The Bulletin of the International Linear Algebra Society

IMAGE

Serving the International Linear Algebra Community
Issue Number 43, pp. 1-44, Fall 2009

Contents continued on page 2

Journal News

ILAS News

Employment

IMAGE Problem Corner: Solutions of Old Problems

IMAGE Problem Corner: New Problems

Other Conferences of Interest

January 13-16, 2010, AMS/MAA Joint Meetings, San Francisco, CA, USA. Includes special sessions on Inverse Problems, Functional Analysis and Operator Theory, and Innovative and Effective Ways to Teach Linear Algebra

July 12-16, 2010, SIAM Annual Meeting, Pittsburgh, PA, USA. Includes a Linear Algebra track; deadline for minisymposia proposals is January 12, 2010

August 19-27, 2010, International Congress of Mathematicians, ICM-2010, Hyderabad, India

July 18-22, 2011, International Conference on Industrial and Applied Mathematics, Vancouver, British Columbia, Canada

August 22 -26, 2011, 17th ILAS Conference, Braunschweig, Germany

June 18-22, 2012, SIAM Conference on Applied Linear Algebra, Valencia, Spain

UPCOMING CONFERENCES AND WORKSHOPS

The 16th ILAS Conference
Pisa, Italy
June 21-25, 2010

The 16th ILAS Conference will be held in Pisa, Italy. It will open at 9:00 on Monday June 21 and close at 18:00 on Friday June 25. The Plenary Speakers will be Rajendra Bhatia, Richard A. Brualdi, Pauline van den Driessche, Nick Higham, Beatrice Meini, Vadim Olshevsky, Zdenek Strakos, Daniel B. Szyld, and Luis Verde Star.

Special Lectures will be given by Oliver Ernst (SIAG/LA), Olga Holtz (Taussky-Todd), Lek-Heng Lim (LAA), Cleve Moler (Hans Schneider Prize), and Beresford N. Parlett (Hans Schneider Prize).

Six minisymposia will highlight the recent developments in specific areas of linear algebra and its applications. The titles and organizers are:

- Structured Matrices (Yuli Eidelman, Lothar Reichel, Marc Van Barel)
- Markov Chains (Steve Kirkland, Michael Neumann)
- Nonnegative Matrices (Judi McDonald, Michael Tsatsomeros)
- Matrix Functions and Matrix Equations (Chun-Hua Guo, Valeria Simoncini)
- Combinatorial Linear Algebra (Shaun Fallat, Bryan Shader)
- Linear Algebra Education (Avi Berman, Steven J. Leon)
- Matrix Inequalities (Chi-Kwong Li and Fuzhen Zhang)

The Scientific Committee consists of Avi Berman, Michele Benzi, Dario A. Bini, Luca Gemignani, Leslie Hogben, Steve Kirkland, Julio Moro, Ilia Spitkovsky, Francoise Tisseur, and Eugene Tyrtyshnikov. The Local Organizing Committee is Dario A. Bini (Chair), Gianna Del Corso, Bruno Iannazzo, Beatrice Meini, Ornella Menchi, and Federico Poloni.

The conference proceedings will appear as a special issue of Linear Algebra and its Applications. The Guest Editors for this issue are: D. A. Bini, A. Boettcher, L. Gemignani, L. Hogben, and F. Tisseur.

The sponsors of this conference are the Universita' di Pisa Dipartimento di Matematica "Leonida Tonelli", Dipartimento di Matematica Applicata "Ulisse Dini", Dipartimento di Informatica, Centro Interdipartimentale "E. Piaggio", and Facolta' di Economia; Taylor & Francis Group; ILAS; and the SIAM Activity Group on Linear Algebra.

The deadline for early registration is January 31, 2010. The registration fee is 220 Euros if paid by January 31 and 260 Euros otherwise. For students, the registration fee is 160 Euros if paid by January 31 and 260 Euros otherwise. For details about registration, accommodations, program, etc., visit the conference website http://www.dm.unipi.it/~ilas2010/index.php?page=proceed. For more information, send email to bini@dm.unipi.it with subject ilas2010.

Applied Linear Algebra Conference in Honor of Hans Schneider
Department of Mathematics, Faculty of Science, Novi Sad, Serbia
May 24-28, 2010

This conference on Applied Linear Algebra, in honor of Hans Schneider, is inspired by the success of two previous conferences, Applied Linear Algebra in honor of Richard Varga, 2005, Palic; and Applied Linear Algebra in honor of Ivo Marek, 2008, Novi Sad. ALA 2010 has a similar purpose, to review the numerous contributions of the honree and to report and discuss recent progress in the field through the participation of international leaders who will gather in his honor. In addition, the 10th GAMM Workshop on Applied and Numerical Linear Algebra with special emphasis on Positivity will be organized as a part of ALA 2010. A special issue of Linear Algebra and its Applications will be devoted to selected papers presented during the conference.

For more information, visit http://www.dmi.uns.ac.rs/events/ala2010.

19th International Workshop on Matrices and Statistics
Shanghai Finance University, Shanghai, China
June 5-8, 2010

The 19th International Workshop on Matrices and Statistics will be held June 5-8, 2010, at Shanghai Finance University, Shanghai. The purpose is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. The workshop will provide a forum through which statisticians and mathematicians may be better informed of the latest theoretical developments and techniques as well as their use in related application areas. Participants will be able to take advantage of the opportunity to exchange ideas with researchers from a number of countries, who are working in these areas.

The conference seeks a wide variety of invited and contributed papers. Special sessions on computational matrix procedures, statistical applications in economics and matrix methods in applied probability are proposed. A list of invited speakers and dates for submission of abstracts and registration will be available shortly on the workshop website: http://www1.shfc.edu.cn/iwms/index.asp. Submission of abstracts and registration will also be done through the website. Publication of the workshop proceedings is being explored. For enquiries regarding local arrangements, use the email address: IWMS2010@shfc.edu.cn

Social activities, including an excursion and workshop banquet, will be held. In addition, a pre-workshop 14-day Tour of China, starting in Beijing on Friday May 21, has been arranged. Details may be found on the workshop website.

The International Organizing Committee (IOC) is George P. H. Styan, Honorary Chair of IOC and Honorary Chair of ISC (Canada)*; Zhong-Ci Shi, Honorary Chair of IOC, Chair of ISC (China)*; Jeffrey J. Hunter, Chair of the IOC (New Zealand)*; Hans Joachim Werner, Vice-Chair of the IOC (Germany)*; S. Ejaz Ahmed (Canada); Zhi-Dong Bai (Singapore)*; Zhong-Zhi Bai (China)*; Guoliang Chen (China)*; Ben-Yu Guo (China)*; Er-Xiong Jiang (China)*; Tian-Gang Lei (China)*; Augustyn Markiewicz (Poland); Simo Puntanen (Finland); Götz Trenkler (Germany); Julia Volaufova (USA); Dietrich von Rosen (Sweden); Musheng Wei (China)*; and Xue-Jun Xu (China)*. *Denotes those who are also members of the International Scientific Committee (ISC).

The Local Organizing Committee is Yonghui Liu (China); Chair, Rong-Xian Yue, Co-Chair (China); Yongge Tian; Baoxue Zhang; Rui Li; Rongqiang Che; Yong Fang; Manrong Wang; Qian Wang; and Lei Fu.

Sponsors of the workshop include Shanghai Finance University, National Natural Science Foundation of China, Shanghai Normal University, and the Division of Scientific Computation E-Institute of Shanghai Universities. The International Linear Algebra Society endorses IWMS-2010.

Society for Industrial and Applied Mathematics

SIAM International Summer School on Numerical Linear Algebra (ISSNLA)
Fasano, Italy
June 7-18, 2010

This Summer School, co-sponsored by SIAM and the Instituto per le Applicazioni del Calcolo, will take place at Selva di Fasano, Brindisi, Italy, during June 7-18, 2010. Fasano is located in southern Italy, very near the Adriatic Sea.

This is the second ISSNLA, which coincides with the first Gene Golub Summer School, funded by Gene Golub's legacy to SIAM. These ISSNLA workshops are organized by the SIAM Activity Group on Linear Algebra. The main goal is to offer accessible courses on current developments in Numerical Linear Algebra and its applications to other disciplines.

The following four courses will be given in 2010:

- Minimizing communication in numerical linear algebra, James Demmel, University of California at Berkeley, USA
- Nonlinear eigenvalue problems: analysis and numerical solution, Volker Mehrmann, Technische Universitaet Berlin, Germany
- From Matrix to Tensor: The Transition to Computational Multilinear Algebra, Charles Van Loan, Cornell University, Ithaca, New York, USA
- Linear Algebra and Optimization, Margaret H. Wright, Courant Institute, New York University, USA

These courses will focus on recent research topics that have reached a significant maturity and whose impact is widely recognized by the community, but that are not usually included in text books or in basic courses at the doctoral level. The workshop is mainly intended for doctoral students in any field where methods and algorithms of Numerical Linear Algebra are used.

The summer school is geared towards graduate students, with a limit of 50 students. There will be no registration fee. Funding for local accommodations and/or local expenses will be available for some of the participants. Limited travel funds may also be available.

The deadline for applications is February 1, 2010. For more details, watch the conference website http://www.ba.cnr.it/ISSNLA2010.

The Advisory Committee consists of:

Dario Bini (Università di Pisa, Italy)
Jim Demmel (University of California at Berkeley, USA)
Iain Duff (Rutherford Appleton Laboratory, UK)
Anne Greenbaum (University of Washington, USA)
Martin Gutknecht (ETH Zurich, Switzerland)
Nick Higham (University of Manchester, UK)
Rich LeHoucq (Sandia National Lab, Albuquerque, USA)
John Lewis (Cray Inc., USA)
Volker Mehrmann (Technische Universität Berlin, Germany)
Dianne O'Leary (University of Maryland, USA)
Michael Overton (New York University, USA)
Lothar Reichel (Kent State University, USA)
Axel Ruhe (KTH Stockholm, Sweden)
Henk van der Vorst (Utrecht University, The Netherlands)
Paul van Dooren (Catholic University of Louvain, Belgium).

The Steering Committee consists of Froilán Dopico (Universidad Carlos III de Madrid, Spain), Andreas Frommer (Bergische Universität Wuppertal, Germany), Ilse Ipsen (North Carolina State University), and Daniel Szyld (Temple University).

The Local Organizing Committee members are Fasma Diele (IAC-CNR), Teresa Laudadio (IAC-CNR), Nicola Mastronardi (IAC-CNR), Filippo Notarnicola (IAC-CNR), Valeria Simoncini (Università di Bologna), and Nicola Fracasso (Administrator/Manager IAC-CNR).

4th Annual International Conference on Mathematics & Statistics Athens, Greece June 14-17, 2010

The Mathematics and Statistics Research Unit of the Athens Institute for Education and Research (ATINER) will hold its 4th International Conference in Athens, Greece, June 14-17, 2010. Papers from all areas of Mathematics, Statistics, Mathematics and Engineering and Mathematics and Education are welcome.

Selected reviewed papers will be published in a special volume of the Conference Proceedings. To contribute, send an abstract of about 300 words by December 14, 2009, to Dr. Vladimir Akis (atiner@atiner.gr). For more information, see the conference website www.atiner.gr/docs/Mathematics.htm.

10th Workshop on Numerical Ranges and Numerical Radii Krakow, Poland June 27-29, 2010

The 10th Workshop on "Numerical Ranges and Numerical Radii" will be held in the beautiful old city Krakow, Poland from June 27-29, 2010, immediately following the 2010 ILAS Conference at Pisa, Italy, June 21-25, 2010. The workshop is endorsed by ILAS.

The purpose of the workshop is to stimulate research and foster interaction of researchers interested in the subject. The informal workshop atmosphere will facilitate the exchange of ideas from different research areas and, hopefully, the participants will leave informed of the latest developments and newest ideas.

The Organizers are Chi-Kwong Li, College of William and Mary; Franciszek Hugon Szafraniec, Uniwersytet Jagiellonski; and Jaroslav Zemanek, Instytut Matematyczny PAN. There will be no registration fee. A special issue of Linear and Multilinear Algebra will be devoted to the papers presented at the workshop. For more information, visit the conference website, http://www.math.wm.edu/~ckli/wonra10.html. To see some background about the subject and previous meetings, visit the website http://www.math.wm.edu/~ckli/wonra.html .

Zero-One Matrix Theory and Related Topics Coimbra, Portugal June 17-19, 2010

This meeting will be held at the University of Coimbra. The invited speakers will be Alexander Barvinok (University of Michigan, USA), Adrian Bondy (Université Claude Bernard Lyon 1, France), Richard A. Brualdi (University of Wisconsin-Madison, USA), Domingos M. Cardoso (Universidade de Aveiro, Portugal), Geir Dahl (University of Olso, Norway), Michael Drmota (Technische Universität Wien, Austria), Catherine Greenhill (University of New South Wales, Australia), Gyula O. H. Katona (Alfréd Rényi Institute of Mathematics, Hungary), Hadi Kharaghani (University of Lethbridge, Canada), Christian Krattenthaler (Universität Wien, Austria), Brendan McKay (Australian National University, Australia), Irene Sciriha (University of Malta, Malta), and Herbert S. Wilf (University of Pennsylvania, USA) - ILAS Lecturer.

Contributed papers are still sought. For more information about the conference, see Image issue 42, p. 8, and visit the conference website http://www.mat.uc.pt/~cmf/01MatrixTheory.

This meeting is endorsed by the International Linear Algebra Society and sponsored by the Center for Mathematics and the Department of Mathematics of the University of Coimbra, and by the American Mathematical Society.

University of Coimbra Library

International Workshop on Accurate Solution of Eigenvalue Problems VIII
Berlin, Germany
June 28-July 1, 2010

IWASEP VIII will be held June 28-July 1, 2010, in cooperation with the GAMM Activity Group Applied and Numerical Linear Algebra, at TU Berlin. Its purpose is to bring together experts on accuracy issues in the numerical solution of eigenvalue problems for research presentations and discussions. There will be no parallel sessions; research presentations will be 30 to 40 minutes each, with sufficient time for discussions. Specially titled discussion/working groups and a poster session will also be organized.

The invited speakers are Rafikul Alam, Indian Inst. of Technology Guwahati; Peter Benner, Chemnitz University of Technology; James Demmel, University of California at Berkeley; Louis Komzsik, Siemens PLM Software; Wen-Wei Lin, National Chiao Tung University (TBC); Julio Moro, Universidad Carlos III de Madrid; Beresford Parlett, University of California at Berkeley; Valeria Simoncini, University of Bologna; Pete Stewart, University of Maryland; Eugen Tyrtyshnikov, Russian Academy of Sciences; Nick Trefethen, Oxford University; and Marc Van Barel, Katholieke Universiteit Leuven.

The organizers are Jesse Barlow, Penn. State Univ., USA; Zlatko Drmac, Univ. of Zagreb, Croatia; Volker Mehrmann, TU Berlin, Germany, (Chair); Ivan Slapnicar, Univ. of Split, Croatia; and Kresimir Veselic, Fernuniversitaet, Hagen, Germany.

Linear Algebra and its Applications has published three special issues related to the previous workshops in this series: 309/1-3 (2000), 358 (2003), and part of 417/2-3 (2006). Also the SIAM Journal on Matrix Analysis and Applications published a special issue in Vol. 28/4 (2006), and reports on those workshops were published in SIAM News.

The deadline for abstracts and for registration is April 1, 2010. The organizers welcome proposals for talks and posters that are consistent with the theme of the meeting. A prize will be given for the best poster. For more information, visit http://www3.math.tu-berlin.de/iwasep8/.

19th International Symposium on
Mathematical Theory of Networks and Systems
Budapest, Hungary
July 5-9, 2010

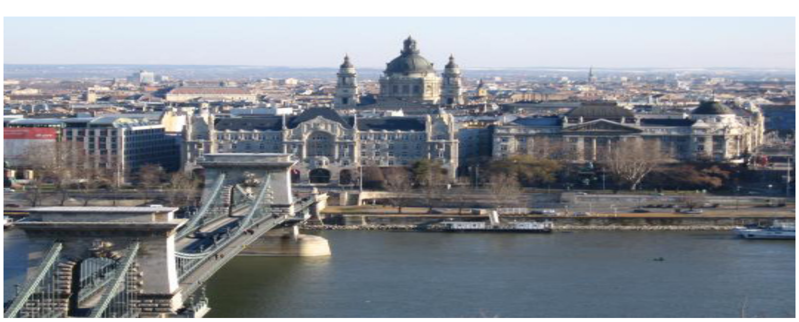
The 19th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2010) will take place at ELTE, the University Congress Center, in Budapest, Hungary, July 5-9, 2010. The Symposium is hosted by the Eötvös Loránd University (ELTE) and the Computer and Automation Research Institute of the Hungarian Academy of Sciences (MTA SZTAKI).

MTNS 2010 is a prime conference in the general area of mathematical system theory. The symposium is interdisciplinary and attracts mathematicians, engineers and researchers working in any aspect of system theory and its applications. A prime objective of the MTNS 2010 Symposium is to explore and present mathematics as a key technology for the 21st century.

The scientific programme will cover 12 main areas: Biological Systems, Communication Systems, Computing, Control and Systems Theory, Cooperative Systems, Economics and Systems Theory, Hybrid Systems, Mechanical Systems, Networked Control, Signal Processing, Stochastic Systems, and Systems Inspired Mathematics.

For further information, please visit the conference web site: www.conferences.hu/mtns2010

The General Chair of the conference is Prof. György Michaletzky and the IPC Chair is Prof. László Gerencsér.



View of Budapest and Chain Bridge across the Danube

19th China Matrix Theory and Applications International Conference Shanghai University, Shanghai, China July 18 - 22, 2010

The purpose of this conference is to stimulate research and to foster the interaction of researchers. The topics cover matrix theory, linear algebra and applications, including traditional linear algebra, combinational linear algebra and numerical linear algebra. The conference will provide a forum through which scientists may be better informed of the latest developments and newest techniques and may exchange ideas with researchers from several different countries.

The invited speakers are: Man-Duen Choi, University of Toronto, Toronto, Canada; Yongdo Lim, Kyungpook National University, Taegu, Republic of Korea; Koenraad M.R. Audenaert, University of London, Egham, United Kingdom; Chi Kwong Li, Ferguson Professor of Mathematics, College of William & Mary, Williamsburg, VA, USA; PeiYuan Wu, National Chiao Tung University, Hsinchu, Taiwan; Zhaojun Bai, University of California at Davis, Davis, CA USA; Qingwen Wang, Shanghai University, Shanghai, China; Ming Gu, University of California at Berkeley, CA USA; Daniel Kressner, ETH Zurich, Switzerland; Biswa Nath Datta, Northern Illinois University, De Kalb, IL USA; Natalia Bebiano, University of Coimbra, Coimbra, Portugal; and Zhinan Zhang, Xinjiang University, China.

The members of the Organizing Committee are: Shufang Xu, Peking University, Beijing, China; Yimin Wei, Fudan University, Shanghai , China; Xingzhi Zhan, East China Normal University, Shanghai, China; Yongge Tian, China Central University of Finance and Economy, Beijing, China; Hongguo Xu, University of Kansas, Lawrence, KS USA; Rencang Li,University of Texas, Arlington, TX USA; Erxiong Jiang (Chair), Shanghai University, Shanghai, China; Chuanqing Gu, Shanghai University, Shanghai, China; Zhongzhi Bai, Chinese Academy of Science, Beijing, China.

