* (Shuangzhe Liu) 6
Dragoš Cvetković, Peter Rowlinson & Slobodan Simić: *Eigenspaces of Graphs* (Russell Merris) . 6
Biswa Nath Datta: *Numerical Linear Algebra and Applications* (James R. Bunch) 7

Reports on Linear Algebra Events Attended

June 1–2, 1998: Madison, Wisconsin (Chi-Kwong Li) 12
June 3–6, 1998: Madison, Wisconsin (Richard A. Brualdi) 13
July 30–31, 1998: Victoria, British Columbia (Stephen J. Kirkland) 16
August 8–13, 1998: Zhang Jia Jie, Hunan, China (Fuzhen Zhang) 16

Forthcoming Linear Algebra Events

December 11–14, 1998: Fort Lauderdale, Florida 18
June 6–9, 1999: Regina, Saskatchewan 19
July 26–29, 1999: Barcelona, Spain 19
August 6–7, 1999: Tampere, Finland 19

Image *Problem Corner*

18-1: 5x5 Complex Hadamard Matrices 21
19-3: Characterizations Associated with a Sum and Product 21
19-4: Eigenvalues of Non-negative Definite Matrices 21
19-5: Symmetrized Product Definiteness? 22
20-1: Eigenvalues 22
20-2: Eigenvalues and Eigenvectors of a Patterned Matrix 23
20-3: When is $\{X|X_n \leq A\}$ Convex? 25
20-4: Continuity of Transposition? 25
20-5: Matrix Similarity 26
20-6: Non-negative Definiteness and Square Roots 27
New Problems 28

Printed in Canada by Copie Nova, Montréal, Québec · (514) 848-0423

user-friendly and modern

TWO PERSPECTIVES ON LINEAR ALGEBRA:

The Geometry Toolbox for Graphics and Modeling

Gerald Farin, Dianne Hansford
Spring 1998

Hardcover, 288 pages
1-56881-074-1, ca. $48.00

Linear Algebra is the mathematical language of the geometry essential for describing computer-modeling, computer graphics, and animation systems. Progressing judiciously from two dimensions to three, this innovative text covers all major geometric concepts: points and vectors, lines and planes, maps and matrices, conics and curves. While the basic theory is completely covered, the emphasis of the book is not on abstract proofs but rather on examples and algorithms.

Matrix Algebra Using MINImal MATlab

Joel W. Robbin
1994

Hardcover, 560 pages
1-56881-024-5, $68.00

This undergraduate text aims to teach mathematical literacy by providing a careful treatment of set theoretic notions and elementary mathematical proofs. It gives a complete handling of the fundamental normal form theorems of matrix algebra. Use of the computer is fully integrated into Robbin's approach; not only does the book describe the basic algorithms in the computer language MATlab and provide unique computer exercises, it also includes an accompanying diskette and tutorial manual.

A K Peters, Ltd. 63 South Avenue, Natick, MA 01760
Phone: (508) 655-9933 Fax: (508) 655-5847
service@akpeters.com http://www.akpeters.com

Please send me the following titles:	Quantity	x Price	Subtotal
____________	______	______	______
____________	______	______	______

Massachusetts residents add 5% sales tax. Tax: ________
Shipping: In the US, add $5.00 for the first title, $2.00 for each additional title.
In Canada, add $10.00 (air) or $5.00 (surface) for each title. Shipping: ________
Total: ________

Name: ____________
Address: ____________

______ MasterCard/Visa ______ Check enclosed
No. ____________
Exp. Date ____________
Tel: ____________ Email: ____________ Signature ____________

The Development of Linear Algebra in Portugal

by Natália Bebiano, University of Coimbra

1. Mathematics in Portugal: An Overview. Linear Algebra research in Portugal was born in the Department of Mathematics of the University of Coimbra. Founded in 1290, the University of Coimbra is one of the oldest universities in Europe. Let us begin by sketching the historical development of mathematics in Portugal.

From its foundation in 1290 until 1537, the University was located alternately in Coimbra and Lisbon. In general, very little is known about academic activity during this period, in particular, about mathematical studies. There is no evidence for a strong connection between Portuguese discoveries and substantial knowledge in the field of mathematics. In 1537, the University moved permanently to Coimbra, and profound changes were instituted. Distinguished masters taught here, among them the celebrated mathematician and royal cosmographer Pedro Nunes:

The *Libro de Algebra en Arithmetica y Geometria* by Pedro Nunes published in 1567 (in Spanish), is one of the most famous treatises in the period between the *Ars Magna* of Cardano (1545) and the *Artem Analyticam Isagoge* of Vièta (1591).

In 1581, the kingdom of Portugal lost its independence and came under the rule of the Spanish kingdom for 60 years. Nunes died without leaving disciples. Only a single mathematical chair (which included studies in astronomy) existed in the whole University, in the faculty of medicine. The chair remained vacant from 1712 to 1772 despite the fact that mathematics was considered "an important science for the benefit of the Kingdom, and Navigation, and an ornament of the University" (*University Statutes*, 1612). The mathematical works of this period (most of them in manuscript form) are predominantly on subjects of applied mathematics: arithmetic applied to financial activities, geometry applied to fortification, records of astronomical observations, atlases and maps, and some works on astronomy, navigation and cartography.

After Nunes, Portuguese scholars seemed to be unaware of the great mathematical advances of the time. The prominence in university curricula of traditional subjects, such as rhetoric and Aristotelian philosophy, reflected the prevailing mentality. From the beginning of the 16th century the kingdom had been under the rule of the Inquisition and education was controlled by the Society of Jesus. In the first half of the 18th century, an evident desire for change developed in the scientific atmosphere. An interest in modern scientific trends motivated some Portuguese people to go abroad to satisfy their thirst for knowledge.

In 1772, under the rule of King D. José, an impressive university reform took place. In Coimbra, the Faculty of Mathematics was created (as well as the Faculty of Philosophy, i.e., natural sciences). Indeed the 1772 reform marks a turning point in the history of mathematical education in Portugal. Mathematics was placed in an exalted position, and a "University devoid of the light of mathematics" was said "to be like a universe without sun." Accordingly a first year mathematics course in geometry was compulsory for all University students. Some foreign books were translated, and (a few) original ones were published. The training of specialists in mathematics in the new Faculty had a notable impact. Frei Alexandre de Gouveia, who later became Bishop of Peking and a member of the Chinese Mathematical Tribunal, was one of the first to earn a doctorate after the 1772 reform. After the death of King D. José, the Inquisition developed new strength, and the university went into decline. There were several cases of political and religious persecution, and victims included university members (such as the mathematician José Anastácio da Cunha).

The isolation of the Portugese mathematical community is one of the relevant aspects of the 19th century. This community and its activity were mostly ignored abroad (the 20th century would mark the breakdown of this isolation). Deep political disturbances (French invasions and the Civil War) affected university life, resulting in the dispersal of vital members from mathematical science activities. The Faculty of Mathematics in Coimbra remained inactive for long periods.

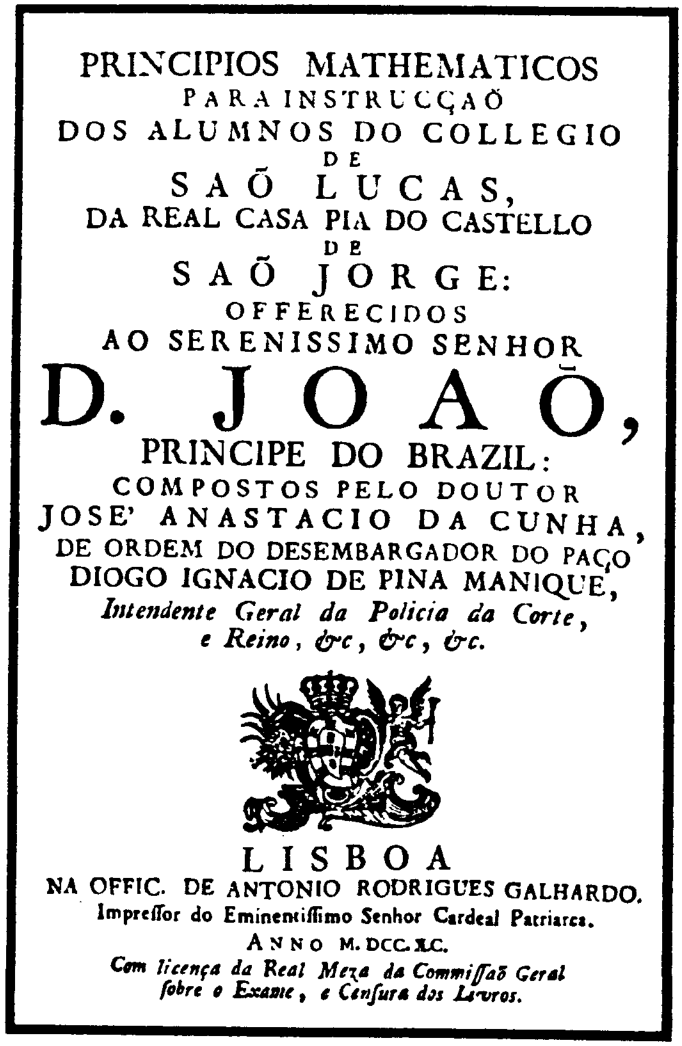
In 1910 Portugal's political regime changed from a monarchy to a republic. The Universities of Lisbon and Oporto (Porto) were created and, in Coimbra, the Faculty of Mathematics and the Faculty of Natural Philosophy merged into the Faculty of Science. During the thirties and forties, new mathematics textbooks were published and the methods of mathematical teaching in the Universities were modernized.

During the forties there was a period of intense mathematical activity. In 1937, a research journal of international calibre was created: *Portugaliae Mathematica*. In 1940 the Portuguese Mathematical Society was founded as well as *Gazeta de Matemática*, an expository journal for the mathematical community. Unfortunately, a great part of this activity was interrupted for political reasons (from 1926 to 1974 Portugal was under a Fascist regime) and most brilliant mathematicians were expelled from their posts and were forced into exile. Such scholars played an important role in developing mathematics in the countries where they sought exile—for example, Antonio Monteiro played a major role in mathematics in Argentina.

Frontispiece (above) of *De Crepuscalis* (1542), from *Principios Mathematicos* by José Anastácio da Cunha (1790), cf. title-page (right).

In the late sixties, important changes in the Portuguese educational system occurred. As a consequence of the so-called democratization of education, the number of undergraduate students and the number of universities increased, as well as the number of research centers and PhDs. Several new research areas were developed and research groups of international stature were founded. Besides the school of algebra, we mention the school of functional analysis and differential equations, the school of statistics and those of dynamical systems, control theory, numerical analysis, theoretical computer science, and geometry. Most students were encouraged to pursue graduate studies in other countries, such as England, USA, and France, and some of them became successful mathematicians, though for various reasons, there were students who preferred to do graduate work in Portugal. This remains the trend up to the present.

2. The Initiator of Linear Algebra in Portugal. Dr. Luís de Albuquerque (1917–1992) was the initiator of the linear algebra research group in Coimbra. Dr. Albuquerque attended the Universities of Coimbra and Lisbon and obtained his first degree from the University of Lisbon in 1939, after finishing high school in Coimbra.



PRINCIPIOS MATHEMATICOS
PARA INSTRUCÇAÕ
DOS ALUMNOS DO COLLEGIO
DE
SAÕ LUCAS,
DA REAL CASA PIA DO CASTELLO
DE
SAÕ JORGE:
OFFERECIDOS
AO SERENISSIMO SENHOR
D. JOAÕ,
PRINCIPE DO BRAZIL:
COMPOSTOS PELO DOUTOR
JOSE' ANASTACIO DA CUNHA,
DE ORDEM DO DESEMBARGADOR DO PAÇO
DIOGO IGNACIO DE PINA MANIQUE,
Intendente Geral da Policia da Corte, e Reino, &c, &c, &c.

LISBOA
NA OFFIC. DE ANTONIO RODRIGUES GALHARDO,
Impressor do Eminentissimo Senhor Cardeal Patriarca.
ANNO M.DCC.XC.
Com licença da Real Meza da Commissaõ Geral sobre o Exame, e Censura dos Livros.

Until the seventies, most Portuguese mathematicians obtained their PhD abroad, and those who earned their doctorate in Portugal did so without the guidance of a supervisor. Dr. Albuquerque was a self-made man: he prepared his PhD by himself, and at the same time taught drawing and mathematics more than thirty hours per week! As a graduate student, Dr. Albuquerque spent one year in Göttingen, studying probability and statistics with Konrad Jacob. At the University of Coimbra in 1959, he presented a thesis entitled *On the Theory of Functional Approximation*. He received his PhD and after that he was appointed as an assistant professor. Then he was promoted twice, becoming a full professor in 1966.

Curriculum modernization of undergraduate studies was one of the main goals of Dr. Albuquerque. In the sixties, he taught algebra and introduced in Coimbra the theory of groups and of linear spaces; he taught probability and introduced measure theory. In a second algebra course, he discussed the theory of matrices. Topics such as nonnegative matrices and Perron-Frobenius theory were covered, arousing the interest of students in the subject. Dr. Albuquerque wrote a set of lecture notes for the courses on analysis (which included elementary mathematical analysis and multivariate calculus), Monge geometry, probability, etc. He had a strong influence on the mathematical teaching of the second half of the century in Coimbra. In fact, he carried out innovation in the curriculum and was a brilliant lecturer.

