The Bulletin of the International Linear Algebra Society

[IL AS] IMAGE [IL AS]

Serving the International Linear Algebra Community

Issue Number 23, pp. 1–28, October 1999

Editor-in-Chief: George P. H. STYAN — styan@total.net, styan@together.net

Dept. of Mathematics & Statistics, McGill University
805 ouest, rue Sherbrooke Street West
Montréal, Québec, Canada H3A 2K6

Editorial Assistants:
Millie Maldé & Evelyn M. Styan

Books

Linear Algebra Events

IMAGE *Problem Corner*

Printed in Canada by Anlo Inc., Westmount, Québec • (514) 937-9346

Results of ILAS Elections

The ballots for this year's ILAS elections have now been counted. On March 1, 2000,

Jeff Stuart will become ILAS Secretary/Treasurer, and
Steve Kirkland and Harm Bart will become
members of the ILAS Board.

We look forward to working with Jeff, Steve, and Harm in service to the linear algebra community.

I want to personally thank the members of the nomination committee (Rajendra Bhatia (chair), Jim Weaver, Wayne Barrett, Judi MacDonald, and Tom Laffey) for their efforts on behalf of ILAS. I also want to thank Rajendra Bhatia and his colleagues T. Parthasarathy and Isha Dewan for counting the ballots in a timely manner.

Richard A. BRUALDI: brualdi@math.wisc.edu
ILAS President: *University of Wisconsin–Madison*

Vlastimil Pták: 1925–1999

The Institute of Mathematics and the Institute of Computer Science of the Czech Academy of Sciences announce with deep sorrow that Professor Vlastimil Pták passed away in Prague on May 9, 1999.

Professor Pták, born 8 November 1925 in Prague, was an outstanding world-known mathematician. Besides functional analysis, his scientific activity involved linear algebra, real analysis and its relation to topology, combinatorics and numerical analysis. A number of world-renowned results are due to him: a generalization of the open mapping principle (the Pták spaces), extensions of separately continuous functions, investigations concerning convergence of iterative processes, theory of critical exponents, theory of Hermitian algebras, the method of continuous induction and its application to iterative processes. In his last years, the activity of Professor Pták concentrated around problems concerning Toeplitz and Hankel operators, functional models and lifting of intertwining relations. He was for a long time the Head of the Department of Functional Analysis in the Mathematical Institute of the Academy, and supervised a number of PhD students. He has published more than 160 mathematical research papers.

Vladimir MÜLLER: mueller@cspgas11.bitnet
Czech Academy of Sciences, Prague, Czech Republic

Editorial Remark. For the biography and interview: "Seventy years of Professor Vlastimil Ptak" in *Mathematica Bohemica*, 121, 315–327 (1996), visit the Web site:

http://www-irma.u-strasbg.fr/EMIS/journals/MB/121.3/9.html

Norman J. Pullman: 1931–1999

The Department of Mathematics and Statistics at Queen's University in Kingston is deeply saddened to announce the death of Professor Emeritus Norman J. Pullman who passed away on 28 May 1999 after a struggle with ALS.

Norm Pullman was born in New York in 1931. After a brief career as a commercial artist, he obtained his MA in mathematics at Harvard and his PhD at the University of Syracuse (1962) under the supervision of Peter Frank. He taught at McGill University from 1962 to 1965, and held a postdoctoral fellowship at the University of Alberta in 1965-66. He then moved to Queen's, where he taught until his retirement in 1994.

Throughout his career, he made significant contributions to the study of powers of nonnegative matrices, as well as to the theory of tournaments, graph decompositions, and linear preserver problems. He published over 80 papers in the areas of matrix theory and graph theory, as well as the book *Matrix Theory and its Applications* (Dekker, 1976). While at Queen's, he supervised 15 graduate students, including 4 doctoral students. His personal and professional contributions were honoured in the festschrift *Graphs, Matrices, and Designs* (Dekker, 1993, Rolf Rees, ed.), which marked the occasion of his 60th birthday.

An insightful researcher, a supportive and respected collaborator, a valued colleague and friend, Norm will be fondly remembered for his generosity, his sound counsel, and above all for his warm and lively sense of humour.

David A. GREGORY: gregoryd@qucdn.queensu.ca
Queen's University, Kingston, Ontario, Canada
& Stephen J. KIRKLAND: kirkland@math.uregina.ca
University of Regina, Regina, Saskatchewan, Canada

Editorial Remark. For more information and many pictures visit the "Norm Pullman Memorial Page" Web site:

http://www.mast.queensu. ca/~pullman/index.html

Mathematics Genealogy Project

The brainchild of retired mathematics Professor Harry B. Coonce, the Web site http://hcoonce.math.mankato.msus.edu/ has the modest goal of listing information about everyone who has earned a doctorate in mathematics during the 20th century. This will allow mathematicians to trace their academic "family tree" and discover their advisor's advisor, their advisor's "siblings" at graduate school, and so on. With over 28,000 names from 380 universities and 30,000 records waiting to be organized, the database is off to a good start. Compiled from the input by various universities or from *Dissertation Abstracts*, the records include name of degree recipient, university, year in which degree was awarded, dissertation title, name of advisor, and a list of the degree recipient's students, if any. –Ed.

A Genealogy of William Spottiswoode: 1825–1883

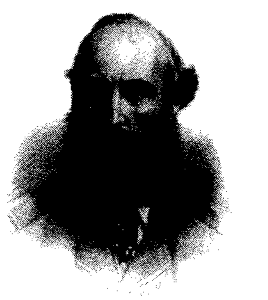
by Richard William FAREBROTHER, Victoria University of Manchester

Unknown SPOTTISWOODE (17??–17??)
William STRAHAN (1715–1785)
Thomas LONGMAN II (1730–1797)
William SLATER (17??–18??)

John SPOTTISWOODE (17??–18??) = (m. 1779) Margaret STRAHAN (b. 1751)
Thomas Norton LONGMAN (1771–1842) = (m. 1799) Mary SLATER

Andrew SPOTTISWOODE (1787–1866) = (m. 1819) Mary LONGMAN (1801–1870)

William SPOTTISWOODE (1825–1883)



The name of William Spottiswoode (1825–1883) is not as well known to linear algebraists as it deserves. William Spottiswoode was the author of the first (elementary) published treatise on determinants in 1851, with a second edition in 1856. Further, in 1861 he published a paper which stimulated Sir Francis Galton's interest in what we now call regression analysis (Stigler 1986, p. 219).

Besides being a partner in the publishing house of Eyre & Spottiswoode, William Spottiswoode was a leading mathematician and physicist who was the President of the mathematics section of the British Association for the Advancement of Science in 1865, the President of the London Mathematical Society, 1870–1872, and the President of the Royal Society of London from 1878 until his death in 1883. He was buried in Westminster Abbey; his memorial is located at the entrance to the South Transcept near the gates by the South Ambulatory.

It is interesting to note that, for family reasons, Spottiswoode's *Elementary Theorems Relating to Determinants* was published by one of the sequence of partnerships that may conveniently be summarised under the title of the 'House of Longman.' This publishing house was founded in 1724 by Thomas Longman I (1699–1755) on the completion of his apprenticeship with John Osborn. Initially he was the sole partner, but he was joined by his former master and father-in-law in 1725. In 1746 Thomas Longman I and other booksellers including Andrew Millar contracted with Samuel Johnson to publish his *Dictionary of the English Language*, a project which came to fruition in 1755.

Thomas Longman I was succeeded by his nephew Thomas Longman II (1730–1797) and he by his son Thomas Norton Longman (1771–1842). Thomas Longman II had been responsible for securing the proprietorship of the *Cyclopaedia of the Arts and Sciences* by Ephraim Chambers, and Thomas Norton Longman for developing the firm's list of works by romantic novelists and poets, including Sir Walter Scott and William Wordsworth. In 1799, Thomas Norton Longman married Mary Slater, the daughter of William Slater and the cousin of Sydney Smith. This connection with the great conversationalist broadened Thomas Norton Longman's literary circle. In 1819, their eldest daughter, Mary (1801–1870) married Andrew Spottiswoode (1787–1866) who, with his brother Robert (1791–1832), had taken effective control of the firm of Eyre and Spottiswoode in 1819.

Andrew Spottiswoode's maternal grandfather, William Strahan (1715–1785), had set up his printing business before 1739. Indeed, the first page of his ledger for this year shows the account for printing Chambers's *Cyclopaedia* for Thomas Longman I. In 1755, William Strahan was awarded the contract to print the first edition of Samuel Johnson's *Dictionary*. He was also one of the joint publishers, with Thomas Longman II, of the second octavo edition of 1760. In later years William Strahan increased his publishing business, having a share with Andrew Millar and others in the publication of books by Edward Gibbon, David Hume, Samuel Johnson, and Adam Smith.

William Strahan was succeeded by his son Andrew (1750–1831) and he by his nephews Andrew and Robert Spottiswoode, and they by Andrew Spottiswoode's sons William (1825–1883) and George (1827–1899). The printing and publishing business must have been financially rewarding, as William Strahan, Andrew Strahan, and Andrew Spottiswoode all became Members of Parliament.

The firm of Eyre and Spottiswoode was formally established in 1812. In its day, it was an important printing and publishing house that had held the honour of Royal Printer from 1766, when William Strahan had bought a one-third share in the patent from its holder, Charles Eyre, who was not himself a printer. The Royal Patent carried with it the exclusive right of printing all Acts of Parliament and the English translation of the Bible authorised by King James I. This patent continued in the hands of Eyre and Spottiswoode until sold to Cambridge University Press in 1989, following the unfortunate outcome of an attempt in 1961 to exercise their supposed right to publish the new English translation of the Bible, as well as the version authorised by King James I.

For histories of the firm of Eyre and Spottiswoode, see Austen-Leigh (1912) and Turner (1991). For similar histories of the House of Longman, see Cox (1924) and Wallis (1974).

Acknowledgements. I am indebted to David Lea of Pearson Education Limited and to Michael Bott and Frances Miller of the University of Reading for supplying copies of the source material employed in this article.

References

R. A. Austen-Leigh (1912). *The Story of a Printing House: Being A Short Account of the Strahans and Spottiswoodes.* Second Edition. Spottiswoode & Co. Ltd., London.

H. Cox (1924). The house of Longman 1724–1924. In *Edinburgh Review*, vol. 240, no. 490. [Reprinted by Longman Green & Company, London, 1925.]

W. Spottiswoode (1851). *Elementary Theorems Relating to Determinants.* Longman, Brown, Green, and Longman, & George Bell, London. [Second revised edition: *Journal für die Reine und Angewandte Mathematik*, 51 (1856), 209–271 & 328–381.]

W. Spottiswoode (1861). On typical mountain ranges: an application of the calculus of probabilities to physical geography. *Journal of the Royal Geographical Society*, 31, 149–154.

S. M. Stigler (1986). *The History of Statistics: The Measurement of Uncertainty before 1900.* Harvard University Press, Cambridge, MA.

J. R. Turner (1991). Eyre and Spottiswoode (London: 1812–1989). In *Dictionary of Literary Biography*, Gale Research Company, Detroit, MI, vol. 106, pp. 133–138.

P. Wallis (1974). *At the Sign of the Ship: Notes on the House of Longman.* Privately printed, London.

Richard William FAREBROTHER: Msrbsrf@fs1.ec.man.ac.uk
Victoria University of Manchester, Manchester, England UK

Editorial Remark. The picture of William Spottiswoode on page 3 was originally "engraved by G. I. Stodart from a photograph by Van der Weyde" in the unsigned obituary "Scientific worthies, XXI.–William Spottiswoode", *Nature*, 27, 597–601 (April 26, 1883).

More about William Spottiswoode

According to the excellent Web site:

http://www-history.mcs.st-andrews.ac.uk/history/BiogIndex.html

created and maintained at the School of Mathematical and Computational Sciences, University of St Andrews (St Andrews, Scotland, UK), by John J. O'Connor: joc@st-andrews.ac.uk and Edmund F. Robertson: efr@st-andrews.ac.uk:

> "William attended school in Laleham, then went to Eton College, one of the most prestigious schools in England situated on the Thames near London. From Eton he went to another top school in Harrow School, another prestigious school in Greater London. From Harrow, Spottiswoode was awarded a Lyon Scholarship to attend Balliol College, Oxford which he entered in 1842. Three years later he graduated with a First Class degree in mathematics. In 1846 and 1847 he was awarded mathematics scholarships at Balliol College where he became a lecturer in mathematics.
>
> "In 1853 Spottiswoode was elected a Fellow of the Royal Society of London. Spottiswoode was appointed president of the mathematical section of the British Association in 1865. Around 1870 there were major changes to the direction of his research. This was a time when he received high office in a number of societies, being president of the London Mathematical Society from 1870 to 1872 and, in 1871, being elected Treasurer of the Royal Society of London. Spottiswoode's research changed to physical topics, and from 1871 he studied the polarisation of light and later he studied electrical discharge in rarefied gases.
>
> "In 1878 Spottiswoode was elected president of the Royal Society of London, and in the same year he was president of the British Association for its Dublin meeting. At the Dublin meeting he gave his presidential address on the growth of mechanised invention applied to mathematics: 'Conterminous with space and coeval with time is the kingdom of Mathematics; within this range her dominion is supreme; otherwise than according to her order nothing can exist, and nothing takes place in contradiction to her laws.'
>
> "Spottiswoode published 99 papers and several books. His interests however were not confined to mathematics and physics since he was also a leading expert on European languages and on oriental languages. He died of typhoid while still President of the Royal Society."