The members of Local Organizing Committee are: Qingwen Wang, (Chair) Shanghai University, Shanghai, China; Xinjian Xu, Shanghai University, Shanghai, China; Qin Zhang, Shanghai University, Shanghai, China; Xiaomei Jia, Shanghai University, Shanghai, China; Jianxin Yang, Shanghai University, Shanghai, China; Jianjun Zhang, Shanghai University, Shanghai, China; Xiaodong Zhang, Jiaotong University, Shanghai, China; Yonghui Liu, Shanghai Finance University, Shanghai, China; Changqing Xu, Zhejiang Forestry University, Hangzhou, China.

For details, visit the conference web page http://math.shu.edu.cn/CLAS2010/Default.aspx.

Participants wishing to present a contributed talk should submit by e-mail an extended abstract (1-2 pages) written in Latex to Prof. Qin Zhang (zhangqin@staff.shu.edu.cn or shug505@163.com). The deadline for submissions is April 30, 2010. Notification of acceptance will be given by June 10, 2010.

Shanghai University Main Gate

The International Conference on Trends and Perspectives in Linear Statistical Inference LinStat2010 Tomar, Portugal July 27-31, 2010

LinStat 2010 will be held in Tomar, Portugal. The purpose of this conference is to bring together researchers to discuss current developments in a variety of aspects of statistics and its applications, and also to encourage young scientists. Prizes will be awarded to graduate students or scientists with a recently completed Ph.D., for the best poster and the best talk.

Young Scientists awarded at Linstat 2008, which was held in Będlewo, Poland, will be Invited Speakers at LinStat 2010.

The conference organizers are Joao T. Mexia (Scientific Committee) and Francisco Carvalho (Organizing Committee).

To view the invited speakers and other information, visit the conference website http://www.linstat2010.ipt.pt.

International Conference on Recent Trends in Graph Theory and Combinatorics (ICRTGC-2010) Cochin, India August 12-15, 2010

The Conference on Recent Trends in Graph Theory and Combinatorics will be held August 12-15, 2010, in Cochin, India. This is a Satellite Conference of the International Congress of Mathematicians, ICM-2010, which will be held at Hyderabad, India on August 19-27, 2010.

The Covener of ICRTGC-2010 is Ambat Vijayakumar, Department of Mathematics, Cochin University of Science and Technology, Cochin-682 022, India. Send email to icrtgc2010@gmail.com or icrtgc2010@cusat.ac.in. For more information, see Image issue 42, p. 9 and visit the conference website http://ams.org/mathcal/info/2010_aug12-15_cochin.html.

Mural
Mattancherry Palace,
Cochin, India

2nd IMA Conference on Numerical Linear Algebra and Optimization University of Birmingham, UK September 13-15, 2010

The Institute of Mathematics and its Applications (IMA) and the University of Birmingham are pleased to announce the Second IMA Conference on Numerical Linear Algebra and Optimization, Sept. 13-15, 2010. Future meetings will be held biennially. The meeting is co-sponsored by SIAM and ILAS, whose members will receive the IMA members' registration rate.

The success of modern codes for large-scale optimization is heavily dependent on the use of effective tools of numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimization problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest.

Conference topics include any subject that could be of interest to both communities, such as direct and iterative methods for large sparse linear systems, eigenvalue computation and optimization, large-scale nonlinear and semidefinite programming, effect of round-off errors, stopping criteria and embedded iterative procedures, optimization issues for matrix polynomials, fast matrix computations, compressed/sparse sensing, PDE-constrained optimization, and applications and real time optimization.

The invited speakers are Larry Biegler (Carnegie Mellon), Nick Higham (University of Manchester), Adrian Lewis (Cornell University), Volker Mehrmann (Technische Universität Berlin), Mike Saunders (Stanford University), Valeria Simoncini (Università di Bologna), Jared Tanner (University of Edinburgh), and Andy Wathen (University of Oxford).

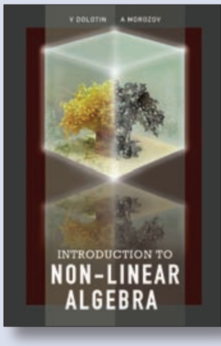
The Organising Committee is Roy Mathias, co-chair (University of Birmingham), Michal Kočvara, co-chair (University of Birmingham), Iain Duff (Rutherford Appleton Laboratory), Nick Gould, (Rutherford Appleton Laboratory), Daniel Loghin (University of Birmingham), Alastair Spence (University of Bath), Zdeněk Strakoš, (Charles University, Prague) and Philippe Toint (University of Namur).

For more information such as how to propose a mini symposium, submit an abstract and register, visit the conference website http://www.ima.org.uk/Conferences/2nd_numerical_linear_algebra.html.

Aston Webb Building, University of Birmingham

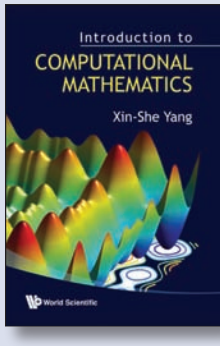
World Scientific
Connecting Great Minds

TEXTBOOKS



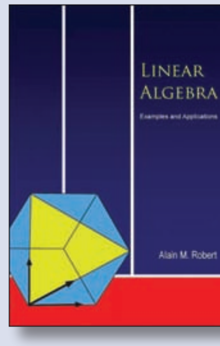
by **V Dolotin** & **A Morozov**
(ITEP, Russia)

288pp; ISBN: 978-981-270-800-7, US$55



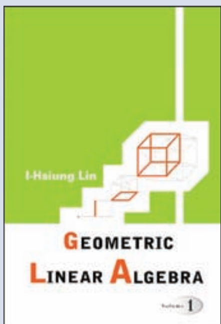
by **Xin-She Yang**
(University of Cambridge, UK)

260pp; ISBN: 978-981-281-817-1, US$69



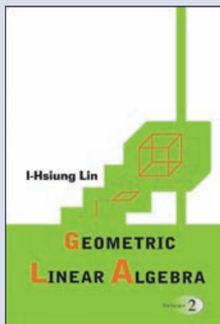
by **Alain M Robert**
(Université de Neuchâtel, Switzerland)

388pp; ISBN: 978-981-256-432-0, US$75;
ISBN: 978-981-256-499-3(pbk), US$39



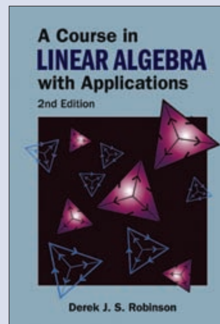
(Volume 1)
by **I-Hsiung Lin**
(National Taiwan Normal University, China)

880pp; ISBN: 978-981-256-087-2, US$109;
ISBN: 978-981-256-132-9(pbk), US$59



(Volume 2)
by **I-Hsiung Lin**
(National Taiwan Normal University, Taiwan)

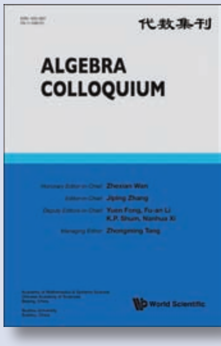
832pp; ISBN: 978-981-270-775-8(pbk), US$99



(2nd Edition)
by **Derek J S Robinson**
(University of Illinois at Urbana-Champaign, USA)

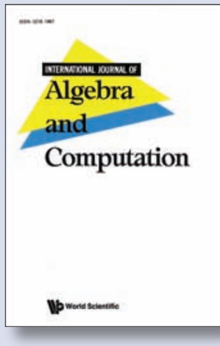
452pp; ISBN: 978-981-270-023-0, US$95;
ISBN: 978-981-270-024-7(pbk), US$47

JOURNALS



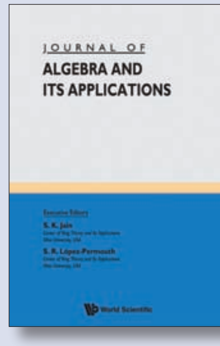
www.worldscinet.com/ac

Editor-in-Chief
Jiping Zhang
(Peking University, China)



www.worldscinet.com/ijac

Managing Editors
S W Margolis
(Bar Ilan University, Israel) &
J Meakin
(University of Nebraska-Lincoln, USA)



www.worldscinet.com/jaa

Executive Editors
S K Jain & **S R López-Permouth**
(Ohio University, USA)

REPORTS ON CONFERENCES AND WORKSHOPS

Workshop on Spectral Graph Theory With Applications to Computer Science, Combinatorial Optimization and Chemistry Rio de Janeiro, Brazil December 1-4, 2008

Report by Vladimir Nikiforov

This workshop was organized by the Production Engineering Program of the Federal University of Rio de Janeiro. The sessions were held at the hospitable Military Institute of Engineering, next to the famous mount Sugar Loaf. The meeting was sponsored and funded by a number of prominent Brazilian organizations and businesses, whose generous contributions are acknowledged at the workshop website: http://www.sgt.pep.ufrj.br/~tegrio/.

The theory of graph spectra is now a well established field of research in mathematics and in several applied sciences like chemistry, computer science and operational research, to name a few. In the last few decades an astonishing body of work has been produced in this area. In recognition of this strong development, the workshop was organized as a truly international forum that was attended by 74 participants from all around the globe.

The workshop brought together leading researchers on graph spectra that presented state of the art results and discussed new challenges. The topics included also applications to computer science, combinatorial optimization, chemistry and other branches of science.

The members of the Scientific Committee, chaired by Dragos Cvetkovic, all renowned specialists in spectral graph theory, delivered 12 plenary presentations of one hour. There were also 9 plenary presentations of 30 min and other 28 short communications, split into two parallel sessions.The topics that received the most attention included graph energy, digraph spectra, eigenvalue bounds and the spectrum of the signless Laplacian.

With ample opportunities for discussions and communication, the meeting was truly rewarding for the participants, among whom were many young researchers. The organizing committee, chaired by Nair de Abreu, did a fantastic job: in a splendid setting, the meeting ran smoothly and efficiently from the registration to the final cocktail. There were several social events, all turned unforgettable by the warm Brazilian hospitability.

The results presented at the workshop will appear, subject to refereeing and approval, in a special volume of Linear Algebra and Its Applications.

Spectral Graph Theory Workshop Participants

15^{th} Haifa Matrix Theory Conference
May 18-21, 2009

Report by Raphael Loewy

The 2009 Haifa Matrix Theory Conference took place at the Technion-Israel Institute of Technology in Haifa, Israel, on May 18-21, 2009. This was the 15^{th} in a sequence of matrix theory conferences held at the Technion since 1984.

More than 80 people participated in the conference, and 59 talks were presented. As in the previous meetings, talks were of 30 minutes duration and covered a wide range of topics related to matrix theory.

All talks on the third day of the conference were dedicated to Professor Leiba Rodman on the occasion of his 60^{th} birthday. This special session began with a warm greeting and appreciation of his work by Israel Gohberg, and was followed by 4 talks. He completed his doctoral studies under Professor Gohberg in 1978.

Professor Rodman works in the general areas of operator theory and matrix analysis. His current research interests include matrix and operator valued functions, linear control systems, differential equations, optimization and interpolation, combinatorial matrix theory, and applications such as control systems and signal processing.

He is the author or co-author of 7 books and about 290 papers. He is Co-Editor-in-Chief of Operators and Matrices and a member of the editorial boards of other journals, including Linear Algebra and Its Applications, Mathematical Inequalities and Applications, Integral Equations and Operator Theory, and Complex Analysis and Operator Theory. He has also been Co-Editor of 7 special volumes in operator theory, matrix analysis and related topics.

One of the topics discussed in the conference was "Teaching of Linear Algebra". Steve Leon described developments in linear algebra education reform, while Gilbert Strang spoke about key points in a linear algebra course. In addition to these plenary talks there was a special session in which three lecturers from the Technion described several initiatives aimed at making the teaching of linear algebra more interesting and more effective. These included a model for evaluating students' understanding of the concept of equivalence relation, the use of clickers to improve interaction between students and teachers, and the matrix theory behind Google's PageRank algorithm, which can be a good motivating introduction to a matrix theory course.

The conference was supported and held under the auspices of the Center for Mathematical Sciences at the Technion. The education lectures were supported by the Ruth and Allen Ziegler Center for Learning Sciences at the Technion's Department of Education.

More information about the conference and photos can be found at its website http://www.math.technion.ac.il/cms/decade_2001-2010/year_2008-2009/matrix/.

Summer School and Advanced Workshop onTrends and Developments in Linear Algebra International Centre for Theoretical Physics, Trieste, Italy, June 22-July 3, 2009

Report by Peter Šemrl

A Summer School and Advanced Workshop on Trends and Developments in Linear Algebra was held at The International Centre for Theoretical Physics (ICTP) in Trieste, Italy, June 22-July 10, 2009.

This event was organized by R. Bhatia, J. Holbrook, and S. Serra Capizzano, with the help of R.T. Ramakrishnan (who works full time at ICTP). During the first two weeks, a summer school was held on "Trends and Developments in Linear Algebra", followed by a one week workshop at the Adriatico Guest House of the ICTP located only a few meters from the Adriatic sea, and next to the beautiful castle Miramare.

Three broad topics were chosen for the program: Matrix Analysis, Matrix Computations, and Quantum Information Theory. During the first two weeks each morning there were three lectures of 60 minutes. After a lunch break there was one more lecture followed by three problem/tutorial sessions.

Besides the organizers, the lecturers at the summer school were L. Elden, R. Horn, D. Kribs, M.B. Ruskai, P. Šemrl, and V. Simoncini. Some of the lecturers prepared lecture notes. Links to these can be found in the programme (http://cdsagenda5.ictp.trieste.it/full_display.php?smr=0&ida=a08167).

J.A. Antezana, J.S. Aujla, and I. Wrobel acted as tutors to help the lecturers with afternoon problem/tutorial sessions.

There were over 70 participants coming from Algeria, Argentina, Azerbaijan, Cameroon, Canada, China, Cuba, Egypt, Georgia, Hungary, India, Iran, Italy, Japan, Korea, Macedonia, Morocco, Nepal, Nigeria, Pakistan, Portugal, Romania, Senegal, Serbia, Singapore, Slovenia, Sudan, Tunisia, Ukraine, United Arab Emirates, United Kingdom, USA, and Vietnam. The lecturers agreed that it was a great joy to work with the enthusiastic participants. It is clear that some of them are very talented and will soon become successful mathematicians.

The workshop speakers were K. Audenaert, J.-Z. Bai, F. Benatti, G. Benenti, D. Bini, H.R. Chan, N. Guglielmi, T. Huckle, R. Kannan, H. Kosaki, J. Lawson, Y. Lim, V. Mehrmann, B. Meini, M. Moakher, D. Petz, P. Šemrl, E. Tyrtyshnikov, J. Zemanek, and X. Zhan. It was a special pleasure to attend shorter talks given by younger colleagues J.A. Antezana, J.S. Aujla, B. Iannazzo, C. Manni, and I. Wrobel, who presented their research results of high quality in a very professional way.

The International Centre for Theoretical Physics in Trieste, Italy was founded by Abdus Salam in 1964. There he instituted the famous "Associateships" which make it possible for talented young scientists from developing countries to spend their vacations at ICTP in close touch with leading experts from all over the world and then return to their own countries for nine months of the academic year.

Abdus Salam

Abdus Salam was a Nobel Laureate in physics in 1979, together with Americans Steven Weinberg and Sheldon Glashow.

He was born in 1926 in Jhang, a small town in what is now Pakistan. After graduating from the University of Punjab he continued his studies in Cambridge, obtaining a PhD in theoretical physics in 1951. He then returned to Pakistan with the intention of founding a high level research school at Punjab University. This turned out to be a difficult mission. The only possible way to continue his outstanding career in theoretical physics was to leave his own country and to work abroad.

Many years later, in 1964, he founded the International Centre for Theoretical Physics in Trieste. Salam died in 1996 after a long illness.

Today ICTP operates under a tripartite agreement among the Italian Government and two United Nations Agencies, UNESCO and IAEA. Its mission is to support the best possible science with special attention to the needs of developing countries. In particular, high-level training courses, workshops, conferences and topical meetings take place throughout the year at ICTP. The participation of scientists from developing countries is fully supported by ICTP. For more information, visit http://www.ictp.trieste.it/.

International Workshop on Scientific Computing and its Applications
Shanghai, China, June 28-30, 2009

Report by Yimin Wei and Fuzhen Zhang

The International Workshop on Scientific Computing and its Applications was held in Shanghai Fudan University, China, from June 28 to 30, 2009. This meeting was dedicated to numerical analysis and related areas. About 30 mathematicians, including some graduate students, attended the conference.

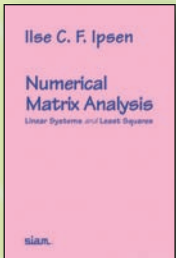
The speakers were:
Wei-wei SUN: Mathematical Modeling, Computation and Analysis for Heat and Moisture Transport in Fibrous Materials
Fu-zhen ZHANG: Recent Results and Research Problems of Hua Matrices
Jian-feng CAI: A Singular Value Thresholding Algorithm for Matrix Completion
Moody T. CHU: Data Mining and Applied Linear Algebra
Hong-gang ZENG: Matrix Computation in Numerical Polynomial Algebra and Algebraic Geometry
Chi-Tien LIN: Numerical Study of Solution to a p-System with Nonlinear Damping
Chun-hua OU: Traveling Waves of Non-local Reaction Diffusion Equations
Xu-zhou CHEN: Discussion on Dimensionality Reduction and Data Representation
Rong-qing JIA: Riesz Bases of Wavelets and Applications to Numerical Solutions of Elliptic Equations
San-zheng QIAO: The LLL Algorithm and Sphere Decoding
Zi-cai LI: New Simple Local Refinements of Finite Element Method for a Corner
Li-hong ZHI: Determining Singular Solutions of Polynomial Systems via Symbolic-Numeric Reduction to Geometric Involutive Form
Xiao-qing JIN: Tri-diagonal Preconditioner for Family of Toeplitz Systems with Financial Applications
Zhong-xiao JIA: A Refined Harmonic Lanczos Bidiagonalization Method and an Implicitly Restarted Algorithm for Computing the Smallest Singular Triplets of Large Matrices
Mu-sheng WEI: A Canonical Decomposition of a Right Invertible System with Applications
Hai-wei SUN: Shift-Invert Arnoldi Approximation to a Toeplitz Matrix
Li QIU: Measure of Instability

The event was organized by Yimin Wei of Fudan University and sponsored by the Shanghai Key Laboratory of Contemporary Applied Mathematics of Fudan University, Shanghai Municipal Science and Technology Committee, Shanghai Municipal Education Committee (Dawn Project), and the National Natural Science Foundation of China.

Participants, International Workshop on Scientific Computing and its Applications

APPLIED MATHEMATICS New Titles! from siam®

Numerical Matrix Analysis: *Linear Systems and Least Squares*



Ilse C. F. Ipsen

This self-contained textbook presents matrix analysis in the context of numerical computation with numerical conditioning of problems and numerical stability of algorithms at the forefront. Using a unique combination of numerical insight and mathematical rigor, it advances readers' understanding of two phenomena: sensitivity of linear systems and least squares problems, and numerical stability of algorithms. The material is presented at a basic level, emphasizing ideas and intuition, and each chapter offers simple exercises for use in the classroom and more challenging exercises for student practice.