In 1962, Albuquerque published a monograph entitled *Nonnegative Matrices: Stochastic Matrices* in the Review of the Faculty of Sciences of Coimbra (volume 31). Written in Portuguese, it was essentially an expository work—remarkably the first one in the world on this topic. It collected existing results which were presented in different papers and included some open problems and and a bibliography. One of these problems (proposed by the Russian mathematician H. R. Suleimanova) was on the characterization of the spectrum of stochastic matrices. This problem attracted the attention of Graciano Neves de Oliveira (a disciple of the first generation) and inspired his PhD Thesis [*Sobre Matrizes Estocásticas e Duplamente Estocásticas*, 224 pp., 1968].

George Styan (left) and Graciano de Oliveira (right): Santa Barbara, 18 June 1981. (Photo: Yasuko Chikuse.)

Dr. Albuquerque tried hard to promote research in the Department of Mathematics in Coimbra and to create a tradition of PhD supervision and training of disciples. With his brillance, creativity, and engaging personality, the program developed. In 1972, Dr. Albuquerque coordinated a research project on *Generalized Matrix Functions*, which included graduate and undergraduate students. He devoted his efforts to directing graduate students and developing the linear algebra group.

Luís de Albuquerque

Dr. Albuquerque was a respected teacher, a key figure in the history of Portuguese culture, and a great human spirit. His interests were diverse, among them literature, music, theatre and cinema. Dr. Albuquerque was a scientist of great accomplishment. In addition to being the founder of the Portugese linear algebra group, he was a famous cartographer of international standing, and he worked in the history of science (mainly, on the 15th and 16th century European navigations).

3. Linear Algebra Research in Portugal. The linear algebra group in Coimbra expanded to other Portuguese universities and also to Spain, e.g., to Vitoria. By the late seventies, Dr. Graciano de Oliveira had been involved in various linear algebra activities (seminars, recruitment of good graduate students, coordination of post-graduate courses, etc.). One of his students, Eduardo Marques de Sá, won the Householder Prize (*exaequo*) for the best PhD thesis in numerical linear algebra—the winning title: *Immersion of Matrices and Invariant Factors Interlacing*.

In the last two decades, there have been many linear algebra events in Portugal. Besides several mini-conferences on the theory of matrices, the *2nd ILAS Meeting* in 1992 took place in Lisbon. Research in linear algebra has become more and more active in Portugal. Various graduate students have obtained their PhD degrees in linear algebra and are now active researchers in Portugal. One of them, Jose Perdigão Dias da Silva, leads a linear algebra research group in Lisbon.

The Portuguese linear algebra group has connections with linear algebra groups in other countries. In Portugal, a great majority of the students are female (in contrast with other countries—the situation may seem rather surprising and deserves some attention, but that is another story). Linear algebra in Portugal will certainly continue to grow and contribute to the linear algebra community throughout the world.

Book Reviews

Badi H. Baltagi: *Econometrics*

Reviewed by Shuangzhe Liu

Econometrics by Badi H. Baltagi [Springer, xiv + 398 pp., ISBN 3-540-63617-X, 1998, $42.00] is a graduate textbook which treats basic econometric methods and some more advanced topics. It contains fourteen chapters covering Simple and Multiple Regression Analysis, Violations of the Classical Assumptions, Distributed Lags and Dynamic Models, General Linear Model (GLM), Regression Diagnostics and Specification Tests (RDST), Generalized Least Squares (GLS), Seemingly Unrelated Regressions (SUR), Simultaneous Equations Model, Pooling Time-Series of Cross-Section Data, Limited Dependent Variables (LDV), and Time-Series Models (TSM). GLM, GLS and SUR make full use of matrix algebra.

Several chapters include results and exercises (based) on matrices. This may be the first book with RDST as an individual chapter in an econometrics text. The treatment of TSM is brief—less than seventeen pages (the main material on TSM, e.g., in Pindyck and Rubinfeld (1998), which is more or less at the same level, covers about one hundred and thirty pages and five chapters). The TSM chapter does, however, introduce some newly popular concepts, e.g., Unit Root, ECR, Cointegration and ARCH.

This book gives special treatment to selected issues such as Likelihood Ratio, Wald and Lagrange Multiplier Tests, Frisch-Waugh Lovell Theorem, Efficiency of Ordinary Least Squares, and Goodness of Fit Measures (even for LDV), most of which involves matrix algebra. Like many other econometrics texts, the book gives notes on several widely used computer packages. It supplies empirical examples (as well as a Numerical Example, §3.9, independent from an Empirical Example, §3.10) and exercises via using the packages.

It is welcome that the text provides applications of matrices in statistics and econometrics with practical (case) studies drawn from recent research publications. The book would benefit, I think, not only students, but also statisticians, econometricians and other readers. There is a *Solutions Manual* [viii + 321 pp., ISBN 3-540-63896-2] to accompany the textbook. An abstract and detailed table of contents of the text and the *Solutions Manual* can be found by visiting `http://www.springer.de`.

Reference

Pindyck, Robert S. and Rubinfeld, Daniel L. (1998). *Econometric Models and Economic Forecasts*. 4th edition, Irwin/McGraw-Hill.

Dragoš Cvetković, Peter Rowlinson and Slobodan Simić: *Eigenspaces of Graphs*

Reviewed by Russell Merris

This outstanding book is volume 66 in the series, Encyclopedia of Mathematics and Its Applications [Cambridge University Press, 1997, viii + 258 pp., ISBN 0-521-57352-1, $69.95].

Let $G = (V, E)$ be a graph with vertex set $V = \{1, 2, ..., n\}$ and edge set E. Its *adjacency matrix* $A = A(G)$ is the $n \times n$ matrix whose (i, j)-entry is 1 if vertices i and j are adjacent, and 0 otherwise. It is easy to show that adjacency matrices of G afforded by different numberings (labelings) of its vertices are permutation similar. Indeed, graphs G_1 and G_2 are isomorphic if and only if there is a permutation matrix P such that $A(G_2) = P^{-1}A(G_1)P$. It follows, of course, that the spectrum, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$ of $A(G)$, is a graph-theoretic invariant.

The central theme of spectral graph theory is the relation between spectral and structural properties of graphs. As evidenced by two special issues of *Linear and Multilinear Algebra* [vol. 28, no. 1–2, 1990 & vol. 39, no. 1–2, 1995] and the bibliographies from the references at the end of this review, spectral graph theory has been a topic of considerable interest over the past three decades. A typical result is the following: Let G be a connected graph. Then G is bipartite (a structural property) if and only if the nonzero eigenvalues of $A(G)$ occur in $\pm$ pairs, i.e., λ is an eigenvalue of $A(G)$ of multiplicity k if and only if $-\lambda$ is an eigenvalue of $A(G)$ of multiplicity k.

Because adjacency matrices can be similar without being permutation similar, nonisomorphic graphs can afford the same spectrum. If, however, along with the eigenvalues we also know the corresponding eigenvectors, then it is a simple matter to reconstruct $A(G)$ and hence G. This brings us to *Eigenspaces of Graphs*.

Notation and terminology are introduced in the first chapter by means of a brief survey of spectral graph theory with special emphasis on the limitations of the spectrum as a graph invariant. While the authors' primary interest is the adjacency spectrum, other graph matrices are touched upon as well. Relationships between eigenvectors and structural properties of graphs are the topic of Chapter 2, and several eigenvector techniques are discussed in Chapter 3.

Consider the spectral decomposition $A(G) = \mu_1 P_1 + \mu_2 P_2 + \cdots + \mu_r P_r$, where $\mu_1 > \mu_2 > \cdots > \mu_r$ are the distinct eigenvalues of $A(G)$ and P_i is the orthogonal projection of $\Re^n$ onto $\mathcal{E}(\mu_i)$, the eigenspace corresponding to μ_i. Let $\alpha_{ij} = \| P_i(e_j) \|$, where e_i, $1 \leq i \leq n$, are the standard ordered basis vectors of $\Re^n$. Then $\beta_{ij} = \arccos(\alpha_{ij})$ is the *angle* between $\mathcal{E}(\mu_i)$ and e_j. Chapters 4 and 5 address properties of these "graph angles".

A "spherical basis" of $\Re^n$ is a set of the form $cB = \{cv : v \in B\}$, where c is positive and B is an orthonormal basis. A *eutactic star* in a nontrivial subspace U of $\Re^n$ is the orthogonal projection of cB onto U. When $c = 1$ and $B = \{e_i : 1 \leq i \leq n\}$, the vectors $P_i(e_j)$, $1 \leq j \leq n$, are the *arms* of a eutactic star in $\mathcal{E}(\mu_i)$, $1 \leq i \leq r$. A partition $X_1 \cup X_2 \cup \cdots \cup X_r$ of the vertex set $V = \{1, 2, ..., n\}$ is a *star partition* of G if $S_i = \{P_i(e_j) : j \in X_i\}$ is a basis of $\mathcal{E}(\mu_i)$, $1 \leq i \leq r$. In this case, $S = S_1 \cup S_2 \cup \cdots \cup S_r$ is a *star basis* of $\Re^n$ (afforded by G). Every graph on n vertices affords a star basis. Moreover, it is possible to associate with each graph a unique *canonical* star basis of $\Re^n$ such that two n-vertex graphs are isomorphic if and only if they afford the same spectrum and the same canonical star basis. These and related results (most of which were recently obtained by the authors) form the substance of Chapters 7–8.

Suppose ij and xy are edges of G while ix and jy are not. Let G' be the graph obtained from G by deleting ij and xy, and adding ix and jy. Then it is easy to see that the perturbed graph G' has the same degree sequence as G. The effects of this (and other) perturbations on the spectrum and angles are addressed in Chapter 6. Finally, Chapter 9 contains several miscellaneous results that do not fit conveniently into any of the earlier chapters. The end material includes tables of angles for the graphs on $n \leq 5$ vertices and a bibliography listing more than 300 entries. Designed as an overview of recent research, the book contains no exercises. Also missing is an index of notation, an inconvenience that is minimized by an especially comprehensive index.

References

Biggs, Norman (1993). *Algebraic Graph Theory*. 2nd Edition, Cambridge University Press.

Cvetković, Dragoš, and Doob, Michael (1985). Developments in the theory of graph spectra. *Linear and Multilinear Algebra* 18, 153–181.

Cvetković, Dragoš; Doob, Michael; Gutman, Ivan, and Torĝasev, Aleksandar (1988). *Recent Results in the Theory of Graph Spectra*. Annals of Discrete Mathematics, vol. 36, North-Holland.

Cvetković, Dragoš; Doob, Michael, and Sachs, Horst (1995). *Spectra of Graphs: Theory and Applications*. 3rd Edition, Johann Ambrosius Barth Verlag, Heidelberg.

Biswa Nath Datta: *Numerical Linear Algebra and Applications*

Reviewed by James R. Bunch

This excellent book [Brooks/Cole, xxii + 680 pp., ISBN 0-534-17466-3] on numerical linear algebra is very well-written and contains a plethora of interesting applications. The applications come from engineering, physics, chemistry, statistics, business, and the biological and biomedical sciences. There are also numerous examples throughout the text illustrating every algorithm and every situation.

When solving problems in science and engineering the very last step is usually the solution of a system of consistent linear equations, the least-squares solution of a system of inconsistent linear equations, the solution of an eigenproblem, or of a singular value problem. These topics come under the heading of *linear algebra*; when solving these problems on a computer these topics come under the heading of *numerical linear algebra*.

The book covers all the topics commonly associated with numerical linear algebra:

Chapter 1: A Review of Required Linear Algebra Concepts
Chapter 2: Floating-Point Numbers and Errors in Computation
Chapter 3: Stability of Algorithms and Conditioning of Problems
Chapter 4: Numerically Effective Algorithms & Mathematical Software
Chapter 5: Some Useful Transformations in Numerical Linear Algebra
Chapter 6: Numerical Solutions of Linear Systems
Chapter 7: Least-Squares Solutions to Linear Systems
Chapter 8: Numerical Matrix Eigenvalue Problems
Chapter 9: The Generalized Eigenvalue Problem
Chapter 10: The Singular Value Decomposition
Chapter 11: A Taste of Roundoff Error Analysis

There are also appendices giving an introduction to Matlab and computer programs in Matlab. Each chapter ends with a review and a summary, followed by numerous exercises. In the preface, suggestions are given for using the book in a one-semester or one-quarter undergraduate course, a beginning graduate course, or a one-semester course for engineers.

Several numerical linear algebra textbooks have appeared in the last few years. This is the one that I have been using for the past three years in our one-quarter undergraduate (upper division) course in numerical linear algebra (Math 170A). In general, I follow the topics suggested in the one-quarter undergraduate course, but I do them in a different order. I prefer to discuss perturbation theory and conditioning first, before introducing computer arithmetic, as perturbation theory and conditioning is fundamental and is independent of whether one uses finite or infinite precision arithmetic. Then I introduce finite precision arithmetic and the stability of algorithms.

I start the first day with two examples, one showing that a small change in the right-hand side can cause a large change in the solution, and the other showing that one can have a small residual but a bad approximate solution. I use these as motivation for the next few weeks. Then I cover the topics in this order:

(1) Review: 1.1–1.4, 1.7
(2) Perturbation theory and conditioning: 6.6–6.7
(3) Floating point numbers: 2.1–2.3
(4) Stability of algorithms: 3.1–3.4
(5) Linear equations: 5.1–5.3, 6.1–6.5
(6) Least squares: 1.4.2, 1.8, 5.4, 5.6, 7.1–7.8
(7) Eigensystems: 8.1–8.9
(8) Singular values: 10.1–10.3, 10.8, 10.9.1

My only criticism is that there is too much material in the book for a one-quarter or one-semester undergraduate course. A smaller (hence cheaper) edition covering only the material suitable in an undergraduate course would be very useful.