From the biographical article by Rix (1898):

> "Of the early days of William Spottiswoode there is but little to tell. His school life began at Laleham under a brother of Dean Buckland. From Laleham he went to Eton, but his stay there was short, as the first recorded development of his scientific tastes resulted in an explosion which, though effecting no damage to his moral reputation, was deemed inconsistent with sound discipline. He was accordingly moved to Harrow, then under Dr. Wordsworth, and was there placed in the upper 'shell'. After continuing three years at Harrow, where he had the reputation for being studious and thoughtful, he in 1842 obtained a Lyon Scholarship and went to Balliol College, Oxford.
>
> "To a mind imbued with the love of mathematical symmetry the study of determinants had naturally every attraction. In 1851, Mr. Spottiswoode published, in the form of a pamphlet [5], an account of some elementary theorems on the subject. This having fallen out of print, permission was sought by the editor of "Crelle" to reproduce it in the pages of that journal. Mr. Spottiswoode granted the request, and undertook to revise his work. 'The subject had, however, been so extensively developed in the interim, that it proved necessary not merely to revise it, but entirely to rewrite the work, which became a memoir [6] of 116 pages. To this, the first elementary treatise on determinants, much of the rapid development of the subject is due. The effect of the study of Mr. Spottiswoode's own methods was most pronounced; there is scarcely a page of his mathematical writings that does not bristle with determinants.
>
> "Two communications made by Mr. Spottiswoode in 1860 and 1863 to the Royal Asiatic Society upon mathematical subjects, should be specially referred to. In a brief note in the *Journal* of that Society (vol. 17, pp. 221–222) he discusses the claims of Bhosharaachary, an Indian astronomer, to the discovery of the principle of the differential calculus; and in a more lengthy article (vol. 20, pp. 345–370 of the same publication), he translates into modern symbols the formulae made use of by the Hindoos in calculating eclipses, contained in the 'Surya Siddhanta'. The acquaintance which he had with this work was formed by reading it in the original tongue, for among his varied acquirements he possessed a remarkable knowledge of several European and Oriental languages.
>
> "The great beauty of the experiments involved in Mr. Spottiswoode's physical researches led to demands from his friends that they should be laid before the public in a popular form. The lectures which he delivered to crowded audiences at the Royal Institution and elsewhere were characterised by a remarkable clearness of exposition, and by a depth of poetic feeling which excited much surprise among those who knew him only as an abstruse mathematician. Perhaps the most interesting example of his powers as a lecturer is to be found in a discourse on 'Sunlight, Sea, and Sky', delivered to working men at the British Association Meeting in Brighton in 1872 (*Nature*, 6, 333–336). The reputation he acquired in these essays excited high expectations with regard to the address which, as President of the British Association, he had to deliver in Dublin in 1878. These expectations were fully justified by the result. The stores of a mind imbued with the spirit of a philosopher, a mathematician, a physicist, and a poet, were drawn upon with no niggard hand, and matters usually regarded as beyond the ken of others than experts were explained to the unversed in language as interesting as it was simple, clear, and precise. The judgement of his fellow-workers could now be unhesitatingly approved by others."

In the unsigned obituary [8] in *Nature* we learn that

> "In 1856 Mr. Spottiswoode made a journey through Eastern Russia; of this he has published a graphic and, in parts, very lively account in his book entitled *A Tarantasse Journey through Eastern Russia in the Autumn of 1856* [7]. 'I neither made the journey, nor do I now write, with any political object, but simply as a traveller to whom every square mile of the earth's surface is interesting, and the more so in proportion as it is less known'. There are several illustrations by the author, and a route map of Russia."

What is a Tarantasse?

We may ask, what is a tarantasse? I had no idea until I found a copy (of the original edition) of this book in the Concordia University Library in Montreal—it is a wonderful book. Spottiswoode purchases his tarantasse in Kazan:

> "Kazan is, amongst other things, celebrated for its manufacture of carriages. The demand for them is considerable, on account of its being the point, on a journey eastward, most convenient for beginning posting. The railway between Petersburg and Moscow, the new chaussée to Nizhni, and the steam communication of the Volga, have all combined to divert traffic between the capital, the Eastern Governments, and Siberia, from the old route by Kostroma and Viatka, to that by Kazan. The travelling carriage used generally throughout Russia is the tarantasse. It is a four-wheeled vehicle,

with a body which has been compared to that of a barouche without doors; but which, at least in the case of mine, more nearly resembles a broad, low-built boat truncated at both ends. It is furnished with a hood, or other leathern apparatus for closing in; and has a coach-box in front, capable of holding one passenger beside the yamstchik or driver. The purchaser who is a novice in the subject, is a little astonished when, having been told that his vehicle is quite ready for use, he finds that there is no seat whatever inside, and that he must in future sit upon his luggage. He however soon finds out that if he is not most conveniently placed on the top of his portmanteau, his portmanteau is at all events most conveniently placed under him, as they are not so likely to part company as if it were outside. But the peculiarity which forces itself particularly on the notice of the traveller using a tarantasse for the first time, is the fact that it has no springs, and that, instead, the body is placed on several long poles which reach from the fore to the aft axletree, and act, or rather are supposed to act, as substitutes.

ТАРАНТАСЪ.

ПУТЕВЫЯ ВПЕЧАТЛѢНІЯ.

СОЧИНЕНІЕ

ГРАФА В. А. СОЛЛОГУБА.

САНКТПЕТЕРБУРГЪ.

ИЗДАНІЕ КНИГОПРОДАВЦА АНДРЕЯ ИВАНОВА.

1845.

Front cover page of [3]:
The Tarantas: Impressions of a Journey (Russia in the 1840s)
[in Russian] by Vladimir Aleksandrovich Sollogub (1845).

"But the play—and at the time, one is inclined to call it by a much more serious name than play—of the wooden poles is so different from that of steel springs, that had we not been told beforehand, we should certainly have failed to recognise any resemblance. There was, of course, a puzzling choice between strength and lightness among the tarantasses which we found ready-made; but, having had some experience of a rather infirm carriage between Warsaw and Petersburg, I decided in favour of the former; and, after the usual amount of bargaining, and a solemn shaking of hands, agreed for an herculean vehicle at the price of 170 roubles. I am inclined to think that a lighter one would have answered quite as well, even in the north; in the south it would certainly have answered better; but we had a long journey before us, and the cost of a few repairs would pay for an extra horse over a considerable distance; besides which the consciousness of strength saved us many an uneasy hour in the cross roads of the Ural.

"While L. was laying in provisions, and making other arrangements for our departure, I went to call on Captain Lan, whose agreeable little wife was in a state of consternation at her husband's return. She was preparing a little surprise in the shape of new paper and whitewash to their house, which would have been complete had he delayed his journey as long as was expected. In spite of the rain, he kindly insisted on taking me on a drive through the Tatar quarter of the town, and showing me some mosques, butcher's shops for horseflesh, and the house of a wealthy Tatar. The latter was filled with a curious mixture of furniture in both the Oriental and French styles. The sitting-rooms were furnished with chairs and tables; the bedrooms with richly coloured divans and huge piles of pillows; while the stoves were constructed in the form of Mecca shrines. The owner of the house insisted on my writing my name and address, although he could not read a letter of it, or in any way realise the position of my country. Beside these acquaintances, Kazan contains among its inhabitants a professor of English, and a free-trade political economist.

"On my return to the inn I found that our preparations were nearly complete. Large quantities of provisions had been laid in: bread, cheese—Cheshire is sold in Kazan—cold chickens, chocolate, tea, sugar, wine, brandy, &c. For on the road nothing can be calculated on but a little black bread, a kind of food decidedly to be eschewed when better can be had. About 6.0 P.M., on the 9th of September [1856], everything being on board, we left Kazan in a cold rain . . . "

Sollogub [3, 4] made an earlier journey in 1845 by "tarantas" through Russia and described the "tarantas" as follows:

> "Outside the tarantas was displayed in all its prairie splendor. But what a tarantas, what an amazing invention of the human mind! Imagine two long poles, two parallel shafts, endless and immeasurable. Slung between them, unexpectedly as it were, an enormous basket, rounded on the sides like a gigantic cup, like a goblet of antediluvian banquets. To the ends of the shafts wheels were attached, and this whole strange creation seems from a distance to be some queer product of a fantastic world, something half-way between a dragon-fly and a kibitka. But what can one say of the art by which in a few minutes the tarantas suddenly disappeared beneath small trunks, small valises, chests, big boxes, little boxes, baskets, casks and all sorts of things of every kind and description? First of all, in this chiselled-out vessel there were no seats; an enormous feath-erbed had been dumped into the expanse, the upper stripes of its coarse huck covering were level with the sloping edges. Next seven feather pillows in cotton cases intentionally dark in preparation for the dirty roads, towered up in a pyramid upon their soft foundation. In the boot were stowed the pie for the road in a pile of bast, a flask with anise-flavored vodka, various roast fowls wrapped in gray paper, cheese cakes, a ham, loaves of white bread, fancy rolls and the so-called cellarette [pogrebets], the indispensable travelling companion of every steppe landowner.
>
> "This cellarette, upholstered on the outside with a sealskin, bristles uppermost, and bound with tin hoops, contains in it a whole tea service—invention of undoubted usefulness, but by no means complicated manufacture. Open it: under the cover a tray, and on the tray before you in all her beauty a sleeping shepherdess under a tree, swiftly sketched in three rosy spots by the decisive brush stroke of a bazaar artist. Inside the wallpaper-covered chest majestically stands a teapot of a dirty white hue with a gilt edge; close beside this a glass carafe with tea, another like it, with rum, two glasses, a milk pitcher, and the appurtenances of tea-time enjoyment. The Russian cellarette, by the by, is fully deserving our respect. Amid the general changes and improvements among us, it alone has not changed its original type, has not been attracted by the allurements of a deceptive beauty, but has gone indifferently and untouched through all the upheavals of time... That's what a Russian cellarette is like! All around the tarantas were strung little sacks and cartons. In one of these was a cap and crimson turban from Mme Lebourg on Kuznetsky Bridge [a fashionable Moscow shopping street], for Vasily Ivanovich's spouse; in others were children's books, dolls and playthings for Vasily Ivanovich's children, and above all two lamps for the house, several vessels for the kitchen, and even some colonial provisions for Vasily Ivanovich's table: everything purchased in accordance with a list given him from the country. Finally, in the rear, three monstrous trunks, stuffed with all sorts of trash and tied around with ropes, towered like an Egyptian obelisk on the hind portion of our travelling chariot. With a look of dissatisfaction the red-haired coachman began to harness three puny horses to the tarantas."

Acknowledgements. I am very grateful to Shane T. Jensen and Millie Maldé for their help in preparing this article.

References

[1] A. B. K. (1884). Untitled. *Proceedings of the Royal Society London*, 38, xxxiv–xxxix.

[2] H[erbert] R[ix] (1898). Spottiswoode, William (1825–1883). *Dictionary of National Biography*, 53, 417–418.

[3] V[ladimir] A[leksandrovich] Sollogub (1845). *Tarantas: Putevye Vlechatleniia* [in Russian]. Gosudarstvennoe Izdatelstvo, Moskva & Petrograd. (Translated into English as [4].)

[4] Vladimir A[leksandrovich] Sollogub (1989). *The Tarantas: Impressions of a Journey (Russia in the 1840s).* In English. Translated (from [3]) with an afterword by William Edward Brown. Ardis, Ann Arbor.

[5] William Spottiswoode (1851). *Elementary Theorems Relating to Determinants.* Longman, Brown, Green, and Longman, & George Bell, London. (Revised and expanded as [6].)

[6] William Spottiswoode (1856). Elementary theorems relating to determinants. *Journal für die Reine und Angewandte Mathematik*, 51, 209–271 & 328–381. (Revised and expanded version of [5].)

[7] William Spottiswoode (1857). *A Tarantasse Journey through Eastern Russia in the Autumn of 1856.* Longman, Brown, Green, Longmans, & Roberts, London. [Reprint Edition: Arno Press, New York, 1970.]

[8] Unsigned Obituary (1883a). Scientific worthies, XXI.–William Spottiswoode. *Nature*, 27, 597–601 (April 26, 1883).

[9] Unsigned Obituary (1883b). William Spottiswoode. *The Times* (June 28, 1883).