2009 · xiv + 128 pages · Softcover · ISBN 978-0-898716-76-4
List Price $59.00 · SIAM Member Price $41.30 · **OT113**

Matrix Polynomials

I. Gohberg, P. Lancaster, and L. Rodman

Classics in Applied Mathematics 58

A comprehensive treatment of the theory of polynomials in a complex variable with matrix coefficients. Basic matrix theory can be viewed as the study of the special case of polynomials of first degree; the theory developed in *Matrix Polynomials* is a natural extension of this case to polynomials of higher degree. It has applications in many areas, such as differential equations, systems theory, the Wiener–Hopf technique, mechanics and vibrations, and numerical analysis.

2009 · xxiv + 409 pages · Softcover · ISBN 978-0-898716-81-8
List Price $92.00 · SIAM Member Price $64.40 · **CL58**

Functions of Matrices: *Theory and Computation*

Nicholas J. Higham

The only book devoted exclusively to matrix functions, this research monograph gives a thorough treatment of the theory of matrix functions and numerical methods for computing them. The author's elegant presentation focuses on the equivalent definitions of $f(A)$ via the Jordan canonical form, polynomial interpolation, and the Cauchy integral formula, and features an emphasis on results of practical interest and an extensive collection of problems and solutions. This book is for specialists as well as anyone wishing to learn about the theory of matrix functions.

2008 · xx + 425 pages · Hardcover · ISBN 978-0-898716-46-7
List Price $59.00 · SIAM Member Price $41.30 · **OT104**

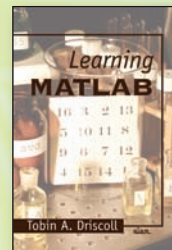
Assignment Problems

Rainer Burkard, Mauro Dell'Amico, and Silvano Martello

A comprehensive treatment of assignment problems from their conceptual beginnings in the 1920s through present-day theoretical, algorithmic, and practical developments. The authors have organized the book into 10 self-contained chapters to make it easy for readers to use the specific chapters of interest to them without having to read the book linearly. This book is a useful tool for researchers, practitioners, and graduate students.

2009 · xx + 382 pages · Hardcover · ISBN 978-0-898716-63-4
List Price $110.00 · SIAM Member Price $77.00 · **OT106**

Learning MATLAB



Tobin A. Driscoll

This engaging book is a concise introduction to the essentials of the MATLAB® programming language and is ideal for readers seeking a focused and brief approach to the software. *Learning MATLAB* contains numerous examples and exercises involving the software's most useful and sophisticated features and an overview of the most common scientific computing tasks for which it can be used. The presentation is designed to guide new users through the basics of interacting with and programming in the MATLAB software, while also presenting some of its more important and advanced techniques.

2009 · xiv + 97 pages · Softcover · ISBN 978-0-898716-83-2
List Price $28.00 · SIAM Member Price $19.60 · **OT115**

Parallel MATLAB for Multicore and Multinode Computers

Jeremy Kepner

Software, Environments, and Tools 21

This is the first book on parallel MATLAB® and the first parallel computing book focused on the design, code, debug, and test techniques required to quickly produce well-performing parallel programs. MATLAB is currently the dominant language of technical computing with one million users worldwide, many of whom can benefit from the increased power offered by inexpensive multicore and multinode parallel computers. MATLAB is an ideal environment for learning about parallel computing, allowing the user to focus on parallel algorithms instead of the details of implementation.

2009 · xxvi + 253 pages · Hardcover · ISBN 978-0-898716-73-3
List Price $65.00 · SIAM Member Price $45.50 · **SE21**

www.siam.org/catalog

siam® Society for Industrial and Applied Mathematics

9/09

ORDER ONLINE: www.siam.org/catalog
Or use your credit card (AMEX, MasterCard, and VISA): Call SIAM Customer Service at +1-215-382-9800 worldwide · Fax: +1-215-386-7999
E-mail: service@siam.org. Send check or money order in US dollars to: SIAM, Dept. BKIL09, 3600 Market Street, 6th Floor, Philadelphia, PA 19104-2688 USA.
Members and customers outside North America can also order SIAM books through Cambridge University Press at *www.cambridge.org/siam*.

10^{th} SIAM Applied Linear Algebra Conference, Monterey CA, Oct. 26-29, 2009

Report by Chen Greif

The 10th SIAM Conference on Applied Linear Algebra took place in Monterey, California, on Oct. 26-29, 2009. It was a record breaking meeting: over 200 minisymposia talks (almost double compared to the 2006 meeting!), along with approximately 80 contributed talks and 11 plenary talks, brought the overall number of presentations beyond 300. The participants were certainly spared of the need to think about how to fill their day. Talks went on from 8 a.m. to as late as 10 p.m. in two of the evenings, featuring no less than six parallel sessions. And there were plenty of other activities: a SIAG/LA business lunch, a conference banquet, a forward looking panel discussion, and much more.

Over 250 participants braved the harsh California weather conditions of approximately 20 degrees (Celsius) and beautiful sunshine, and spent the week at the glamorous Monterey Bay of California, located about 100 miles south of San Francisco. The Embassy Hotel, where the meeting took place, was pleasant, and the facilities were excellent.

The plenary talks were remarkably diverse and of a very high quality. The speakers, in order of their appearance, were: Andy Wathen, Oxford (preconditioners for PDE-constrained optimization), Michele Benzi, Emory (matrix functions), Shaun Fallat, Regina (ILAS speaker - matrix factorizations and total positivity), Karen Wilcox, MIT (model reduction), Françoise Tisseur, Manchester (quadratic eigenvalue problems), Christian Mehl, TU Berlin (ILAS speaker - indefinite linear algebra), Hugo Zaragoza, Yahoo! (link graphs), Michael Ng, Hong Kong (data mining and image processing), Zlatko Drmač, Zagreb (SIAG/LA Best Paper Prize - see more below), Dianne O'Leary, Maryland (confidence in image reconstruction), Michael Friedlander, UBC (sparse optimization), and Dan Spielman, MIT (combinatorial solution of systems with M-matrices).

There were two history sessions, featuring four speakers in each, over the course of two evenings. Drawing lessons from the previous meeting in Dusseldorf, where those talks were a huge draw but were scheduled in parallel with other talks, this time the history sessions stood alone as single sessions. They were very well attended despite the late hour of 8-10 p.m.

The SIAG/LA business lunch meeting was led by the activity group's Chair, Daniel Szyld, who updated the SIAG members with the state of affairs in the group. Daniel described the activities that the SIAG has been involved with (such as the summer school in Spain in 2007), some statistics about the attendance of SIAG members in recent meetings, and the upcoming election of new officers. The SIAG/LA best paper prize was also announced: Z. Drmač and K. Veselić, for their paper, *New Fast and Accurate Jacobi SVD Algorithm*, published in SIMAX in 2008.

Just to make sure no time was wasted on activities not involving talking linear algebra, a panel on the role of linear algebra in industrial applications took place on the third day, over lunch time. The session was moderated by Mark Embree, along with the panelists Iain Duff, Roger Grimes, Volker Mehrmann, and Rosemary Renaut.

The conference banquet featured a choice of salmon or chicken, but no alcohol unless you wanted to add more to the already hefty dinner fee. (The hotel bar must have been grateful for that -- everybody flocked to the lobby after dinner to get their fix.) The after-dinner speaker, Beresford Parlett from Berkeley, did not spare words to share with us a few well kept secrets from his days as a graduate student. We would share those with you, but in the spirit of "what happens in SIAM ALA stays in SIAM ALA", let us leave it at this...

Altogether, it was a most enjoyable meeting, with excellent talks and a wonderful diversity of topics. Special thanks go to Esmond Ng, the meeting's Chair, for successfully taking on the momentous task of planning and running the meeting. Daniel Szyld, the SIAG/LA Chair, also played an important role in assuring the success of the meeting.

The next SIAM Applied Linear Algebra conference is scheduled to take place in Valencia, Spain, on June 18-22, 2012. The local organizers are Rafael Bru and José Mas.

Shaun Fallat, ILAS Speaker
"Matrix Factorizations and Total Positivity"

Christian Mehl, ILAS Speaker
"Definite versus Indefinite Linear Algebra"

OBITUARY

Israel Gohberg: August 23, 1928 - October 12, 2009

By Leiba Rodman, College of William and Mary, Williamsburg, VA 23188

Israel Gohberg

With deep sadness I inform that Israel Gohberg passed away on Monday, October 12, 2009.

Israel Gohberg received his mathematical education in the Soviet Union. After holding several teaching and research positions in Moldova, Gohberg emigrated to Israel in 1974, where he was appointed Professor of Mathematics at Tel Aviv University and (later) incumbent of the Nathan and Lily Silver Chair in Mathematical Analysis and Operator Theory. He held also part time positions for many years at Weizmann Institute of Science (Israel), Vrije Universiteit, Amsterdam (The Netherlands), and the University of Maryland, College Park (USA).

Israel Gohberg leaves an enormous, lasting legacy. MathSciNet lists 573 items for him as of October 26, 2009, including 90 books (authored or edited). He supervised 40 Ph.D. students (I am one of them), many of whom went on to build successful careers in mathematics.

In 1978 he established *Integral Equations and Operator Theory*, a premier journal in the area of operator theory and its applications, and since then has been Editor-in-Chief of this journal. He also established *Operator Theory: Advances and Applications*, a book series published by Birkhauser Verlag, which has about 200 volumes in print so far.

Israel Gohberg received numerous honors and awards, including 6 honorary doctoral degrees from universities in Germany, Austria, Moldova, Romania, and Israel. He was a corresponding member of the Academy of Science of Moldova and Foreign Member of the Royal Netherlands Academy of Arts and Sciences.

Israel Gohberg was a great research mathematician, educator, and expositor. His visionary ideas and charismatic leadership inspired many in our research community. He will be dearly missed.

Ed. Note: Professor Rodman and other colleagues of Israel Gohberg are preparing an extensive tribute to him, which will appear in Linear Algebra and Its Applications.

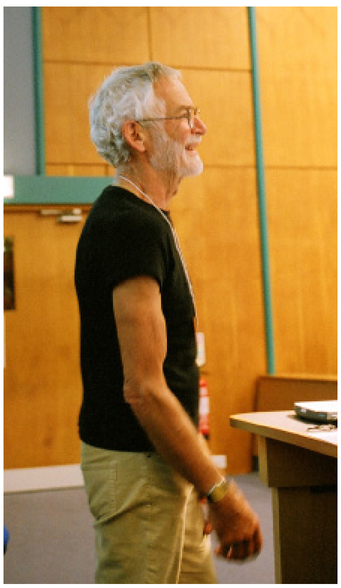
ARTICLES

Addendum to "Linear Algebra in Iran", IMAGE Issue 42

We should have mentioned in our history of linear algebra in Iran (among other self-imposed restrictions on the list of articles there) that we included only those papers in linear algebra that had primary designation 15xx in Mathematical Reviews. Otherwise, our list would have been almost twice as long, as pointed out to us by M. S. Moslehian. Mehdi Radjabalipour and Heydar Radjavi.

Finding John Francis Who Found QR Fifty Years Ago

By Frank Uhlig, Auburn
Thanks to the Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849-5310, uhligfd@auburn.edu



John Francis in Glasgow, 2009

Half a century ago, John Francis developed his QR algorithm to compute the eigenvalues of matrices. He submitted his first QR paper on October 29, 1959 to the *Computer Journal* and his second one on June 6, 1961. Both appeared in October 1961 without any modification. The referee of the first paper, Velvel Kahan, had held its publication back while trying to understand why the algorithm converged.

The first paper contains a proof of convergence, but the question of why QR converges has occupied mathematicians ever since. In the meantime, John expanded, implemented and tested his algorithm. His second paper explains shifts; in fact the one later named after Jim Wilkinson first appears there. The theory of implicit shifts and the Implicit Q Theorem also appear in paper 2. A wee bit later, Vera Kublanovskaya explored a QR type matrix eigenvalue algorithm, but without the implementation, tests and enhancements that Francis' papers had already offered.

Once the two Francis papers had appeared in the *Computer Journal* of 1961/62, John left the field and was soon lost to and forgotten by the numerical analysis and computations community. Here is the story of how he was located recently and came to speak at a minisymposium in June 2009, in honor of him and his remarkable algorithm.

After his papers appeared, Francis' algorithm became the mainstay of dense matrix eigenvalue and singular value computations, the latter through a genial trick of Golub and Kahan to represent

Minisymposium 'The QR Algorithm, 50 Years Later,' Glasgow, June 23-25, 2009
One side of the auditorium. Recognizable from left: Volker Mehrmann, Vladimir Khazanov, David Watkins, Charlie van Loan, Daniel Kressner, Françoise Tisseur, Chris Paige, Ilse Ipsen, Martin Gutknecht, Beresford Parlett, Andy Wathen, Chris Fuller, Nick Trefethen

the singular values of a matrix inside an equivalent bi-diagonal matrix and then to use Francis' Implicit Q Theorem to shift implicitly to obtain the singular values.

Finding matrix eigenvalues had long been one of the holy grails of numerical analysis. In the era following Abel's 1805 discovery that polynomial equations generally do not admit closed form solutions, more than 8,000 papers and well over 300 books have been published with ideas and algorithms to find the roots of polynomials numerically. Why polynomials? Because matrix eigenvalues were defined theoretically by Cauchy via the characteristic polynomial in much the same way as many of us still teach the eigenvalue concept in elementary linear algebra classes today. As matrix models arose for solving ODEs and PDEs, matrix eigenvalues came to the fore and needed reliable computational methods. None of the polynomial root methods fulfilled that need of applied mathematics.

And thus Francis' QR was included in the turn of the century list of the ten most important algorithms of the 20th century by Dongarra and Sullivan in 2000. Adding Gaussian elimination and the Euclidean algorithm to that list of most important algorithms of all mankind places Francis' achievement in the elite dozen category of human history. Yet in 2000, no living mathematician knew whether John Francis was alive or where he lived. None of the then working mathematicians had met him before he had vanished in the early 1960s.

Soon after Gene Golub and this author began to search for John Francis, completely independently. I had started a project to develop software and adapt algorithms for use by chemical and biological engineers. Within this project, I became aware of many an engineering equation, constant, and apparatus that was associated with a person's name and I searched far and wide for usable records of the name-givers. Eigenvalues and John Francis' role in their computation led me to discover his whereabouts and part of his history. Similarly, Gene used internet searches and suggestions from our collective memory to find John.

Looking back at history, it appears that John's whereabouts were actually found through his own famed algorithm: In the early, Bay area garage days of google, its founders looked for ways to search the internet effectively. They contacted Gene at Stanford who advised them of the page rank algorithm. This uses the singular value decomposition and helps google sort web data. Of course the SVD algorithm is just a specialization of Francis' QR to bi-diagonal matrices and it also uses Francis' Implicit Q Theorem. Searching for `John Francis' in google brought both Gene and me a treasure of information on John. I enquired at the Museum of Science and Industry in Manchester that holds the files of Ferranti Ltd where John was employed after 1961. The enquiries gave me John's middle name and his birthdate before the communications were abruptly terminated at the museum's archive end for `privacy' concerns. But this sufficed to locate John's public school, his short Cambridge University student stay, and - last but not least - a recent church petition signed in his full name in Hove on the English Channel coast. That gave me all the information I would use in the footnote on Francis in the chemical engineering numerics book. And I left it at that.

And then chance took over. At an Operator Theory meeting in Moscow in July 2007, I happened to mention the often lost name-givers of quoted scientific entities, even in our own times. And Gene blurted out: "What about Francis?" to which I answered with

From the left: Martin Gutknecht, Chris Paige, David Watkins, John Francis, Volker Mehrmann, Beresford Parlett, and Andy Wathen

John's birthdate, his full name, schools/years attended, work at Ferranti etc. Gene was stunned. We, the only two who knew about John Francis had met! Gene invited me to join him in visiting with John on August 7, 2007, but I had to decline. We wondered what it would be like to write John Francis' biography for the community. Andy Wathen had intended to go with Gene that day, but at the last minute he was otherwise detained and Gene drove the 100 miles from Oxford to the coast by himself. Gene and John spent a couple of hours together and talked. Six days later Gene wrote a short e-mail to a number of friends about the visit. I received a copy, but was busy with the new semester at Auburn and did not communicate with Gene again. Then in November I learnt of Gene's death in Stanford.

Now I was stunned. Again, no living mathematician had met or seen John Francis. Besides, I was apparently the only one who knew John's whereabouts. I vowed to complete the task of bringing John back into the numerical analysts' realm, into our community and consciousness.

To meet with John, I detoured through London's Gatwick airport in July 2008 and spent 4 nights at a B&B two blocks from where John lives. We met, talked, ate out, walked, swam in the Channel together and had an interview in which John reminisced on the times and ideas around QR and I tried to give impromptu insights into the subsequent uses and impact of John's seminal work. On the night before the interview I had given John a list of items and dates from his life that I knew about, as well as a list of possible topics and questions for the interview. The next morning, before the actual interview, John seemed very upset about the personal data I had shown him. How had I obtained that information? My honest answer of 'from the internet' was unbelievable to him. He had no idea how google worked nor how his Implicit Q Theorem and Gene and Velvel's application of it had given me this information. An hour and a half later John understood and we were friends.

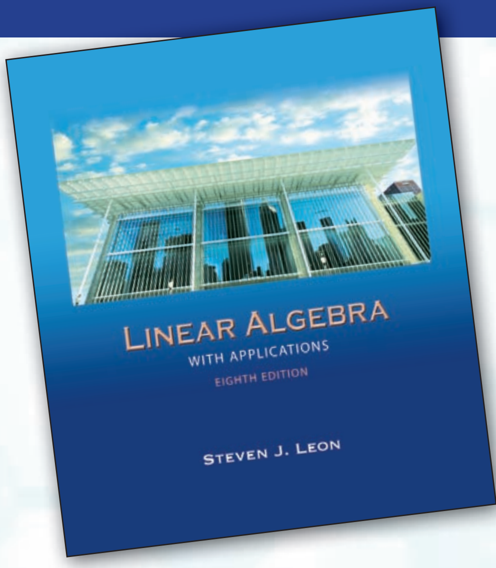
The next day I mentioned that I might try to organize a meeting on the history and recent advances of QR and asked John if he would come. John is a very private, unassuming and quiet person. After a couple of minutes and a block or two of walking down the road, he answered 'I might'. I thanked him and we parted. Now I was left to re-introduce John Francis to our community. I chose the British Isles and contacted friends who all unfortunately were too engaged in research, editing, conference preparations etc. to be able to help until I convinced Andy Wathen that we owed John and our students and research community a chance to meet.