Overall, Biswa Datta has made an excellent presentation of numerical linear algebra in his very readable and interesting book. I find it to be the best book on the market at this time for an undergraduate course in numerical linear algebra.

New & Forthcoming Books on Linear Algebra & Related Topics: 1997–1999

Listed here are some new and forthcoming books on linear algebra and related topics that have been published in 1997 or in 1998, or are scheduled for publication in 1998 or in 1999. This list updates and augments our previous list in *Image* 20 (April 1998), 4–7. The references to CMP = *Current Mathematical Publications* and MR = *Mathematical Reviews* were found using MathSciNet and to Zbl = *Zentralblatt für Mathematik/MATH Database* by visiting `http://www.emis.de/cgi-bin/MATH` (three free hits are allowed for non-subscribers).

Althaus, G. Winter & Spedicato, E., eds. (1998). *Algorithms for Large Scale Linear Algebraic Systems: Applications in Science and Engineering. Proceedings, Gran Canaria, Spain, June 23 – July 6, 1996.* NATO ASI Series C (Mathematics and Physical Sciences), vol. 508. Kluwer Academic, viii + 409 pp., ISBN 0-7923-4975-X, ISSN 0258-2023 [Zbl 889.00027].

Boldin, M. V.; Simonova, G. I. & Tyurin, Yu. N. (1997). *Sign-based Methods in Linear Statistical Models.* Translated from the Russian by D. M. Chibisov. Translations of Mathematical Monographs, vol. 162. AMS, xii + 234 pp., ISBN 0-8218-0371-9 [MR 98f:62189, Zbl 873.62064].

Borghesani, Craig. *Linear Algebra.* Mathematics Series. PWS, ISBN 0-534-95249-6, in press.

Brezinski, Claude (1997). *Projection Methods for Systems of Equations.* Studies in Computational Mathematics, vol. 7. Elsevier, vii + 400 pp., ISBN 0-444-82777-3 [CMP 1 616 573, Zbl 980.06355].

Cameron, Peter J. (1998). *Introduction to Algebra.* Oxford Science Publications. Oxford University Press, x + 295 pp., ISBN 0-19-850194-3; 0-19-850195-1 [Zbl 980.20594].

Cartier, Pierre & Chevalley, Catherine, eds. (1997) *Claude Chevalley Collected Works, Vol. 2: The Algebraic Theory of Spinors and Clifford Algebras.* With a Postface by J.-P. Bourguignon. Springer, xiv + 214 pp., ISBN 3-540-57063-2 [CMP 1 636 473, Zbl 970.06870].

Castillo, Enrique; Cobo, Angel; Jubete, Francisco & Pruneda, Eva (1998). *Orthogonal Sets and Polar Methods in Linear Algebra: Applications to Matrix Calculations, Systems of Equations, Inequalities, and Linear Programming.* Wiley, 425 pp., ISBN 0-4713-2889-8, in press (expected November 1998).

Christensen, Ronald (1997). *Log-linear Models and Logistic Regression.* 2nd edition. Springer Texts in Statistics. Springer, xv + 483 pp. ISBN 0-387-98247-7 [CMP 1 633 357, Zbl 880.62073]

Cloud, Michael J. & Drachman, Byron C. (1998). *Inequalities With Applications to Engineering.* Springer, viii + 150 pp., ISBN 0-387-98404-6 [CMP 1 622 226].

Cohen, Joel L.; Kemperman, J. & Zbăganu, Gh. (1998). *Comparisons of Stochastic Matrices with Applications in Information Theory, Statistics, Economics and Population Sciences.* Birkhäuser, viii + 158 pp., ISBN 0-8176-4082-7.

Coriand, Andrea (1997). *Verfahren für Dünnbesetzte Lineare Systeme.* In German. DLR-Forschungsbericht, vol. 97-31. Deutsche Forschungsanstalt für Luft- und Raumfahrt, 162 pp. [CMP 1 617 150].

de Olazábal, Juan Manuel (1998). *Procedimientos Simbólicos en Algebra Lineal.* In Spanish. With a prologue by Juan Miguel Gracia Melero. Servicio de Publicaciones de la Universidad de Cantabria, Santander, xviii + 287 pp., ISBN 84-8102-195-4 [CMP 1 625 356].

Dietrich, Volker; Habetha, Klaus & Jank, Gerhard, eds. (1998). *Clifford Algebras and Their Application in Mathematical Physics: Proceedings of the 4th Conference, Aachen, Germany, May 28–31, 1996.* Fundamental Theories of Physics, vol. 94. Kluwer Academic, xxxii + 441 pp, ISBN 0-7923-5037-5 [CMP 1 627 070, Zbl 889.00021].

Dongarra, Jack J.; Duff, Iain S.; Sorensen, Danny C. & van der Vorst, Henk A. *Numerical Linear Algebra for High-Performance Computers.* Software, Environment, and Tools, vol. 7. SIAM, approx. 400 pp., ISBN 0-89871-428-1, in press (expected November 1998).

Douglas, Ronald G. (1998). *Banach Algebra Techniques in Operator Theory.* 2nd edition. Graduate Texts in Mathematics, vol. 179. Springer, xvi + 194 pp., ISBN 0-387-98377-5 [CMP 1 634 900, Zbl 980.27940]

Draper, Norman R. & Smith, Harry (1998). *Applied Regression Analysis.* 3rd Edition. Wiley Series in Probability and Statistics: Texts and References Section. Wiley, xx + 706 pp., ISBN 0-471-17082-8 [CMP 1 614 335, Zbl 980.20263].

Duff, Iain; Gould, Nick; Douglas, Craig & Giraud, Luc, eds. (1997). *Direct Methods, Linear Algebra in Optimization, Iterative Methods: Proceedings of the Workshop on Direct Methods, September 26–29, 1995; the Workshop on Linear Algebra in Optimization, April 22–25, 1996; and the Workshop on Iterative Methods, June 10–13, 1996, held in Toulouse as part of the International Linear Algebra Year (ILAY).* BIT, vol. 37., pp. i–ii and 473–769 [MR 98f:65005].

Gentle, James E. (1998). *Numerical Linear Algebra for Applications in Statistics.* Statistics and Computing. Springer, xiv + 221 pp., ISBN 0-3879-8542-5 (H) [CMP 1 638 791].

Grosse-Erdmann, Karl-Goswin (1998). *The Blocking Technique, Weighted Mean Operators and Hardy's Inequality.* Lecture Notes in Mathematics, vol. 1679. Springer, x + 114 pp., ISBN 3-540-63902-0 [CMP 1 611 898, Zbl 980.01382].

Grosshans, Frank D. (1997). *Algebraic Homogeneous Spaces and Invariant Theory.* Lecture Notes in Mathematics, vol. 1673. Springer, vi + 148 pp., ISBN 3-540-63628-5 [CMP 1 489 234, Zbl 886.14020].

Gunawardena, Ananda D. & Deeba, Elias Y. (1998). *Interactive Linear Algebra With Maple V: A Complete Software Package for Doing Linear Algebra.* Textbooks in Mathematical Sciences. Springer, xiii + 317 pp. ISBN 0-387-98240-X [Zbl 980.28233].

Gurlebeck, Klaus & Sproessig, Wolfgang (1997). *Quarternionic and Clifford Calculus for Physicists and Engineers.* Wiley, 365 pp., ISBN 0-471-9620-0 [Zbl 970.51553].

Gustafson, Karl E. (1998). *Lectures on Computational Fluid Dynamics, Mathematical Physics, and Linear Algebra.* Edited and with a foreword by T. Abe and K. Kuwahara. With comments by T. Abe, K. Kuwahara, Izumi Ojima and Mitsuharu Otani. World Scientific, x + 169 pp., ISBN 981-02-3213-6 [CMP 1 609 640].

Hartfiel, Darald J. (1998). *Markov Set-Chains.* Lecture Notes in Mathematics, vol. 1695. Springer, viii + 131 pp., ISBN 3-540-647759.

Hubbard, John H. & Hubbard, Barbara Burke (1998). *Vector Calculus, Linear Algebra and Differential Forms; A Unified Approach.* Prentice Hall, 608 pp., ISBN 0-1365-7446-7.

John, Peter W. M. (1998). *Statistical Design and the Analysis of Experiments.* Classics in Applied Mathematics, vol. 22. SIAM, xxiv + 356 pp., ISBN 0-89871-427-3 [CMP 1 639 721, Zbl 980.35762].

Johnson, Richard A. & Wichern, Dean W. (1998). *Applied Multivariate Statistical Analysis.* Fourth Edition. Prentice Hall, 704 pp., ISBN: 0-13-834194-X.

Katerinochkina, N. N. (1997). *Methods for the Separation of a Maximal Joint Subsystem of a System of Linear Inequalities.* In Russian. Soobshcheniya po Prikladnoj Matematike, Vychislitel'nyj Tsentr RAN, 16 pp. [Zbl 980.10525].

Khuri, André I.; Mathew, Thomas & Sinha, Bimal K. (1998). *Statistical Tests for Mixed Linear Models.* Wiley Series in Probability and Statistics: Applied Section. Wiley, xvi + 352 pp., ISBN 0-471-15653-1 [CMP 1 601 351, Zbl 980.13558].

Lang, Bruno (1997). *Effiziente Orthogonal Transformationen bei der Eigen- und Singulärwertberechnung.* In German. FB Mathematik, x + 163 pp. [Zbl 885.65040]

Latouche, G. & Ramaswami V. *Introduction to Matrix Analytic Methods in Stochastic Modeling.* ASA-SIAM Series on Statistics and Applied Probability. SIAM, approx. 350 pp. ISBN 0-89871-425-7, in press (expected February 1999).

Lindsey, James K. (1997). *Applying Generalized Linear Models.* Springer Texts in Statistics. Springer, xiii + 256 pp., ISBN 0-387-98218-3 [Zbl 883.62074].

Lounesto, Pertti (1997). *Clifford Algebras and Spinors.* London Mathematical Society Lecture Note Series, no. 239, Cambridge University Press, ix + 306 p., ISBN 0-521-59916-4 [MR 98k:15045, Zbl 887.15029].

Marques de Sá, Eduardo (1998). *Interlacing Problems for Invariant Factors.* Textos de Matemática, Série B, no. 2. Departamento de Matemática, Universidade de Coimbra, iv + 44 pp. [CMP 1 622 732].

Miller, Rupert G., Jr. (1997). *Beyond ANOVA: Basics of Applied Statistics.* Chapman and Hall Texts in Statistical Science Series, Chapman and Hall, xviii + 317 pp., ISBN 0-412-07011-1 [Zbl 885.62081].

Moore, Hal G. & Yaqub, Adil (1998). *A First Course in Linear Algebra with Applications.* 3rd edition. Academic Press, xvi + 752 pp., ISBN 0-12-505760-1 [CMP 1 631 193].

Nabebin, A. A. (1997). *Analytical Geometry and Linear Algebra: Compulsory Homework and Examples of Solution.* In English. Moscow Power Engineering Institute, 60 pp. [Zbl 868.15003].

Nabebin, A. A. (1997). *Linear Algebra.* In English. Moscow Power Engineering Institute, 130 pp. [Zbl 868.15002].

Nencka, Hanna & Bourguignon, Jean-Pierre, eds. (1997). *Geometry and Nature: In Memory of W. K. Clifford. A Conference on New Trends in Geometrical and Topological Methods, July 30–August 5, 1995, Madeira, Portugal.* Contemporary Mathematics, vol. 203, AMS, xvi + 296 pp., ISBN 0-8218-0607-6 [MR 97k:00024, Zbl 864.00055].

Pindyck, Robert S. & Rubinfeld, Daniel L. (1998). *Econometric Models and Economic Forecasts.* Irwin/McGraw-Hill, xx + 634 pp., ISBN 0-0791-3292-8.

Rao, C. Radhakrishna & Rao, M. Bhaskara (1998). *Matrix Algebra and Its Applications to Statistics and Econometrics.* World Scientific, xix + 535 pp., ISBN 981-02-3268-3.

Ryan, Thomas P. (1997). *Modern Regression Methods.* With disk. Wiley Series in Probability and Statistics: Applied Section. Wiley, xix + 515 pp., ISBN 0-471-52912-5 [MR 97k:62002, Zbl 885.62074].

Sarkar, Tapan K.; Hu, F. D.; Hua, Y. B. & Wicks, M. (1998). *Real Time Approximation of a Function by a Sum of Complex Exponentials: Utilizing the Matrix Pencil Approach.* World Scientific, 300 pp., ISBN 9-8102-1819-2.

Schmidt, Karsten & Trenkler, Götz (1998). *Moderne Matrix-Algebra: Mit Anwendungen in der Statistik.* In German. Springer, xi + 247 pp., ISBN 3-540-64194-7.

Solow, Daniel (1998). *The Keys to Linear Algebra:Applications, Theory, and Reasoning.* Books Unlimited,548 pp.,ISBN 0-9644-51921.

Sontag, Eduardo D. (1998). *Mathematical Control Theory: Deterministic Finite Dimensional Systems.* 2nd edition. Texts in Applied Mathematics, vol. 6. Springer, xvi + 531 pp., ISBN 0-387-98489-5, ISSN 0939-2475 [Zbl 980.28023].

