



Serving the International Linear Algebra Community
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Editor-in-Chief: Jane M. Day, Dept. of Mathematics, San Jose State University, San Jose, CA, USA 95192-0103; day@math.sjsu.edu
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UPCOMING CONFERENCES AND WORKSHOPS

Information Retrieval by Matrix Methods on Supercomputer Infrastructures Part of the ACM 24th International Conference on Supercomputing (ICS'10) Tsukuba, Japan, June 1-4, 2010

Information retrieval (IR) over large data sets, is an important application domain for high-performance computing. The algorithmic foundations of IR depend on linear and multilinear algebra theory and algorithms. The goal of this workshop is to bring together scientists engaged in the development of algorithms and tools for high-performance IR on state-of-the-art systems (multicores, GPUs and Grid environments) in order to present their latest results in this exciting area of research.

Workshop topics include Innovative Matrix and Tensor-Based Models for IR, Novel Fast Linear Algebra Solvers and Environments for IR, and Formal Comparisons of Matrix-Based to Matrix-Free Methods. For more information, visit <http://www.cosy.sbg.ac.at/events/ws-irmm10.html>, <http://www.c.csce.kyushuu.ac.jp/~inoue/ICS2010WorkshopTutorial.html>, or <http://www.ics-conference.org/>.



The purpose of this workshop, as for earlier workshops in this series, is to stimulate research and to foster the interaction of researchers in the interface between statistics and matrix theory. It will provide a forum through which statisticians and mathematicians may be better informed of the latest theoretical developments and techniques as well as their use in related application areas. The conference will have a wide variety of invited and contributed papers and special sessions on matrix analysis, matrix computations, statistical applications in economics, and matrix methods in applied probability. For details, visit <http://www1.shfc.edu.cn/iwms/index.asp>. The International Linear Algebra Society endorses IWMS-2010.

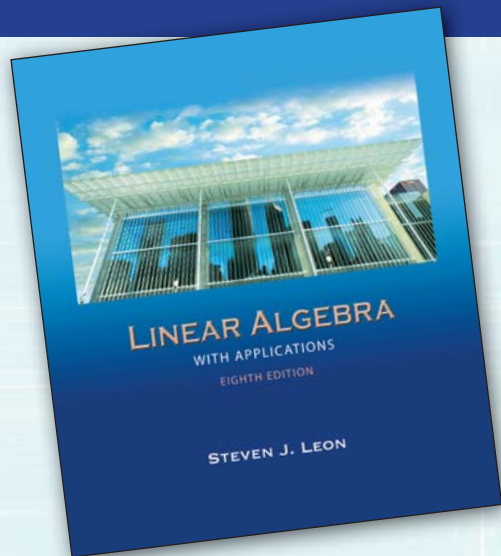
4th Annual International Conference on Mathematics and Statistics ATINER, Athens, Greece June 14-17, 2010

The Mathematics and Statistics Research Unit of the Institute for Education and Research (ATINER) will hold its 4th International Conference in Athens, Greece, June 14-17, 2010.

Topics cover all areas of Mathematics, Statistics, Mathematics and Engineering, and Mathematics and Education. For details, visit www.atiner.gr/docs/Mathematics.htm.

ATINER, established in 1995, is an independent academic organization that is a forum where academics and researchers from all over the world can meet to exchange ideas on their research and discuss the future developments of their discipline. It has held more than 100 international conferences and at least 80 books have resulted. The Institute has four research divisions and nineteen research units. Each research unit organizes at least one annual conference and undertakes various small and large research projects.

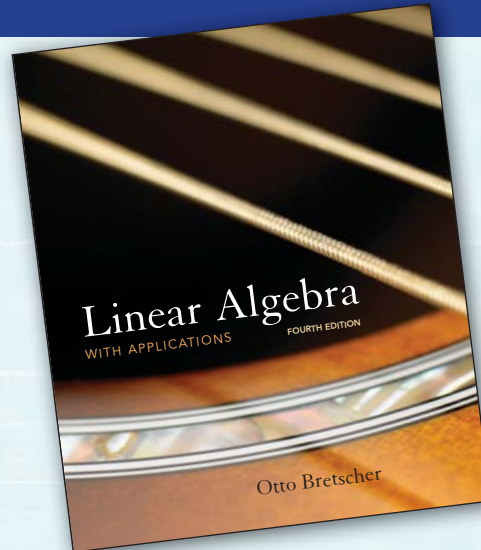
AT THE FOREFRONT OF LINEAR ALGEBRA



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Gene Golub SIAM International Summer School on Numerical Linear Algebra Brindisi, Italy, June 7-18, 2010

This Summer School 2010 is the second in a series of periodic International Summer Schools on Numerical Linear Algebra (ISSNLA) being organized by the SIAM Activity Group on Linear Algebra (SIAG/LA). The first ISSNLA took place in 2008. The 2010 program will be supported in part by generous contribution from the Gene Golub Summer School fund, that was established with a bequest from the estate of Gene Golub. Attendance will be limited to 50 graduate students, mainly doctoral students in fields where methods and algorithms of Numerical Linear Algebra are used.