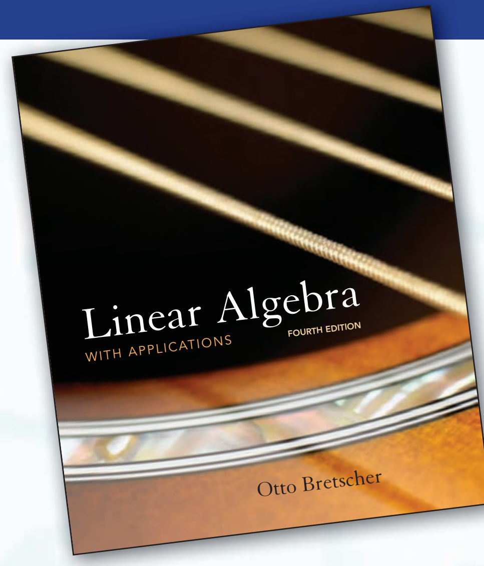
With Andy's help, the 23rd Biennial Conference on Numerical Analysis at Strathclyde University in Glasgow June 23-25, 2009 became the venue for a mini-symposium on 'The QR Algorithm, 50 Years Later, its Genesis by John Francis, and Subsequent Developments'. More than 100 participants listened to John Francis' opening talk. He was followed by David Watkins on the perennial question of why, titled 'Understanding the QR Algorithm, Part X'. I followed with applications of the Implicit Q Theorem and the tridiagonal DQR algorithm and its O^{n2} polynomial root finder. Talks on recent extensions of QR were given by Chris Fuller on the nature of the general orthogonal group and Householder factorizations, by Karen Braman on multishift QR, by Daniel Kressner on parallel versions of QR and QZ, by Luca Gemignani on fast QR, and by Volker Mehrmann on structure preserving QR like algorithms. Another section of the mini dealt with its history in talks by Vladimir Khazanov, stepping in for Vera Kublanovskaya whose health prevents travel, on Vera's work from QR to AB for matrix pencils. Beresford Parlett traced the antecedents of QR and LR to much earlier work on differential equations from the 1930s and Martin Gutknecht elaborated on how Rutishauser, whose 1958 LR paper inspired both John and Vera to discover QR, might have found qd and LR.

A historical scientific paper by Gene Golub and Frank Uhlig appeared in time for the Glasgow conference in the IMA Journal of Numerical Analysis, 29 (2009), 467-485 on June 8, 2009.

Andy Wathen, (Jen Pestana, hidden), and John Francis
at dinner in Glasgow, 2009

AT THE FOREFRONT OF LINEAR ALGEBRA





NEW! ***Linear Algebra with Applications,*** **Eighth Edition**
by Steven J. Leon
© 2010 • 552 pages • Cloth
ISBN-13: 978-0-13-600929-0 • ISBN-10: 0-13-600929-8

This thorough and accessible text—from one of the leading figures in the use of technology in linear algebra—gives students a challenging and broad understanding of the subject. Leon infuses key concepts with their modern practical applications to offer students examples of how mathematics is used in the real world.

NEW! ***Linear Algebra with Applications,*** **Fourth Edition**
by Otto Bretscher
© 2009 • 504 pages • Cloth
ISBN-13: 978-0-13-600926-9 • ISBN-10: 0-13-600926-3

Offering the most geometric presentation available, this book emphasizes linear transformations as a unifying theme. This elegant textbook combines a user-friendly presentation with straightforward, lucid language to clarify and organize the many techniques and applications of linear algebra.

ALSO AVAILABLE FOR YOUR LINEAR ALGEBRA COURSE

INTRODUCTORY LINEAR ALGEBRA

Linear Algebra and Its Applications with CD-ROM, Update, **Third Edition**
David C. Lay • © 2006
ISBN-13: 978-0-321-28713-7 • ISBN-10: 0-321-28713-4

Elementary Linear Algebra with Applications, **Ninth Edition**
Kolman • Hill • © 2008
ISBN-13: 978-0-13-229654-0 • ISBN-10: 0-13-229654-3

Introduction to Linear Algebra, **Fifth Edition**
Johnson • Riess • Arnold • © 2002
ISBN-13: 978-0-201-65859-0 • ISBN-10: 0-201-65859-3

Elementary Linear Algebra, **Second Edition**
Spence • Insel • Friedberg • © 2008
ISBN-13: 978-0-13-187141-0 • ISBN-10: 0-13-187141-2

DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

NEW! ***Differential Equations and Linear Algebra,*** **Third Edition**
Edwards • Penney • © 2010
ISBN-13: 978-0-13-605425-2 • ISBN-10: 0-13-605425-0

Differential Equations and Linear Algebra, **Third Edition**
Goode • Annin • © 2008
ISBN-13: 978-0-13-045794-3 • ISBN-10: 0-13-045794-9

Differential Equations and Linear Algebra, **Second Edition**
Farlow • Hall • McDill • West • © 2007
ISBN-13: 978-0-13-186061-2 • ISBN-10: 0-13-186061-5

ADVANCED LINEAR ALGEBRA

Linear Algebra, **Fourth Edition**
Friedberg • Insel • Spence • © 2003
ISBN-13: 978-0-13-008451-4 • ISBN-10: 0-13-008451-4

FIND YOUR FIT

Find the book that's right for you!
Visit **www.pearsonhighered.com/findyourfit**

PEARSON

Symmetric Sudoku Matrices and Magic Squares

By Richard William Farebrother

2. Symmetric Sudoku Matrices

For m odd, Farebrother [2, 3] has introduced a family of nonlinear transformations that carries each column of an $m \times m$ matrix into a primary diagonal, each primary diagonal into a row, each row into a secondary diagonal, and each secondary diagonal into a column. Further, he has shown [2, p.30] that the 'prescription matrices' associated with these nonlinear transformations may be used to convert an arbitrary $m \times m$ 'seed matrix' B containing m^2 distinct symbols into an $m \times m$ (generalised) Sudoku matrix by replacing the cell marked (i,j) in the prescription matrix by the $m \times m$ matrix $P^{\,i-1}BP^{1-j}$ where P is an $m \times m$ permutation matrix of full cycle length such as that which carries the mth row into the $(m\text{-}1)$th row, ..., the second row into the first row, and the first row into the mth row.

It is immediately apparent that any such $m \times m$ matrix will satisfy a natural generalisation to $n = m^2$ of the six sets of $n = 9$ constraints employed by Bailey, Cameron & Connelley (henceforth BCC) [1, p.390] in their definition of a 9×9 'symmetric Sudoku matrix':

> Now a symmetric Sudoku solution is an arrangement of the symbols 1,...,9 in a 9×9 grid in such a way that each symbol occurs once in each row, column, subsquare, broken row, broken column, and location.

Typical examples of such 9×9 Sudoku matrices have been identified independently by BCC [1, p.391] and Farebrother [2, p.30]. Readers are invited to use the method outlined in Farebrother [2, 3] to construct the corresponding 25×25, 49×49, 81×81, ... matrices using sets of 25, 49, 81, ... distinct symbols. They may then check whether the matrices so obtained are 'orthogonal' to their transposes in the sense understood in combinatorics. That is, whether, for each symbol σ in the original matrix and for each symbol τ in its transpose, the ordered combination (σ,τ) occurs in only one location. If so, then such matrices may be employed as the basis of Graeco-Latin squares. Farebrother & Styan [4, p.23] have shown that this is true of the 9×9 Sudoku matrices identified above.

2. Magic Squares

The structures of such $m^2 \times m^2$ Sudoku matrices are particularly interesting when the symbols in the $m \times m$ seed matrix B are replaced by a sequence of numbers chosen in such a way that the elements in each of the rows, columns and primary or secondary cyclic-diagonals sum to the same fixed value.

A matrix with these properties may readily be obtained when $m \geq 5$ by converting one of Farebrother's [3] $m \times m$ ('bishop's move') prescription matrices into a 'knight's move' matrix by interleaving the last (first) $(m\text{-}1)/2$ rows (columns) amongst the first (last) $(m+1)/2$ rows (columns) before identifying the typical element (i,j) with the value $(i\text{-}1)m + j$.

For instance, when $m = 5$, we have to interleave the last two rows of Farebrother's 5×5 prescription matrix [3, p.33] into the first three rows in the order 1, 4, 2, 5, 3:

$$\begin{bmatrix} (1,3) & (4,5) & (2,2) & (5,4) & (3,1) \\ (2,1) & (5,1) & (3,3) & (1,5) & (4,2) \\ (3,5) & (1,2) & (4,4) & (2,1) & (5,3) \\ (4,1) & (2,3) & (5,5) & (3,2) & (1,4) \\ (5,2) & (3,4) & (1,1) & (4,3) & (2,5) \end{bmatrix}$$

before identifying the typical element (i,j) with the value $5(i\text{-}1) + j$ to obtain the matrix with the required properties:

$$\begin{bmatrix} 3 & 20 & 7 & 24 & 11 \\ 9 & 21 & 13 & 5 & 17 \\ 15 & 2 & 19 & 6 & 23 \\ 16 & 8 & 25 & 12 & 4 \\ 22 & 14 & 1 & 18 & 10 \end{bmatrix}$$

References

[1] R. A. Bailey, Peter J. Cameron, & Robert Connelly. Sudoku, gerechte designs, revolutions, affine space, spreads, reguli, and Hamming codes. *The American Mathematical Monthly*, 115: (2008) 383--404.

[2] Richard William Farebrother. A general matrix transformation: problem and solution. *Image* No. 37, Fall 2006, p.39 and No.39, Fall 2007, pp.28--30.

[3] Richard William Farebrother. A simple procedure for a general matrix transformation: problem and solution. *Image* No.39, Fall 2007, p.32 and No.41, Fall 2008, pp.32--34.

[4] Richard William Farebrother & George P. H. Styan. Double Sudoku Graeco-Latin Squares. *Image* No. 41, Fall 2008, pp.22--24.

11 Castle Road, Bayston Hill, Shrewsbury, England SY3 0NF

Ancient Approximations To Pi

Richard William Farebrother

As noted in both solutions to Problem 36-1 (Image No.37, Fall 2006, pp.27–30), the simplest nontrivial solution in integers to the indeterminate equation $x^2-10y^2 = 1$ is $x_1 = 19$, $y_1 = 6$. These values serve to define the simple iterative procedure:

$$x_{n+1} = 19x_n + 60y_n$$
$$y_{n+1} = 6x_n + 19y_n$$

which gives rise to two sequences of ever closer approximations to the square root of 10, namely $x_1/y_1 = 19/6 = 3.1667$, $x_2/y_2 = 721/228 = 3.1623$, ... and $10y_1/x_1 = 60/19 = 3.1579$, $10y_2/x_2 = 2280/721 = 3.1623$, ...

Although, strictly speaking, these expressions represent rational approximations to the square root of 10, we may follow ancient tradition and use them as the basis of rational approximations to the value of $\pi = 3.1416$. Restricting ourselves to the ratio $10y_2/x_2 = 2280/721$, we obtain a familiar expression for π by approximating this ratio by 2288/728 = 22/7=3.1429. Alternatively, we may approximate 2280/721 = 2560/809.54 by 2560/810 to obtain a value of 4(8/9)2 =3.1605, employed in some of the calculations featured in the so-called Rhind Mathematical Papyrus; see Midonick [2] or Joseph [1] for details.

Readers will be surprised to learn that this last approximate of π is also to be found amongst the cake recipes given in some English and American cookery books where it appears in the form: ”pour the mixture into a square tin of width 8 inches or a circular tin of diameter 9 inches” although most chefs prefer the less accurate value implied by the statement: ”pour the mixture into a square tin of width 7 inches or a circular tin of diameter 8 inches”. Assuming, in each case, that the area of the square tin is roughly the same as that of the circular tin (so that both cakes have the same depth), we obtain 4(8/9)2 = 3.1605 and $4(7/8)^2 = 3.0625$ as culinary approximations to the value of π.

References

[1] George Gheverghese Joseph. *The Crest of the Peacock: Non-European Roots of Mathematics*, Penguin Books, Harmondsworth, 1990. Second edition Princeton University Press, Princeton, N.J., 2000.

[2] Henrietta Midonick. *The Treasury of Mathematics*, Penguin Books, Harmondsworth, 1968.

11 Castle Road, Bayston Hill, Shrewsbury, England SY3 0NF

The GAMM Activity Group *Applied and Numerical Linear Algebra*

Peter Benner* Daniel Kressner†

The GAMM (Gesellschaft für Angewandte Mathematik und Mechanik) is a German organization that promotes scientific development in all areas of applied mathematics and mechanics. The GAMM was founded in 1922 by Ludwig Prandtl and Richard von Mises. Nowadays, it is a society with a distinctly international organization and currently more than 2500 members. Traditionally, the GAMM cultivates contacts to scientists in Eastern Europe, which today also shows in a high membership in the corresponding countries. It is a member society of the ICIAM.

The GAMM advances the international cooperation in applied mathematics as well in all areas of mechanics and physics relating to the foundations of the engineering sciences. For this purpose, it constitutes and supports several activity groups in scientific fields of current interest.

The GAMM Activity Group *Applied and Numerical Linear Algebra* (GAMM-ANLA) aims at fostering scientific activities and collaboration of all researchers in the field in Germany and Europe. It recognizes the importance of taking concepts from other fields into account and takes an active role in supporting cross-disciplinary activities in linear algebra, numerical analysis, matrix theory, and applications. Any person (including non-GAMM members!) interested in applied and numerical linear algebra can join the activity group by filling in the membership form at the web page of GAMM-ANLA: `http://www.sam.math.ethz.ch/GAMM-ANLA/`.

History. GAMM-ANLA was founded in 2001 on the initiative of Volker Mehrmann, who also served as chair from 2001 to 2003. Heike Faßbender chaired GAMM-ANLA from 2004 to 2008. Since January 2009, Peter Benner and Daniel Kressner act as chair and vice chair, respectively. GAMM-ANLA currently has more than 60 members, mostly from Germany, but also from many other European countries including Belgium, Croatia, Czech Republic, France, Ireland, Poland, Romania, Serbia, Slovakia, Switzerland, and UK. Currently, one member is US-based.

Workshops. The organization of annual GAMM-sponsored workshops and conferences belongs to the core activities of GAMM-ANLA. The workshops usually emphasize a particular topic around which the invited talks are centered. The contributed talks, however, can be in any area of applied and numerical linear algebra. Ample room is given to PhD students and beginning PostDocs, who are strongly encouraged to present their work at GAMM-ANLA workshops.

The 2009 workshop took place in Zurich (see Figure 1 for some impressions) and the 2010 workshop will take place May 26–27 in Novi Sad, Serbia, embedded into the Applied Linear Algebra conference ALA 2010[1]. See Table 1 for a list of all workshops and their special topics. It happens quite frequently that workshops are joint events with established conference series in linear algebra and computational mathematics. It certainly is of interest to IMAGE readers that the 2011 workshop in Braunschweig will take place jointly with the ILAS conference! Also, the 2006 Joint GAMM-SIAM Conference on Applied Linear Algebra in Düsseldorf (in cooperation with ILAS) was co-organized by GAMM-ANLA. This is testimonial to the aim of GAMM-ANLA to activate and maintain close contact to ILAS and to the SIAM Special Activity Group on Applied Linear Algebra.

2001	Berlin	*Structured Matrices.*
2002	Bielefeld	*Multilevel and Krylov Subspace Methods.*
2003	Braunschweig	*Mathematical Biology.*
2004	Hagen	*Linear Algebra in Systems and Control.* Joint conference (IWASEP).
2005	Dresden	*Large Scale Eigenvalue and Singular Value Computations.*
2006	Düsseldorf	Joint conference (SIAM).
2007	Harrachov	Joint conference (Computational Methods with Applications).
2008	Hamburg-Harburg	*Regularization of Ill-posed Problems.*
2009	Zurich	*Preconditioning.*
2010	Novi Sad	*Positivity.* Joint conference (Applied Linear Algebra).
2011	Braunschweig	Joint conference (ILAS).

Table 1: GAMM-ANLA workshops, with their special topics and joint conferences in the 3rd column.

The Activity Group within GAMM. The GAMM annual meeting is the single most important event organized by GAMM and belongs to one of the largest European mathematics conferences. The meeting is unique in the sense that it hosts plenary talks, minisymposia and contributed talks in both disciplines, applied mathematics and mechanics. GAMM-ANLA submits proposals for plenary speakers and invited minisymposia for the annual meetings. Successfully proposed plenary speakers include Jim Demmel, Martin Gander, Zdeněk Strakoš, and Heinrich Voß. Moreover, GAMM-ANLA is responsible for organizing a special session on Applied and Numerical Linear Algebra, which hosts contributed talks in the field. The 2010 GAMM annual meeting will take place March 22–26 in Karlsruhe, with the special session organized by Lars Grasedyck and Christian Mehl.

*Fakultät für Mathematik, TU Chemnitz, Germany; `benner@mathematik.tu-chemnitz.de`.

†Seminar für angewandte Mathematik, ETH Zurich, Switzerland; `daniel.kressner@sam.math.ethz.ch`.

[1] See `http://ala2010.pmf.uns.ac.rs/`.

Figure 2: Group picture taken at the ”German-Polish Workshop for Young Researchers in Applied and Numerical Linear Algebra”, Mathematical Research and Conference Center in Będlewo, Poland, February 2–5, 2006.

Figure 1: Pierre-Antoine Absil giving the Stiefel Centennial Lecture (left), Urs Hochstrasser (who coded the CG algorithm on a Z4 in 1952/54) and Eduard Stiefel Jr. during the “Apero” at the GAMM-ANLA workshop 2009 in Zurich. Pictures courtesy of Simon Gutknecht.

The 2010 meeting also features a so called Young Researchers’ Minisymposia on “Numerical Linear Algebra and Optimization” organized by the GAMM-ANLA members Melina Freitag and Martin Stoll.

The activity group regularly contributes to the GAMM newsletter and the scientific journal “GAMM-Mitteilungen”; both are distributed to more than 2000 GAMM members and to libraries. In 2005–2006, the “GAMM-Mitteilungen” published two “Themenhefte” (special issues) on *Applied and Numerical Linear Algebra*, edited by Heike Faßbender, with invited survey articles covering a wide range of topics in the field. Despite the German name of the journal, all articles are in English!

Support for Young Scientists. From its beginning, the activity group was intended to establish a forum for information and identification for young scientists in the field. As already mentioned above, GAMM-ANLA strongly encourages young scientists to contribute talks to workshops and proposals for Young Researchers’ Minisymposia at GAMM annual meetings. In 2006, the activity group organized a “German-Polish Workshop for Young Researchers in Applied and Numerical Linear Algebra” in Będlewo, Poland[2], see Figure 2 for a group picture taken there. A very intense program with talks by selected speakers and extensive poster sessions led to a stimulating atmosphere which gave rise to countless discussions among the participants. The prize for the best poster was awarded to Marta Markiewicz for her poster on ”Numerical Simulation of Quantum Dots”. It is planned to organize a similar workshop or summer school in 2011.

Outlook. The future of GAMM-ANLA will see a continuation of current activities, with more pronounced efforts to increase the involvement and identification of young researchers, not only with GAMM-ANLA, but also with the GAMM in general. The activity group also seeks to intensify its collaboration with ILAS. Suggestions and contributions are always most welcome!

[2] See `http://www.icm.tu-bs.de/ag_numerik/bedlewo/`.

BOOK REVIEW

Matrix Mathematics: Theory, Facts, and Formulas

By Dennis S. Bernstein

Princeton University Press, Princeton, New Jersey, 2009, 1137 pages.