Spottiswoode Lodge in Singapore

"Tucked away in a serene and historic part of Singapore, every unit is self contained with attached bathroom and furnished in antique Oriental style. Painstakingly restored to its former glory and is a conservation landmark. Intricately carved swing doors, corbels, decorative air vent panels & light fittings complement the antique Oriental Setting inside the building. An air well rises above the Koi pond surrounded by lush potted plants. Lively gold and silver Koi of every colour ensures Feng-shui for good fortune, health and prosperity for all occupants ... For enquiries, please contact: Spottiswoode Lodge, Bukit Merah Lane 1, Block 125, #03-160, Singapore 150125: spottiswoode@netv.com.sg."

IMAGE Philatelic Corner

While the first book on determinants is by William Spottiswoode (1825–1883), according to the O'Connor-Robertson Web site:

http://www-history.mcs.st-andrews.ac.uk/history/BiogIndex.html

> "it appears that Takakazu Seki Kôwa was the first person to study determinants—in 1683. Ten years later Leibniz, independently, used determinants to solve simultaneous equations although Seki's version was the more general".

Moreover, there is a postage stamp of Seki[1] (Scott #2147, 1992):

According to the O'Connor-Robertson Web site:

> "Takakazu Seki Kôwa was born in Fujioka, Kozuke, Japan, in March 1642 and died in Edo (now Tokyo), Japan, on 24 October 1708. He was born into a samurai warrior family. However at an early age he was adopted by a noble family named Seki Gorozayemon. The name by which he is now known, Seki, derives from the family who adopted him rather than from his natural parents."

We find, however, on page 117 of Abbott [1] that:

> "Seki was born probably in Huzioka in about the year 1642—the exact date is unknown, and even his birthplace is the subject of some doubt."

For more about "The determinant theory of Seki Kôwa and subsequent commentaries and corrections" see the article by Mikami [2].

An extremely useful and very extensive checklist of mathematicians on postage stamps has been prepared by Monty Strauss: m.strauss@ttu.edu (President: Mathematical Study Unit of The American Topical Association & The American Philatelic Society). Four other useful sources of information about mathematical stamps are the books by Schaaf [3] and by Schreiber [4], as well as the regular column by Wilson [5] and the journal edited by Woodward [6] for the Mathematical Study Unit of The American Topical Association & The American Philatelic Society.

[1] But apparently no postage stamp of Spottiswoode.

Acknowledgements. I am very grateful to Sanjo Zlobec for drawing to my attention Seki's work on determinants and the article by Mikami [2]. Many thanks go also to Monty Strauss for making available to me his checklist of mathematical stamps as an Excel spreadsheet, and to Milton Sobel for drawing my attention to the book by Schreiber [4] and to Friedrich Pukelsheim for providing me with a copy of it. In addition I am indebted to Shane T. Jensen for his help.–Ed.

References

[1] David Abbott, editor: *The Biographical Dictionary of Scientists: Mathematicians*, pub. Blond Educational (Muller, Blond & White): London, 1985, iii + 175 pp.

[2] Yoshio Mikami: "On the Japanese theory of determinants", in *Science & Technology in East Asia* by Nathan Sivin (editor), pub. Science History Publications: New York, 1977, pp. 3–30.

[3] William L[eonard] Schaaf: *Mathematics and Science: An Adventure in Postage Stamps*, pub. National Council of Teachers of Mathematics, Reston, Virginia, 1978, xiv + 152 pp.

[4] Peter Schreiber: *Die Mathematik und ihre Geschichte im Spiegel der Philatelie* [in German]. Mathematische Schülerbücherei 68, pub. BSB B. G. Teubner Verlagsgesellschaft, Leipzig, 1980, 101 pp.

[5] Robin J. Wilson, editor: "Stamp Corner" column in the *Mathematical Intelligencer*. [According to *PhilaMath*, vol. 19, no. 1, July 1997, page 4, Robin Wilson is "doing a mathematical stamps book, in color, for the Mathematical Association of America".]

[6] Randy Woodward, editor: rswoodward@theriver.com. *Philamath: A Journal of Mathematical Philately*, pub. Mathematical Study Unit of The American Topical Association & The American Philatelic Society. [Four issues per year: mimeographed. Latest issue: vol. XXI, No. 2 (October 1999), 12 pp.]

The Root of the Matter

In their article on Matrix Theory for the *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences* (Routledge, London, 1994), I. Grattan-Guinness and W. Ledermann bemoan (on p. 785):

> "... the popularization of the appalling non-words 'eigenvalue' and 'eigenvector' created out of (absurdly partial) translations of the German words *Eigenwert* and *Eigenvektor.*"

Readers who are concerned with this problem, but who are willing to permit 'Eigenvector' as a necessary amendment of the original German spelling of *Eigenvektor* to suit the needs of English orthography, may also be willing to accept 'Eigenwort' as a similar amendment of the German *Eigenwert*, where the second element of this word is familiar in the names of such plants as *bladderwort, liverwort, ragwort, rupturewort, Saint John's wort, sneezewort,* and *spiderwort*, where the first element of the first, second, fourth, and sixth of these words indicates their supposed areas of medical efficacy.

Richard William FAREBROTHER: Msrbsrf@fs1.ec.man.ac.uk
Victoria University of Manchester, Manchester, England UK

New & Forthcoming Books on Linear Algebra & Related Topics

Per Christian HANSEN: *Rank-Deficient and Discrete Ill-Posed Problems*

Reviewed by Steven J. Leon

In recent years, SIAM has published a host of inexpensive paperback editions on the subjects of applied and computational linear algebra. One of the nicest of these books is *Rank-Deficient and Discrete Ill-Posed Problems* by Per Christian Hansen, Technical University of Denmark, Lyngby, Denmark [SIAM, Philadelphia, 1998, 247 pp., ISBN 0-89871-403-6].

A problem is ill-posed if its solution does not depend continuously on its data. Ill-posed problems arise in the solution of inverse problems. Typically the measurable data is insufficient to uniquely determine a function to model the phenomena in question. A standard technique is to discretize the problem and then add regularization conditions in order to uniquely determine a solution. The basic tools for doing this are matrix decompositions. The author provides a nice survey of some of the basic matrix decompositions and then shows how they are used in solving discrete ill-posed problems.

Hansen begins the book with a general discussion of ill-posed and inverse problems in Chapter 1. In the next three chapters, he discusses the matrix decompositions that are necessary for the treatment of these problems. Chapter 2 covers the singular value decomposition and its generalizations, the GSVD or QSVD and the product form PSVD. It also discusses rank revealing factorizations, such as the rank revealing QR and the URV and ULV factorizations introduced and popularized by G. W. Stewart. Chapter 3 is devoted to numerical rank and truncated decompositions. Chapter 4 discusses problems with ill-determined rank when the singular values decay gradually to zero with no clear gap in their range of values. Chapters 5 and 6 deal with regularization methods, and Chapter 7 covers parameter choice methods.

This book is certainly a 'must have' for anyone working on inverse problems. Other researchers in matrix theory will find the book a valuable reference for generalized singular value decompositions and rank revealing matrix factorizations. The book is well written and does a beautiful job of providing insight into the subject matter.

Frank UHLIG: *Transform Linear Algebra*

Available for test teaching starting December 1999, approx. 400 pp. The author uhligfd@auburn.edu "is looking for a handful of college math teachers who are scheduled to teach elementary Linear Algebra courses in Winter/Spring 2000 and who would want to try this locally-tested and new approach to teaching linear algebra. The book is based on the fundamental concepts of Linear Algebra and Matrix Theory, such as 'Linear Transformations', 'Subspaces', 'Basis Representation', etc. It develops all standard subjects of a first undergraduate Linear Algebra course from these first principles. The conceptual approach benefits from the synergy of going back to the fundamental notions in each instance and leads students to a more thoughtful mastery of the subject than with our current example driven textbooks. There are 14 chapters, each made up of one basic (one to three) hour lecture, followed by sections on theory and on applications. The book currently contains 20 figures, 66 examples and about 570 problems".

Hard Copy of the *Electronic Journal of Linear Algebra*

Production of a hard copy (soft cover) of volumes 1–4 of the *Electronic Journal of Linear Algebra* (ELA)—about 320 pages—is being planned. It is expected to be sold to ILAS members, other linear algebraists, interested mathematicians, and libraries at a cost of US$20, with a 20% discount to ILAS members. We shall soon take orders, with expected delivery in early February 2000. Our plan is to make hard copies of future volumes of ELA available to those who wish to purchase them. To access ELA visit the Web site:

http://www.math.technion.ac.il/iic/ela/

New and Forthcoming Books: 1999–2000

Listed here are some new and forthcoming books on linear algebra and related topics that have been published in 1998–1999 or are scheduled for publication in 1999–2000. This list updates and augments our previous lists in IMAGE 22 (April 1999), pp. 10–12, and in "A Third Guide to Books on Matrices and Books on Inequalities" by Simo Puntanen, George P. H. Styan & Shane T. Jensen (74 pp.), prepared for the Seventh International Workshop on Matrices and Statistics (Fort Lauderdale, Florida, 1998).

E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, & D. Sorensen (1999). *LAPACK Users' Guide,* 3rd Edition. Software, Environments, and Tools 9. SIAM, xxii + 407 pp., ISBN 0-89871-447-8.

Howard Anton & Chris Rorres (2000). *Elementary Linear Algebra.* 8th Edition. Wiley, New York, ISBN 0-471-17055-0, in press (expected January 2000).

Howard Anton & Chris Rorres (2000) *Elementary Linear Algebra With Applications: Applications Version.* 8th Edition. Wiley, New York, ISBN 0-471-17052-6, in press (expected January 2000).

Peter Arbenz, Marcin Paprzycki, Ahmed H. Sameh & Vivek Sarin (1998). *High Performance Algorithms for Structured Matrix Problems.* Advances in the Theory of Computation and Computational Mathematics 2. Nova Science Publishers, Inc., Commack, NY, xii + 203 pp., ISBN 1-56072-594-X [MR 99k:65004].

R. B. Bapat (1999). *Linear Algebra and Linear Models.* 2nd Edition. Universitext. Springer-Verlag, New York, in press, ISBN 0-387-98871-8. [Original version: Hindustan Book Agency, viii + 124 pp., ISBN 81-85931-00-3. Review: IMAGE 11 (1993), p. 16.]

David J. Bartholomew & Martin Knott (1999). *Latent Variable Methods and Factor Analysis.* Kendall's Library of Statistics 7. Arnold, London, 232 pp., ISBN 0-340-69243-X.

David M. Bressoud (1999). *Proofs and Confirmations: The Story of the Alternating Sign Matrix Conjecture.* Mathematical Association of America & Cambridge University Press, xv + 274 pp.

David Carlson & Guershon Harel. *A Linear Algebra Companion.* Cogito Media, CD Study Guide.

Enrique Castillo, Angel Cobo, Francisco Jubete & Rosa Eva Pruneda (1999). *Orthogonal Sets and Polar Methods in Linear Algebra: Applications to Matrix Calculations, Systems of Equations, Inequalities, and Linear Programming.* Wiley, New York, xii + 422 pp., ISBN 0-471-32889-8.

Patrick Dewilde & Alle-Jan van der Veen (1998). *Time-Varying Systems and Computations.* Kluwer, 459 pp., ISBN 0-7923-8189-0.

Laurent El Ghaoui & Silviu-Iulian Niculescu (1999). *Advances in Linear Matrix Inequality Methods in Control.* Advances in Control and Design 2. SIAM, xxvii + 375 pp., ISBN 0-89871-438-9.

Michael W. Frazier & R. Meyer-Spasche (1999). *An introduction to Wavelets through Linear Algebra.* Springer, xvi + 501 pp., ISBN 0-387-98639-1.

Solomo Friedberg, Stephen H. Friedberg, Arnold J. Insel & Lawrence E. Spence (1999). *Elementary Linear Algebra: A Matrix Approach.* Prentice Hall, ca. 477 pp., in press (expected September 1999), ISBN 0-13-71672-29.

Stephen W. Goode (1999). *Differential Equations and Linear Algebra.* 2nd Edition. Prentice Hall, 703 pp., ISBN 0-13-263757-X.

Hans Grauert & Hans-Chistroph Grunau (1999). *Lineare Algebra und Analytische Geometrie.* In German. Oldenbourg Verlag, München, xi + 273 pp., ISBN 3-486-24739-5.

Eugene Herman *et al.* (1999). *Linear Algebra: Modules for Interactive Learning Using Maple.* Updated Preliminary Version. Addison-Wesley, in press, ISBN 0-201-64846-6.

C. Y. Hsiung & G.Y. Mao (1999). *Linear Algebra.* World Scientific, ISBN 981-023092-3.

John Hamal Hubbard & Barbara Burke Hubbard (1999). *Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach.* Prentice Hall, Upper Saddle River, NJ, xvi + 687 pp., ISBN 0-13-657446-7 [MR 99k:00002].

Vlad Ionescu, Cristian Oară & Martin Weiss (1999). *Generalized Riccati Theory and Robust Control: A Popov Function Approach.* Wiley, Chichester, xxii + 380 pp., ISBN 0-471-97147-2.

S. K. Jain *et al.* (1999). *Basic Linear Algebra With Matlab.* Textbooks in Mathematical Sciences. Springer, in press, ISBN 0-387-989269.