The Summer School courses will focus on recent research topics that have reached a significant maturity and whose impact is widely recognized by the community, but that are not usually included in text books, or in basic courses at the doctoral level:

* Applications are quickly expanding from the classical ones (numerical solution of partial differential equations, statistics, optimization, control...) to new areas as data mining, pattern recognition, image processing, web search engines, particle physics.

* Fundamental problems are still the subject of intense research. For instance, structured algorithms for different classes of matrices and the corresponding structured perturbation theory, distance problems in matrix computations, high relative accuracy algorithms and the corresponding roundoff error analysis, fast matrix multiplication and its application to develop fast and stable algorithms in Numerical Linear Algebra, convergence of iterative methods, stability analysis of iterative methods, combinatorial algorithms, algorithms for matrix functions.

* Development of reliable and efficient software is the final goal of Numerical Linear Algebra. In this context, there exist well known packages that are frequently revised and improved by adding new routines and functions, and new packages are constantly appearing.

Coimbra Meeting on 0-1 Matrix Theory and Related Topics Department of Mathematics, University of Coimbra, Portugal June 17-19, 2010



University of Coimbra Library

This conference is cosponsored by the University of Coimbra and the American Mathematical Society, and endorsed by ILAS.

The invited speakers are Alexander Barvinok, USA; Adrian Bondy, France; Richard A. Brualdi, USA; Domingos M. Cardoso, Portugal; Geir Dahl, Norway; Michael Drmota, Austria; Catherine Greenhill, Australia; Gyula O. H. Katona Hungary; Hadi Kharaghani, Canada; Christian Krattenthaler, Austria; Brendan McKay, Australia; Irene Sciriha, Malta; and Herbert S. Wilf, USA, ILAS Lecturer.

There will be a peer-refereed special issue of Linear Algebra and Its Applications devoted to the papers presented at the conference. The special editors of the issue are C. M. da Fonseca, J. A. Dias da Silva, Natália Bebiano and Geir Dahl. Manuscripts must be submitted to LAA by November 1, 2010. For more information, visit <http://www.mat.uc.pt/~cmf/01MatrixTheory.html>.



The conference will provide a broad view of the state of the art in both theoretical and applied linear algebra. The Organizing Committee is Michele Benzi (SIAM representative member), Avi Berman, Dario A. Bini (Chair), Luca Gemignani, Leslie Hogben, Steve Kirkland, Julio Moro, Ilya Spitkovsky, Françoise Tisseur, and Eugene Tyrtyshnikov.

The Plenary Lectures will be:

- Rajendra Bhatia, Indian Statistical Institute, New Delhi, India. Loewner Matrices
- Richard A. Brualdi, University of Wisconsin - Madison, USA. Matrices and Indeterminates
- Pauline van den Driessche, University of Victoria, B.C. Canada. Potential Stability and Related Spectral Properties of Sign Patterns
- Nicholas J. Higham, Univ. of Manchester, UK. Computing the Action of Matrix Exponential, with an Application to Exponential Integrators
- Beatrice Meini, Università di Pisa, Italy. Nonsymmetric Algebraic Riccati Equations Associated with M-matrices: Theoretical Results and Algorithms
- Vadim Olshevsky (LAMA speaker), University of Connecticut, USA. Potpourri of Quasiseparable Matrices
- Zdenek Strakos, Charles University, Prague, Czech Republic. Moments, Model Reduction and Nonlinearity in Solving Linear Algebraic Problems
- Daniel B. Szyld, Temple University, Philadelphia, USA. Modifications to Block Jacobi with Overlap to Accelerate Convergence of Iterative Methods for Banded Matrices.
- Luis Verde Star, Universidad Autónoma Metropolitana, Mexico City, Mexico. Linear Algebraic Foundations of the Operational Calculi.