Reviewed by Helmut Lütkepohl
Florence, Italy

This is the second edition of a book that first appeared in 2005. I did not take note of the first edition but according to the author it was less than two thirds of the present version. The previously included material is updated and new material is added in many parts of the new edition. It is a remarkable source of matrix results. I will put it on the shelf nearest to my desk so that I have quick access to it. The book is an impressive accomplishment by the author.

He has structured the content in 12 chapters. The first chapter reviews some basic terminology and mathematical background material such as functions, relations and graphs and even more basic things like integers, real and complex numbers. It also presents a large number of inequalities and identities. Chapter 2 contains basic concepts and results related to matrices. For example, it includes results on ranks, determinants, traces, products, inverses, partitioned matrices and the like. Also important mathematical concepts such as subspace, null space, range space, cones and convex hulls are covered.

In Chapter 3 many useful results on a range of special matrices are presented. Matrices covered include Hermitian, symmetric, skew-Hermitian, skew-symmetric, idempotent, nilpotent, normal, projector, reflector, Hankel and Toeplitz matrices plus many more. Also Lie algebras and groups are discussed in this chapter.

Chapter 4 is devoted to polynomial matrices or matrix polynomials and rational transfer functions and covers among many other things the Smith and Smith-McMillan forms. Matrix decompositions such as companion, Jordan, Schur and singular value decompositions and related results are treated in Chapter 5. Chapter 6 deals with generalized inverses. Kronecker and Schur or Hadamard products are considered in Chapter 7. Positive-semidefinite matrices are the topic of Chapter 8. Vector and matrix norms are discussed in Chapter 9 and matrix derivatives are treated in Chapter 10. Matrix exponentials and stability theory follow in Chapter 11, while the final chapter deals with linear systems and control theory.

The dimensions of the book are impressive. The chapters cover 879 pages which gives an idea of the number of results summarized in this volume. There is also a breathtaking bibliography with 1540 items and information on where the references are cited in the text. Moreover, there is a detailed author index stretching out over more than 10 pages.

The book concludes with a most remarkable index of 161 pages. This index is extremely valuable for locating specific results. Clearly, not many people want to read through hundreds of pages of matrix results to find a specific property of a matrix, if they want to use the book as a reference volume. For them the index is of great help. It is detailed and informative at the same time by indicating precisely what can be found for a specific term on a particular page. For example, for a controllability matrix the index tells us that we find a theorem on a controllable pair on p. 815, the definition on p. 809, a result on the rank on p. 809 and Sylvester's equation on p. 869. The book also has an excellent listing of special symbols at the beginning. Again this is a great help for someone wanting to use the book as a reference.

The style is very concise, as one would expect from a mathematics book. It lists definitions and results. The latter are often called facts while more important results are signified as theorems or propositions. There are also many short remarks and very occasional examples and exercises. Despite the length of the book, proofs are not given for all results but often a reference is given. There are also many results that I would classify as nontrivial for which no proofs or references are provided. In general, the style is very efficient. In other words, the number of facts or results per page is rather large. Given the size of this volume, the reader may infer from that what a valuable source of matrix results it contains. Indeed, for some of the topics I would not be able to name a more comprehensive collection of results. So there is no doubt that this is an enormously valuable reference book.

Are there any features that I do not like? For me some of the remarks are just too short and uninformative. Too often they just point to other results without telling me why. For example, following Fact 2.11.11, there is the remark "See Fact 6.5.6." This kind of remark is not untypical. It has its value because it refers to a related result. Still a brief explanation would be helpful for me. In some cases, I did not find a result that is useful for me. For example, the collection of results related to vector and matrix derivatives is not as comprehensive as in some other collections, although many important results are there. Also, the half-vectorization operator is not covered, although mentioned, and discrete time state-space models would be useful for me.

Given the impressive size of the book it is perhaps not too surprising that some results are not easy to locate, despite the features that make it easier, e.g., the excellent index. For example, I was expecting to find the fact that the rank of an idempotent matrix is equal to its trace in Section 3.12, Facts on Idempotent Matrices. I found it only in Section 5.8 entitled Facts on Inertia and later again in Chapter 6 on Generalized Inverses. Finally, I would have liked to see more proofs or at least easily accessible references to proofs.

In summary, the main use of this impressive volume is clearly that of a reference book and for that purpose it is excellent. In the preface the author mentions different potential uses of the book, the main one being as a reference. He clearly achieves this goal. I can enthusiastically recommend it to anyone who uses matrices. The author has to be applauded for the accomplishment of putting together this impressive volume.

QUICK NOTES ABOUT BOOKS

New Books

Numerical Matrix Analysis: Linear Systems and Least Squares, by Ilse C. F. Ipsen, SIAM, 2009. This textbook presents matrix analysis in the context of numerical computation with special focus on conditioning of problems and stability of algorithms.

Applications of Linear Algebra to DNA Microarrays, by Amir Niknejad and Shmuel Friedland, VDM Verlag Dr. Müller, 2009. The purpose of this monograph is to condense the information that arises in molecular biology in general, and in gene expression data embedded in DNA microarrays in particular, and to use approximation by low rank matrices to complete missing data.

Matrix Methods: Applied Linear Algebra, by Richard Bronson and Gabriel B. Costa, 3rd ed., 2009. This text is suitable for a one semester first linear algebra course or a two semester course when supplemented with outside reading. It offers a balance between theory and matrix techniques. An appendix discusses use of technology.

List of Books on the ILAS-IIC Website

A list list of books about linear algebra and its applications is maintained by Oskar Baksalary, the Book Editor for IMAGE. This list is stored on the ILAS-IIC website (http://www.ilasic.math.uregina.ca/iic/). When you find new titles that would be good to include, or informative reviews that would be appropriate to reference, please send those suggestions to Prof. Baksalary (baxx @amu.edu.pl).

JOURNAL NEWS

Call for Papers: Special Issue of LAA on Tensors and Multilinear Algebra

Submission Deadline: September 30, 2010

Tensors are increasingly ubiquitous in various areas of applied, computational, and industrial mathematics because multilinear algebra arises naturally in many contexts. The purpose of this special issue is to bring together three lines of research -- multilinear algebraic techniques in multilinear algebra, linear algebraic techniques in multilinear algebra, and multilinear algebraic techniques in linear algebra.

The issue will attract the attention of mathematicians to challenges and recent findings in the rapidly developing field of applied and numerical multilinear algebra. It is open to all papers with significant new results where either multilinear algebraic methods play an important role or new tools and problems of multilinear algebraic nature are presented. Survey papers that illustrate common themes across disciplines and application areas, and especially where multilinear algebraic or tensor techniques play a central role, are also welcomed.

Papers must meet the publication standards of Linear Algebra and Its Applications and will be refereed in the usual way. For details on submitting papers, see http://www.stanford.edu/group/multilinear/laa.html.

Editors for this issue are Shmuel Friedland, University of Illinois at Chicago; Tamara G. Kolda, Sandia National Laboratories; Lek-Heng Lim, University of California, Berkeley; and Eugene E. Tyrtyshnikov, Russian Academy of Sciences.

Call for Papers: Special Issue of LAA on 1st Montreal Workshop on Idempotent and Tropical Mathematics

Deadline for Submissions: December 31, 2009

The 1st Montreal Workshop on Idempotent and Tropical Mathematics was held June 29-July 3, 2009, at the University of Montreal, Canada. A special issue of LAA will be devoted to papers within the scope of this workshop, Tropical Algebra and Mathematical Physics and their Applications, and Tropical Geometry and its Applications. The Editor-in-Chief for this special issue is Hans Schneider. Topics include:

Idempotent analysis;
Idempotent functional analysis;
Tropical geometry and algebra;
Tropical and idempotent convex geometry;
Mathematics of idempotent semirings and their applications;
Linear algebra over semirings;
Quantization, dequantization, and related topics of mathematical physics.

The deadline for submission of papers is December 31, 2009. Submissions will be refereed according to LAA standards. Send to any of the following special editors, preferably as PDF files in email attachments:

Peter Butkovic, School of Mathematics, Univ. of Birmingham, Edgbaston, Birmingham B15 2TT, United Kingdom (p.butkovic@bham.ac.uk).

Alexander Guterman, Faculty of Algebra, Dept. of Math and Mechanics, Moscow State Univ. GSP-1, 119991, Moscow, Russia (alexander.guterman@gmail.com).

Jean Jacques Loiseau, IRCCyN, UMR CNRS 6597, Ecole Centrale de Nantes, 1 rue de la Noe, BP 92101 44321 Nantes cedex 3, France (loiseau@irccyn.ec-nantes.fr).

William M. McEneaney, Dept. of Mechanical and Aerospace Engineering, MC 0411, Univ. of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0411, USA (wmceneaney@ucsd.edu).

Edouard Wagneur, GERAD, HEC Montréal, 3000, chemin de la Côte-Sainte-Catherine Montréal (Québec) Canada, H3T 2A7 (Edouard.Wagneur@gerad.ca).

Call for Papers: JP Journal of Algebra, Number Theory and Applications

The Pushpa Publishing House invites original research papers and critical survey articles for possible publication after peer review in its journal JP Journal of Algebra, Number Theory and Applications (JPANTA). This journal provides a common forum for significant research both on theoretical and on applied aspects of current interest in algebra and number theory and innovative links between these and various fields of applications. It is published in three volumes annually, each having two issues. All articles are reviewed in Mathematical Reviews, MathSciNet and Zentralblatt für Mathematik. Published articles and information about submissions are available at http://pphmj.com/journals/jpanta.htm.

Call for Papers Journal of Algebra, Number Theory: Advances and Applications

The Scientific Advances Publishers, Allahabad, India, invite original research papers and critical survey articles in all areas of algebra, number theory and their applications for possible publication in their new journal, Journal of Algebra, Number Theory: Advances and Applications (JANTAA).

JANTAA is being published in two volumes annually, with two issues in each. To see the contents of Volume 1, issue 2, visit http://scientificadvances.org/journals4P5.htm. For details about submissions, please visit the website http://scientificadvances.org/journals.

ILAS NEWS

ILAS President-VicePresident Annual Report

Submitted July 10, 2009

1. The following were elected in the ILAS 2008 elections to offices with terms that began on March 1, 2009 and end on February 29, 2012:

Secretary-Treasurer: Leslie Hogben
Board of Directors: Dario Bini and Shaun Fallat

The following continue in ILAS offices to which they were previously elected:
Vice-President: Chi-Kwong Li (term ends February 28, 2010)

Board of Directors:
Wayne Barrett (term ends February 28, 2010)
Judi McDonald (term ends February 28, 2011)
Andre Ran (term ends February 28, 2011)
Bryan Shader (term ends February 28, 2010)

James R. Weaver continues his appointment as Outreach Director until February 28, 2011.

Shmuel Friedland and William Watkins completed three-year terms on the ILAS Board of Directors on February 28, 2009. We thank both for their valuable contributions as Board members; their willingness to serve ILAS is most appreciated.

On February 28, 2009, Jeff Stuart completed his most recent term as ILAS Secretary-Treasurer, a position he held since 2000. ILAS extends its deep appreciation to Jeff for his years of careful management of ILAS's finances and membership records. In accordance with ILAS bylaws, Jeff will serve for one year as a voting member of the ILAS Board, until February 28, 2010.

We thank the members of the Nominating Committee - Rajendra Bhatia, Jane Day, Christian Mehl, Michael Neumann (Chair), and Michael Tsatsomeros - for their work on behalf of ILAS, and to all of the candidates that agreed to have their names stand for election.

2. The following ILAS-endorsed meetings have taken place since our last report:

5th International Workshop on Parallel Matrix Algorithms and Applications (PMAA'08), Neuchatel, Switzerland, June 20-22, 2008

9th Workshop on Numerical Ranges and Numerical Radii, Williamsburg, USA, July 19-21, 2008

International Workshop on Operator Theory and its Applications (IWOTA 2008), Williamsburg, USA, July 22-26, 2008

18th International Workshop on Matrices and Statistics, Smolenice Castle, Slovakia, June 23-27, 2009

3. ILAS has endorsed the following upcoming non-ILAS conferences of interest to ILAS members:

3rd International Workshop on Matrix Analysis and Applications, Zhejiang Forestry University, Hangzhou/Lin'An, China, July 9-13, 2009

3rd International Symposium on Positive Systems: Theory and Application (POSTA09), Valencia, Spain, September 2-4, 2009

Applied Linear Algebra Conference in honor of Hans Schneider, Novi Sad, Serbia, May 24-28, 2010

Coimbra Meeting on 0-1 Matrix Theory and Related Topics, Department of Mathematics, University of Coimbra, Coimbra, Portugal, June 17-19, 2010

10th Workshop on Numerical Ranges and Numerical Radii, Krakow, Poland, June 27-29, 2010

4. The following ILAS Lecture at a non-ILAS conference has been delivered since the last report:

Hans Schneider spoke at the XIXth International Workshop on Operator Theory and its Applications, College of William and Mary, Williamsburg, VA, USA, July 22-26, 2008.

5. The following ILAS conferences are scheduled:

16th ILAS Conference, Pisa, Italy, June 21-25, 2010. The Organizing Committee consists of: Michele Benzi, Avi Berman, Dario Bini (Chair), Luca Gemignani, Leslie Hogben, Steve Kirkland, Julio Moro, Ilya Spitkovsky, Françoise Tisseur, and Eugene Tyrtyshnikov.

17th ILAS Conference, Braunschweig, Germany, 2011 (contact person is Heike Fassbender).

6. The Electronic Journal of Linear Algebra (ELA) is now compiling its 18th volume. Its Editors-in-Chief are Ludwig Elsner and Danny Hershkowitz. Vol. 17 (2008) was ELA's largest ever, containing 45 papers (698 pages, a 51% increase over Vol. 16). Vol. 18 (2009) contains 26 papers so far (301 pages). There are 83 pending papers at present. The acceptance rate is 47%.

7. IMAGE, the semi-annual newsletter of ILAS has Chief Editor Jane Day. Several Consulting Editors are responsible for specific parts: Fuzhen Zhang (Problems Section), Peter Semrl (History of Linear Algebra), Oskar Baksalary (Book Reviews), Steve Leon (Education) and James Weaver (Advertisements).

Each issue is posted on the ILAS-IIC website http://www.ilasic.math.uregina.ca/iic/IMAGE/ and these are searchable PDF files. About 150 printed copies of each issue are actually mailed, but most ILAS members have opted to read it online only. The due dates for posting online are June 1 and December 1 each year. As of June 1, 2009, 42 issues have appeared. With issue 41, Hans Joachim Werner completed his service as an Editor-in-Chief of IMAGE, a position he held since 2000. ILAS extends its thanks to Professor Werner for his contributions to the newsletter.

8. ILAS-NET is managed by Sarah Carnochan Naqvi. This year the ILAS-Net distribution list is being reconciled with the membership list to ensure that all members are receiving ILAS-Net messages. There are approximately 600 email addresses on the current list.

9. The primary site of the ILAS INFORMATION CENTER (IIC) is at the University of Regina. Mirror sites are located

at the Technion, Temple University, the University of Chemnitz, and the University of Lisbon. This year, an online membership renewal form for ILAS is being developed. Also the Table of Contents of most IMAGE issues have been added in order to assist with searching the large IMAGE files.

10. The Linear Algebra Education Committee is chaired by Steve Leon. This committee published a survey about second courses in linear algebra in Image issue 41. There was a very limited response. The Education Committee needs to gather more information on this matter before it can write a report on the findings.

Linear algebra education was an area of emphasis at the May 2009 Haifa Matrix Theory Conference, with invited education talks by Gil Strang and Steve Leon as well as three contributed education talks. The Linear and Numerical Linear Algebra Conference to be held in August 2009 at Northern Illinois University will also feature linear algebra education as an area of emphasis. Linear algebra education minisymposia are being planned for the SIAM Applied Linear Algebra Conference in October 2009 and for the ILAS Annual Conference in Pisa, Italy in June 2010.

Respectfully submitted,
Steve Kirkland, ILAS President, stephen.kirkland@nuim.ie
Chi-Kwong Li, ILAS Vice-President, ckli@math.wm.edu

Increase in ILAS Dues

After careful consideration and extensive discussion, the ILAS Board of Directors has approved an increase to the annual dues, effective immediately. The new dues structure is as follows: $50 (US) per year, with a $10 (US) reduction for those members that choose not to have the print version of IMAGE mailed to them.

Note that complete PDF versions of all issues of IMAGE are available at http://www.ilasic.math.uregina.ca/iic/IMAGE/ and that each new issue is posted promptly.

The Board has also agreed that there will be no further dues increases for the next five years. In keeping with past practice, dues can be waived annually for students, postdoctoral fellows, and people undergoing financial hardship. Those who have already renewed their memberships at the old dues level will still have their memberships honoured.

The scope of ILAS activities has expanded considerably since the Society's founding. Recent years have seen ILAS providing partial support for students to attend ILAS annual conferences, as well as support for an increasing number of ILAS Lectures at non-ILAS conferences. Additional costs are anticipated in order to provide adequate technical support for the editing and production of the Electronic Journal of Linear Algebra (ELA), which has grown dramatically and is now receiving recognition for its high quality. Thus, the Board has approved the dues increase with the goals of strengthening ILAS, and ensuring that the Society can continue to play an active role in the growth of linear algebra.

Steve Kirkland, President
Chi-Kwong Li, Vice-President
Leslie Hogben, Secretary-Treasurer

Nominations for 2009 ILAS Elections

Submitted by Steve Kirkland, ILAS President

The Nominating Committee for the 2009 ILAS elections has completed its work.

Nominated for a three year term, beginning March 1, 2010, as ILAS Vice-President is Chi-Kwong Li.

Nominated for the two open three-year terms, beginning March 1, 2010, as "at-large" members of the ILAS Board of Directors are:
Christian Mehl, Berlin, Germany;
Naomi Shaked-Monderer, Haifa, Israel;
Jorma Merikoski, Tampere, Finland;
Tin-Yau Tam, Auburn, US.

According to ILAS By-Laws, additional nominations may be made by electronic or regular mail by any three members of ILAS; prior approval of the nominee is required. All such nominations should be sent to the chair of the Nominating Committee, Rajendra Bhatia <rbh@isid.ac.in>, by November 4, 2009. Ballots will be sent out towards the end of 2009.

Thanks to the nominees for agreeing to stand for election, and to the Nominating Committee for their important service to ILAS: Rajendra Bhatia (Chair), Dario Bini, Roger Horn, Tom Laffey, and Francoise Tisseur.

New Address for ILAS President Steve Kirkland

As of July 1, 2009, Steve Kirkland's new postal and email addresses are:

Steve Kirkland
Hamilton Institute
National University of Ireland, Maynooth
Co. Kildare
Ireland
email: stephen.kirkland@nuim.ie

ILAS Members Named SIAM Fellows

The Society for Industrial and Applied Mathematics has inaugurated a Fellows Program. This honor is bestowed for outstanding contributions to applied mathematics and computational science, and these first honorees will be recognized during the 2009 SIAM Annual Meeting in Denver, Colorado. There are 191 members in this first group, and ten members of ILAS are among them:

Carl de Boor, University of Wisconsin, Madison
Nicholas J. Higham, University of Manchester
Thomas Kailath, Stanford University
Peter Lancaster, University of Calgary
Cleve Moler, MathWorks, Inc.
Michael L. Overton, Courant Institute of Mathematical Sciences, New York University
Seymour V. Parter, University of Wisconsin, Madison
G.W. Stewart, University of Maryland, College Park
Gilbert Strang, Massachusetts Institute of Technology
Paul M. Van Dooren, Université Catholique de Louvain

The full list can be found at http://fellows.siam.org.