T. Kailath & A. H. Sayed, eds. (1999). *Fast Reliable Algorithms for Matrices with Structure.* SIAM, xvi + 342 pp., ISBN 0-89871-4311.

Aslam Kassimali (1999). *Matrix Analysis of Structures.* Brooks/Cole, Pacific Grove, CA, xii + 592 pp., ISBN 0-534-20670-0

Karl-Heinz Kiyek & Friedrich Schwarz (1999). *Lineare Algebra.* In German. Teubner Studienbuecher: Mathematik. B. G. Teubner, Stuttgart, ISBN 3-519-02390-3.

Steven J. Leon (1999). *Àlgebra Linear com Aplicações.* 4th Edition. In Portuguese (translated from the English by Valeria de Magalhães Iorio). LTC Livros Tècnicos E Cientificos Editora, Rio de Janeiro, 390 pp.

Chris J. Lloyd (1999). *Statistical Analysis of Categorical Data.* Wiley, New York, xii + 468 pp., ISBN 0-471-29008-4.

Christian Mehl (1999). *Compatible Lie and Jordan algebras and Applications to Structured Matrices and Pencils.* In English. Logos-Verlag, Berlin, 109 pp., ISBN 3-89722-173-X.

Jorma Merikoski, Keijo Väänänen, Teuvo Laurinolli & Timo Sankilampi (1998). *Matematikkan Taito 15: Lineaarialgebra.* In Finnish. WSOY-yhtymä: Weilin+Göös, Porvoo, 167 pp., ISBN 951-35-6339-1.

G. Meurant (1999). *Computer Solution of Large Linear Systems.* Studies in Mathematics and its Applications, 28. North-Holland, Amsterdam, xxii + 753 pp., ISBN 0-444-50169-X.

Kurt Meyberg & Peter Vachenauer (1999). *Höhere Mathematik 1: Differential- und Integralrechnung, Vektor- und Matrizenrechnung.* In German. Springer-Lehrbuch. Springer-Verlag, xvi + 530 pp., ISBN 3-540-66148-4.

Roger B. Nelsen (1999). *An Introduction to Copulas: Properties and Applications.* Lecture Notes in Statistics 139. Springer, New York, xi + 216 pp., ISBN 0-387-98623-5.

Frederic Pham & Herve Dillinger (1999). *Découvrir l'algebre linéaire.* In French. Bibliothèque des Sciences. Diderot Éditeur, Paris.

Themistocles M. Rassias & Hari M. Srivastava, eds. (1999). *Analytic and Geometric Inequalities and Applications.* Kluwer, vii + 378 pp., ISBN 0-7923-5690-X.

Christof Sinn (1999). *Zur verifizierten Lösung großer linearer Gleichungssysteme mit spärlich besetzten Matrizen beliebiger Bandbreite.* In German. Shaker Verlag, Aachen, xviii + 129 pp., ISBN 3-8265-4736-5.

Uwe Storch & Hartmut Wiebe (1999). *Lehrbuch der Mathematik, Band 2: Lineare Algebra.* 2nd Edition. In German. Spektrum Akademischer Verlag, Heidelberg, ISBN 3-8274-0359-6.

Szabo (2000). *Lab Manual for Linear Algebra With Maple.* Academic Press, ISBN 0-12-68014-28, 0-12-68014-01 in press (expected February 2000).

Szabo (2000). *Lab Manual for Linear Algebra With Mathematica.* Academic Press, in press (expected January 2000).

John W. Van Ness & Chi-Lun Cheng (1999). *Statistical Regression with Measurement Error.* Kendall's Library of Statistics 6. Arnold, London, 288 pp., ISBN 0-340-61461-7.

Fuzhen Zhang (1999). *Matrix Theory: Basic Results and Techniques.* Universitext. Springer-Verlag, New York, xiii + 277 pp., ISBN 0-387-98696-0.

Andreas Ziegler (1999). *Pseudo Maximum Likelihood Methode und Generalised Estimating Equations zur Analyse korrelierter Daten.* In German. Anwendungsorientierte Statistik 3. Peter Lang, Frankfurt am Main, ii + 117 pp., ISBN: 3-361-34240-3.)

The Eighth International Workshop on Matrices and Statistics—Tampere, Finland: August 6–7, 1999

Selected Linear Algebra Events Attended

The Eighth ILAS Conference Barcelona, Spain: July 19–22, 1999

Report by Richard A. BRUALDI

The Eighth Conference of the International Linear Algebra Society (ILAS) was held at the Universitat Politècnica de Catelunya in Barcelona, Spain, July 19–22, 1999. All of the scientific sessions were held in the building of the Departament de Matemática Aplicada I. The number of (registered) conference participants was 197 representing thirty different countries. Many thanks to Manuel Martin (Fotografo, Barcelona) for the photo on pp. 14–15.

There were fourteen invited speakers each of whom gave a very stimulating and well-organized lecture. Among these invited talks were the Olga Taussky–John Todd Lecture on *Recent Studies on the Numerical Range* by Chi-Kwong Li (College of William & Mary), and the Hans Schneider Prize Lecture on *The Power Method in Max Algebra* given by Ludwig Elsner (Universität Bielefeld). There were five minisymposia on (1) *Total Positivity*, (2) *Parametrization Problems in Linear Algebra and Systems Theory*, (3) *Parallel Asynchronous Methods*, (4) *Combinatorial Matrix Theory* (In honor of Richard A. Brualdi's 60th birthday), and (5) *Minisymposium in Memory of Robert C. Thompson*. There were a total of 39 presentations in the minisymposia, and over 100 contributed talks.

A conference tour showed participants the sights of Barcelona; included was a reception at City Hall where they were treated to a very enjoyable welcome by a (mathematically informed) minister from the Mayor's office, and glasses of cava. A banquet was held at the Hotel Hilton Barcelona on July 21. At the banquet, Hans Schneider presented the 1999 Linear Algebra Prize to Ludwig Elsner, and John Todd gave a rousing speech. The conference concluded with thanks to all of those who made the conference such a wonderful success, especially Ferran Puerta, and with more cava.

A special issue of *Linear Algebra and its Applications* (LAA), edited by Nick Higham, Roger Horn, Tom Laffey and Ferran Puerta, containing papers from the conference is being assembled, The deadline for submission of papers to one of the special editors is November 30, 1999.

The Eighth International Workshop on Matrices and Statistics Tampere, Finland: August 6–7, 1999

Report by Simo PUNTANEN

The Eighth International Workshop on Matrices and Statistics was held at the University of Tampere on Friday, August 6, and Saturday, August 7, 1999. This Workshop was a Satellite Conference of the 52nd Session of the International Statistical Institute being held in Helsinki August 10–18, 1999.

The International Organizing Committee comprised R. William Farebrother, Simo Puntanen (chair), George P. H. Styan (vice-chair) & Hans Joachim Werner. The Local Organizing Committee at the University of Tampere comprised Riitta Järvinen, Erkki Liski, Jyrki Ollikainen, Tapio Nummi, and Simo Puntanen (chair), with the assistance of Poika Isokoski, Petri Latva-Rasku, Riina Metsänoja, and Anne Puustelli. The photo on page 12 was taken by Erkki Karen.

The Workshop, getting together 95 participants from 22 countries, included sessions of invited and contributed talks and talks by students, as well as a poster session. All Workshop abstracts appear on the Workshop Web site:

http://www.uta.fi/laitokset/mattiet/workshop99/index.html

Selected refereed papers from this Workshop will be published in a Special Issue on Linear Algebra and Statistics of *Linear Algebra and its Applications.*

In the opening session, a silent moment was held for Bernhard Flury (1951–1999), who was killed in July 1999 in a mountain-climbing accident near Trento, Italy.

The Keynote Speaker was T. W. Anderson, with the title "Canonical analysis and reduced rank regression in autoregressive models". The ILAS Lecturer was Gene H. Golub, who spoke on "Reconstruction of a polygon from its moments". The invited Speakers were T. Ando, Ronald Christensen, Seppo Mustonen, Friedrich Pukelsheim, Alastair J. Scott, Shayle R. Searle, Bimal K. Sinha, and George P. H. Styan. Eleven talks were presented in two plenary Graduate Student sessions.

An informal reception was held on Thursday evening, August 5. The reception speech "Why do statisticians kiss my toes, or how close can a statistician get to a Nobel Prize?" was given by Dr. Martin Rasmussen[1] of the Tampere City Health Care and Social Welfare Office. We hope that this excellent talk will soon be published.

A Sauna Party in Finnish style (Finns continue to take partying very seriously) was held on the Friday evening on Viikinsaari Island. The long-awaited soccer game Finland vs Dream Team International was held in the Viikinsaari Olympic Stadium. The highlights of the event were Dorothy Anderson (scored beautifully 3 penalty kicks), Shayle Searle (extremely clever use of stick), and George Styan (excellent utilization of the opportunity). The result of the match is immaterial (but let it be mentioned that it was 6 to 7 or 7 to 6). Dr. Tarmo Pukkila, Ex-Rector of the University of Tampere, gave an After-Sauna Speech entitled: "Is there life outside the universities?"

[1] Dr. Rasmussen is a grandson of Harald Cramér (1893–1985).

The Eighth ILAS Conferenc

elona, Spain: July 19–22, 1999

Selected Forthcoming Linear Algebra Events

We have selected 7 forthcoming linear algebra events, scheduled as follows:

- December 10, 1999: Columbia, South Carolina
- March 16–19, 2000: San Antonio, Texas
- June 26–28, 2000: Nafplio, Greece
- July 12–14, 2000: Leuven, Belgium
- August 9–12, 2000: Lund, Sweden
- August 11–14, 2000: Kunming, China
- December 9–13, 2000: Hyderabad, India.

For more details on these and information on other linear algebra events please visit the ILAS/IIC Web site:

http://www.math.technion.ac.il/iic/conferences.html

Matrix Theory Symposium in Honor of Tom Markham

Columbia, South Carolina: December 10, 1999

In honor of his 60th birthday and planned retirement, the University of South Carolina (Columbia) will host a one-day matrix theory symposium for Tom Markham on December 10, 1999, preceded by a reception for Tom on Thursday, December 9, 5–7 pm. The scientific program begins at 9 am (December 10); both events will be held the Daniel Management Center on campus. This should be a valuable program highlighting important recent work in the field. All interested parties are invited to attend. For further information contact:

Charles Johnson: crjohnso@math.wm.edu
Ronald Smith: rsmith@cecasun.utc.edu

Reflections on the Past & Visions for the Future: An International Conference in Honor of Professor C. R. Rao on the Occasion of his 80th Birthday

San Antonio, Texas: March 16–19, 2000

This conference will be held at the University of Texas–San Antonio, March 16–19, 2000. The primary focus of the conference will be to provide a forum for participants to discuss the changing face of statistical research and to highlight the contributionsof C. R. Rao to different areas of Probability and Statistics.

There will be several invited presentations by renowned statisticians and numerous technical sessions of contributed talks. The organizing committee consists of N. Balakrishnan (McMaster University), N. Kannan (University of Texas–San Antonio), J. P. Keating (Univ. of Texas–San Antonio), R. Khattree (Oakland University), T. Nayak (George Washington University), and S. Peddada (Univ. of Virginia). For more information on registration and other details, please contact Nandini Kannan: kannan@sphere.math.utsa.edu or visit the Web site

http://www.math.utsa.edu/~kannan/conf

The Fifth Workshop on Numerical Ranges and Numerical Radii

Nafplio, Greece: June 26–28, 2000

The 5th Workshop on Numerical Ranges and Numerical Radii will be held in the historical and enjoyable town of Nafplio (Nauplia) in the Peloponnese, Greece, from Monday, June 26 through Wednesday, June 28, 2000, starting with a reception on Sunday evening, June 25. The purpose of the workshop is to stimulate research and to foster the interaction of researchers on the subject. The informal workshop atmosphere will guarantee the exchange of ideas from different research areas and in particular, the researchers may be better informed on the latest developments and the newest techniques.

Topics: Matrix Analysis, Matrix Polynomials, computation of Numerical Ranges and Radii, applications in Stability Theory, Perturbation Theory and in Discrete Mathematics.

There will be no registration fees and it is not possible to provide financial support to the participants. All participiants though will have the chance, if they wish, to attend the social events during the workshop without charge. Information about accommodation will be available by December 1999.

If you plan to attend the workshop or if you want to receive more information, please contact the organizer : John Maroulas, Dept. of Mathematics, National Technical University, Zografou Campus, Athens 15781, Greece; maroulas@math.ntua.gr. You can register online at the Web site

http://www.math.uregina.ca/ tsat/nr/nr.html

Deadlines: To confirm your participation at the workshop : 1 January 2000. To reserve your accomodation : 1 March 2000. To submit the title and absract of your talk : 1 April 2000.

The Third International Conference on Matrix-Analytic Methods in Stochastic Models

Leuven, Belgium: July 12–14, 2000

This conference will provide an international forum for the presentation of recent results on matrix-analytic methods in stochastic models. Its scope includes development of the methodology, as well as the related algorithmic implementations and applications in communications, production and manufacturing engineering; it also includes computer experiments in the investigation of specific probability models.