Additional Special Lectures:

- Oliver Ernst (SIAG/LA speaker), TU Bergakademie Freiberg, Germany. Krylov Subspace Approximations of the Action of Matrix Functions for Large-Scale Problems
- Olga Holtz (Tausky-Todd Lecture), University of California, Berkeley, USA. Zeros of entire functions: from René Descartes to Mark Krein and Beyond
- Lek-Heng Lim (LAA Lecture), University of California, Berkeley, USA. Multilinear Algebra and its Applications
- Cleve Moler (Hans Schneider Prize), The MathWorks. Evolution of MATLAB
- Beresford N. Parlett (Hans Schneider Prize), University of California, Berkeley, USA. Linear Algebra Meets Lie Algebra

There will be 17 minisymposia plus daily sessions for contributed papers. The Minisymposia topics are: Combinatorial Linear Algebra, Linear Algebra Education, Markov Chains, Matrix Functions and Matrix Equations, Nonnegative Matrices, Structured Matrices, Application of Linear and Multilinear Algebra in Life Sciences and Engineering, Generalized Inverses and Applications, Linear Algebra and Inverse Problems, Linear Algebra in Curves and Surfaces Modeling, Linear Algebra in Quantum Information Theory, Matrix Inequalities - In Memory of Ky Fan, Matrix Means, Max Algebras, Nonlinear Eigenvalue Problems, Spectral Graph Theory, and Tensor Computations in Linear and Multilinear Algebra.

Social Events will include an excursion to tour the city, a concert by piano, violin and cello in the Church of San Francesco, and a dinner in the same location. For details, visit <http://www.dm.unipi.it/~ilas2010/index.php>.



10th WONRA Workshop, Numerical Ranges and Numerical Radii
Instytut Matematyki, Uniwersytet Jagiellonski, Krakow, Poland
June 27-29, 2010



University Museum

The 10th Workshop on Numerical Ranges and Numerical Radii will be held at the University Museum of the Jagiellonian University, in the beautiful old city of Krakow, Poland from June 27-29, 2010. Tentatively, the scientific program will be on June 28 and June 29. A workshop dinner will be held on June 28.

The purpose of the workshop is to stimulate research and foster interaction of researchers interested in the subject. The informal workshop atmosphere will facilitate the exchange of ideas from different research areas and, hopefully, the participants will leave informed of the latest developments and newest ideas. The workshop is endorsed by ILAS and held in conjunction with the 2010 ILAS Conference that will occur in Pisa, Italy, the following week, June 27-29.

The conference organizers are Chi-Kwong Li, College of William and Mary (ckli at math.wm.edu); Franciszek Hugon Szafraniec, Uniwersytet Jagiellonski (umszafra at cyf-kr.edu.pl); and Jaroslav Zemanek, Instytut Matematyczny PAN (zemanek at impan.pl). For details, visit <http://www.math.wm.edu/~ckli/wonra10.html>.

The 5th Workshop on Matrices and Operators
(MAO2010)
Taiyuan, Shanxi, P. R. China, July 12-15, 2010

This workshop is co-sponsored by Taiyuan University of Technology and the Shanxi Association for Science and Technology. The purpose of the workshop is to stimulate research and foster interaction of researchers all over the world interested in matrix theory, operator theory, and their applications.

We expect there will be over 100 mathematicians participating in the workshop. Hopefully, the workshop will ensure the exchange of ideas from different research areas.

For more information, visit <http://www.sxkp.com/mao> or email the secretary of the workshop, Dr. Runling An (mao2010ty@163.com or runlingan@yahoo.com.cn).

The Conference Co-chairs are Jinchuan Hou, Taiyuan University of Technology, China, and Chi-Kwong Li, College of William and Mary, US.



View of Taiyuan

The Mutually Beneficial Relationship of Matrices and Graphs
2010 NSF-CBMS Regional Conference
Iowa State University, Ames, Iowa, U.S.
July 12-16, 2010



Richard Brualdi

The principal speaker will be Richard Brualdi, University of Wisconsin-Madison. He is one of the founders and continuing leaders in combinatorial matrix theory, author of more than 200 papers, and an award-winning teacher.

The conference will discuss the symbiotic relationship between matrices and graphs and the significant role that they jointly play in pure and applied mathematics, science, and technology. In addition to Prof. Brualdi's lectures, there will be collaborative research groups with leading experts in combinatorial matrix theory that will begin work on research problems. These collaborative groups will be in the style of those at AIM, the American Institute of Mathematics.



Iowa State University Center

The conference is being organized by Leslie Hogben of Iowa State University and Bryan Shader, University of Wyoming. It is supported by the National Science Foundation through DMS 0938261 and by the Institute for Mathematics and its Application (IMA) through its Participating Institution (PI) Program. For more information, visit <http://orion.math.iastate.edu/lhogben/CBMS/>.

The 9th China Matrix Theory and Applications International Conference
Shanghai University, Shanghai, China, July 18-22, 2010

This conference aims to promote academic research, exchange and collaboration among researchers both home and abroad. The topics cover matrix theory, linear algebra, and applications, including traditional linear algebra, combinatorial linear algebra and numerical linear algebra. A forum will be provided during the conference to get participants better informed of the latest development and research achievements of these relevant areas.