Send News for IMAGE Issue 44 by April 1, 2010

All items of interest to the linear algebra community are welcome, including:

Announcements and reports of conferences and workshops
News about striking developments in linear algebra and its applications
Honors and awards
Articles on history
Survey articles
Books, websites, funding sources
News about journals
Employment and other funding opportunities
Problems and solutions
Transitions: new appointments, responsibilities, deaths
Linear algebra education
Possible new corporate sponsors

Send material to the appropriate Editor by April 1, 2010:

Problems and solutions to Fuzhen Zhang (zhang@nova.edu).
Book news and reviews to Oskar Baksalary (baxx@amu.edu.pl).
History of linear algebra to Peter Šemrl (peter.semrl@fmf.uni-lj.si).
Linear algebra education to Steve Leon (sleon@umassd.edu).
Advertisements to Jim Weaver (jweaver@uwf.edu).
All other material and ideas to Jane Day (day@math.sjsu.edu)

Photos are welcome. Historical tidbits and other short items as well as longer articles are welcome.

If you wish to submit an article after April 1, let Jane Day know as soon as possible when to expect your item, and it may be possible to include it.

Send material in plain text, Word, or both Latex and PDF (Article.sty with no manual formatting is preferred). Send photos in JPG format. Issue 44 will be published on June 1, 2010.

EMPLOYMENT

Research Positions at SISTA: Signals, Identification, System Theory and Automation Research Division, University of Leuven, Belgium

The research division SISTA of the Department of Electrical Engineering (http://www.esat.kuleuven.be/scd/) is one of the largest research groups of the University of Leuven (<http://www.kuleuven.be/>).

Research activities involve numerical linear algebra, system identification and control, algorithms for numerical optimization and machine learning (support vector machines), and signal processing. Numerous applications in industrial process control, biomedical signal processing, bio-informatics, health decision support systems and telecommunications are studied.

SISTA is looking for well motivated researchers to start a 4 year PhD or for experienced postdocs to pursue research activities in these fields and application areas. On the website (http://www.esat.kuleuven.be/scd/), one can find a description of more than 20 different specific research topics, in which there are openings for PhDs and postdocs. SISTA can offer a competitive salary and a stimulating research environment, settled in the nice historic city of Leuven and its surroundings, within a university whose origins go back to 1425 (http://www.leuven.be/).

Electronic applications, including a CV, a list of publications, names of two possible references, and a brief description of your research interests and motivation, should be submitted to bart.demoor@esat.kuleuven.be.

IMAGE Problem Corner: Old Problems With Solutions

We present solutions to Problems 41-4 and 42-1 through 42-6. Seven new problems are on the back cover; solutions are invited.

Problem 41-4: Integer Matrices with Unit Determinants II

Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England,* R.W.Farebrother@Manchester.ac.uk

(i) For positive integer values of d, f and g the equation $(f^2 - dg^2)^2 + d(2fg)^2 = (f^2 + dg^2)^2$ supplies sets of three integers satisfying the generalised Pythagorean equation $x^2 + dy^2 = z^2$. Outline a matrix proof of this result.

(ii) In IMAGE Problem 36-1 [see IMAGE **37** (Fall 2006), pp. 20-23 for the problem and its solutions by Farebrother and by Werner] we were concerned with the construction of a 2×2 matrix with a determinant of plus one, and thus with the solution of Pell's Equation $f^2 - dg^2 = +1$ where f and g are nonzero integers and d is a fixed positive integer (though not a square). If instead we assume that the matrix has a determinant of minus one, then we are concerned with the solution of a variant of Pell's Equation (which, for obvious reasons, we shall name Nell's Equation) $f^2 - dg^2 = -1$. Identify integer solutions to this equation for certain choices of d. Use the result established in Part (i) to obtain the corresponding solutions to Pell's Equation.

Solution 41-4 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

By using the product rule, taking determinants of the matrices on both sides of the following matrix identity

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} f^2 - dg^2 & 2dg^2 \\ -2f^2 & f^2 - dg^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} f^2 + dg^2 & 2dg^2 \\ 0 & f^2 + dg^2 \end{pmatrix}$$

yields $(f^2 - dg^2)^2 + d(2fg)^2 = (f^2 + dg^2)^2$. So $\tilde{f}^2 - \tilde{d}\tilde{g}^2 = (f^2 - dg^2)^2$, where $\tilde{f} := f^2 + dg^2$, $\tilde{g} := (2fg)^2$ and $\tilde{d} := d$. It is now clear that if $(f,\, d,\, g) \in \mathbb{N}^3$ solves Nell's equation, i.e., if $f^2 - dg^2 = -1$, then $(\tilde{f},\, \tilde{d},\, \tilde{g})$ solves Pell's equation, i.e., $\tilde{f} - \tilde{d}\tilde{g}^2 = 1$.

Next, we are interested in positive integer solutions to Nell's equation

$$f^2 - dg^2 = -1. \tag{1}$$

Clearly, for any positive integer f, the triplet $(f,\, d,\, g) \in \mathbb{N}^3$ with $d := f^2 + 1$ and $g := 1$ trivially satisfies eqn. (1). Therefore, we list in the following table only all those integers f between 1 and 1000 for which there also exists a nontrivial solution to eqn. (1). Our table contains for these f-values only all the nontrivial (positive integer) solutions $(f,\, d,\, g) \in \mathbb{N}^3$.

f	7	18	32	38	41	43	57	68	70	82	93	99	107	117	118	132	143	157	168
d	2	13	41	5	2	74	130	185	29	269	346	58	458	10	557	697	818	986	1129
g	5	5	5	17	29	5	5	5	13	5	5	13	5	37	5	5	5	5	5

f	182	182	193	207	218	232	239	239	243	251	257	268	268	268	282	293
d	53	1325	1490	1714	1901	2153	2	338	2362	218	2642	17	425	2873	3181	3434
g	25	5	5	5	5	5	169	13	5	17	5	65	13	5	5	5

f	307	318	327	332	343	357	368	378	382	393	407	408	418	432	437
d	3770	4045	370	4409	4706	5098	5417	85	5837	6178	6626	985	6989	7465	1130
g	5	5	17	5	5	5	5	41	5	5	5	13	5	5	13

f	443	443	457	468	482	493	500	507	515	518	532	540	543	557	568
d	314	7850	8354	8761	9293	9722	89	10282	26	10733	11321	1009	11794	12410	12905
g	25	5	5	5	5	5	53	5	101	5	5	17	5	5	5

f	577	582	593	606	607	616	618	632	643	657	668	682	682	682
d	1970	13549	14066	2173	14738	1313	15277	15977	16538	17266	17849	5	125	18605
g	13	5	5	13	5	17	5	5	5	5	5	305	61	5

f	693	707	718	732	743	746	757	768	775	776	782	793	800
d	19210	19994	20621	21433	22082	3293	22922	23593	3554	113	24461	25154	761
g	5	5	5	5	5	13	5	5	13	73	5	5	29

f	807	807	818	829	832	843	857	868	882	882	882	893
d	1042	26050	26765	2378	27689	28426	29378	30137	37	925	31117	31898
g	25	5	5	17	5	5	5	5	145	29	5	5

f	905	907	915	918	932	943	944	957	968	982	993
d	2834	32906	4954	33709	34745	35570	5273	36734	37481	38573	39442
g	17	5	13	5	5	5	13	5	5	5	5

Many results in this table can be explained as follows. If $(f,\, d,\, g) \in \mathbb{N}^3$ is a solution to eqn. (1), then, for each $k \in \mathbb{N}$, the triplets

$$(f + kg^2,\, d + k(2f + kg^2),\, g), \quad (kg^2 - f,\, d + k(kg^2 - 2f),\, g) \tag{2}$$

also represent solutions to eqn. (1). For observe that if $(f,\, d,\, g) \in \mathbb{N}^3$ satisfies $\frac{f^2+1}{g^2} = d$, then $\frac{(f+x)^2+1}{g^2} = \frac{f^2+1}{g^2} + \frac{x(x+2f)}{g^2} \in \mathbb{N}$ if and only if $\frac{x(x+2f)}{g^2} \in \mathbb{N}$, which in turn happens whenever (a) $x = kg^2$ or (b) $x = kg^2 - 2f$ for some $k \in \mathbb{N}$. In case (a), $f + x = f + kg^2$, so

$$\frac{(f+x)^2+1}{g^2} = \frac{(f+kg^2)^2+1}{g^2} = \frac{f^2+1}{g^2} + \frac{2fkg^2 + k^2g^4}{g^2} = d + k(2f + kg^2) \in \mathbb{N}.$$

This shows that if $(f,\, d,\, g)$ solves eqn. (1), then triplet $(f + kg^2,\, d + k(2f + kg^2),\, g)$ also solves eqn. (1). Likewise, in case (b), $f + x = kg^2 - f$, so

$$\frac{(f+x)^2+1}{g^2} = \frac{(kg^2-f)^2+1}{g^2} = \frac{f^2+1}{g^2} + \frac{k^2g^4 - 2fkg^2}{g^2} = d + k(kg^2 - 2f),$$

which shows that if $(f,\, d,\, g)$ solves eqn. (1), then triplet $(kg^2 - f,\, d + k(kg^2 - 2f),\, g)$ also solves eqn. (1). Since $(7,\, 2,\, 5)$ represents a solution to Nell's equation (with $g = 5$), all other solutions with $g = 5$ can be reproduced by means of this method.

In what follows, let $\mathcal{D}_{\mathbb{N}}$ denote the set of all positive integers d satisfying $\frac{f^2+1}{d} \in \mathbb{N}$ for some number $f \in \mathbb{N}$, and let $\mathcal{D}$ denote the set of all those positive integers d for which there exists a triplet $(f,\, d,\, g) \in \mathbb{N}^3$ representing a solution to Nell's equation (1). Needless to say, $\mathcal{D}$ is a proper subset of $\mathcal{D}_{\mathbb{N}}$. The elements of $\mathcal{D}_{\mathbb{N}}$ which are less than or equal to 200 are as follows: 2, 5, 10, 13, 17, 25, 26, 29, 34, 37, 41, 50, 53, 58, 61, 65, 73, 74, 82, 85, 89, 97, 101, 106, 109, 113, 122, 125, 130, 137, 145, 146, 149, 157, 169, 170, 173, 178, 181, 185, 193, 194, 197. As mentioned before, not all of these integers can be expected to be also elements of $\mathcal{D}$. So, of course, all listed squares fail. Some other non-squares, such as the integer 34, also fail. Astonishingly, 34 is the only non-square element of $\mathcal{D}_{\mathbb{N}}$ that is ≤ 100 and has this property. We conclude our answer to this interesting problem by listing in our final table nontrivial solutions to equation (1) whose d-values are between 2 and 100.

f	18	70	32	182	99	29718	1068	43	378	500	5604
d	13	29	41	53	58	61	73	74	85	89	97
g	5	13	5	25	13	3805	125	5	41	53	569

Also solved by the proposer.

Problem 42-1: Square Root of a Product of Two Orthogonal Projectors

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

Let P, Q be $n \times n$ orthogonal projectors (i.e., Hermitian idempotent matrices) such that PQ is nonzero. Find a square root of PQ.

Solution 42-1.1 by Johanns de Andrade Bezerra, *Campina Grande, PB, Brazil,* pav.animal@hotmail.com

We prove a more general result.

Let A and B be two positive semidefinite matrices of the same order. Then AB is diagonalizable and has nonnegative eigenvalues [1, Corollary 2.3]. If $C^2 = AB$, then C is a square root of AB.

Let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be the distinct eigenvalues of AB. Then $AB = \lambda_1 E_1 + \lambda_2 E_2 + \cdots + \lambda_k E_k$, where

$$E_i E_j = 0,\, i \neq j, \quad E_i^2 = E_i,\, i = 1, \ldots, k, \quad E_1 + E_2 + \cdots + E_k = I, \quad rangeE_i = ker(\lambda_i I - AB).$$

Thus, $(AB)^* = \bar{\lambda_1} E_1^* + \bar{\lambda_2} E_2^* + \cdots + \bar{\lambda_k} E_k^*$, and $rangeE_i^* = ker(\bar{\lambda_i} I - (AB)^*)$. It follows that

$$(rangeE_i^*)^\perp = (ker(\bar{\lambda_i} I - (AB)^*))^\perp = kerE_i = range(\lambda_i I - AB).$$

Therefore, $C = \sqrt{\lambda_1} E_1 + \sqrt{\lambda_2} E_2 + \cdots + \sqrt{\lambda_k} E_k$, $C^2 = AB$, with $rangeE_i = ker(\lambda_i I - AB)$ and $kerE_i = range(\lambda_i I - AB)$.

Reference

[1] Y.-P. Hong and R.A. Horn, The Jordan canonical form of the product of a Hermitian matrix and a positive semidefinite matrix, *Linear Algebra Appl.* 147(**1991**)373–386.

Solution 42-1.2 by by Richard William Farebrother, *Bayston Hill, Shrewsbury, England,* R.W.Farebrother@Manchester.ac.uk

Let P have rank r and Q have rank s and let m satisfy $max(r, s) \leq m \leq n$, then the $n \times n$ idempotent Hermitian matrices P and Q may be written in the form $P = XX^*$ and $Q = YY^*$ where X is an $n \times m$ matrix with $m - r$ zero columns satisfying $X^*X = diag\{I_r,\, 0_{m-r}\}$ and where Y is an $n \times m$ matrix with $m - s$ zero columns satisfying $Y^*Y = diag\{I_s,\, 0_{m-s}\}$.

Let the $m \times m$ matrix X^*Y have singular value decomposition $X^*Y = UDV^*$ where U and V are $m \times m$ orthonormal matrices satisfying $U^*U = I_m$ and $V^*V = I_m$ and where D is a *real* $m \times m$ diagonal matrix with nonnegative elements on its diagonal and zeros elsewhere [as any negative or complex factors $d_j/|d_j|$ can be absorbed into the jth columns of U or V].

Clearly $R = XUV^*Y^*$ is the required square root of PQ as it satisfies $R^2 = XUV^*Y^*XUV^*Y^* = XUV^*VD^*U^*UV^*Y^* = XUDV^*Y^* = XX^*YY^* = PQ$.

Also solved by Eugene A. Herman, Hans J. Werner, and the proposers.

Problem 42-2: Properties of a Certain Matrix Product

Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England,* R.W.Farebrother@Manchester.ac.uk

Given an $n \times n$ nonzero matrix A with complex elements. Suppose that B is a second $n \times n$ matrix with complex elements satisfying $AB = BA$ and $ABA = A$. Investigate the properties of the $n \times n$ matrix $C = BAB$.

Solution 42-2.1 by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Dortmund University of Technology, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

The solution to the problem is given in what follows.

Proposition. Let $A, B \in \mathbb{C}_{n,n}$ be such that $AB = BA$ and $ABA = A$. Then $C = BAB$ is the group inverse of A.

Proof. The assertion will be established by straightforward verifications of the three conditions constituting the definition of the group inverse. Recall that the group inverse of $A \in \mathbb{C}_{n,n}$ is the unique matrix $A^\# \in \mathbb{C}_{n,n}$ satisfying the equations

$$AA^\# A = A, \quad A^\# AA^\# = A^\#, \quad AA^\# = A^\# A; \tag{3}$$

see e.g., Section 4.4 in Ben-Israel & Greville (2003).

On account of $ABA = A$, we have

$$ACA = ABABA = ABA = A$$

and

$$CAC = BABABAB = BABAB = BAB = C,$$

what shows that C is a reflexive inverse of A, satisfying the first two conditions in (3).

The satisfaction of the third conditions in (3) is implied by $AB = BA$, which ensures that

$$CA = BABA = ABAB = AC.$$

Thus, $C = A^{\#}$. $\square$

Reference

A. Ben-Israel and T.N.E. Greville, *Generalized Inverses: Theory and Applications* (2nd ed.), Springer-Verlag, New York, 2003.

Solution 42-2.2 by Susana Furtado, *Faculdade de Economia do Porto, Portugal*, sbf@fep.up.pt
and Maria Isabel Bueno, *University of California, Santa Barbara, USA*, mbueno@math.ucsb.edu

Theorem. Let $A, B \in \mathbb{C}^{n\times n}$. If $AB = BA$ and $ABA = A$ then A, B and BAB are simultaneously similar to matrices of the forms $0_p \oplus J_A$, $B_{11} \oplus J_A^{-1}$ and $0 \oplus J_A^{-1}$, respectively, where $0 \le p \le n$, $J_A \in \mathbb{C}^{(n-p)\times(n-p)}$ is nonsingular and $B_{11} \in \mathbb{C}^{p\times p}$.

Proof. If A is nonsingular the result is trivial. Now suppose that A is singular, $AB = BA$ and $ABA = A$. Let $Q \in \mathbb{C}^{n\times n}$ be a nonsingular matrix such that

$$A' := QAQ^{-1} = J_0 \oplus J_A,$$

where $J_0 \in \mathbb{C}^{p\times p}$ is nilpotent and $J_A \in \mathbb{C}^{(n-p)\times(n-p)}$ is nonsingular. W.l.g., suppose that $J_0 = J_{p_1}(0) \oplus \cdots \oplus J_{p_k}(0)$, where $J_{p_i}(0)$ is the nilpotent Jordan block of size p_i and $p_1 \ge \cdots \ge p_k > 0$ ($p = p_1 + \cdots + p_k$). Let

$$B' := QBQ^{-1} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

where $B_{11} \in \mathbb{C}^{p\times p}$. The equality $AB = BA$ implies that $J_A B_{21} - B_{21} J_0 = 0$ and $J_0 B_{12} - B_{12} J_A = 0$.

Since J_A is nonsingular and J_0 is nilpotent, $B_{21} = 0$ and $B_{12} = 0$. Therefore $B' = B_{11} \oplus B_{22}$. Then, the equality $ABA = A$ implies that $B_{22} = J_A^{-1}$ and

$$J_0 B_{11} J_0 = J_0. \tag{4}$$

If

$$B_{11} = \begin{bmatrix} B'_{11} & * \\ * & * \end{bmatrix},$$

with $B'_{11} \in \mathbb{C}^{p_1\times p_1}$, the equality $AB = BA$ implies that $J_{p_1}(0)B'_{11} - B'_{11}J_{p_1}(0) = 0$. If $p_1 > 1$, a calculation shows that B'_{11} is upper triangular. From (4) it follows that

$$J_{p_1}(0)B'_{11}J_{p_1}(0) = J_{p_1}(0). \tag{5}$$

Then $p_1 = 1$ as, if $p_1 > 1$, because B'_{11} would be upper triangular, the entry in position $(1, 2)$ of the matrix in the left hand side of (5) would be 0 while the corresponding entry of the matrix in the right hand side is 1. Hence, $p_1 = \cdots = p_k = 1$, i.e., $J_0 = 0$. Then,

$$QAQ^{-1} = 0 \oplus J_A,\ QBQ^{-1} = B_{11} \oplus J_A^{-1} \text{ and } Q(BAB)Q^{-1} = 0 \oplus J_A^{-1}. \quad \square$$

Remark. It is a simple consequence of the previous theorem that if $AB = BA$ and $ABA = A$ then BAB is the Drazin inverse of A. We also note that if A and B are simultaneously similar to matrices of the forms described in the statement of the theorem then $AB = BA$ and $ABA = A$.