Sparse Matrices In Industry. Papers from the 2nd Symposium on New Techniques for the Treatment of Sparse Matrices for Industrial Problems held at the Université des Sciences et Technologies de Lille, Lille, April 28–29, 1997 (1998). Numerical Algorithms, vol. 16 (1). Baltzer Science, vi + 94 pp. [CMP 1 623 627].

Special Issue: R. C. Thompson 1931–1995 (1997). Linear and Multilinear Algebra, vol. 43, no. 1–3, 326 pp., ISSN 0308-1087 [Zbl 885.00038].

Springer, Tony [Tonny Albert] (1998). *Linear Algebraic Groups.* 2nd Edition. Progress in Mathematics, vol. 9. Birkhäuser, x + 334 pp., ISBN 0-8176-4021-5.

Srivastava, Hari M. & Rassias, Themistocles M., eds. *Analytic and Geometric Inequalities and Their Applications.* Kluwer, in press (expected 1999).

Strang, Gilbert (1998). *Introduction to Linear Algebra.* 2nd Edition. Wellesley-Cambridge Press, vii + 503 pp., ISBN 0-9614-0885-5.

Strang, Gilbert (1998). *Linear Algebra and Its Applications.* 4th Edition. Harcourt Brace College Publishers, ISBN 0-03-010567-6, in press (expected December 1998).

Strang, Gilbert & Borre, Kai (1997). *Linear Algebra, Geodesy, and GPS.* Wellesley-Cambridge Press, xvi + 624 pp., ISBN 0-9614-0886-3.

Szirtes, Thomas (1998). *Applied Dimensional Analysis and Modeling.* With a chapter on mathematical preliminaries by Pál Rózsa. McGraw-Hill, xxv + 790 pp., ISBN 0-07-062811-4.

Verbeke, Geert & Molenberghs, Geert eds. (1997). *Linear Mixed Models in Practice: A SAS-oriented Approach.* Lecture Notes in Statistics, vol. 126. Springer, xiii + 306 pp., ISBN 0-387-98222-1 [Zbl 882.62064]

Voiculescu, Dan-Virgil, ed. (1997). *Free Probability Theory: Papers from the Workshop on Random Matrices and Operator Algebra Free Products held during the Special Year on Operator Algebra at the Fields Institute for Research in Mathematical Sciences, Waterloo, ON, March 1995.* Fields Institute Communications, vol. 12. AMS, vii + 312 pp., ISBN 0-8218-0675-0 [MR 97g:46002, Zbl 859.00025].

Vonesh, Edward F. & Chinchilli, Vernon M. (1997). *Linear and Nonlinear Models for the Analysis of Repeated Measurements.* Includes one disk. Statistics: Textbooks and Monographs, vol. 154. Marcel Dekker, xii + 560 pp. ISBN 0-8247-8248-8 [Zbl 970.29971].

Wright, Donald J. *Introduction to Linear Algebra.* McGraw-Hill, ISBN 0-07-072098-3, in press.

Zieschang, Heiner (1997). *Lineare Algebra und Geometrie.* In German. Mathematische Leitfäden. Teubner, ix + 654 pp., ISBN 3-519-02230-3 [MR 98j:51002, Zbl 970.09258].

Zurmühl, Rudolf & Falk, Sigurd (1997). *Matrizen und ihre Anwendungen. 1. Grundlagen. Für Ingenieure, Physiker und Angewandte Mathematiker.* In German. 7th Edition. Springer, xiv + 496 pp., ISBN 3-540-61436-2 [Zbl 859.15001].

FORTHCOMING TITLES IN THE ALGEBRA, LOGIC AND APPLICATIONS SERIES FROM GORDON AND BREACH

BILINEAR ALGEBRA
An Introduction to the Algebraic Theory of Quadratic Forms

Algebra, Logic and Applications, *Volume 7*

Kazimierz Szymiczek

Giving an easily accessible elementary introduction to the algebraic theory of quadratic forms, this book covers both Witt's theory and Pfister's theory of quadratic forms.

Leading topics include the geometry of bilinear spaces, classification of bilinear spaces up to isometry depending on the ground field, formally real fields, Pfister forms, the Witt ring of an arbitrary field (characteristic two included), prime ideals of the Witt ring, Brauer group of a field, Hasse and Witt invariants of quadratic forms, and equivalence of fields with respect to quadratic forms. Problem sections are included at the end of each chapter. There are two appendices: the first gives a treatment of Hasse and Witt invariants in the language of Steinberg symbols, and the second contains some more advanced problems in 10 groups, including the u-invariant, reduced and stable Witt rings, and Witt equivalence of fields.

1997 • 496pp • Cloth ISBN 90-5699-076-4 • US$84 / £55 / ECU70 • Gordon and Breach

MULTILINEAR ALGEBRA

Algebra, Logic and Applications, *Volume 8*

Russell Merris

The prototypical multilinear operation is multiplication. Indeed, every multilinear mapping can be factored through a tensor product. Apart from its intrinsic interest, the tensor product is of fundamental importance in a variety of disciplines, ranging from matrix inequalities and group representation theory, to the combinatorics of symmetric functions, and all these subjects appear in this book.

Another attraction of multilinear algebra lies in its power to unify such seemingly diverse topics. This is done in the final chapter by means of the rational representations of the full linear group. Arising as characters of these representations, the classical Schur polynomials are one of the keys to unification.

1997 • 396pp • Cloth ISBN 90-5699-078-0 • US$90 / £59 / ECU75 • Gordon and Breach

ADVANCES IN ALGEBRA AND MODEL THEORY

Algebra, Logic and Applications, *Volume 9*

Edited by **M. Droste** and **R. Göbel**

Contains 25 surveys in algebra and model theory, all written by leading experts in the field. The surveys are based around talks given at conferences held in Essen, 1994, and Dresden, 1995. Each contribution is written in such a way as to highlight the ideas that were discussed at the conferences, and also to stimulate open research problems in a form accessible to the whole mathematical community.

The topics include field and ring theory as well as groups, ordered algebraic structure and their relationship to model theory. Several papers deal with infinite permutation groups, abelian groups, modules and their relatives and representations. Model theoretic aspects include quantifier elimination in skew fields, Hilbert's 17th problem, (aleph-0)-categorical structures and Boolean algebras. Moreover symmetry questions and automorphism groups of orders are covered.

1997 • 500pp • Cloth ISBN 90-5699-101-9 • US$95 / £62 / ECU79 • Gordon and Breach

TO ORDER

North/South America
International Publishers Distributor • Tel: + 1 800 545 8398 • Fax: + 1 973 643 7676
Europe/Middle East/Africa
Marston Book Services, Ltd. • Tel: + 44 (0) 1235 465 500 • Fax: + 44 (0) 1235 465 555
Asia
International Publishers Distributor • Tel: + 65 741 6933 • Fax: + 65 741 6922
Australia/New Zealand
For a separate Australian dollar price please contact our distributor:
Fine Arts Press • Tel: + 61 2 9878 8222 • Fax: + 61 2 9878 8122
Japan
For yen price please contact our distributor:
YOHAN (Western Publications Distribution Agency)
Tel: + 81 03 3208 0186 • Fax: + 81 03 3208 5308

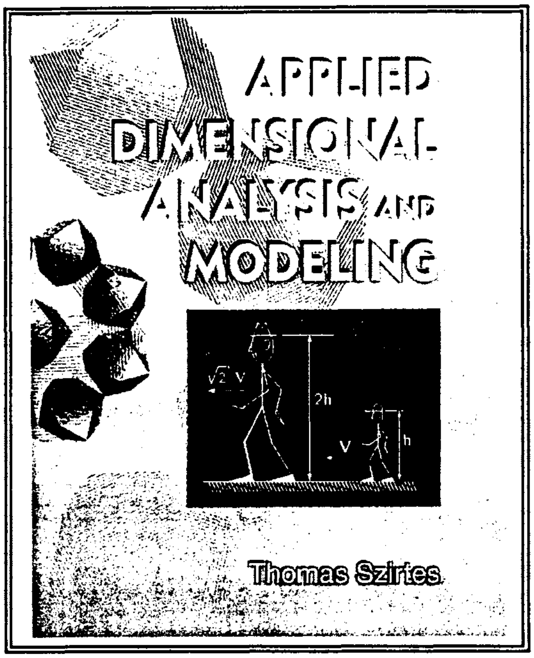
GB
Gordon and Breach
http://www.gbhap.com

APPLIED DIMENSIONAL ANALYSIS AND MODELING

by Thomas Szirtes

McGraw-Hill, New York, 1998; ISBN: 007-062811-4

815 pages, 200 illustrations, 250 + worked-out examples and case studies; US$99.95

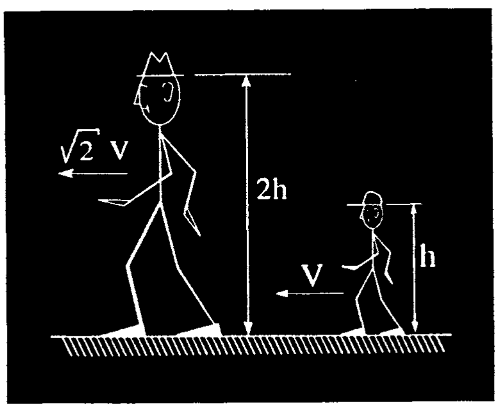


With a most creative use of linear algebra - specifically matrix arithmetic - this book presents the "*dimensional method*", which is an enormously potent tool in design, analyses and tests of engineering and scientific systems and products. The method is also efficient in forming relations among variables by non-analytic means, in verifying analytical formulas, and in the logistics of graphical presentations of multivariable relations.

The most important application of the dimensional method, however, is *model experimentation*, for which the determination of case-specific *Model Laws* and *Scale Factors* are mandatory. These laws and factors are all based on *dimensionless variables*. The book presents a highly elegant and fast method, based on matrix algebra, by which dimensionless variables are generated, and the relevant Model Laws and Scale Factors easily determined.

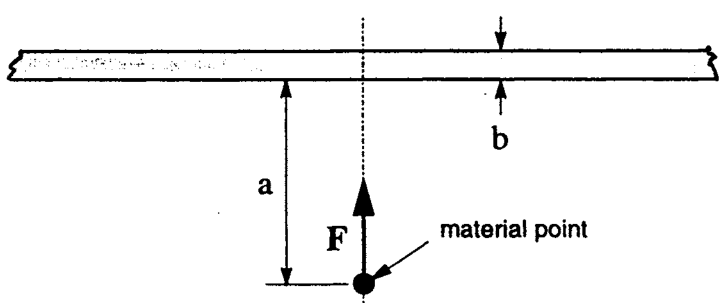
This in-depth landmark reference work also includes over 250 numerical examples and case-studies drawn from the theory and practical utilisation of dimensional method in applied science, engineering, biomechanics, astronomy, geometry and economics.

Did you know that a man *100 %* taller than a child, walks 'most comfortably' a mere *41.4* % faster?



Did you know that the *more* the air in a soap bubble, the *smaller* its internal pressure?

Did you know that the *distance* 'a' of a material point to a large plate has *no effect* on the gravitational force F that the plate imparts on the material point?



These startling facts are all true, as you can easily convince yourself without any analysis, only by some nice matrix arithmetic explained fully in

APPLIED DIMENSIONAL ANALYSIS AND MODELING.

ILAS5.SAM

Reports on Meetings Attended

Fourth Workshop on Numerical Ranges and Numerical Radii Madison, Wisconsin, USA: June 1–2, 1998

Report by Chi-Kwong Li

The fourth workshop on "Numerical Ranges and Numerical Radii" was held on June 1–2, 1998, in conjunction with the Seventh International Linear Algebra Conference, June 3–6, 1998, dedicated to Hans Schneider. The workshop took place on the ninth floor of Van Vleck Hall of the University of Wisconsin–Madison, which has a magnificent view of beautiful Lake Mendota.

The one-and-a-half day meeting began with an opening remark by the local organizer, Richard Brualdi, who is also the Chair of the Department of Mathematics of the University of Wisconsin–Madison. Then fourteen talks were given in three sessions with coffee breaks sponsored by the International Linear Algebra Society. The meeting ended around noon, June 2, after a closing remark by Hans Schneider. There were about three dozen participants for the workshop and a workshop photo (below) was taken around noon on Monday, June 1. Unfortunately, some participants from Europe were not included in the workshop photo due to delayed arrivals caused by the bad weather in the New York area on June 1.

The participants at the fourth workshop on "Numerical Ranges and Numerical Radii" were: T. Ando, Hokusei Gakuen University, Sapporo; Olga Azenhas, University of Coimbra; A. Berman, Technion, Haifa; R. A. Brualdi (organizer), Univ. of Wisconsin–Madison; B. Cain, Iowa State University, Ames; M. T. Chien, Soochow University; C. Davis, Univ. of Toronto; L. M. DeAlba, Drake University, Des Moines, Iowa; J. Dias da Silva, Univ. of Lisbon; Antonio Leal Duarte, Univ. of Coimbra; D. Farenick, Univ. of Regina; M. Fiedler, Praha, Czech Republic; C. R. Johnson, College of William and Mary; Yueh-er Kuo, Univ. of Tennessee–Knoxville; Tom Laffey, University College Belfield; C. K. Li (organizer), College of William and Mary; Raphael Loewy, Technion, Haifa; John Maroulas, National Technical University, Athens, Greece; R. McEachin, Univ. of the West Indies; Dobovisek Mirko, Univ. of Ljubljana; H. Nakazato, Hirosaki University; K. Okubo, Hokkaido Univ. of Education; Andre Ran, Vrije Universiteit; L. Rodman, College of William and Mary; H. Schneider, Univ. of Wisconsin–Madison; Fernando C. Silva, Univ. of Lisbon; Wasin So, Sam Houston State University; I. Spitkovsky, College of William and Mary; B. S. Tam, Tamkang University; T. Y. Tam, Auburn University; P. Y. Wu, National Chiao Tung University; David Yopp, Idaho State University; Xingzhi Zhan, Peking University; and Hongding Zhou, Univ. of Regina.