The topics of interest include but are not limited to: methodology, general theory, computational methods, computer experimentation, queueing models, telecommunications modeling, spatial processes, reliability problems, risk analysis, and production and inventory models.

The organizers wish to encourage students to attend the conference. To that effect, financial assistance will be made available on a limited basis and a streamlined submission procedure will be implemented. Details will be published on the conference Web page: http://www.econ.kuleuven.ac.be/mam3. Queries should be addressed to MAM3@econ.kuleuven.ac.be.

International Conference on Numerical Mathematics on the 40th Anniversary of the Journal *BIT*

Lund, Sweden: August 9–12, 2000

Topics central to *BIT* will be emphasized. Several invited talks will survey important recent developments in areas including numerical solution of odes, numerical linear algebra. The conference is hosted by the Department of Mathematics of the University of Lund located in southern Sweden. The Organizing Committee comprises: Åke Björck (Linköping University), Gustaf Söderlind (Lund University), and Kaj Madsen (Technical University of Denmark). The Program Committee comprises: Per Christian Hansen (Lyngby), Olavi Nevanlinna (Helsinki), Syvert Nørsett (Trondheim), and Axel Ruhe (Gothenburg).

The following speakers have been invited: G. H. Golub (Stanford University), G. W. Stewart (University of Maryland), E. Hairer (Université de Genève), and H. Van der Vorst (Utrecht University).

Minisymposia on the following topics central to *BIT* will be organized: Geometric integration (Syvert Nørsett), Generalized eigenproblems (Axel Ruhe), Inverse and illposed problems (Per Christian Hansen), Iterative methods (Olavi Nevanlinna), & Ordinary differential equations (Gustaf Söderlind).

Participants wishing to present a contributed talk should submit by e-mail an extended abstract (1–2 pages) written in LaTeX to Prof. Gustaf Söderlind, Gustaf.Soderlind@na.lu.se. Deadline for submission is March 1, 2000. Notification of acceptance will be given by April 15, 2000. Participants should register by June 15, 2000, for their talk to be included in the Conference program.

The conference registration fee is US$175 for early registration until May 31, 2000, and US$200 after this date or on site registration. The fee includes: a welcoming reception, coffee breaks, an afternoon excursion over the new bridge from Malmö to Copenhagen, where the conference dinner will take place. Additional information about the Conference may be obtained by writing to BIT Conference 2000, Center for Mathematical Sciences, Lund University, Box 118, SE-221 00 Lund, Sweden, or by visiting the Web site: http://www.maths.lth.se/na/.

The Fourth China Matrix Theory Conference

Kunming, China: August 11–14, 2000

The Fourth China Matrix Theory Conference will be held at Yunnan University in Kunming, Yunnan Province, China, August 11–14, 2000. With the support of the Chinese Linear Algebra Society, the conference is being organized by Yunnan University and Fudan University. The conference registration fee is US$150, and the daily expenses for accommodation and food at the Guest House of Yunnan University is about US$20. The conference proceedings (in English) will be published. We have no funding to support participants. For further information, please contact: Bit-Shun Tam, Department of Mathematrics, Tamkang University, Tamsui, Taiwan 25137; bsm01@mail.tku.edu.tw, bsm01@mail.tku.edu.tw.

The Ninth International Workshop on Matrices and Statistics in Celebration of C. R. Rao's 80th Birthday

Hyderabad, India: December 9–13, 2000

The Ninth International Workshop on Matrices and Statistics, in Celebration of C. R. Rao's 80th Birthday, will be held in the historic walled city of Hyderabad, in Andra Pradesh, India, on December 9–13, 2000. The program will start with a two-day course on recent advances in Matrix Theory, with Special Reference to Applications in Statistics, on Saturday, December 9, and Sunday, December 10, 2000. This will be followed by the presentation of research papers, which will be published in a professional journal after refereeing.

The International Organizing Committee for this Workshop comprises R. W. Farebrother (Manchester), S. Puntanen (Tampere; vice-chair), G. P. H. Styan (McGill), and H. J. Werner (Bonn; chair). For further information, e-mail the Local Organizing Committee in India: P. Bhimasankaram: isihyd@ap.nic.in, R. J. R. Swamy: nhasan@ouastr.ernet.in, or K.Viswanath: kvsm@uohyd.ernet.in.

IMAGE Problem Corner: Problems and Solutions

We are still hoping to receive solutions to Problems 18-1, 19-3b & 21-2, which are repeated below; we present solutions to Problems 19-4 and 22-1 through 22-7. In addition, we introduce 7 new problems (pp. 28 & 27) and invite readers to submit solutions, as well as new problems, for publication in IMAGE. Please send all material in LaTeX — (a) embedded as text in an e-mail to styan@total.net *and* (b) nicely printed (2 copies please) by p-mail to George Styan, PO Box 270, Franklin, VT 05457-0270, USA.

Problem 18-1: 5×5 Complex Hadamard Matrices

Proposed by S. W. DRURY: drury@math.mcgill.ca; *McGill University, Montréal, Québec, Canada.*

Show that every 5×5 matrix U with complex entries $u_{j,k}$ of constant absolute value one that satisfies $U^*U = 5I$ can be realized as the matrix $(\omega^{jk})_{j,k}$ where ω is a complex primitive fifth root of unity by applying some sequence of the following: (1) A rearrangement of the rows, (2) A rearrangement of the columns, (3) Multiplication of a row by a complex number of absolute value one, (4) Multiplication of a column by a complex number of absolute value one.

The Editor has not yet received a solution to this problem—indeed even the Proposer has not yet found a solution!

Problem 19-3: Characterizations Associated with a Sum and Product

Proposed by Robert E. HARTWIG: hartwig@math.ncsu.edu; *North Carolina State University, Raleigh, North Carolina, USA,*
Peter ŠEMRL: peter.semrl@fmf.uni-lj.si; *University of Maribor, Maribor, Slovenia &*
Hans Joachim WERNER: werner@united.econ.uni-bonn.de; *Universität Bonn, Bonn, Germany.*

(19-3a) Characterize square matrices A and B satisfying $AB = pA + qB$, where p and q are given scalars.

(19-3b) More generally, characterize linear operators A and B acting on a vector space $\mathcal{X}$ satisfying
$ABx \in \mathrm{Span}(Ax, Bx)$ for every $x \in \mathcal{X}$.

The Editor has not yet received a solution to Problem 19-3b. The solution by the Proposers to Problem 19-3a appeared in IMAGE 22 (April 1999), p. 25. *We look forward to receiving a solution to Problem 19-3b.*

Problem 19-4: Eigenvalues of Positive Semidefinite Matrices

Proposed by Fuzhen ZHANG: zhang@polaris.ncs.nova.edu; *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

Show that there are constants $\gamma \in (0, \frac{1}{2}]$ and $\varepsilon \in (0, \frac{1}{2})$ such that if $A_1, \ldots, A_n$ are non-negative definite $r \times r$ matrices of rank one satisfying

(a) $A_1 + \cdots + A_n = I_r$

(b) $\mathrm{trace}(A_i) < \gamma$ for each $i = 1, \ldots, n$,

then there is a subset σ of $\{1, 2, \ldots, n\}$ such that the eigenvalues of $A_\sigma = \sum_{i \in \sigma} A_i$ all lie in the interval $(\varepsilon, 1 - \varepsilon)$.

Solution 19-4.1 by Alexander KOVAČEC: kovacec@gentzen.mat.uc.pt; *Universidade de Coimbra, Coimbra, Portugal.*

We reformulate Problem 19-4 as follows:

> Show that there are constants $\gamma \in]0, \frac{1}{2}[$ and $\varepsilon \in]0, \frac{1}{2}]$ such that if $A_1, \ldots, A_n$ are non-negative definite $r \times r$ matrices satisfying $A_1 + \ldots + A_n = I_r$ and $\mathrm{trace}(A_i) < \gamma$ for all $i = 1, \ldots, n$, then there there exists a set $I \subset [n] := \{1, 2, \ldots, n\}$ such that the eigenvalues of $A(I) = \sum_{i \in I} A_i$ all lie in $]\varepsilon, 1 - \varepsilon[$.

In this formulation it is not as clear as it should be whether γ and ε are meant to be independent of r. We admit dependency on r and prove for this case a stronger result. This is based on a result of Bourbaki [BTG, ch. 5-8, p. 87], cf. Mitrinović [MAI, §3.9.33, p. 355].

THEOREM (Bourbaki). *Assume $\{p_i\}_1^m$ are m vectors in the Euclidean unit $n-$ball: i.e. that sum to a: $p_i \in \mathbb{R}^n$ with $|p_i| \leq 1$, $\sum_{i=1}^m p_i = a$. Then there exists a $\sigma \in S_m$ (symmetric group of m elements) such that for all $k \in [m]$*

$$\left| \sum_{i=1}^k p_{\sigma(i)} \right| \leq (1+a) \cdot 5^{(n-1)/2}.$$

We write $I \sqsubset I'$ to say $I' = I \cup \{i\}$ with $i \notin I$. An obvious corollary to this theorem of Bourbaki's is:

COROLLARY. *If $\sum_{i=1}^m p_i = 0$, then there exists a chain $\mathbf{C} = \{\emptyset = I_0 \sqsubset I_1 \sqsubset \ldots \sqsubset I_m = [m]\}$ such that for all $I \in \mathbf{C}$ $\left|\sum_{i\in I} p_i\right| \leq 5^{(n-1)/2}$.*

THEOREM. *Given any $r \in \mathcal{N}$, and any $u \in]0,1[$ there exist constants $\varepsilon, \gamma = \gamma(\varepsilon) > 0$ arbitrarily near to 0 such that for all sets $\{A_1, \ldots, A_n\}$ of $r \times r$ non-negative definite matrices of rank one for which (a) $A_1 + \ldots + A_n = I_r$, (b) trace$(A_i) < \gamma$, there is a set $I \subset [n]$ such that the eigenvalues of $A(I) = \sum_{i\in I} A_i$ all lie in $]u-\varepsilon, u+\varepsilon[$.*

Proof. For simplicity of notation let us assume $r = 3$. It will be clear how the reasoning can be generalized to arbitrary r. From the rank one hypothesis and [HJ, Problem 7.1.4, p. 400], we see that for adequate $a_i, b_i \in \mathbf{C}$, $i = 1, 2, \ldots, n$, any $A(I)$ can be written

$$A(I) = \begin{pmatrix} \sum_I |a_i|^2 & \sum_I a_i\bar{b}_i & \sum_I a_i\bar{c}_i \\ \sum_I \bar{a}_i b_i & \sum_I |b_i|^2 & \sum_I b_i\bar{c}_i \\ \sum_I \bar{a}_i c_i & \sum_I \bar{b}_i c_i & \sum_I |c_i|^2 \end{pmatrix}.$$

The remaining hypotheses transform into

$$(\mathrm{a}_1)\text{:} \quad \sum_{i\in[n]} |a_i|^2 = \sum_{i\in[n]} |b_i|^2 = \sum_{i\in[n]} |c_i|^2 = 1, \qquad (\mathrm{a}_2)\text{:} \sum_{i\in[n]} a_i\bar{b}_i = \sum_{i\in[n]} a_i\bar{b}_i = \sum_{i\in[n]} b_i\bar{c}_i = 0,$$

$$\text{and (b):} \quad |a_i|^2 + |b_i|^2 + |c_i|^2 < \gamma, \quad i \in [n].$$

Since $A(I)$ is positive semidefinite [HJ, §7.1.3], we find by Gershgorin's theorem [HJ, §6.1.1] that all the eigenvalues of $A(I)$ are contained in the union

$$B\left(\sum_I |a_i|^2, R_{ab}(I) + R_{ac}(I)\right) \cup B\left(\sum_I |b_i|^2, R_{ab}(I) + R_{bc}(I)\right) \cup B\left(\sum_I |c_i|^2, R_{ac}(I) + R_{bc}(I)\right),$$

where $B(a,r)$ is the interval $]a-r, a+r[$, $a \in \mathbb{R}$, $r > 0$, and $R_{ab}(I) = |\sum_I a_i\bar{b}_i|$ etc. Define point $p_i \in \mathbb{R}^8$ by

$$p_i = (|a_i|^2 - |b_i|^2, |a_i|^2 - |c_i|^2, \Re a_i\overline{b_i}, \Im a_i\overline{b_i}, \Re a_i\overline{c_i}, \Im a_i\overline{c_i}, \Re b_i\overline{c_i}, \Im b_i\overline{c_i}).$$

From hypothesis (b), we find that each component of p_i is in modulus $< \gamma$, so $|p_i| < \sqrt{8}\gamma$. Also (a) implies $\sum_{[n]} p_i = 0$. Thus there exists a chain $\mathbf{C} = \{\emptyset = I_0 \sqsubset I_1 \sqsubset \ldots \sqsubset I_n = [n]\}$ of sets such that for all $I \in \mathbf{C}$, $|\sum_I p_i|$, and hence each of the components of $\sum_I p_i$ is in modulus$< c(\gamma) := \sqrt{8} \cdot 5^{7/2}\gamma$.