Also solved by Johanns de Andrade Bezerra, Eugene A. Herman, Hans J. Werner, and the proposer.

Problem 42-3: Commutators and Nonderogatory Matrices

Proposed by Geoffrey Goodson, *Towson University, Towson, USA,* goodson@towson.edu

Let A, B and D be n-by-n matrices. It is known that if $AB - BA = D$, where $AD = DA$, then D is nilpotent; and if A is diagonalizable, then $D = 0$. If instead $AB - BA^T = D$ and $AD = DA^T$, then D need not be nilpotent, but it is also known that if A is diagonalizable then $D = 0$.

More generally, show that if $AB - BA^T = D$ and $AD = DA^T$, where A is nonderogatory, then D is symmetric, singular and the eigenvalue $\lambda = 0$ of D has a geometric multiplicity greater than or equal to the number of distinct eigenvalues of A.

Solution 42-3 by Johanns de Andrade Bezerra, *Campina Grande, PB, Brazil,* pav.animal@hotmail.com

Let λ_i be any eigenvalue of A. For $c \in \mathbf{C}$, with $c \neq \lambda_i$ for any i, obviously, $A - cI$ is non-singular, nonderogatory, $(A-cI)B - B(A-cI)^T = D \Leftrightarrow AB - BA^T = D$ and $(A-cI)D = D(A-cI)^T \Leftrightarrow AD = DA^T$. Moreover, the number of distinct eigenvalues of A equals the number of distinct eigenvalues of $A - cI$. Hence, for simplicity, we consider non-singular A.

Since A and A^T are similar, it follows that A and A^T have the same characteristic polynomials and minimal polynomials, and so A^T is also nonderogatory; hence $A^Tv_i = \lambda_i v_i$ for some eigenvector of A^T related with λ_i, and so $DA^Tv_i = ADv_i = \lambda_i Dv_i$. In fact, as A^T is nonderogatory, that is, each eigenspace of A^T has dimension equal to one, it follows that it is enough to show that $Dv_i = 0$ to prove that D is singular and that the eigenvalue $\lambda = 0$ of D has a geometric multiplicity greater than or equal to the number of distinct eigenvalues of A.

Thus, let $AB - BA^T = D$, then $ABv_i - BA^Tv_i = ABv_i - \lambda_i Bv_i = Dv_i$, hence $\lambda_i ABv_i - \lambda_i^2 Bv_i = \lambda_i Dv_i = ADv_i = A^2Bv_i - \lambda_i ABv_i$, which implies $A^2Bv_i - 2\lambda_i ABv_i + \lambda_i^2 Bv_i = 0$, and so $(A-\lambda_i I)^2 Bv_i = 0$. If $Bv_i = 0$, then $Dv_i = 0$. Thus, consider $Bv_i \neq 0$ for some i. Suppose that $(A - \lambda_i I)Bv_i = Dv_i \neq 0$, then Bv_i and $(A-\lambda_i I)Bv_i$ are linearly independent, and so $Dv_i \neq kBv_i$, k constant, that is, $Dv_i - kBv_i = x_k$, with $x_k \neq 0$ (obviously, if $x_k = 0$, then $Dv_i = 0$ since Dv_i and Bv_i are linearly dependent). Hence, $ADv_i - kABv_i = DA^Tv_i - kABv_i = \lambda_i Dv_i - kABv_i = \lambda_i Dv_i - k(Dv_i + \lambda_i Bv_i) = (\lambda_i - k)Dv_i - \lambda_i kBv_i = Ax_k$. Consider $k = \lambda_i$, and so $x_k = x$, then $-\lambda_i^2 Bv_i = Ax \Rightarrow -\lambda_i^2 ABv_i = A^2x = -\lambda_i^2(Dv_i + \lambda_i Bv_i) = -\lambda_i^2(x + 2\lambda_i Bv_i) = -\lambda_i^2(x + 2\lambda_i \frac{Ax}{-\lambda_i^2}) = -\lambda_i^2 x + 2\lambda_i Ax \Rightarrow A^2x - 2\lambda_i Ax + \lambda_i^2 x = 0 \Rightarrow (A - \lambda_i I)^2 x = 0$.

Let $x_k = Dv_i - kBv_i = ABv_i - (\lambda_i + k)Bv_i$. Now consider $k = -\lambda_i$, and so $x_k = y$, then $y = Dv_i + \lambda_i Bv_i = ABv_i \Rightarrow ADv_i + \lambda_i ABv_i = DA^Tv_i + \lambda_i ABv_i = \lambda_i Dv_i + \lambda_i y = Ay \Rightarrow \lambda_i ADv_i + \lambda_i Ay = A^2y = \lambda_i^2 Dv_i + \lambda_i Ay \Rightarrow \lambda_i(Ay - \lambda_i y) + \lambda_i Ay = A^2y \Rightarrow (A - \lambda_i I)^2 y = 0$.

$y = sx$, s constant, $\Leftrightarrow Dv_i + \lambda_i Bv_i = sDv_i - s\lambda_i Bv_i \Leftrightarrow (1-s)Dv_i + (1+s)\lambda_i Bv_i = 0 \Leftrightarrow Dv_i = \frac{1+s}{s-1}\lambda_i Bv_i$, but this contradicts that $Dv_i \neq kBv_i$ for any $k \in \mathbf{C}$; so x and y are linearly independent, with $s \neq \pm 1$ (if $s = \pm 1$, then $Dv_i = 0$).

Since A is nonderogatory, it follows that $dimker(A - \lambda_i I)^2 = 2$, and so if $(A - \lambda_i I)x \neq 0$, then $(A - \lambda_i I)y = 0$ since x and y are linearly independent. Suppose that $(A - \lambda_i I)x \neq 0$, as $(A - \lambda_i I)^2 x = 0$, then $(A - \lambda_i I)x \neq ax$, a constant, which implies $Ax - (\lambda_i + a)x = z = A(Dv_i - \lambda_i Bv_i) - (\lambda_i + a)(Dv_i - \lambda_i Bv_i) \Rightarrow$

$$ADv_i - \lambda_i ABv_i - (\lambda_i + a)Dv_i + (\lambda_i^2 + a\lambda_i)Bv_i = z \tag{6}$$

Let $(A - \lambda_i I)y = 0 \Rightarrow (A - \lambda_i I)(Dv_i + \lambda_i Bv_i) = 0 \Rightarrow$

$$ADv_i + \lambda_i ABv_i - \lambda_i Dv_i - \lambda_i^2 Bv_i = 0 \tag{7}$$

Adding (6) and (7), we have $2ADv_i - (2\lambda_i + a)Dv_i + a\lambda_i Bv_i = z \Rightarrow 2\lambda_i Dv_i - (2\lambda_i + a)Dv_i + a\lambda_i Bv_i = -aDv_i + a\lambda_i Bv_i = z = -a(Dv_i - \lambda_i Bv_i) = -ax$, hence $Ax - (\lambda_i + a)x = -ax$, this yields $(A - \lambda_i I)x = 0$. However, since A is nonderogatory, it follows that $dimker(A - \lambda_i I) = 1$, and as x and y are linearly independent, there exists clearly a contradiction, and so the supposition $Dv_i \neq 0$ is false, and therefore $Dv_i = 0$.

Now we will show that D is symmetric.

Let $D = AB - BA^T$ and $D^T = B^TA^T - AB^T$, then

$$D - D^T = A(B + B^T) - (B + B^T)A^T = X, \tag{8}$$

hence $AD - AD^T = DA^T - AD^T = DA^T - (DA^T)^T = AX$. Clearly, $X^T = -X$ and $(AX)^T = -AX = X^TA^T = -XA^T$, and so

$$AX = XA^T \tag{9}$$

From equations (8) and (9), we can conclude that $Xv_i = 0$.

Let $v_1^i, v_2^i, \ldots, v_{r_i}^i$ be a Jordan chain of A^T corresponding to λ_i. If $v_1^i = v_i \neq 0$, then the following relations hold:

$$\left\{\begin{array}{lcr} A^Tv_1^i = \lambda_i v_1^i & \Rightarrow & XA^Tv_1^i = \lambda_i Xv_1^i \\ A^Tv_2^i - \lambda_i v_2^i = v_1^i & \Rightarrow & XA^Tv_2^i - \lambda_i Xv_2^i = Xv_1^i \\ A^Tv_3^i - \lambda_i v_3^i = v_2^i & \Rightarrow & XA^Tv_3^i - \lambda_i Xv_3^i = Xv_2^i \\ \cdots\cdots\cdots\cdots & \cdot\cdot & \cdots\cdots\cdots\cdots\cdots \\ A^Tv_{r_i}^i - \lambda_i v_{r_i}^i = v_{r_i-1}^i & \Rightarrow & XA^Tv_{r_i}^i - \lambda_i Xv_{r_i}^i = Xv_{r_i-1}^i \end{array}\right\} \Rightarrow \left\{\begin{array}{l} AXv_2^i = \lambda_i Xv_2^i \\ AXv_3^i - \lambda_i Xv_3^i = Xv_2^i \\ \cdots\cdots\cdots\cdots\cdots \\ AXv_{r_i}^i - \lambda_i Xv_{r_i}^i = Xv_{r_i-1}^i \end{array}\right\}.$$

Since A and A^T are similar and nonderogatory, it follows that the Jordan chain of A and A^T corresponding to λ_i have the same quantity of vectors. Hence, suppose that there exists a vector $v^i_{r_i+1}$ such that

$$AXv^i_{r_i+1} - \lambda_i Xv^i_{r_i+1} = Xv^i_{r_i}. \tag{10}$$

Thus, $v^i_{r_i+1} = a_1 v^i_1 + \cdots + a_{r_i} v^i_{r_i}$, $a_1, \ldots, a_{r_i} \in \mathbf{C}$, which implies $Xv^i_{r_i+1} = a_1 Xv^i_1 + a_2 Xv^i_2 + \cdots + a_{r_i} Xv^i_{r_i}$ and $AXv^i_{r_i+1} = a_1 AXv^i_1 + a_2 AXv^i_2 + a_3 AXv^i_3 + \cdots + a_{r_i} AXv^i_{r_i}$, substituting in (10), we have that $a_2\lambda_i Xv^i_2 + a_3(Xv^i_2 + \lambda_i Xv^i_3) + \cdots + a_{r_i}(Xv^i_{r_i-1} + \lambda_i Xv^i_{r_i}) - \lambda_i a_2 Xv^i_2 - \lambda_i a_3 Xv^i_3 - \cdots - \lambda_i a_{r_i} Xv^i_{r_i} = Xv^i_{r_i} \Rightarrow a_3 Xv^i_2 + a_4 Xv^i_3 + \cdots + a_{r_i} Xv^i_{r_i-1} = Xv^i_{r_i}$, but this represents a contradiction since $Xv^i_2, Xv^i_3, \ldots, Xv^i_{r_i}$ are linearly independent, hence $Xv^i_2 = Xv^i_3 = \cdots = Xv^i_{r_i} = 0$, and therefore $X = 0$, that is, D is symmetric.

Also solved by the proposer.

Problem 42-4: Eigenvalue Interlacing for Normal Matrices

Proposed by Roger A. Horn, *Department of Mathematics, University of Utah, Salt Lake City, Utah, USA,* rhorn@math.utah.edu

Prove the following generalizations of eigenvalue interlacing for bordered or additively perturbed Hermitian matrices.

1. Let $n \geq 3$, $A \in M_{n-1}$, $x, y \in \mathbb{C}^{n-1}$, $\beta \in \mathbb{C}$, and $k \in \{2, \ldots, n-1\}$ be given and let

$$C = \begin{bmatrix} A & y \\ x^* & \beta \end{bmatrix} \in M_n.$$

   Suppose that A and C are normal and let $\mathcal{D} \subset \mathbb{C}$ be a given closed disk that is not a point. If there are k eigenvalues of A in $\mathcal{D}$, then there are at least $k-1$ eigenvalues of C in $\mathcal{D}$. If there are p eigenvalues of C in $\mathcal{D}$, then there are at least $p-2$ eigenvalues of A in $\mathcal{D}$.

2. Let $A \in M_n$ and $x, y \in \mathbb{C}^n$ be given and let $C = A + yx^*$. Suppose that A and C are normal and let $\mathcal{D} \subset \mathbb{C}$ be a given closed disk that is not a point. Then the numbers of eigenvalues of A and C in $\mathcal{D}$ differ by no more than 1.

Solution 42-4 by the proposer Roger A. Horn, *Department of Mathematics, University of Utah, Salt Lake City, Utah, USA,* rhorn@math.utah.edu

We invoke the following *residual theorem* to prove both claims.

Theorem. Let $C \in M_n$ be normal, let $\mathcal{S} \subset \mathbb{C}^n$ be a given k-dimensional subspace, and let $\gamma \in \mathbb{C}$ and $\delta > 0$ be given. Suppose that $||Cx - \gamma x|| \leq \delta$ for every unit vector $x \in \mathcal{S}$. Then there are at least k eigenvalues of C in the disk $\{z \in \mathbb{C} : |z - \gamma| \leq \delta\}$.

Proof. Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of C. The eigenvalues of the positive semidefinite matrix $G = (C - \gamma I)^*(C - \gamma I)$ are $|\lambda_1 - \gamma|^2, \ldots, |\lambda_n - \gamma|^2$. We have assumed that $x^* G x \leq \delta^2 x^* x$ for all $x \in \mathcal{S}$, so at least k eigenvalues of G are not greater than δ^2 [1, Theorem 4.3.21]. □

For a stronger form of the residual theorem, see [2, Theorem 4.1].

In the following, let $\{\xi_1, \ldots, \xi_k\}$ be an orthonormal set of eigenvectors of a normal matrix A corresponding to its eigenvalues $\lambda_1, \ldots, \lambda_k$ in a closed disk $\mathcal{D}$, which has center γ and radius $\delta > 0$. Let $\mathcal{E} = \operatorname{span}\{\xi_1, \ldots, \xi_k\}$ and check that $z \in \mathcal{E} \Rightarrow \|(A - \gamma I)z\| \leq \delta \|z\|$ (write z as a linear combination of $\xi_1, \ldots, \xi_k$).

Proof of #1. We have $\dim \mathcal{E} + \dim\{x^\perp\} = k + (n-2) = (n-1) + (k-1)$, so the dimension of $\mathcal{E} \cap \{x^\perp\}$ is at least $k-1$. Thus, the dimension of

$$\mathcal{S} = \left\{ \zeta = \begin{bmatrix} z \\ 0 \end{bmatrix} \in \mathbb{C}^n : z \in \mathcal{E} \cap \{x^\perp\} \right\}$$

is also at least $k-1$. Then $\zeta \in \mathcal{S} \Rightarrow ||(C - \gamma I)\zeta|| = ||(A - \gamma I)z|| \leq \delta ||z||$. Apply the residual theorem.

Proof of #2. We have $\dim \mathcal{E} + \dim\{x^\perp\} = k + (n-1) = n + (k-1)$, so the dimension of $\mathcal{S} = \mathcal{E} \cap \{x^\perp\}$ is at least $k-1$. Then $z \in \mathcal{S} \Rightarrow \|(C - \gamma I)z\| = \|(A - \gamma I)z\| \leq \delta ||z||$. Apply the residual theorem to conclude that if A has k eigenvalues in $\mathcal{D}$, then C has at least $k-1$ eigenvalues there. Now interchange A and C in the preceding argument to show that if C has p eigenvalues in $\mathcal{D}$, then A has at least $p-1$ eigenvalues there. Conclude that $k+1 \geq p \geq k-1$.

References

[1] R. Horn and C. Johnson, *Matrix Analysis*, Cambridge University Press, New York, 1985.

[2] R. Horn, N. Rhee, and W. So, Eigenvalue inequalities and equalities, *Linear Algebra Appl.* 270(**1998**)29-44.

Problem 42-5: Obtuse Basis and Acute Dual Basis for Euclidean Space

Proposed by Tin-Yau Tam, *Auburn University, Auburn, USA,* tamtiny@auburn.edu

If $\{e_1, \ldots, e_n\}$ is an obtuse basis of the Euclidean space E (i.e., $(e_i, e_j) \le 0$ for all $i \neq j$), prove that the dual basis $\{e_1^*, \ldots, e_n^*\}$ (i.e., $(e_i^*, e_j) = \delta_{ij}$) is acute (i.e., $(e_i^*, e_j^*) \ge 0$ for all $i \neq j$).

Solution 42-5.1 by Eugene A. Herman, *Grinnell College, Grinnell, IA,* herman@math.grinnell.edu

We prove the following equivalent statement: If $\{u_1, \ldots, u_n\}$ and $\{v_1, \ldots, v_n\}$ are bases of $\mathbb{R}^n$ such that $u_j \cdot v_k = \delta_{jk}$ for all j, k and $u_j \cdot u_k \le 0$ for all $j \neq k$, then $v_j \cdot v_k \ge 0$ for all $j \neq k$. We also use another equivalent statement: If P is a positive definite $n \times n$ matrix whose off-diagonal elements are nonpositive, then the off-diagonal elements of P^{-1} are nonnegative. Here is why these two statements are equivalent. If $\{u_1, \ldots, u_n\}, \{v_1, \ldots, v_n\}$ are given as above, let $P = U^{\mathsf{T}}U$, where $U = [u_1 \ \cdots \ u_n]$, and note that $P^{-1} = V^{\mathsf{T}}V$, where $V = [v_1 \ \cdots \ v_n]$. Conversely, if P is a given positive definite matrix, then $P = U^{\mathsf{T}}U$ for some nonsingular matrix U.

We give a proof by induction on n. The $n = 1$ case is trivial. For the $n = 2$ case we use the second of our two equivalent statements: $P^{-1} = \left[\begin{smallmatrix} a & b \\ b & c \end{smallmatrix}\right]^{-1} = \frac{1}{ac-b^2}\left[\begin{smallmatrix} c & -b \\ -b & a \end{smallmatrix}\right]$. Since P is positive definite, its determinant, $ac - b^2$, is positive; hence the conclusion follows.