Similar to the previous three meetings, participants exchanged many ideas, results and problems on the subject in a very friendly atmosphere. Furthermore, it was announced that there will be a special session on Numerical Ranges, organized by Pei-Yuan Wu [pywu@cc.nctu.edu.tw], in the International Conference on Mathematical Analysis and its Applications to be held in Taiwan, January 17–21, 2000. The Fifth Workshop on Numerical Ranges and Numerical Radii will be organized by John Maroulas, National Technical University, Athens [maroulas@math.ntua.gr], in Greece, June 2000.

The "Hans Schneider Linear Algebra" Conference
Seventh ILAS Conference
Madison, Wisconsin, USA: June 3–6, 1998

Report by Richard A. Brualdi

The Seventh Conference of the International Linear Algebra Society (ILAS) was held at the University of Wisconsin (UW)–Madison in Madison, Wisconsin on June 3–6, 1998. All of the scientific sessions were held in E. B. Van Vleck Hall, the home of the Department of Mathematics. The conference was dedicated to Hans Schneider in recognition of his enormous contributions to linear algebra and the linear algebra community.

The number of conference participants was 212 (cf. photo on pp. 14–15) representing 25 different countries. There were 12 invited speakers each of whom gave a very stimulating and well-presented lecture; many of the lectures described the contributions of Hans Schneider to different parts of linear algebra. On Wednesday, June 3, the first ILAS-LAA lecture (sponsored by Elsevier Science, Inc.) was given by Carl de Boor of UW–Madison on the topic "Linear algebra issues in approximation theory." This special lecture was followed by a lavish reception in the 9th floor conference room of Van Vleck Hall, with a wonderful panoramic view of Madison and its lakes. In addition, there were 8 invited minisymposia featuring nearly 60 talks. Over 100 contributed talks were presented, covering almost all aspects of linear algebra. The conference was noteworthy for the mixture of core linear algebraists, numerical linear algebraists, and applied and computational linear algebraists present. All registered participants were given a high quality, multi-purpose shoulder bag featuring the ILAS logo and the words "The Hans Schneider Linear Algebra Conference." An ILAS Business meeting was held during the afternoon of Thursday, June 4.

A gala banquet was held at the UW–Madison Memorial Union on Friday evening, June 5, with 200 partygoers present. Following a delicious dinner, there were tributes to Hans Schneider given by Richard A. Brualdi, David H. Carlson, Richard S. Varga, and John Todd. Daniel Hershkowitz presented Hans with a preliminary version of the issue of the *Electronic Journal of Linear Algebra* (ELA) that is being dedicated to Hans. In addition, a tribute to Hans's wife Miriam was given by Richard A.'s wife Mona, and flowers were presented to Miriam. All of Hans's children Barbara, Peter, and Michael (and Barbara's son David) were at the banquet and a family tribute was given by Peter. The evening closed with remarks and thanks by Hans. A disposable camera was placed on each of the tables, and a photo album is being constructed to give to Hans as a remembrance of the meeting.

On Saturday afternoon, June 6, there was an excursion to the House on the Rock in nearby Spring Green. Participants were treated to a remarkable house and an amazing collection of "unimaginables" including a not-to-be-believed carousel. It took nearly three hours to wander the 2 1/2 miles through the connected rooms containing the collection.

A special issue of *Linear Algebra and Its Applications* (LAA), edited by B. Cain, B. N. Datta, M. Goldberg, U. Rothblum, and D. Szyld, containing papers from the conference will be dedicated to Hans Schneider. Funding for the conference was given by the National Science Foundation and the Institute for Mathematics and its Applications (IMA) in Minnesota through its Participating Institution Conference Fund.

From left to right: Jochen Werner, George Styan, Fuzhen Zhang, Chi-kwong Li, Steve Kirkland, and Steve leon. *Image* Editorial Board lunch, Madison, Wisconsin: June 4, 1998. (Photo via Chi-Kwong Li.)

The "Hans Schneider Linear Algebra" Conference, S

nth ILAS Conference, Madison, Wisconsin: June 3–6, 1998.

Western Canada Linear Algebra Meeting
Victoria, British Columbia: July 30–31, 1998

Report by Stephen J. Kirkland

The Western Canada Linear Algebra Meeting (WCLAM) provides an opportunity for mathematicians in western Canada working in linear algebra and related fields to meet, present some of their recent research, and exchange ideas in an informal atmosphere. The WCLAM in Victoria was the fourth in the ongoing series: previous meetings were held in Regina (1993), Lethbridge (1995) and Kananaskis Village (1996).

There were 37 participants at the Victoria meeting, and while many were from WCLAM's natural base in western Canada, the meeting also attracted people from Australia, Greece, Holland, Poland, and the U.S.A. The program featured two invited talks by Stephen Boyd and Jennifer Seberry, as well as lectures by H. Bart, T. Bhattacharyya, R. Craigen, J. Drew, R. Edwards, S. Fallat, D. Farenick, C-H. Guo, R. Hryniv, H. Kharaghani, P. Lancaster, J. Muldowney, P. Psarrakos, B. Watson, and K. Wood. The presentations spanned a number of research areas, including matrix theory, operator theory, linear algebra, combinatorics, matrix functions, designs, differential equations, optimization, and Markov chains. In addition to the talks, there was also an informal open-problem session.

The organizing committee is happy to thank both the University of Victoria and the Pacific Institute for the Mathematical Sciences for providing partial support for the meeting. In light of the level of interest displayed at the Victoria gathering, a WCLAM in 2000 is being contemplated.

Third Chinese Conference on Matrix Theory and Applications
Zhang Jia Jie, Hunan, China: August 8–13, 1998

Report by Fuzhen Zhang

The Third Chinese Conference on Matrix Theory and Applications was held on August 8–13, 1998, in Zhang Jia Jie, Hunan Province, China. The Welcome Reception was presided over by Professor E. X. Jiang (Chairman of the Chinese Linear Algebra Society) and by Professor J. S. Li and Professor L. P. Huang.

The conference featured fifteen 45-minutes talks by Professors T. Ando (Japan), H. L. Chen, L. P. Huang, E. X. Jiang, G. Y. Lei, T. G. Lei, J. S. Li, Z. Li (USA), J. Z. Liu, X. P. Liu, M. X. Peng, B-S Tam (Taiwan), S. J. Yang, X. Zhan, and F. Zhang (USA), followed by presentations in parallel sessions on Matrix Algebra, Matrix Computation, and Combinatorial Matrix Theory. There were 85 papers submitted to the conference, and 45 were published as the Proceedings of the Conference in Vol. 18, No. 3, 1998, of the *Hunan Annals of Mathematics* (ISSN 1006-8074).

This Third Chinese Conference on Matrix Theory and Applications was supported by the Chinese Mathematical Society, Xiangtang Polytechnic Institute, and Xiangtang Teachers' College.

The Fourth Chinese Conference on Matrix Theory and Applications will be held in Kunming, Yunnan Province, China, in 2000.

From left to right: Rob Craigen, Harm Bart, Hadi Kharaghani, Jan Okninski, Shaun Fallat, Jennifer Seberry, James Muldowney, Peter Lancaster, Tobias Krahn, Dick Phan, Pauline van den Driessche, Panos Psarrakos, Stephen Boyd, Paul Binding, Steve Kirkland, Michael Tsatsomeros, Kathryn Wood, Dale Olesky, John Drew, Kelly Choo, Bruce Watson, Rostysdlav Hryniv, Jeff Heo, Wai-Sun Cheung, Doug Farenick, Rod Edwards, and Chun-Hua Guo.
WCLAM'98, University of Victoria: July 30–31, 1998. (Photo: David Leeming.)

Third Chinese Conference on Matrix Theory and Applications, Zhang Jia Jie, Hunan, China: August 8–13, 1998.

Patricia James Eberlein: 1923–1998

Patricia James Eberlein, professor emerita and former chair of the Department of Computer Science at the University at Buffalo, New York, died in her Buffalo home on August 11, 1998, from lung cancer. She was 75 years old.

Pat Eberlein was one of the first faculty members, and the only woman, hired when the department was formed in 1967. She served as chair of the department from 1981–84 and was acting chair from 1971–72, the only woman to hold those positions in the department. While she was chair, Eberlein significantly increased the amount of sophisticated computer equipment in the department, propelling it — and the university — into the technological forefront. Before coming to Buffalo, she was a mathematician at the Institute for Advanced Study in Princeton, where she worked on the "Electronic Computer Project" that led to the development of one of the first computers ever constructed. Eberlein also was an assistant professor at the University at Rochester and Associate Director of its Computing Center. Her research interests focused on algorithms for linear algebra, parallel algorithms for numerical computations and combinatorial and graph algorithms.

Described by a colleague as "the most fascinating person I've ever met," Pat did not limit herself to the academic world. She had a stint as a fashion model and piloted airplanes to military bases from their manufacturing plants during World War II. She also lived for a time on an Indian reservation. Eberlein often spoke on panels about the need to attract more women to study mathematics and computer science. She held visiting professorships at Oxford and Cornell and was a visiting senior scientist at the Argonne National Laboratory. A member of numerous professional societies, she was active in the Association for Computing Machinery, the American Mathematical Society, the American Association for the Advancement of Science, and the Association for Women in Mathematics. A graduate of the University of Chicago, she earned master's and doctoral degrees from Michigan State University.

Pat Eberlein is survived by seven children, 13 grandchildren and one great-grandchild. Pat was the widow of William Eberlein, who was a member of the mathematics faculty at the University of Rochester. [Many thanks go to Joseph Straight for leading us to the website `www.buffalo.edu/news/CampusNews/EberleinObit.html` and for his permission to use the obituary published in *The Seaway Current*, vol. 22, no. 1, Fall 1998, p. 11. —Ed.]

Forthcoming Meetings

Seventh International Workshop on Matrices and Statistics: in Celebration of T. W. Anderson's 80th Birthday

Fort Lauderdale, Florida: December 11–14, 1998

The Seventh International Workshop on Matrices and Statistics, in celebration of T. W. Anderson's 80th birthday, will be held at Nova Southeastern University, Fort Lauderdale, Florida, on the weekend from Friday, December 11 through Monday, December 14, 1998. The meeting will begin in the early evening of the Friday with a reception and end on the Monday with an excursion and a gourmet dinner; a banquet is planned for the Saturday evening (December 12). For their support of this 7th Workshop we are very grateful to the International Linear Algebra Society (ILAS), Nova Southeastern University (NSU), and the Statistical Society of Canada (SSC). The International Organizing Committee for this Workshop comprises R. William Farebrother (Victoria University of Manchester, England), Simo Puntanen (University of Tampere, Tampere, Finland), George P. H. Styan (McGill University, Montréal, Québec, Canada; chair), and Hans Joachim Werner (University of Bonn, Bonn, Germany; vice-chair). The Local Organizing Committee at Nova Southeastern University comprises Naomi D'Alessio, William D. Hammack, David Simon, and Fuzhen Zhang (chair). This Workshop is the seventh in a series. The previous six Workshops were held as follows: (1) Tampere, Finland: August 1990, (2) Auckland, New Zealand: December 1992, (3) Tartu, Estonia: May 1994, (4) Montréal, Québec, Canada: July 1995, (5) Shrewsbury, England: July 1996, and (6) İstanbul, Turkey: August 1997. The 8th Workshop will be held in Tampere, Finland, on Friday, August 6 and Saturday, August 7, 1999.

The purpose of this Seventh Workshop, the first to be held in the United States, is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between matrix theory and statistics. This Workshop will provide a forum through which scientists working in the areas of linear algebra/matrix theory and/or statistics may be better informed of the latest developments and newest techniques and may exchange ideas with researchers from several different countries. Selected fully-refereed papers from this Workshop will be published in a Special Issue on Linear Algebra and Statistics of *Linear Algebra and Its Applications.*

The Workshop will include invited and contributed talks. These talks and the informal workshop atmosphere will guarantee an intensive exchange of ideas. We expect special coverage of some of the topics that have highlighted Ted Anderson's research: multivariate statistical analysis and econometrics, as well as matrix inequalities.

As of October 24, 1998, the following persons are scheduled to give talks: Gülhan Alpargu, Yasuo Amemiya, Theodore W. Anderson, Natália Bebiano, Philip V. Bertrand, James V. Bondar, Theophilos Cacoullos, N. Rao Chaganty, Xiao-Wen Chang, Pinyuen Chen, John S. Chipman, Knut Conradsen, Carles M. Cuadras, James Durbin, Friedhelm Eicker, Richard William Farebrother, Donniell E. Fishkind, Nancy Flournoy, Bernard Flury, Jürgen Groß, Valia Guerra Ones, Steve Halitsky, Frank J. Hall, Robert E. Hartwig, David A. Harville, Berthold Heiligers, Erin M. Hodgess, Shane T. Jensen, André Klein, Tõnu Kollo, Zhongshan Li, Erkki Liski, Shuangzhe Liu, Augustyn Markiewicz, Jorma Kaarlo Merikoski, Tapio Nummi, Ingram Olkin, Marianna Pensky, Michael D. Perlman, Friedrich Pukelsheim, Simo Puntanen, C. Radhakrishna Rao, Shayle R. Searle, Peter Šemrl, Bimal Kumar Sinha, Michael A. Stephens, George P. H. Styan, Gerald E. Subak-Sharpe, Yoshio Takane, Yongge Tian, Imbi Traat, Götz Trenkler, Michel Van De Velden, Humberto Vaquera-Huerta, Hele-Liis Viirsalu, Hrishikesh D. Vinod, Heinrich Voss, Tonghui Wang, Hans Joachim Werner, Haruo Yanai, Fuzhen Zhang, and Yijun Zuo.