Given any $\varepsilon > 0$ and any $u \in]0,1[$, choose γ such that $4c(\gamma) < \varepsilon$. By (a_1) and (b), there is an $I \in \mathbf{C}$ such that $\sum_I |a_i|^2 \in]u-\gamma, u+\gamma[$. Hence we have $\sum_I |b_i|^2, \sum_I |c_i|^2 \in]u-\gamma-c(\gamma), u+\gamma+c(\gamma)[$. Also, the radii of the balls $B(\ldots)$ are $< 2c(\gamma)$. This proves the theorem. □

References

[BTG] N. Bourbaki (1955). *Topologie Générale*, 2e édition. Paris.

[HJ] Roger A. Horn & Charles R. Johnson (1985). *Matrix Analysis.* Cambridge Univ. Press. [Reprinted with corrections 1990.]

[MAI] D. S. Mitrinović (1970). *Analytic Inequalities.* Springer-Verlag, New York.

Problem 21-2: The Diagonal of an Inverse

Proposed by Beresford PARLETT: parlett@math.berkeley.edu; *University of California, Berkeley, California, USA,*
via Roy MATHIAS: mathias@math.wm.edu; *College of William and Mary, Williamsburg, Virginia, USA.*

Let J be an invertible tridiagonal $n \times n$ matrix that permits triangular factorization in both increasing and decreasing order of rows: $J = L_+ D_+ U_+$ and $J = U_- D_- L_-$. (Here the L's are lower triangular, the U's are upper triangular, and the D's are diagonal.) Show that $(J^{-1})_{kk} = [(D_+)_{kk} + (D_-)_{kk} - J_{kk}]^{-1}$.

We look forward to receiving a solution to this problem!

Problem 22-1: Powers, g-Inverses and Core Matrices

Proposed by Gülhan ALPARGU: alpargu@math.mcgill.ca; *McGill University, Montréal, Québec, Canada,*
George P. H. STYAN: styan@total.net; *McGill University, Montréal, Québec, Canada &*
Hans Joachim WERNER: werner@united.econ.uni-bonn.de; *Universität Bonn, Bonn, Germany.*

For a real matrix A, let A^- denote an arbitrary g-inverse of A satisfying $AA^-A = A$, and let $\{A^-\}$ denote the set of all g-inverses of A. Show that for a real square matrix A the following conditions are equivalent:

(a) $\sum_{i=0}^{h} A^i \in \{(I - A)^-\}$ for some nonnegative integer h.

(b) $A^s = A^{s+1}$ for some nonnegative integer s.

(c) The core part $C(A)$ of A is idempotent. [We recall that a square matrix A has index $\operatorname{ind}(A) = k$ iff there exist a nilpotent matrix N_A of degree k (i.e., $N_A^k = 0$ while $N_A^{k-1} \neq 0$) and a core matrix C_A (i.e., $\operatorname{ind}(C_A) = 1$) such that $A = C_A + N_A, C_A N_A = N_A C_A = 0$; cf. e.g., pp. 175–177 in Adi BEN-ISRAEL & Thomas N. E. GREVILLE, *Generalized Inverses: Theory and Applications*, Wiley, New York, 1974; corrected reprint edition: Krieger, Huntington, NY, 1980.]

Solution 22-1.1 by the Proposers: Gülhan ALPARGU, George P. H. STYAN & Hans Joachim WERNER.

Clearly, $(I - A)\sum_{i=0}^{h} A^i = I - A^{h+1}$. Consequently, $(I - A)\left(\sum_{i=0}^{h} A^i\right)(I - A) = (I - A)$ if and only if $(I - A^{h+1})(I - A) = (I - A)$ or, equivalently, $A^{h+1} = A^{h+2}$. That (a) is equivalent to (b) with $s = h + 1$ is hence plain. Next, let A be of index k. Consider the "core-nilpotent" decomposition $A = C_A + N_A$, where C_A stands for the core part $C(A)$ of A. Then, whenever $s \geq k$, $N_A^s = 0$. Since $N_A C_A = C_A N_A = 0$, for each $s \geq k$ therefore $A^s = C_A^s$. Hence, in particular, $A^k = C_A^k$ and $A^{k+1} = C_A^{k+1}$. So we have $A^k = A^{k+1}$ if and only if $C_A^k = C_A^{k+1}$. Since C_A is of index 1, the group inverse $C_A^{\#}$ of C_A does exist and is unique. Moreover, recall that this inverse satisfies simultaneously the three matrix equations $C_A C_A^{\#} C_A = C_A$, $C_A^{\#} C_A C_A^{\#} = C_A^{\#}$, and $C_A^{\#} C_A = C_A C_A^{\#}$. Therefore $C_A^k (C_A^{\#})^{k-1} = C_A$ and $C_A^{k+1} (C_A^{\#})^{k-1} = C_A^2$, and it is clear that (c) is equivalent to (b) with $s = k$. With the foregoing observations in mind it should further be clear that if (a) and (b) hold for h and s, respectively, then (a) and (b) hold for each $h \geq k - 1$ and $s \geq k$, where k is the index of A. □

Solution 22-1.2 by Tian-Gang LEI: leitg@rose.nsfc.gov.cn; *National Natural Science Foundation of China, Beijing, China.*

The equivalence of (a) and (b) follows from the fact that for any nonnegative integer h, $\sum_{i=0}^{h} A^i \in \{(I - A)^-\}$ if and only if $(I - A)(\sum_{i=0}^{h} A^i)(I - A) = (I - A)$ if and only if $A^{h+1} = A^{h+2}$.

To prove the equivalence of (b) and (c), let A have index $\operatorname{ind}(A) = k$, and let A_d denote the Drazin inverse of A. Then $A^{k+1}A_d = A^k$, $AA_d^2 = A_d$, $AA_d = A_d A$, and $A^2 A_d = C_A$, the core part of A. It follows that $C_A^2 = A^2 A_d A^2 A_d = A^4 A_d^2 = A^3 A_d$. Suppose $A^s = A^{s+1}$. If $s \leq 2$, then $C_A^2 = A^{s+1} A^{2-s} A_d = A^s A^{2-s} A_d = C_A$. If $s \geq 3$, then $C_A^2 = A^{s+1} A_d^{s-2} A_d = A^s A_d^{s-2} A_d = C_A$. Conversely, suppose $C_A^2 = C_A$. If $k \geq 1$, then $A^{k+1} = A^{k+2} A_d = A^{k-1} A^3 A_d = A^{k-1} A^2 A_d = A^k$. If $k = 0$, then $A_d = A^{-1}$ and $A^2 = A$. □

A solution was also received from Taosheng LI, *Central Normal University, Wuhan, China.*

[*Editorial Remark:* See also "A generalized inverse for quasi-idempotent matrices A when $A(A - I)^k = 0$ for some positive integer k": Problem 619 (proposed by Götz Trenkler and solved by Nora S. Thornber) in *The College Mathematics Journal*, vol. 29 (1998), page 66 & vol. 30 (1999), page 65.]

Problem 22-2: Square Complex Matrices, Linear Maps and Eigenvalues

Proposed by Ludwig ELSNER: elsner@mathematik.uni-bielefeld.de; *Universität Bielefeld, Bielefeld, Germany.*

(a) Let M_n denote the set of $n \times n$ complex matrices. Find a *linear* map $S : M_n \longrightarrow M_{n^2}$ with the following property: If $A \in M_n$ has the (not necessarily real) eigenvalues $\lambda_i, i = 1, \ldots, n$, then $S(A)$ has the eigenvalues $\lambda_i, i = 1, \ldots, n$, as well as $\sqrt{\lambda_i \lambda_j}$ and $-\sqrt{\lambda_i \lambda_j}$ for all pairs $i < j$.

(b) Show that there is no linear map $S : M_2 \longrightarrow M_2$ satisfying the following condition: If $A \in M_2$ has the eigenvalues λ, μ, then $S(A)$ has the eigenvalues $\pm\sqrt{\lambda\mu}$.

Solution 22-2.1 by the Proposer: Ludwig ELSNER.

(a) Let P denote the permutation in $\mathbb{C}^{n^2}$ characterized by $P(x \otimes y) = y \otimes x$; $x, y \in \mathbb{C}^n$ and define $S(A) = P(I \otimes A)$, where $I \in M_n$ is the identity. If $Ax = \mu x$, $Ay = \lambda y$, we find by elementary calculations the relation $S(A)(\sqrt{\mu}(x \otimes y) \pm \sqrt{\lambda}(y \otimes x)) = \pm\sqrt{\lambda\mu}(\sqrt{\mu}(x \otimes y) \pm \sqrt{\lambda}(y \otimes x))$, where it is understood that we use always the upper or always the lower sign. The claim follows now by standard arguments.

(b) Such an $S(A)$ has to satisfy $\mathrm{tr}(S(A)) = 0$ and $\det(S(A)) = -\det(A)$. Describe the matrix $A = (a_{ij})_{i,j=1,2}$ by $a = (a_{11}, a_{21}, a_{12}, a_{22})^T$ and $S(A)$ accordingly by a vector $s(a) \in \mathbb{C}^4$. Then due to the linearity of S there are vectors $x, y, z \in \mathbb{C}^4$ such that $s(a) = (x^T a, z^T a, y^T a, -x^T a)^T$. We find $-2\det(S(A)) = a^T(2xx^T + zy^T + yz^T)a$, while $2\det(A) = a^T Da$ with

$$D = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

It follows that $D = 2xx^T + yz^T + zy^T$. Comparing ranks leads to a contradiction. □

Problem 22-3: The Rank of a Matrix Difference

Proposed by Yongge TIAN: tiany@alcor.concordia.ca; *Concordia University, Montréal, Québec, Canada.*

Let the $n \times n$ matrices A and B be idempotent. Show that $\mathrm{rank}(A - B) = \mathrm{rank}(A - AB) + \mathrm{rank}(B - AB)$.

Solution 22-3.1 by R. B. BAPAT: rbb@isid.ernet.in; *Indian Statistical Institute–Delhi Centre, New Delhi.*

Let $X = A - AB, Y = AB - B$. Since A is idempotent, $AX = X$ and $AY = 0$. Suppose u is in the column space of both X and Y. Then there exist v and w such that $u = Xv = Yw$. Then $Au = AXv = Xv = u$, since $AX = X$. Now $u = Au = AYw = 0$ since $AY = 0$. Thus the column spaces of X and Y have only the null vector in common. It can be shown similarly, using the fact that B is idempotent, that the row spaces of X and Y have only the null vector in common. It follows that $\mathrm{rank}(X+Y) = \mathrm{rank}(X)+\mathrm{rank}(Y)$ and this is clearly equivalent to the assertion we wish to prove. □

Solution 22-3.2 by Tain-Gang LEI: leitg@rose.nsfc.gov.cn; *National Natural Science Foundation of China, Beijing, China.*

$$\begin{aligned}
\mathrm{rank}(A - AB) + \mathrm{rank}(B - AB) &= \mathrm{rank}(A^2 - AB) + \mathrm{rank}(B^2 - AB) \\
&= \mathrm{rank}\big(A(A - B)\big) + \mathrm{rank}\big((B - A)B\big) \\
&= \mathrm{rank}\big(A(A - B)\big) + \mathrm{rank}\big((A - B)B\big) \\
&\leq \mathrm{rank}\big(A(A - B)B\big) + \mathrm{rank}(A - B) \\
&= \mathrm{rank}(A^2B - AB^2) + \mathrm{rank}(A - B) \\
&= \mathrm{rank}(A - B) \\
&\leq \mathrm{rank}(A - AB) + \mathrm{rank}(AB - B) = \mathrm{rank}(A - AB) + \mathrm{rank}(B - AB).
\end{aligned}$$

Here the first inequality follows from the Frobenius inequality: $\mathrm{rank}(PQ) + \mathrm{rank}(QR) \leq \mathrm{rank}(Q) + \mathrm{rank}(PQR)$. □

Solution 22-3.3 by Steven J. LEON: SLEON@umassd.edu; *University of Massachusetts–Dartmouth, North Dartmouth, Massachusetts, USA.*

For any $n \times n$ matrix C, we will denote the column space of C by $\mathcal{R}(C)$ and note that $\mathcal{R}(C) = \{y = Cx | x \in \mathbb{R}^n\}$. To prove the result about the ranks, it suffices to show that $\mathcal{R}(A - B)$ is a direct sum of $\mathcal{R}(A - AB)$ and $\mathcal{R}(B - AB)$. If $y \in \mathcal{R}(A - B)$. then $y = (A - B)x$ for some $x \in \mathbb{R}^n$. If we then set $c = (A - AB)x$ and $d = (B - AB)(-x)$, then $c \in \mathcal{R}(A - AB)$ and $d \in \mathcal{R}(B - AB)$ and

$$y = (A - B)x = (A - AB)x - (B - AB)x = c + d, \tag{1}$$

so $\mathcal{R}(A - B) = \mathcal{R}(A - AB) + \mathcal{R}(B - AB)$. To show that this is a direct sum we must show that the representation (1) is unique. To see this suppose $y = e + f$ where $e \in \mathcal{R}(A - AB)$ and $f \in \mathcal{R}(B - AB)$. Then

$$(A - B)x = y = e + f = (A - AB)z + (B - AB)w \tag{2}$$

for some vectors z and w in $\mathbb{R}^n$. Multiplying both sides of (2) by A and using the idempotency of A, we see that $\mathcal{R}(B - AB)$.