Now assume $m \ge 3$ and that the statement is true for all $n < m$. By symmetry, to prove the first of our two equivalent statements, we need only prove that $v_1 \cdot v_2 \ge 0$. We use the fact that $\mathbb{R}^m = \text{Span}\{v_1, v_2\} \oplus W$, where $W = \text{Span}\{u_3, \ldots, u_m\}$ and $\oplus$ denotes an orthogonal direct sum. Thus

$$u_1 = av_1 + bv_2 + w \quad \text{for some } a, b \in \mathbb{R}, w \in W \tag{11}$$

Multiply equation (11) by v_2^{T}, v_1^{T}, and u_2^{T}, in turn:

$$0 = v_2 \cdot u_1 = a(v_1 \cdot v_2) + b|v_2|^2 \Rightarrow b = -a(v_1 \cdot v_2)/|v_2|^2$$

$$1 = v_1 \cdot u_1 = a|v_1|^2 + b(v_1 \cdot v_2) = a\frac{|v_1|^2|v_2|^2 - (v_1 \cdot v_2)^2}{|v_2|^2}$$

and so $a > 0$ by Cauchy-Schwarz.

$$0 \ge u_2 \cdot u_1 = b + u_2 \cdot w = -\frac{a(v_1 \cdot v_2)}{|v_2|^2} + u_2 \cdot w$$

and so

$$u_2 \cdot w \le \frac{a(v_1 \cdot v_2)}{|v_2|^2} \tag{12}$$

Now let $P = A^{\mathsf{T}}A$, where $A = [u_3 \ \cdots \ u_m]$. Since P is a $(m-2) \times (m-2)$ positive definite matrix whose off-diagonal entries are nonpositive, the off-diagonal entries of $P^{-1} = (A^{\mathsf{T}}A)^{-1}$ are nonnegative. The matrix $A(A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}$ projects orthogonally onto $W = \text{Span}\{u_3, \ldots, u_m\}$, and so $w = A(A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}u_1$ by (1). Note that $A^{\mathsf{T}}u_1$ has all nonpositive entries and thus so does $x = (A^{\mathsf{T}}A)^{-1}A^{\mathsf{T}}u_1$. Hence $w = Ax = x_3u_3 + \cdots + x_mu_m$, where $x_3, \ldots, x_m$ are the entries of x. Therefore, since $x_3 \le 0, \ldots, x_m \le 0$,

$$u_2 \cdot w = u_2 \cdot (x_3u_3 + \cdots + x_mu_m) = x_3(u_2 \cdot u_3) + \cdots + x_m(u_2 \cdot u_m) \ge 0$$

By equation (12), we have $v_1 \cdot v_2 \ge 0$ since $a > 0$.

Solution 42-5.2 by the proposer Tin-Yau Tam, *Auburn University, Auburn, AL,* tamtiny@auburn.edu

If $\{f_1, \ldots, f_m\}$ is a set of vectors in E such that $(f_i, f_j) \le 0$, $i \neq j$, then for any real scalars a_i

$$(\sum_i |a_i|f_i, \sum_i |a_i|f_i) = \sum_{i,j} |a_ia_j|(f_i, f_j) \le \sum_{i,j} a_ia_j(f_i, f_j) = (\sum_i a_if_i, \sum_i a_if_i).$$

Let $\{e_1^*, \ldots, e_n^*\}$ be the dual basis of $\{e_1, \ldots, e_n\}$. For each $j = 1, \ldots, n$, write $e_j^* = \sum_i a_ie_i$ since $e_1, \ldots, e_n$ is a basis of E. Consider $-e_j^*, e_1, \ldots, e_n$, $(m = n+1)$ and the sum $0 = -e_j + \sum_i a_ie_i$. So $0 = e_j + \sum_i |a_i|e_i$ and thus $0 = \sum_i(a_i + |a_i|)e_i$. Hence $a_i \ge 0$ for all i. Finally $(e_j^*, e_k^*) = (\sum_i a_ie_i, e_k^*) = a_k \ge 0$.

Solution 42-5.3 by Hans Joachim Werner, *University of Bonn, Bonn, Germany:* hjw.de@uni-bonn.de

Let $(\mathbb{V}, \langle \cdot, \cdot \rangle)$ be a Euclidean space of finite dimension n and let $\mathcal{B} = \{v_1, \ldots, v_n\}$ be an ordered basis of $\mathbb{V}$. The transformation

$$\begin{aligned} T : \mathbb{V} &\to \mathbb{R}^n \\ x &\mapsto T(x) =: \tilde{x} = (\tilde{x}_i)_{i=1,2,\ldots,n}, \end{aligned}$$

where $\tilde{x} = (\tilde{x}_i)_{i=1,2,\ldots,n}$ denotes the coordinate vector of x with respect to the given ordered basis $\mathcal{B}$ (i.e., $x = \sum_{i=1}^{n} \tilde{x}_i v_i$), is an isomorphism, i.e., T is linear, one-to-one and onto. We further note that the matrix

$$G := G(v_1, v_2, \ldots, v_n) := \begin{pmatrix} \langle v_1, v_1\rangle & \langle v_2, v_1\rangle & \cdots & \langle v_n, v_1\rangle \\ \langle v_1, v_2\rangle & \langle v_2, v_2\rangle & \cdots & \langle v_n, v_2\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle v_1, v_n\rangle & \langle v_2, v_n\rangle & \cdots & \langle v_n, v_n\rangle \end{pmatrix}$$

is called the GRAMIAN MATRIX or the GRAM of the vectors $v_1, v_2, \ldots, v_n$. Since $\mathcal{B}$ is a basis for $\mathbb{V}$, this Gramian matrix G is positive definite and symmetric. In what follows, let $\mathcal{B} = \{v_1, v_2, \ldots, v_n\}$ be an obtuse basis for $\mathbb{V}$ and let $\mathcal{B}^* = \{w_1, w_2, \ldots, w_n\}$ be the dual basis, i.e., let $\langle w_i, v_j\rangle = \delta_{ij}$ for all $i, j = 1, 2, \ldots, n$. In this case, it is clear that G is a Z-MATRIX (i.e., a real matrix with nonpositive off-diagonal elements) because $\mathcal{B}$ is an obtuse basis. We know that every positive definite symmetric Z-matrix is an M-MATRIX. So the implication $Gx \geq 0 \Rightarrow x \geq 0$ holds true. This implication is equivalent to $G^{-1} \geq 0$, i.e., the nonsingular inverse G^{-1} of G is nonnegative. In order to complete our proof, it now suffices to show that

$$G^{-1} = \left(\langle w_i, w_j\rangle\right)_{j,i=1,\ldots,n}.$$

For convenience, let $\tilde{w}_i$ denote the coordinate vector of the dual basis vector w_i $(i = 1, 2, \ldots, n)$ with respect to the given ordered obtuse basis $\mathcal{B} = \{v_1, \ldots, v_n\}$. Notice that $\langle w_i, v_j\rangle = e_j^t G \tilde{w}_i$, where e_j denotes the j^{th} UNIT COLUMN that contains a 1 in the j^{th} position and zeros everywhere else; observe that this unit vector e_j is trivially the coordinate vector of v_j with respect to $\mathcal{B}$. Letting $\tilde{W} := (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)$ produces $G\tilde{W} = I_n$, where I_n is the identity matrix of order n, and thus $\tilde{W} = G^{-1}$. Next notice that

$$\left(\langle w_i, w_j\rangle\right)_{j,i=1,\ldots,n} = \tilde{W}^t G \tilde{W} \tag{13}$$

and substitute $\tilde{W} = G^{-1}$ in (13). Because G^{-1} is a symmetric nonnegative matrix, it follows that (13) can be rewritten as

$$\left(\langle w_i, w_j\rangle\right)_{j,i=1,\ldots,n} = G^{-1} \geq 0.$$

This says that the dual basis is acute and so our solution is complete.

Problem 42-6: Positive Semidefinite Matrices

Proposed by Fuzhen Zhang, *Nova Southeastern University, Fort Lauderdale, USA,* zhang@nova.edu

Let a and b be (positive) real numbers such that $a^3 + b^3 = 2$. Show that $a + b \leq 2$. Discuss the analog of this for matrices: If A and B are $n \times n$ positive semidefinite matrices (i.e. $A, B \geq 0$) such that $A^3 + B^3 = 2I_n$, does it necessarily follow that $A + B \leq 2I_n$?

Solution 42-6.1 by Jan Hauke, *Adam Mickiewicz University, Poznań, Poland,* jhauke@amu.edu.pl

OBSERVATION 1. Let a, b, c, r be real positive, and $r \geq 1$. Then $a^r + b^r = 2c^r$ implies $a + b \leq 2c$.

Proof. For $r = 1$ the result is trivial. For $r > 1$ due to the symmetric role of a and b we could assume that $a \geq b$, which implies $c \leq a < 2^{1/r}c$. Let us analyze the function $f(a) = a + (2c^r - a^r)^{1/r}$. Observe that the derivative $f'(a) = 1 - [2(c/a)^r - 1]^{1-1/r}]$ is negative for $c < a < 2^{1/r}c$. So, it is enough to note that the function f(a) has a maximum $f(c) = 2c$. □

OBSERVATION 2. For positive semidefinite matrices A and B, $A^r + B^r = 2cI$ implies $AB = BA$.

Proof. Let the spectral decomposition of matrix $B = U\Lambda U^*$. Then the spectral decomposition of $A = UDU^*$, with $D = (2cI - \Lambda^r)^{1/r}$, and as a result the commutativity of A and B is obvious [a matrix function is understood as in [1, p. 526]. □

OBSERVATION 3. For positive semidefinite matrices A and B, $A^r = B^r = 2cI$ implies $A + B \leq 2cI$.

Proof. By Observation 2 the commutativity of A and B implies that it is enough to compare diagonal elements of diagonal matrices in their spectral decompositions. Therefore by Observation 1 the result is obvious. □

Remark. For $r = 3$ and $c = 1$ we obtain the version of Problem 42-6.

Reference

[1] C.D. Meyer, *Matrix Analysis and Applied Linear Algebra*, Society for Industrial and Applied Mathematics, Philadelphia, 2000.

Solution 42-6.2 by Eugene A. Herman, *Grinnell College, Grinnell, IA,* herman@math.grinnell.edu

More generally, if a and b are non-negative real numbers such that $a^p + b^p = 2$ for some integer $p > 1$, one may readily show that $a + b \leq 2$. We show that if A and B are $n \times n$ positive semidefinite matrices such that $A^p + B^p = 2I_n$ for some integer $p > 1$, then $A + B \leq 2I_n$.

A is unitarily similar to a diagonal matrix with non-negative entries along the diagonal. By applying the same similarity transformation to B, we may assume that A is this diagonal matrix. Let $E_1, \ldots, E_m$ be the distinct eigenspaces of B and let $\lambda_1, \ldots, \lambda_m$ be the corresponding eigenvalues of B. Since B is positive semidefinite, we have $E_1 \oplus \cdots \oplus E_m = \mathbb{C}^n$, $E_j \perp E_k$ whenever $j \neq k$, and $\lambda_j \geq 0$ for $j = 1, \ldots, m$. Thus $E_1, \ldots, E_m$ are the eigenspaces of B^p, and the corresponding eigenvalues of B^p are $\lambda_1^p, \ldots, \lambda_m^p$. Since $A^p + B^p = 2I_n$ and A^p is diagonal, B^p is diagonal as well, and the eigenspaces of A^p (and hence A) are also $E_1, \ldots, E_m$. Let $\mu_1, \ldots, \mu_m$ be the corresponding eigenvalues of A. If $\mathbf{v} \in E_j$ for some $j, 1 \leq j \leq m$, then

$$(A + B)(\mathbf{v}) = (\mu_j + \lambda_j)\mathbf{v}, \quad \text{and} \quad 2\mathbf{v} = (A^p + B^p)(\mathbf{v}) = (\mu_j^p + \lambda_j^p)(\mathbf{v})$$

Since $\mu_j^p + \lambda_j^p = 2$ and μ_j, λ_j are non-negative, we know that $\mu_j + \lambda_j \leq 2$. Thus, $A + B \leq 2I_n$.

Note: The above proof extends to all real powers $p > 1$. Here we assume the usual definition of M^p for noninteger powers p when M is Hermitian: For some unitary matrix U, we have $U^*MU = \text{diag}(d_1, \ldots, d_n)$; define M^p to be $U\text{diag}(d_1^p, \ldots, d_n^p)U^*$.

Solution 42-6.3 by Zejun Huang and Xingzhi Zhan, *East China Normal University, Shanghai, China,* huangzejun@yahoo.cn, zhan@math.ecnu.edu.cn

We prove a more general result. We need the following lemma.

Lemma. (Löwner-Heinz, [1]). If $A \geq B \geq 0$ and $0 \leq r \leq 1$, then $A^r \geq B^r$.

Theorem. Let A and B be positive semidefinite matrices of the same order such that $A^k + B^k = tI$, where k is a positive integer, $t \geq 2$ is a real number and I is the identity matrix. Then $A + B \leq tI$.

Proof. The case $k = 1$ is trivial. Next we suppose $k \geq 2$. First we prove that for real numbers $t \geq 2$, we have

$$f(x) = (t - x)^k + x^k - t \geq 0 \quad \text{for all } x \geq 0.$$

In fact, considering the derivative

$$f'(x) = k[x^{k-1} - (t - x)^{k-1}] < 0, \text{ if } \quad x < t/2; \quad = 0, \text{ if } \quad x = t/2; \quad > 0, \text{ if } \quad x > t/2,$$

and

$$f(t/2) \quad = \quad \frac{t^k}{2^{k-1}} - t = [(t/2)^{k-1} - 1]t \geq 0,$$

we obtain $f(x) \geq 0$. Hence

$$(tI - B)^k + B^k - tI = f(B) \geq 0, \quad i.e., \quad (tI - B)^k \geq tI - B^k = A^k.$$

Applying Lemma with $r = 1/k$ we get $tI - B \geq A$, i.e., $A + B \leq tI$. $\square$

We remark that the theorem does not hold when $t < 2$ even for scalars. For a given integer $k \geq 2$ and a positive real number $t < 2$, let $a = b = (t/2)^{1/k}$. Then $a^k + b^k = t$, but $a + b = 2(t/2)^{1/k} > t$.

Reference

[1] X. Zhan, *Matrix Inequalities*, LNM 1790, Springer, Berlin, 2002.

Solution 42-6.4 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

The first assertion follows from the power mean inequality $\frac{a+b}{2} \leq (\frac{a^3+b^3}{2})^{\frac{1}{3}}$, for $a, b \geq 0$.

Since $A^3 + B^3 = 2I_n$, pre and post multiply the equality by A^3 respectively, we see that $A^3B^3 = B^3A^3$, that is, A^3 and B^3 commutes. Since A and B are positive semidefinite matrices, then A and B commutes [1, Theorem 2.5.15]. Therefore, A, B are simultaneously diagonalizable. There exists a unitary matrix U such that $2I_n = U^*(A^3 + B^3)U = diag(\lambda_1^3(A) + \lambda_{i_1}^3(B), \lambda_2^3(A) + \lambda_{i_2}^3(B), \cdots, \lambda_n^3(A) + \lambda_{i_n}^3(B))$. By the scalar case, we have $U^*(A+B)U = diag(\lambda_1(A) + \lambda_{i_1}(B), \lambda_2(A) + \lambda_{i_2}(B), \cdots, \lambda_n(A) + \lambda_{i_n}(B)) \leq diag(2, 2, \cdots, 2) = 2I_n$. This completes the proof.

Reference

[1] R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, New York, 1985.

Editorial note: The statements in fact hold for real numbers a, b, odd integer k in place of 3, and Hermitian matrices A, B.

Also solved by Oskar Maria Baksalary and Götz Trenkler, Richard William Farebrother, Carols M. da Fonseca, Heinz Neudecker, Alicja Smoktunowicz, Hans J. Werner, Iwona Wróbel, and the proposer.

IMAGE Problem Corner: New Problems

Problems: We introduce 7 new problems in this issue and invite readers to submit solutions for publication in IMAGE. Meanwhile we still look forward to receiving solutions to unsolved problems in the past issues of IMAGE: Problems 36-3 [IMAGE 36 (Spring 2006), p. 36] and 39-4 [IMAGE 39 (Fall 2007), p. 32]. Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to IMAGE Problem Corner editor Fuzhen Zhang (zhang@nova.edu). The working team of the Problem Corner consists of Dennis S. Bernstein, Nir Cohen, Shaun Fallat, Dennis Merino, Edward Poon, Peter Šemrl, Wasin So, Nung-Sing Sze, and Xingzhi Zhan.

NEW PROBLEMS:

Problem 43-1: Characterization of EP Matrices

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

Let A be an $n \times n$ complex matrix. Show that A is EP (i.e., $AA^\dagger = A^\dagger A$, where $A^\dagger$ is the Moore-Penrose inverse of A) if and only if the column space of A^2 coincides with the column space of the conjugate transpose of A.

Problem 43-2: Square Roots of (Skew-)Involutory Matrices

Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England,* R.W.Farebrother@Manchester.ac.uk

Identify a family of square matrices with integral elements (few of them zeros) whose squares are nonzero multiples of involutory or skew-involutory matrices. [Recollect that an $n \times n$ matrix A is said to be involutory if it satisfies $A^2 = I_n$ and skew-involutory if it satisfies $A^2 = -I_n$.]

Problem 43-3: Reconstructing a Positive Semidefinite Matrix

Proposed by Bryan T. Kelly, *NYU Stern School of Business, New York, USA*
and Chi-Kwong Li, *College of William and Mary, Williamsburg, USA,* ckli@wm.edu

Suppose $A = (A_{rs})_{1 \le r,s \le k}$ is a block complex Hermitian (or real symmetric) positive semidefinite matrix such that A_{rr} is n_r-by-n_r. Show that $\tilde{A}$ is positive semidefinite if $\tilde{A}$ is obtained from A as follows:

(a) For $r \neq s$, replace each entry of A_{rs} by the average of its entries, and

(b) Replace each diagonal entry of each diagonal block A_{rr} by the average of its diagonal entries, and replace each off-diagonal entry of A_{rr} by the average of its off-diagonal entries.

Problem 43-4: A Trace Inequality for Positive Semidefinite Matrices

Proposed by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Let A be positive semidefinite and B be positive definite. Show that $tr(A^{p+1}B^{-p}) \geq (trA)^{p+1}(trB)^{-p}$ for any positive integer p.

Problem 43-5: The Matrix Equation $YTY = Y$

Proposed by Heinz Neudecker, *University of Amsterdam, Amsterdam, The Netherlands,* ericaengels173@live.nl

Let Y and T be $n \times n$ real matrices and $\mathbf{1}$ be the n-column vector of ones. Assume that Y is positive semidefinite, of rank $n-1$, $Y\mathbf{1} = 0$, $YTY = Y$, and that T is symmetric and has full rank. Find a relationship between $\mathbf{1}'T^{-1}\mathbf{1}$ and $\det T$ that shows they have the same sign. As usual, $\det T$ is the determinant of the matrix T.

Problem 43-6: Matrix Similarity

Proposed by Xingzhi Zhan, *East China Normal University, Shanghai, China,* zhan@math.ecnu.edu.cn

Let A and B be complex matrices of the same order. If A and B are similar and *-congruent and have the same value of Frobenius norm, then does it follow that A and B are unitarily similar?

Problem 43-7: Three Positive Definite Matrices

Proposed by Fuzhen Zhang, *Nova Southeastern University, Fort Lauderdale, USA,* zhang@nova.edu

It is known that any two positive semidefinite matrices of the same size are simultaneously $*$-congruent to diagonal matrices. Can this be generalized to three positive semidefinite matrices? That is, if A, B, and C are positive semidefinite matrices, does there always exist an invertible matrix P such that P^*AP, P^*BP, and P^*CP are all diagonal?

Solutions to Problems 41-4 and 42-1 through 42-6 are on page 35.