The Registration fees are: Regular Participants US$75; Students/Retired Persons US$35. No refund of the registration fee will be given after December 1, 1998. All money transfer fees are at the sender's expense.

Accommodation, at very favorable rates ($50 per night for single or double room, plus 11% tax), has been arranged at the Rolling Hills Hotel & Golf Resort, which will be the Workshop official home. Rolling Hills is the nearest hotel to Nova Southeastern University (NSU) and it takes about 10 minutes to walk (with excellent bird spotting) to the NSU Medical School where the Workshop will be held. Rolling Hills is the official hotel and resort of the Miami Dolphins and was the film location for the hit comedy "Caddyshack". There are many other hotels and resorts in the Fort Lauderdale area, particularly on the beach, with prices in the $70–$100 range (before December 21); these hotels are about 20 minutes drive to the NSU Medical School. The Rolling Hills Hotel & Golf Resort is located at 3501 West Rolling Hills Circle, Fort Lauderdale, Florida 33328, USA; tel. 1-800-327-7735 or (1-954) 475-0400, FAX (1-305) 474-9967, `sales@rollinghillsresort.com`, `http://rollinghillsresort.com`.

There are free van services between the Fort Lauderdale International Airport (FLL) and Rolling Hills. Call Rolling Hills: 1-800-327-7735 or (1-954) 475-0400 for pick-up. It costs about $25 (one-way) by taxi. There is a regular "Super Shuttle" service between Miami International Airport (MIA) and Rolling Hills: this takes about 40 minutes and costs about $28 (one-way); tel. (1-954) 964-1700.

Details of local as well as central and south Florida attractions and excursions are available from the Workshop website: `http://www.polaris.nova.edu/MST/conf/FMW/`, where all available Workshop information is being updated frequently. Or contact George Styan, Dept. of Mathematics and Statistics, McGill University, 805 ouest, Sherbrooke Street West, Montréal, Québec, Canada H3A 2K6; styan@Math.McGill.CA, FAX (1-514) 398-3899 or Fuzhen Zhang, Math, Science & Technology, Farquhar Center for Undergraduate Studies, Nova Southeastern University, 3301 College Avenue, Fort Lauderdale, Florida 33314-7796, USA; zhang@polaris.nova.edu, FAX (1-954) 262-3931, tel. 1-800-338-4723, ext. 8317, or (1-954) 262-8317.

Special Session on Matrices and Statistics

Regina, Saskatchewan: June 6–9, 1999

Matrix methods in statistics have received some attention in recent years; witness the ongoing series of International Workshops on Matrices and Statistics, which has been running since 1990. The Statistical Society of Canada will hold its 1999 meeting in Regina, and at this annual meeting there will be a session on Matrices and Statistics which will feature eight 30-minute lectures on a broad spectrum of topics. The aims of the session are to highlight the interaction between statistics and matrix theory, and to promote dialogue between researchers in both areas. For further information please contact Ejaz Ahmed: ahmed@math.uregina.ca, Doug Farenick: farenick@math.uregina.ca, or Steve Kirkland: kirkland@math.uregina.ca.

Eighth ILAS Conference

Barcelona, Spain: July 19–22, 1999

The eighth of the linear algebra conferences organized by the International Linear Algebra Society (ILAS) will be held in Barcelona, Spain, July 19–22, 1999. Contributions in all areas of linear algebra (core, numerical, computational, systems and control theory, ...) are welcome. The chair of the organizing committee is Ferran Puerta of Barcelona: puerta@ma1.upc.es. The co-chair for programs is Tom Laffey and the co-chair for local arrangements is Mª Isabel Garcia-Planas. The other members of the organizing committee are: Raphael Bru, Luz DeAlba, J. M. Gracia, V. Hernández, Nick Higham, Roger A. Horn, F. Mateos, Paul Van Dooren, and Richard A. Brualdi–*ex officio*.

At present, the following speakers have agreed to participate: Z. Bai, J. Ferrer, D. Hinrichsen, V. Kaashoek, S. Kirkland, C-K. Li, N. Mackey, E. Marques de Sá, K. Murota, V. Pták, F. Silva Leite, A. Urbano, and I. Zaballa. The program will include invited talks (50 and 30 minutes) and several minisymposia about different topics, as well as opportunities for contributed talks and posters. The Conference Proceedings will be published in a special issue of *Linear Algebra and its Applications*. A second announcement will contain further information about the program, registration procedures and instructions for submissions.

Eighth International Workshop on Matrices and Statistics

Tampere, Finland: August 6–7, 1999

The Eighth International Workshop on Matrices and Statistics will be held at the University of Tampere, Finland, on Friday, August 6 and Saturday, August 7, 1999 (starting with a reception on the Thursday evening August 5). This Workshop is a Satellite of the 52nd Session of the International Statistical Institute (ISI) to be held in Helsinki from Tuesday, August 10 through Wednesday, August 18, 1999. The purpose of this Eighth Workshop is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between matrix theory and statistics. The sponsors are: Academy of Finland, Finnair – The Official Carrier of the Workshop, Foundation of the Promotion of the Actuarial Profession, International Linear Algebra Society (ILAS) – support awarded for the ILAS Lecturer, Nokia, and the Tampere University Foundation.

The Keynote Speakers will be: T. W. Anderson (Stanford University, USA) and C. Radhakrishna Rao (PennState University, USA). The ILAS Lecturer will be: Gene H. Golub (Stanford University, USA). The Invited Speakers are: T. Ando (Hokusei Gakuen University, Sapporo, Japan); Ronald Christensen (University of New Mexico, Albuquerque, USA); Seppo Mustonen (University of Helsinki, Finland); Alastair J. Scott (University of Auckland, New Zealand); Shayle R. Searle (Cornell University, USA); Bimal K. Sinha (University of Maryland–Baltimore, USA); and George P. H. Styan (McGill University, Canada).

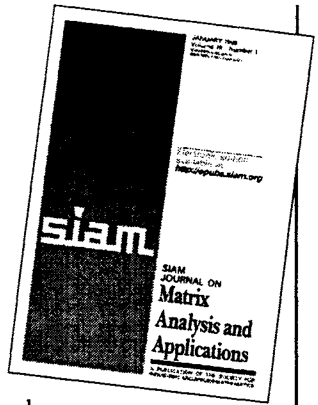
The International Organizing Committee for this Workshop comprises R. William Farebrother (Victoria University of Manchester, England), Simo Puntanen (University of Tampere, Tampere, Finland; chair), George P. H. Styan (McGill University, Montréal, Québec, Canada; vice-chair), and Hans Joachim Werner (University of Bonn, Bonn, Germany). The Local Organizing Committee at the University of Tampere comprises Riitta Järvinen, Erkki Liski, Tapio Nummi, and Simo Puntanen (chair).

A Sauna Party in Finnish style has been arranged for the Friday evening (August 6), on Viikinsaari Island. This evening will include a chance to sit for four hours in heat of 80° C — in which case one loses a lot of sweat, misses the flamed salmon dinner, an international soccer match, and a pleasant stroll around the island. The After-Sauna part of the evening is hosted by Pentti Hietanen, a well-known Finnish singer.

Deadlines: Submission of a preliminary registration form and a tentative title of a contributed paper: 10 January 1999; Abstract of a contributed paper: 10 April 1999; Early registration : 1 June 1999; Registration fee FIM 400 (= ca. US$ 80)—after 1 June 1999, add FIM 100 (= ca. US$ 20); Full-time students: FIM 50 (= ca. US$ 10). For further information, contact: The Workshop Secretary, Dept. of Mathematics, Statistics & Philosophy, University of Tampere, PO Box 607, FIN-33101 Tampere, Finland; FAX +358-3-215-6157; workshop99@uta.fi, http://www.uta.fi/~sjp/workshop99.html.

SIAM Journal on

Matrix Analysis and Applications



Contains research articles in matrix analysis and its applications and papers of interest to the numerical linear algebra community. Applications include such areas as signal processing, systems and control theory, statistics, Markov chains, and mathematical biology. Also contains papers that are of a theoretical nature but have a possible impact on applications. *Quarterly.*

Editor-in-Chief: P. Van Dooren, *Université Catholique de Louvain*

Associate Editors
G. H. Golub, *Stanford University*
J. Varah, *University of British Columbia*

Editorial Board

R. Bhatia	A. Edelman	N. J. Higham	A. C. M. Ran
R. Brualdi	M. Eiermann	I. C. F. Ipsen	L. Reichel
A. Bunse-Gerstner	L. Eldén	V. Mehrmann	Z. Strakoš
D. Calvetti	R. Freund	C. D. Meyer	C. Tomasi
M. Chu	P. Gill	E. Ng	S. Van Huffel
R. Cottle	A. Greenbaum	D. P. O'Leary	S. A. Vavasis
M. Deistler	U. Helmke	M. L. Overton	

Major Mathematical Abstracting and Indexing Sources
ACM Computing Reviews; Current Contents; INSPEC; Mathematical Reviews; Science Citation Index; Zentralblatt für Mathematik.

Impact Factor: 0.713

1999 List Price $332 USA, Canada, and Mexico • $380 overseas
1999 SIAM Individual Member Price $60 USA, Canada, and Mexico • $64 overseas
Electronic access is free with print • Electronic only $298 (worldwide)

Representative Papers

Some Recent Results on the Linear Complementarity Problem
G. S. R. Murthy, T. Parthasarathy, B. Sriparna

Statistical Condition Estimation for Linear Least Squares
C. S. Kenney, A. J. Laub, M. S. Reese

Global Block-Similarity and Pole Assignment of Class C^p
Josep Ferrer, Ferran Puerta

Eigenvalue Locations of Generalized Companion Predictor Matrices
Licio H. Bezerra, Fermin S. V. Bazán

Approximate Semidefinite Matrices in a Linear Variety
Charles R. Johnson, Pablo Tarazaga

Kronecker Stratification of the Space of Quadruples of Matrices
Mª I. Garcia-Planas

On Hyperbolic Triangularization: Stability and Pivoting
Michael Stewart, G. W. Stewart

A Padé Approximation Method for Square Roots of Symmetric Positive Definite Matrices
Ya Yan Lu

Inertias of Block Band Matrix Completions
Nir Cohen, Jerome Dancis

Cone Inclusion Numbers
Geir Nævdal, Hugo J. Woerdeman

A Schur–Fréchet Algorithm for Computing the Logarithm and Exponential of a Matrix
C. S. Kenney, A. J. Laub

Choosing Poles So That the Single-Input Pole Placement Problem Is Well Conditioned
Volker Mehrmann, Hongguo Xu

Node Selection Strategies for Bottom-Up Sparse Matrix Ordering
Edward Rothberg, Stanley C. Eisenstat

Perturbation Results for Projecting a Point onto a Linear Manifold
Jiu Ding

On the Quality of Spectral Separators
Stephen Guattery, Gary L. Miller

Stability of Conjugate Gradient and Lanczos Methods for Linear Least Squares Problems
Åke Björck, Tommy Elfving, Zdenek Strakoš

Partial Superdiagonal Elements and Singular Values of a Complex Skew-Symmetric Matrix
Tin-Yau Tam

Extended Krylov Subspaces: Approximation of the Matrix Square Root and Related Functions
Vladimir Druskin, Leonid Knizhnerman

Other SIAM Journals of Interest...

SIAM Journal on Numerical Analysis
Impact Factor: 1.175 • Bimonthly
1999 List Price $382 (domestic) $448 (overseas)
1999 SIAM Individual Member Price $72 (domestic) $78 (overseas)
Electronic only: Members $35 • Nonmembers $343

SIAM Journal on Scientific Computing
Impact Factor: 1.023 • Bimonthly
1999 List Price $432 (domestic) $510 (overseas)
1999 SIAM Individual Member Price $72 (domestic) $78 (overseas)
Electronic only: Members $35 • Nonmembers $388

TO ORDER
215-382-9800
(worldwide)
800-447-SIAM
(USA/CAN)

For complete information:
www.siam.org

siam.
Society for Industrial and Applied Mathematics

Image Problem Corner

We look forward to receiving solutions to Problems 18-1, 19-3, and 19-4, which we repeat below; we also present some comments concerning our solutions to Problem 19-5 in *Image* 20 (April 1998), pp. 28–31. We present solutions to Problems 20-1 through 20-6, which appeared in *Image* 20 (April 1998), p. 32. In addition, we introduce 5 new problems (page 28) and invite readers to submit solutions, as well as new problems, for publication in *Image*. Please send all material both in LaTeX code embedded as text only by e-mail to `styan@Math.McGill.CA` and by regular air mail (nicely-printed copy, please!) to George P. H. Styan, Editor-in-Chief: *Image*, Dept. of Mathematics & Statistics, McGill University, 805 ouest, rue Sherbrooke Street West, Montréal, Québec, Canada H3A 2K6, or by FAX to (1-514) 398-3899.