Solution 22-3.4 by Hans Joachim WERNER: werner@united.econ.uni-bonn.de; *Universität Bonn, Bonn, Germany.*

Two $m \times n$ matrices C and D, whose row spaces and whose column spaces do have only the respective origin in common, are said to be *weakly bicomplementary* to each other; see [WGI]. A pair of weakly bicomplementary matrices is also often said to be a pair of *disjoint* matrices (also written $A + B = A \oplus B$; cf. [MFR]). According to [JMW, Th. 2.3] , $A + B = A \oplus B$ if and only if $\mathrm{rank(A + B)} = \mathrm{rank(A)} + \mathrm{rank(B)}$. Now, let the $n \times n$ matrices A and B be idempotent. Then, in view of $I_n = A + (I - A) = A \oplus (I - A)$ and $I_n = B + (I - B) = B \oplus (I - B)$, clearly $A - B = (A - B)B + (A - B)(I - B) = (AB - B) + A(I - B) = (A - I)B + A(I - B) = (A - I)B \oplus A(I - B)$. Therefore, in particular, $\mathrm{rank}(A - B) = \mathrm{rank}((A - I)B) + \mathrm{rank}(A(I - B))$, and our claim is plain. □

References

[JMW] S. K. Jain, S. K. Mitra & H. J. Werner (1996). Extensions of $\mathcal{G}$-based matrix partial orders. *SIAM Journal on Matrix Analysis and Applications*, 17, 834–850.

[MFR] S. K. Mitra (1972). Fixed rank solutions of linear matrix equations. *Sankhyā Series A*, 34, 387–392.

[WGI] H. J. Werner (1986). Generalized inversion and weak bi-complementarity. *Linear and Multilinear Algebra*, 19, 357–372.

Solution 22-3.5 by Jürgen GROß: gross@amadeus.statistik.uni-dortmund.de *Universität Dortmund, Dortmund, Germany* & Götz TRENKLER: trenkler@amadeus.statistik.uni-dortmund.de; *Universität Dortmund, Dortmund, Germany.*

From Marsaglia & Styan (1974, Th. 8), we have

$$\mathrm{rank}(A - B) = \mathrm{rank}\begin{pmatrix} 0 & A & -B \\ A & A & 0 \\ -B & 0 & -B \end{pmatrix} - \mathrm{rank}(A) - \mathrm{rank}(B).$$

Using the factorization

$$\begin{pmatrix} I_n & 0 & 0 \\ 0 & -I_n & 0 \\ -B & 0 & I_n \end{pmatrix}\begin{pmatrix} 0 & A & -B \\ A & A & 0 \\ -B & 0 & -B \end{pmatrix}\begin{pmatrix} -I_n & -A & 0 \\ 0 & I_n & 0 \\ 0 & 0 & -I_n \end{pmatrix} = \begin{pmatrix} 0 & A & B \\ A & 0 & 0 \\ B & 0 & 0 \end{pmatrix}$$

we see that

$$\mathrm{rank}(A - B) = \mathrm{rank}\begin{pmatrix} A \\ B \end{pmatrix} + \mathrm{rank}(A, B) - \mathrm{rank}(A) - \mathrm{rank}(B).$$

Furthermore, by the identities

$$\mathrm{rank}\begin{pmatrix} A \\ B \end{pmatrix} - \mathrm{rank}(B) = \mathrm{rank}(A(I_n - B)) \qquad \text{and} \qquad \mathrm{rank}(A, B) - \mathrm{rank}(A) = \mathrm{rank}((I_n - A)B),$$

cf. e.g., Marsaglia & Styan (1974, Th. 19), we obtain

$$\mathrm{rank}(A - B) = \mathrm{rank}(A - AB) + \mathrm{rank}(B - AB),$$

which is the asserted identity.

Addendum: From Marsaglia & Styan (1974, Cor. 6.2), it follows that

$$\operatorname{rank}[A(I_n - B)] = \operatorname{rank}(I_n - B) - \dim[\mathcal{N}(A) \cap \mathcal{C}(I_n - B)] = n - \operatorname{rank}(B) - \dim[\mathcal{C}(I_n - A) \cap \mathcal{C}(I_n - B)]$$

and

$$\operatorname{rank}[(I_n - A)B] = \operatorname{rank}(B) - \dim[\mathcal{N}(I_n - A) \cap \mathcal{C}(B)] = \operatorname{rank}(B) - \dim[\mathcal{C}(A) \cap \mathcal{C}(B)],$$

which gives

$$\operatorname{rank}(A - B) = n - \dim[\mathcal{C}(I_n - A) \cap \mathcal{C}(I_n - B)] - \dim[\mathcal{C}(A) \cap \mathcal{C}(B)],$$

where $\mathcal{C}(\cdot)$ and $\mathcal{N}(\cdot)$ stand for the column space and the nullspace of a matrix, respectively.

Since $\dim[\mathcal{C}(C) \cap \mathcal{C}(D)] = \operatorname{rank}(C) + \operatorname{rank}(D) - \operatorname{rank}(C, D)$, where C and D are matrices with the same number of rows, after some simple calculations, we get the additional result $\operatorname{rank}(A - B) = \operatorname{rank}(A, B) + \operatorname{rank}(I_n - A, I_n - B) - n$.

In conclusion we remark that similar results can be found in Groß & Trenkler (1999). □

References

J. Groß & G. Trenkler (1999). Nonsingularity of the difference of two oblique projectors. *SIAM Journal on Matrix Analysis and Applications*, in press.

G. Marsaglia & G. P. H. Styan (1974). Equalities and inequalities for ranks of matrices. *Linear and Multilinear Algebra*, 2, 269–292.

A solution was also received from Chang-yu LU: cylu@nenu.edu.cn; *Northeast Normal University, Changchun, Jilin, China.*

Problem 22-4: Determinant of a Certain Patterned Matrix

Proposed by William F. TRENCH: wtrench@trinity.edu, *Woodland Park, Colorado, USA.*

Let a, b, c and d be complex numbers. Find the determinant of $G_n = \{\, a(i-j) + b\min(i,j) + c\max(i,j) + d \,\}_{i,j=1}^{n}$. [See also IMAGE Problems 18-5, 20-2 and 21-3 for other matrices of this form.]

Solution 22-4.1 by the Proposer: William F. TRENCH.

Write $G_n = aA_n + bB_n + cC_n + dD_n$, where $A_n = (i-j)_{i,j=1}^{n}$, $B_n = (\min(i,j))_{i,j=1}^{n}$, $C_n = (\max(i,j))_{i,j=1}^{n}$, and $D_n = (1)_{i,j=1}^{n}$. Let T_n be the $n \times n$ tridiagonal Toeplitz matrix with 2's on the main diagonal and -1's above and below the main diagonal. Then

$$A_nT_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & -n \\ 2 & 0 & 0 & \cdots & 0 & -n+1 \\ 3 & 0 & 0 & \cdots & 0 & -n+2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & 0 & 0 & \cdots & 0 & -2 \\ n & 0 & 0 & \cdots & 0 & -1 \end{pmatrix}, \qquad B_nT_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 1 & 0 & \cdots & 0 & 2 \\ 0 & 0 & 1 & \cdots & 0 & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & n-1 \\ 0 & 0 & 0 & \cdots & 0 & n+1 \end{pmatrix},$$

$$C_nT_n = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & n+1 \\ 2 & -1 & 0 & \cdots & 0 & n+1 \\ 3 & 0 & -1 & \cdots & 0 & n+1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ n-1 & 0 & 0 & \cdots & -1 & n+1 \\ n & 0 & 0 & \cdots & 0 & n \end{pmatrix} \quad \text{and} \quad D_nT_n = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 1 \end{pmatrix}.$$

Therefore,

$$G_nT_n = \begin{pmatrix} a+b+d & 0 & 0 & \cdots & 0 & -na+b+(n+1)c+d \\ 2(a+c)+d & b-c & 0 & \cdots & 0 & -(n-1)a+2b+(n+1)c+d \\ 3(a+c)+d & 0 & b-c & \cdots & 0 & -(n-2)a+3b+(n+1)c+d \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (n-1)(a+c)+d & 0 & 0 & \cdots & b-c & -2a+(n-1)b+(n+1)c+d \\ n(a+c)+d & 0 & 0 & \cdots & 0 & -a+(n+1)b+nc+d \end{pmatrix}.$$

If $a+b+d \neq 0$ we can subtract appropriate multiples of the first row of $\det(G_nT_n)$ from the other rows to reduce it to upper triangular form. This yields

$$\det(G_nT_n) = (n+1)(b-c)^{n-2}\left[(n-1)a^2 + b(b+d) - c(nc+d)\right]. \tag{3}$$

However, since both sides of (3) are continuous functions of (a,b,c,d) on $\mathbb{C}^4$, (3) is also valid if $a+b+d=0$.

It is easy to verify that

$$\det(T_n) - 2\det(T_{n-1}) + \det(T_{n-2}) = 0, \quad n \geq 3,$$

so $\det(T_n) = a + bn$ (a, b constant). Since $\det(T_1) = 2$ and $\det(T_2) = 3$, $\det(T_n) = n+1$. Now (3) implies that

$$\det(G_n) = (b-c)^{n-2}\left[(n-1)a^2 + b(b+d) - c(nc+d)\right]. \qquad \square$$

Problem 22-5: Eigenvalues and Eigenvectors of a Certain Toeplitz Matrix

Proposed by William F. TRENCH: wtrench@trinity.edu, *Woodland Park, Colorado, USA.*

Find the eigenvalues and eigenvectors of the Toeplitz matrix $T = \{\alpha\cos(r-s)t + \beta\sin(r-s)t\}_{r,s=1}^{n}$, with $\alpha^2+\beta^2 \neq 0$, $n \geq 3$, and $t \neq k\pi$ with k an integer.

Solution 22-5.1 by the Proposer: William F. TRENCH.

Since $\cos(n+1)t - 2\cos t\cos nt + \cos(n-1)t = 0$ and $\sin(n+1)t - 2\cos t\sin nt + \sin(n-1)t = 0$, $\lambda = 0$ is an eigenvalue of T with multiplicity $n-2$, with associated eigenvectors

$$\begin{pmatrix} 1 \\ -2\cos t \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ -2\cos t \\ 1 \\ \vdots \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \dots, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \\ -2\cos t \\ 1 \end{pmatrix}.$$

To obtain the other eigenvalues and eigenvectors, we use the identity

$$\sum_{s=1}^{n} \sin(s-(n+1)/2)t\cos(s-(n+1)/2)t = 0, \tag{4}$$

and define

$$\sigma = \sum_{s=1}^{n}\cos^2(s-(n+1)/2)t = \frac{1}{2}\left(n + \frac{\sin nt}{\sin t}\right) \quad \text{and} \quad \tau = \sum_{s=1}^{n}\sin^2(s-(n+1)/2)t = \frac{1}{2}\left(n - \frac{\sin nt}{\sin t}\right). \tag{5}$$

Let $u = [u_1\ u_2\ \cdots\ u_n]'$ and $v = [v_1\ v_2\ \cdots\ v_n]'$, with $u_k = \cos(k-(n+1)/2)t$ and and $v_k = \sin(k-(n+1)/2)t$ $(k = 1,2,\dots,n)$. By writing

$$\cos(r-s)t = \cos(r-(n+1)/2)t\cos(s-(n+1)/2)t + \sin(r-(n+1)/2)t\sin(s-(n+1)/2)t,$$

$$\sin(r-s)t = \sin(r-(n+1)/2)t\cos(s-(n+1)/2)t - \cos(r-(n+1)/2)t\sin(s-(n+1)/2)t,$$

and using (4) and (5), we see that $Tu = \sigma(\alpha u + \beta v)$ and $Tv = \tau(-\beta u + \alpha v)$. Therefore, $au+bv$ is a λ-eigenvector of T if $(a\quad b)'$ is a λ-eigenvector of $A = \begin{pmatrix} \alpha\sigma & -\beta\tau \\ \beta\sigma & \alpha\tau \end{pmatrix}$. The eigenvalues of A are

$$\lambda_m = \frac{1}{2}\left\{n\alpha + (-1)^m\sqrt{-n^2\beta^2 + \frac{(\alpha^2+\beta^2)\sin^2 nt}{\sin^2 t}}\right\}, \quad m = 1,2.$$

These are also eigenvalues of T. If $\beta = 0$, then $\lambda_1 = \tau\alpha$, with associated eigenvector v, and $\lambda_2 = \sigma\alpha$, with associated eigenvector u. If $\beta \neq 0$ then $\beta\tau u + (\alpha\sigma - \lambda_m)v$ is a λ_m-eigenvector, $m = 1, 2$. □

Problem 22-6: Idempotency of a Certain Matrix Quadratic Form

Proposed by Michel VAN DE VELDEN: mvdv@fee.uva.nl; *University of Amsterdam, Amsterdam, The Netherlands*,
Shuangzhe LIU: liu@iso.iso.unibas.ch; *Universität Basel, Basel, Switzerland* &
Heinz NEUDECKER: heinz@fee.uva.nl; *Cesaro, Schagen, The Netherlands.*

Let A be an $n \times n$ symmetric idempotent matrix and let U be an $n \times k$ ($k < n$) matrix such that $U'U = I_k$. Characterize those matrices U for which $U'AU$ is idempotent.