Problem 18-1: 5×5 **Complex Hadamard Matrices**
Proposed by S. W. DRURY, *McGill University, Montréal, Québec, Canada.*

The Editor has not yet received a solution to this problem — indeed even the Proposer has not yet found a solution!

Show that every 5×5 matrix U with complex entries $u_{j,k}$ of constant absolute value one that satisfies $U^*U = 5I$ can be realized as the matrix $(\omega^{jk})_{j,k}$ where ω is a complex primitive fifth root of unity by applying some sequence of the following:

1. A rearrangement of the rows,
2. A rearrangement of the columns,
3. Multiplication of a row by a complex number of absolute value one,
4. Multiplication of a column by a complex number of absolute value one.

Problem 19-3: Characterizations Associated with a Sum and Product
Proposed by ROBERT E. HARTWIG, *North Carolina State University, Raleigh, North Carolina, USA*, PETER ŠEMRL, *University of Ljubljana, Ljubljana, Slovenia* & HANS JOACHIM WERNER, *Universität Bonn, Bonn, Germany.*

The Editor has so far received only one partial solution (in addition to a solution by the Proposers) to this problem—we look forward to receiving further solutions!

1. Characterize square matrices A and B satisfying $AB = pA + qB$, where p and q are given scalars.
2. More generally, characterize linear operators A and B acting on a vector space $\mathcal{X}$ satisfying $ABx \in \mathrm{Span}(Ax, Bx)$ for every $x \in \mathcal{X}$.

Problem 19-4: Eigenvalues of Positive Semidefinite Matrices
Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

The proposer has pointed out to us that this is a matrix version of a matroid problem by Charles A. Akemann (Univ. of California, Santa Barbara). We look forward to receiving a matrix-theoretic solution to this Problem!

Show that there are constants $\gamma \in (0, \frac{1}{2}]$ and $\varepsilon \in (0, \frac{1}{2})$ such that, if $A_1, \ldots, A_n$ are non-negative definite $r \times r$ matrices of rank one satisfying

1. $A_1 + \cdots + A_n = I_r$ and
2. $\mathrm{trace}(A_i) < \gamma$ for each $i = 1, \ldots, n$,

then there is a subset σ of $\{1, 2, \ldots, n\}$ such that the eigenvalues of $A_\sigma = \sum_{i \in \sigma} A_i$ all lie in the interval $(\varepsilon, 1 - \varepsilon)$.

Problem 19-5: Symmetrized Product Definiteness?
Proposed by INGRAM OLKIN, *Stanford University, Stanford, California, USA.*

Let A and B be real symmetric matrices. Then prove or disprove:

1. If A and B are both positive definite (non-negative definite) then $AB + BA$ is positive definite (non-negative definite).
2. If A and $AB + BA$ are both positive definite (non-negative definite) then B is positive definite (non-negative definite).

In *Image* 20 (April 1998), pp. 28–31, we published five solutions to this Problem. We recently received the following comments, which we feel are of interest:

Comments on Solutions 19-5.1–19-5.5 by KARL E. GUSTAFSON, *University of Colorado, Boulder, Colorado, USA.*

Let A and B be given real symmetric matrices. One of the tasks in Problem 19-5 is to prove or disprove that if A and B are both positive definite, then $AB + BA$ is positive definite.

About 30 years ago I found the sufficient condition

$$\sin B < \cos A,$$

cf. [1, 2, 3] (see also [4] —Ed.) For A and B real symmetric matrices the relation $\sin B < \cos A$ becomes

$$\frac{\lambda_{\max}(B) - \lambda_{\min}(B)}{\lambda_{\max}(B) + \lambda_{\min}(B)} < \frac{2(\lambda_{\max}(A)\lambda_{\min}(A))^{1/2}}{\lambda_{\max}(A) + \lambda_{\min}(A)}.$$

It is easy to check that each of the counterexamples in Solutions 19-5.1 through 19-5.5 violates the above sufficient condition. For example, for Solution 19-5.1, $\sin B = \sqrt{2}/2$ and $\cos A = 2/3$, for Solution 19-5.2, $\sin B = 3/5$ and $\cos A = 2\sqrt{2}/5$, and similarly for the others. □

References

[1] Karl E. Gustafson (1997). *Lectures on Computational Fluid Dynamics, Mathematical Physics, and Linear Algebra: Kaigai, Tokyo, 1996*. World Scientific, Singapore (cf. p. 140).

[2] Karl E. Gustafson (1997). Operator trigonometry of iterative methods. *Numer. Linear Algebra Appl.* 4:333–347 (cf. pp. 338–339).

[3] Karl E. Gustafson and K. M. Duggirala Rao (1997). *Numerical Range: The Field of Values of Linear Operators and Matrices*. Springer-Verlag, New York (cf. p. 84).

[4] Peter Lax (1997). *Linear Algebra*. Wiley, New York (cf. pp. 120–121).

Problem 20-1: Eigenvalues
Proposed by BENITO HERNÁNDEZ–BERMEJO, *Universidad Nacional de Educación a Distancia. Madrid, Spain.*

Let R, P and K be $n \times n$ real matrices, where P is diagonal and invertible, K is skew-symmetric and $R = PK$. Prove that:

1. The eigenvalues of R appear in pairs of opposite sign of the form $\pm\sqrt{\xi}$, where ξ is a real number which may be positive, zero or negative.

2. If the diagonal entries of P are all positive or all negative, then the eigenvalues of R appear in pairs of opposite sign of the form $\pm\sqrt{\xi}$, where ξ is nonpositive, i.e., all eigenvalues are zero or imaginary.

Solution 20-1.1 by ROY MATHIAS and CHI-KWONG LI, *College of William and Mary, Williamsburg, Virginia, USA* & XINGZHI ZHAN, *Peking University, Beijing, China.*

Part 1 of the problem is only correct if $n \leq 3$, and Part 2 of the problem is true in general. To avoid trivial consideration, we assume $n \geq 2$.

Suppose P is real diagonal and K is real skew symmetic. Let D be a (possibly complex) diagonal matrix such that $D^2 = P$.

Then $PK = D^2K$ and DKD have the same eigenvalues. Since $X = DKD$ is skew symmetric, $\det(\lambda I - X) = \det(\lambda I - X^t) = \det(\lambda I + X)$. Thus, the eigenvalues of X occur in $\pm\mu$ pairs. Also, since PK is a real matrix, the eigenvalues occur in complex conjugate pairs.

If $n = 2$, the two eigenvalues ξ_1 and ξ_2 satisfy $\xi_1 = -\xi_2$ and $\xi_1 = \bar{\xi}_2$, and the conclusion holds. If $n = 3$, then K is singular, and hence PK has only two nonzero eigenvalues ξ_1 and ξ_2. Using the arguments in the previous case, we get the conclusion.

For $n \geq 4$, let $P = P_1 \oplus I_{n-4}$ and $K = K_1 \oplus 0_{n-4}$, where $P_1 = \operatorname{diag}(-1, 1, -1, 1)$ and

$$K_1 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{pmatrix}.$$

One easily checks that the nonzero eigenvalues of PK are $\pm\xi$ and $\pm\bar{\xi}$ with $\xi = 1 + i$, which are not of the asserted form.

Part 2: If P is positive, then we may assume that D is nonnegative. Thus DKD is real skew symmetric, which is orthogonally similar to a matrix of the form

$$\begin{pmatrix} 0 & a_1 \\ -a_1 & 0 \end{pmatrix} \oplus \cdots \oplus \begin{pmatrix} 0 & a_k \\ -a_k & 0 \end{pmatrix} \oplus 0_{n-2k}.$$

Hence PK has eigenvalues of the form $\pm\sqrt{\xi}$ with $\xi \leq 0$.

REMARK: One needs only assume P to be a positive semi-definite real symmetric matrix to get the conclusion of Part 2. □

Problem 20-2: Eigenvalues and Eigenvectors of a Patterned Matrix

Proposed by EMILIO SPEDICATO, *University of Bergamo, Bergamo, Italy.*

Find the eigenvalues and eigenvectors of the $n \times n$ patterned matrix $A = \{a_{ij}\} = \{|i - j|\}$ $(i, j = 1, 2, \ldots, n)$. [For the eigenvalues and eigenvectors of $A = \{a_{ij}\} = \{i - j\}$ $(i, j = 1, 2, \ldots, n)$, see Problem 18-5 and its Solutions in *Image* 19 (Summer/Fall 1997), 28–30.]

Solution 20-2.1 by WILLIAM F. TRENCH, *Trinity University (Emeritus), San Antonio, Texas, USA.*

It is simpler to solve the eigenvalue problem for A^{-1}. Note that A and A^{-1} have the same eigenvectors, and if $\alpha \neq 0$ is an eigenvalue of A^{-1} then $\lambda = 1/\alpha$ is an eigenvalue of A.

It is straightforward to verify that

$$A^{-1} = \begin{bmatrix} -\frac{n-2}{2(n-1)} & \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 & 0 & \frac{1}{2(n-1)} \\ \frac{1}{2} & -1 & \frac{1}{2} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & -1 & \frac{1}{2} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \frac{1}{2} & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \frac{1}{2} & -1 & \frac{1}{2} \\ \frac{1}{2(n-1)} & 0 & 0 & 0 & \cdots & 0 & 0 & \frac{1}{2} & -\frac{n-2}{2(n-1)} \end{bmatrix}.$$

If $x_0, x_1, \ldots, x_{n+1}$ satisfy the difference equation

$$x_{r-1} - 2(\alpha + 1)x_r + x_{r+1} = 0, \quad 1 \leq r \leq n, \tag{1}$$

and the boundary conditions

$$nx_1 + x_n = (n-1)x_0 \quad \text{and} \quad nx_n + x_1 = (n-1)x_{n+1}, \tag{2}$$

then $x = [x_1\, x_2\, \cdots\, x_n]^T$ satisfies $A^{-1}x = \alpha x$; therefore, α is an eigenvalue of A^{-1} if and only if (1) has a nontrivial solution satisfying (2), in which case x is an α-eigenvector of A^{-1}.

The solutions of (1) are given by

$$x_r = c_1\zeta^r + c_2\zeta^{-r} \tag{3}$$

where ζ is a zero of the reciprocal polynomial

$$p(z) = z^2 - 2(\alpha + 1) + 1.$$

Since

$$p(z) = (z - \zeta)(z - 1/\zeta) = z^2 - (\zeta + 1/\zeta)z + 1,$$

it follows that if (3) determines an eigenvector of A^{-1} then the corresponding eigenvalue of A^{-1} is

$$\alpha = -1 + \frac{1}{2}\left(\zeta + \frac{1}{\zeta}\right). \tag{4}$$

An arbitrary $n \times n$ symmetric Toeplitz matrix A has $\lceil n/2 \rceil$ symmetric eigenvectors $[x_1\, x_2 \cdots x_n]^T$ satisfying $x_{n-r+1} = x_r$, $r = 1, \ldots, n$ and $\lfloor n/2 \rfloor$ skew-symmetric eigenvectors satisfying $x_{n-r+1} = -x_r$, $r = 1, \ldots, n$. In seeking skew symmetric eigenvectors we can rewrite (3) as

$$x_r = c\left(\zeta^r - \zeta^{n-r+1}\right). \tag{5}$$

In this case $x_n = -x_1$ and $x_{n+1} = -x_0$, so conditions (2) both reduce to $x_1 = x_0$. Therefore (5) implies that

$$1 - \zeta^{n+1} = \zeta - \zeta^n,$$

so (5) defines an eigenvector if and only if $\zeta^n = -1$, which is equivalent to $\zeta = e^{i(2k+1)\pi/n}$ for some integer k. Therefore, by choosing c in (5) appropriately we can write the components of an eigenvector determined by ζ as

$$x_r = \sin\left[(n + 1 - 2r)(2k + 1)\pi/2n\right], \quad r = 1, 2, \ldots, n.$$

From (4), the corresponding eigenvalue of A^{-1} is

$$\alpha = -1 + \cos[(2k + 1)\pi/n].$$

By taking $k = 0, \ldots, \lfloor n/2 \rfloor - 1$ we obtain $\lfloor n/2 \rfloor$ distinct eigenvalues of A^{-1} of this form.
In seeking symmetric eigenvectors we can rewrite (3) as

$$x_r = c\left(\zeta^r + \zeta^{n-r+1}\right). \tag{6}$$

In this case $x_n = x_1$ and $x_{n+1} = x_0$, so conditions (2) both reduce to $(n + 1)x_1 = (n - 1)x_0$, and (6) implies that

$$(n + 1)(\zeta + \zeta^n) - (n - 1)(1 + \zeta^{n+1}) = 0. \tag{7}$$

The situation is now more complicated, since it seems impossible to solve this equation explicitly. However, since the reciprocal polynomial on the left changes sign on $(0, 1)$, (7) has zeros ζ_0 and $1/\zeta_0$ where $0 < \zeta_0 < 1$. By choosing c in (6) suitably, we can write the components of an eigenvector determined by ζ_0 as

$$x_r = \zeta_0^{(n+1-2r)/2} + \zeta_0^{-(n+1-2r)/2}, \quad r = 1, \ldots, n.$$

From (4), the corresponding eigenvalue of A^{-1} is

$$\alpha = -1 + \frac{1}{2}\left(\zeta_0 + \frac{1}{\zeta_0}\right).$$

It is straightforward to show that $\zeta = e^{i\theta}$ satisfies (7) if

$$\cos(n\theta/2)\cos(\theta/2) - n\sin(n\theta/2)\sin(\theta/2) = 0. \tag{8}$$

Since the function on the left changes sign on the intervals $((2k + 1)\pi/n, (2k + 3)\pi/n)$, $k = 0, \ldots, \lceil n/2 \rceil - 1$, it follows that each of these intervals contains a θ_k satisfying (8). By choosing c appropriately in (6), we can write the components of an eigenvector determined by θ_k as

$$x_r = \cos\left[(n+1-2r)\theta_k/2\right], \quad r = 1, \ldots, n.$$

The corresponding eigenvalue of A^{-1} is $\alpha = -1 + \cos\theta_k$. □

Problem 20-3: When is $\{X | X^n \le A\}$ Convex ?
Proposed by G. E. TRAPP & BOHE WANG, *West Virginia University, Morgantown, West Virginia, USA.*
For a Hermitian non-negative definite matrix A and positive integer n, let $\mathcal{C}(n, A)$ be the following set of Hermitian matrices:

$$\mathcal{C}(n, A) = \{X | X = X^*, \quad X^n \le A\}.$$

For what values of $n = 1, 2, \ldots$ is the set $\mathcal{C}(n, A)$ convex? Here $\le$ denotes the Löwner matrix partial ordering.