Solution 22-6.1 by Hans Joachim WERNER: werner@united.econ.uni-bonn.de; *Universität Bonn, Bonn, Germany.*

We prove the following:

THEOREM. Let A be an $n \times n$ (real) symmetric idempotent matrix and let U be an $n \times k$ ($k < n$) matrix such that $U'U = I_k$. The following are then equivalent:

(i) $U'AU$ is idempotent.

(ii) $U'A$ is an inner inverse of AU, that is, $AUU'AAU = AU$.

(iii) $U'A$ is an outer inverse of AU, that is, $U'AAUU'A = U'A$.

(iv) $(AU)^\dagger = U'A$; $(\cdot)^\dagger$ indicating the Moore-Penrose inverse of $(\cdot)$.

(v) AUU' is idempotent.

(vi) $AUU' = UU'AUU'$.

(vii) AUU' is symmetric.

(viii) $\mathcal{R}(AU) \subseteq \mathcal{R}(U)$; $\mathcal{R}(\cdot)$ indicating the range (column space) of $(\cdot)$.

(ix) $\mathcal{R}(AU) = \mathcal{R}(U) \cap \mathcal{R}(A)$.

(x) $\mathcal{R}(U) = [\mathcal{R}(U) \cap \mathcal{R}(A)] \oplus [\mathcal{R}(U) \cap \mathcal{N}(A)]$ with $\oplus$ indicating direct sum and $\mathcal{N}(\cdot)$ indicating the null space of $(\cdot)$.

(xi) $\mathcal{N}(U') = [\mathcal{N}(U') \cap \mathcal{R}(A)] \oplus [\mathcal{N}(U') \cap \mathcal{N}(A)]$.

(xii) $\mathcal{R}(A) = [\mathcal{R}(A) \cap \mathcal{R}(U)] \oplus [\mathcal{R}(A) \cap \mathcal{N}(U')]$.

(xiii) $\mathcal{N}(A) = [\mathcal{N}(A) \cap \mathcal{R}(U)] \oplus [\mathcal{N}(A) \cap \mathcal{N}(U')]$.

(xiv) U consists only of eigenvectors of A.

(xv) Each eigenvector of UU' is also an eigenvector of A and vice versa.

Proof. We first show that (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (i). Since $U'AU$ is symmetric, it is idempotent if and only if it is the *orthogonal* projector onto $\mathcal{R}(U'AU) = \mathcal{R}(U'A)$. Then $U'AUU'A = U'A$. Transposing this equation and exploiting $A^2 = A$ results in $AUU'AAU = AU$, thus showing that $U'A$ is an inner inverse (g-inverse) of AU. So we have (i) $\Rightarrow$ (ii). Since $U'A$ is an inner inverse of AU if and only if $U'A$ is an outer inverse of AU [transpose the corresponding defining equations], it is also clear that (ii) implies (iii). If (iii) holds, then $U'AAUU'A = U'A$. Postmultiplication with U results in (i); notice that $A^2 = A$. Next, (i) $\Longleftrightarrow$ (iv) will be shown. If (i) holds, then from the foregoing, we have (ii) and (iii). Since, in addition, $AUU'A$ and $U'AAU = U'AU$ are evidently both symmetric matrices, $U'A$ satisfies the four Penrose equations, and we arrive at (iv). Conversely, if $U'A = (AU)^\dagger$, then $U'A$ is an inner inverse of AU. That is, we have (ii) and so (i). That (i) $\Rightarrow$ (v) $\Rightarrow$ (viii) $\Rightarrow$ (vi) $\Rightarrow$ (vii) $\Rightarrow$ (i) is seen as follows. Let (i) hold. Then $U'AUU'A = U'A$ [see the above proof of (i) $\Rightarrow$ (ii)]. Postmultiplying the transposed equation by U' results in

$AUU'AUU' = AUU'$, which is (v). From (v), we get $AUU'AU = AU$ or, equivalently, $A(I - UU')AU = 0$, since $U'U = I$ and $A^2 = A$. This, of course, implies $U'A(I - UU')AU = 0$. Since $U'A(I - UU')AU$ is a nonnegative definite and symmetric matrix, the latter happens if and only if $(I - UU')AU = 0$ and we arrive at (viii). From (viii) we get (vi), since UU' is the orthogonal projector onto $\mathcal{R}(U)$. Trivially, (vi) $\Rightarrow$ (vii). And (vii) clearly implies $U'AUU' = U'AAUU' = U'AUU'AUU'$ and thus $U'AU = (U'AU)^2$, which is (i). Since A is the orthogonal projector onto $\mathcal{R}(A)$ along $\mathcal{R}(I - A) = \mathcal{N}(A)$ and since $U = AU + (I - A)U$, we have (viii) $\iff$ (ix) $\iff$ (x). Since, in view of (vii) and (ix), $\mathcal{R}(UU'A) = \mathcal{R}(AUU') = \mathcal{R}(AU) = \mathcal{R}(A) \cap \mathcal{R}(U)$, and since UU' is the orthogonal projector onto $\mathcal{R}(U)$ along $\mathcal{N}(U') = \mathcal{R}(I - UU')$, we get (viii) $\iff$ (xii) on similar lines. Next check that AUU' is idempotent and symmetric if and only if $A(I - UU')$ is so. With this in mind, it is now clear that (v) is equivalent to (xi). Since AUU' is idempotent and symmetric if and only if $UU'(I - A)$ is so and since $\mathcal{N}(A) = \mathcal{R}(I - A)$, (v) $\iff$ (xiii) should also be plain. That (xiv) and (xv) are both equivalent to (x) [or (xii)], is now evident. This completes the proof. □

A solution was also received from the Proposers: Michel VAN DE VELDEN, Shuangzhe LIU & Heinz NEUDECKER.

Problem 22-7: Characterization of a Square Matrix in an Inner-product Inequality

Proposed by Fuzhen ZHANG,: zhang@polaris.ncs.nova.edu; *Nova Southeastern University, Fort Lauderdale, Florida, USA.*

Let A be an $n \times n$ complex matrix. If, for some real number α, $|(Ay, y)| \leq (y, y)^\alpha$ for all n-column complex vectors y, what can be said about A?

Solution 22-7.1 by the Proposer: Fuzhen ZHANG.

Either $A = 0$, or $\alpha = 1$ and A is a matrix with numerical radius less than or equal to 1. To see this, if $A = 0$ then we have nothing to show. So assume $A \neq 0$. Then α has to equal 1. Otherwise let y_0 be a vector such that $(Ay_0, y_0) \neq 0$ and replace y with ty_0. Letting $t \to 0^+$ or $+\infty$ will yield a contradiction to the inequality. When $\alpha = 1$, the given inequality says that A is a matrix of numerical radius no more than 1. □

Remark. The problem may be restated as: $|(Ay, y)| \leq (y, y)^\alpha$ for all y if and only if $|(Ay, y)| \leq (y, y)$ for all y.

Two More New Problems

Problem 23-6: Linear Combinations and Eigenvalues

Proposed by Jos M. F. TEN BERGE: j.m.f.ten.berge@ppsw.rug.nl; *Rijksuniversiteit Groningen, Groningen, The Netherlands.*

Suppose we have two real matrices of order $p \times p$, with p even, and with all eigenvalues imaginary. Is it possible to find p linear combinations of the matrices that have at least one real-valued eigenvalue?

I expect that this is not always possible and that the set of matrix pairs that does allow real eigenvalues for linear combinations has positive measure. Is there a place in the literature where I can find such things?

Problem 23-7: An Inequality Involving Rank and Matrix Powers

Proposed by Yongge TIAN, *Concordia University, Montréal, Québec, Canada.*

Let A be a square matrix of size $n \times n$. Show that there are n vectors $x_0, x_1, \ldots, x_{n-1}$ such that the square matrix $(x_0, Ax_1, A^2x_2, \cdots, A^{n-1}x_{n-1})$ is nonsingular if and only if

$$\min\{1 + \operatorname{rank} A, \quad 2 + \operatorname{rank} A^2, \quad \cdots, \quad n - 1 + \operatorname{rank} A^{n-1}\} \geq n.$$

Problems 23-1 through 23-5 are on page 28. Please submit solutions, as well as new problems, in LaTeX — (a) embedded as text in an e-mail to styan@total.net *and* (b) nicely printed (2 copies please) by p-mail to George Styan, PO Box 270, Franklin, VT 05457-0270, USA. We look forward particularly to receiving solutions to Problems 18-1, 19-3b & 21-2!

IMAGE Problem Corner: New Problems

Please submit solutions, as well as new problems, both in LaTeXcode embedded as text only by e-mail to styan@total.net and by regular air mail to George Styan, PO Box 270, Franklin, VT 05457-0270, USA. We look forward particularly to receiving solutions to Problems 18-1, 19-3b & 21-2!

Problem 23-1: The Expectation of the Determinant of a Random Matrix

Proposed by Moo Kyung CHUNG: chung@math.mcgill.ca; *McGill University, Montréal, Québec, Canada.*

Let the $m \times n$ random matrix X be such that $\mathrm{vec}(X)$ is distributed as multivariate normal $N(0, A \otimes I_n)$ where 'vec' indicates the vectorization operator for a matrix, the $m \times m$ matrix A is symmetric non-negative definite, $\otimes$ stands for the Kronecker product, $m > n$, and I_n is the $n \times n$ identity matrix. For a given $m \times m$ symmetric matrix C, find $\mathrm{E}\det(X'CX)$ in closed form involving only C and A. Is this possible? (Finite summation would also be fine.)

Problem 23-2: The Equality of Two 4×4 Determinants

Proposed by S. W. DRURY: drury@math.mcgill.ca; *McGill University, Montréal, Québec, Canada.*

Show that

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ a_1b_1 & a_2b_2 & a_3b_3 & a_4b_4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ a_1 & a_2 & a_3 & a_4 \\ b_2 & b_1 & b_4 & b_3 \\ a_1b_2 & a_2b_1 & a_3b_4 & a_4b_3 \end{vmatrix}.$$

The solution of the proposer, who has no imagination, is to expand each determinant and to match the resulting expansions term for term. The proposer has "first dibs" on this solution. Respondents are therefore asked to provide a more elegant solution.

Problem 23-3: An Inequality Involving a Special Hadamard Product

Proposed by Shuangzhe LIU: liu@iso.iso.unibas.ch; *Universität Basel, Basel, Switzerland.*

Let $A > 0$ be an $n \times n$ positive definite Hermitian matrix with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_n$, A^T be the transpose of A, A^{-T} be the inverse of A^T, I_n be an $n \times n$ identity matrix and $\odot$ denote the Hadamard product. Show that then, in the Löwner ordering,

$$A \odot A^{-T} \leq \frac{\lambda_1^2 + \lambda_n^2}{2\lambda_1\lambda_n} I_n.$$

Problem 23-4: Trace and A Partitioned Matrix

Proposed by Heinz NEUDECKER: heinz@fee.uva.nl; *Cesaro, Schagen, The Netherlands.*

Consider the $p \times (p+1)$ matrix $X' = (x : Y')$, where Y is $p \times p$ nonsingular and $x \neq \frac{1}{p}Y'e_p$ with e_p the $p \times 1$ vector with each element equal to 1. Let $M_p := I_p - \frac{1}{p}e_pe_p'$ denote the $p \times p$ centering matrix. Prove then that the trace

$$\mathrm{tr}\big((X'M_{p+1}X)^{-1}Y'M_pY\big) = p - 1.$$

Problem 23-5: An Inequality Involving Diagonal Elements and Eigenvalues

Proposed by Alicja SMOKTUNOWICZ: smok@im.pw.edu.pl; *Warsaw University of Technology, Warsaw, Poland.*

Prove that each eigenvalue λ of $A \in C^{n \times n}$ such that $\lambda \neq a_{i,i}$ for all $i = 1, \ldots, n$, satisfies

$$\sum_{k=1}^{n} \frac{{r_k}^2}{|\lambda - a_{k,k}|^2} \geq \frac{n}{n-1}, \qquad \text{where} \qquad r_k = \sqrt{\sum_{j=1, j\neq k}^{n} |a_{k,j}|^2}.$$

$\Longleftarrow$ *Please find two more new problems at the bottom of page 27.*