Solution 20-3.1 by G. E. TRAPP & BOHE WANG, *West Virginia University, Morgantown, West Virginia, USA.*

When $N = 1$ or 2, C(N,A) is convex for all A. When $N \ge 3$, C(N, A) need not be convex. The case N=1 is obvious. For $N = 2$, notice $X^2 \le A$ is equivalent to saying

$$\begin{pmatrix} A & X \\ X & I \end{pmatrix}$$

is HSD. Thus averaging the X and Y compound matrices gives the result. For $N \ge 3$, the following two counter-examples suffice; we leave it to readers to check the details themselves or see the details in the Latex file at `http://www.cs.wvu.edu/~trapp/convex.html`.

Example 1: For odd $N \ge 3$ consider

$$A = \begin{pmatrix} 4^N & 0 \\ 0 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad Y = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}.$$

Example 2: For even $N \ge 4$ consider

$$A = \begin{pmatrix} 16^N & 0 \\ 0 & 4^N \end{pmatrix}, \quad X = \begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad Y = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}.$$

□

Problem 20-4: Continuity of Transposition?
Proposed by GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let A be a real $m \times n$ matrix. Find matrices B_{ij} depending only on the type of A such that $A^T = \sum_{j=1}^{m} \sum_{i=1}^{n} B_{ij} A B_{ij}$, where A^T is the transpose of A. [If such matrices existed, by the continuity of addition and multiplication of matrices, it would prove also the continuity of transposing.]

Solution 20-4.1 by ROGER HORN, *University of Utah, Salt Lake City, UT, USA.*

The desired representation is discussed on p. 259 in Roger Horn & Charles R. Johnson [*Topics in Matrix Analysis, Cambridge University Press, 1991*]. The n-by-m matrices B_{ij} have a 1 in position j, i and all other entries are zero. □

Solution 20-4.2 by ROY MATHIAS and CHI-KWONG LI, *College of William and Mary, Williamsburg, Virginia, USA* & XINGZHI ZHAN, *Peking University, Beijing, China.*

Let $\{E_{11}, E_{12}, \ldots, E_{nm}\}$ be the standard basis for $n \times m$ real matrices. Then we have $B_{ij} = E_{ij}$ for all i, j. In fact, one easily checks that

$$E_{ij} E_{rs}^t E_{ij} = \begin{cases} E_{ij} & \text{if } (i,j) = (r,s); \\ 0 & \text{otherwise.} \end{cases}$$

Hence for any $m \times n$ matrix $A = \sum_{i,j} a_{ji} E_{ij}^t$,

$$\sum_{i,j} E_{ij} A E_{ij} = \sum_{i,j} E_{ij} (\sum_{i,j} a_{ji} E_{ij}^t) E_{ij} = \sum_{i,j} a_{ji} E_{ij} = A^t.$$

□

Solutions were also received from ULRICH ELSNER, *Technische Universität Chemnitz, Chemnitz, Germany; and from the proposer* GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Problem 20-5: Matrix Similarity

Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

As is well known, if two real matrices are similar over the complex numbers, then they are similar over the real numbers. Show that this assertion holds for unitary similarity. Precisely, if two real matrices are unitarily similar over the complex numbers, then show that they are orthogonally similar over the real numbers.

Solution 20-5.1 by DENNIS I. MERINO, *Southeastern Louisiana University, Hammond, Louisiana, USA.*

It is well known that if two real matrices are similar over the complex number field, then they are also similar via a real matrix. We will show that the statement also holds for unitary similarity, that is, if two real matrices are unitarily similar, then they are real orthogonally similar. We will also show that the same is true if the real matrices are complex orthogonally similar.

LEMMA 1: *Let U be a symmetric unitary matrix, i.e., $U^T = U$ and $U^* = U^{-1}$. Then U has a symmetric unitary square root that commutes with every matrix commuting with U.*

Proof. Since U is unitary, it is normal, and hence unitarily diagonalizable. Let $U = VDV^*$, where V is unitary and $D = a_1 I_1 \oplus \cdots \oplus a_k I_k$ with all a_j's distinct. Since U is unitary, all its eigenvalues have modulus 1. So we may write each $a_j = e^{i\theta_j}$ for some θ_j real. Now let $S = V(b_1 I_1 \oplus \cdots \oplus b_k I_k)V^*$, where $b_j = e^{i\theta_j/2}$. Obviously S is unitary and $S^2 = U$. If A is a matrix commuting with U, then D commutes with V^*AV. It follows that $V^*AV = A_1 \oplus \cdots \oplus A_k$, with each A_i having the same size as I_i. It is easy to see that S commutes with A. Since U is symmetric, $U = U^T$ implies that $V^T V$ commutes with D, so that $V^T V$ also commutes with $b_1 I_1 \oplus \cdots \oplus b_k I_k$. It follows that S is symmetric. □

THEOREM 1: *Let A and B be real matrices of the same size. If $A = UBU^*$ for some unitary matrix U, then there exists a real orthogonal matrix Q such that $A = QBQ^T$.*

Proof. Since A and B are real, we have

$$UBU^* = A = \bar{A} = \bar{U}BU^T.$$

This gives $U^T U B = B U^T U$. Now that $U^T U$ is symmetric unitary, by the lemma it has a symmetric unitary square root, say S, that commutes with B. Let $Q = US^{-1}$ or $U = QS$. Then Q is also unitary. Note that

$$Q^T Q = (US^{-1})^T (US^{-1}) = S^{-1} U^T U S^{-1} = I.$$

Hence Q is orthogonal. Since $Q^T = Q^{-1} = Q^*$ implies $Q = \bar{Q}$, Q is also real. Note that S and B commute, S is unitary, and Q is real orthogonal. Now,

$$A = UBU^* = (US^{-1})(SB)U^* = Q(BS)U^* = QB(S^{-1})^* U^* = QBQ^* = QBQ^T.$$

□

The following can also be proven using a similar approach. However, we can also use Theorem 1.

COROLLARY 1: *Let A and B be real matrices of the same size. If A and B are complex orthogonally similar, then they are also real orthogonally similar.*

Proof. If A and B are complex orthogonally similar, then every word

$$W(A, A^*) = W(A, A^T)$$

in two noncommuting variables, is similar to

$$W(B, B^T) = W(B, B^*).$$

Using Specht's theorem, A and B are unitarily similar. Using Theorem 1, A and B are real orthogonally similar. □

Problem 20-6: Non-negative Definiteness and Square Roots
Proposed by FUZHEN ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

Let $A \geq 0$, $B \geq C \geq 0$. Prove or disprove

$$B^{1/2}AB^{1/2} \geq C^{1/2}AC^{1/2};$$

here $\geq$ is the Löwner matrix partial ordering.

Solution 20-6.1 by JÜRGEN GROß, *Universität Dortmund, Dortmund, Germany.*

The asserted inequality is disproved by

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

□

Solution 20-6.2 by ROY MATHIAS and CHI-KWONG LI, *College of William and Mary, Williamsburg, Virginia, USA* & XINGZHI ZHAN, *Xingzhi Zhan, Peking University, Beijing, China.*

The inequalities $A \geq 0$ and $B \geq C \geq 0$ do not imply $B^{1/2}AB^{1/2} \geq C^{1/2}AC^{1/2}$. Here is a counter-example. Take

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

Then one easily checks that $B \geq C \geq 0$. Now one can check that $B^{1/2}AB^{1/2} \not\geq C^{1/2}AC^{1/2}$ by direct computation (noting that B and C are their own square roots). Alternatively observe that since A is rank one so are the non-zero matrices $B^{1/2}AB^{1/2}$ and $C^{1/2}AC^{1/2}$. Since they are not multiples of each other we cannot have $B^{1/2}AB^{1/2} \geq C^{1/2}AC^{1/2}$. □

Solution 20-6.3 by DENNIS I. MERINO, *Southeastern Louisiana University, Hammond, Louisiana, USA.*

The following counter-example disproves the claim. Take

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 20 \end{pmatrix}, \quad B = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}.$$

One checks that

$$B^{1/2} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \quad \text{and} \quad C^{1/2} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

Hence,

$$B^{1/2}AB^{1/2} - C^{1/2}AC^{1/2} = \begin{pmatrix} -19 & -19 \\ -19 & 57 \end{pmatrix},$$

and its determinant is negative (-1444), so it cannot be positive semidefinite. □

New Problems

Problem 21-1: Square Complex Matrix of Odd Order

Proposed by LUDWIG ELSNER, *Universität Bielefeld, Germany.*

Let A denote an $n \times n$ complex matrix, where n is odd. Show that there exist numbers μ_i, $i = 1, 2$ on the unit circle and nonzero vectors $x_1, x_2 \in \mathbb{C}^n$ satisfying

$$A^H x_1 = \mu_1 A x_1, \quad \bar{A} x_2 = \mu_2 A x_2.$$

Problem 21-2: The Diagonal of an Inverse

Proposed by BERESFORD N. PARLETT, *University of California, Berkeley, California, USA* (via ROY MATHIAS, *College of William and Mary, Williamsburg, Virginia, USA*).

Let J be an invertible tridiagonal $n \times n$ matrix that permits triangular factoriziation in both increasing and decreasing order of rows:

$$J = L_+ D_+ U_+ \quad \text{and} \quad J = U_- D_- L_-.$$

(Here the L's are lower triangular, the U's are upper triangular, and the D's are diagonal.). Show that

$$(J^{-1})_{kk} = [(D_+)_{kk} + (D_-)_{kk} - J_{kk}]^{-1}.$$

Problem 21-3: Eigenvalues and Eigenvectors of Two Symmetric Matrices

Proposed by WILLIAM F. TRENCH, *Trinity University (Emeritus), San Antonio, Texas, USA.*

Find the eigenvalues and eigenvectors of the $n \times n$ symmetric matrices

$$A = [\min(i,j)]_{i,j=1}^n \quad \text{and} \quad B = [\min(2i-1, 2j-1)]_{i,j=1}^n .$$

Problem 21-4: Square Complex Matrix, its Moore-Penrose Inverse and the Löwner Ordering

Proposed by GÖTZ TRENKLER, *Universität Dortmund, Dortmund, Germany.*

Let A be a square matrix with complex entries and Moore-Penrose inverse $B = A^+$. Show that $AA^* \leq A^*A$ if and only if $BB^* \leq B^*B$, where $\leq$ denotes the Löwner-ordering of matrices. Does this result generalize to operators on a Hilbert space?

Problem 21-5: Determinants and G-Inverses

Proposed by SIMO PUNTANEN, *University of Tampere, Tampere, Finland,* GEORGE P. H. STYAN, *McGill University, Montréal, Québec, Canada* & HANS JOACHIM WERNER, *University of Bonn, Bonn, Germany.*

For a real matrix B, let B^- denote an arbitrary g-inverse of B satisfying $BB^-B = B$, and let $\{B^-\}$ denote briefly the set of all g-inverses of B. Now let $A \in \mathbf{R}^{n\times n}$ and $X \in \mathbf{R}^{n\times k}$ be two given matrices such that A is non-negative definite and symmetric, $X'X = I_k$ and $\mathcal{R}(X) \subseteq \mathcal{R}(A)$; here $\mathcal{R}(\cdot)$ denotes the range (column space) of $(\cdot)$. Define the matrix function $M(\cdot)$ from $\{A^-\}$ into $\mathbf{R}^{n\times n}$ according to

$$M(A^-) := I_n - XX' + A^-XX'A.$$

1. Does $M(A^-)$ depend in general on the choice of $A^- \in \{A^-\}$? And if it is so, characterize the restrictive case of invariance.
2. Does $|M(A^-)|$, with $|\cdot|$ indicating determinant, depend on the choice of $A^- \in \{A^-\}$?
3. For each $A^- \in \{A^-\}$, determine explicitly $\{M(A^-)^-\}$ in terms of A, X, A^-.

Please submit solutions, as well as new problems, both in LaTeX code embedded as text only by e-mail to `styan@Math.McGill.CA` and by regular air mail (nicely printed copy, please!) to George P. H. Styan, Editor-in-Chief: *Image*, Dept. of Mathematics & Statistics, McGill University, 805 ouest, rue Sherbrooke Street West, Montréal, Québec, Canada H3A 2K6. We look forward particularly to receiving solutions to Problems 18-1, 19-3 and 19-4!