The Organizing Committee consists of Erxiong Jiang, Shanghai University, China, Chair; Zhongzhi Bai, Chinese Academy of Science, China; Chuanqing Gu, Shanghai University, China; Rencang Li, University of Texas, Arlington, USA; Yongge Tian, China Central University of Finance and Economy, China; Yimin Wei, Fudan University, China; Hongguo Xu, University of Kansas, Lawrence, USA; Shufang Xu, Peking University, China; and Xingzhi Zhan, East China Normal University, China.

The Invited Speakers are Koenraad M.R. Audenaert, University of London, Egham, United Kingdom; Zhaojun Bai, University of California at Davis; USA Natalia Bebiano, University of Coimbra, Coimbra, Portugal; Delin Chu, National University of Singapore, Singapore; Man-Duen Choi, University of Toronto, Toronto, Canada; Biswa Nath Datta, Northern Illinois University, Illinois, USA; Ming Gu, University of California at Berkeley, USA; Daniel Kressner, ETH Zurich, Switzerland; Chi Kwong Li, College of William & Mary, USA; Yongdo Lim, Kyungpook National University, Taegu, Republic of Korea; Qing-Wen Wang, Shanghai University, Shanghai, China; Pei-Yuan Wu, National Chiao Tung University, Hsinchu, Taiwan; and Zhinan Zhang, Xinjiang University, Urumqi, China.

There will be an excursion to visit the 2010 Shanghai World Expo. The conference proceedings will be published by World Academic Press, with all articles indexed by ISTEP. For details, visit <http://math.shu.edu.cn/CLAS2010>.



Shanghai University Main Gate

Numerical Linear Algebra: Perturbation, Performance, and Portability
A conference in honor of G.W. (Pete) Stewart
The University of Texas at Austin, July 19-20, 2010



Pete Stewart

The catalyst for this conference is the recent retirement and upcoming 70th birthday of Prof. G.W. (Pete) Stewart, a world-renowned expert in computational linear algebra. The meeting will focus on the state of numerical linear algebra by examining past and present accomplishments as well as opportunities for future research. We would appreciate contributions of pictures and anecdotes related to Pete Stewart for the conference wiki. Please contact Robert van de Geijn (rvdg@cs.utexas.edu) for instructions on how to upload materials.

The format of the conference consists of eighteen 30 minute invited talks and discussion periods. There will be a Poster Session on the first day. If you wish to present a poster, please indicate that on the registration form. A dinner in honor of Pete Stewart will be held the first evening of the conference, and the conference will conclude with a panel discussion on the future of numerical linear algebra.

There will be no conference fee for this event, thanks to generous support from the National Science Foundation and various entities at the University of Texas, acknowledged on the conference wiki. Graduate and Postdoctoral researchers presenting posters who wish to apply for partial travel funding should submit an abstract and a letter of recommendation to Misha Kilmer (misha.kilmer@tufts.edu).

The Organizing Committee is Ake Bjorck, Linkoping University; Jack Dongarra, University of Tennessee; Howard Elman, University of Maryland; Misha Kilmer (Co-Chair) Tufts University; Dianne O'Leary, University of Maryland; Danny Sorensen, Rice University; Xiaobai Sun, Duke University; and Robert van de Geijn (Co-Chair), The University of Texas at Austin. The program schedule and registration details are available at <http://z.cs.utexas.edu/wiki/stewart2010.wiki/>.

SIAG/LA: 16th Graduate Student Modeling Workshop (IMSM10)
North Carolina State University, July 19-27, 2010

This workshop is sponsored by the Statistical and Applied Mathematical Science Institute (SAMSI) and the Center for Research in Scientific Computation (CRSC). Here graduate students in mathematics, engineering, and statistics will be exposed to exciting real-world problems from industry and government. Besides giving students experience in the team approach to problem solving, the IMSM workshop can help them to decide what kind of professional career they want. The projects will be:

- * The Mathematics of Popcorn: John Peach (Lincoln Laboratory) and Cammey Cole (Meredith College)
- * Forecast Verification for Extreme Storms using Large-Scale Indicators: Harold Brooks (National Severe Storm Laboratory), and Elizabeth Mannshardt-Shamseldin (SAMSI/Duke University)
- * Spatial Forecast Verification: Eric Gilleland (National Center for Atmospheric Research), Emily Lei Kang (SAMSI), and Richard Smith (University of North Carolina)
- * Uncertainty-Enabled Design of a Switch: Jordan Massad (Sandia National Laboratories)
- * Issues in Commodities Pricing Strategies: Albert Hopping (Progress Energy) and Jeff Scroggs (North Carolina State University)
- * Brain Gate: Chad Bouton (Battelle Memorial Institute), Kazi Ito (North Carolina State University), and Ralph Smith (North Carolina State University):

For details, visit <http://www.ncsu.edu/crsc/events/imsm10/> or email imsm_10@ncsu.edu. The Program Organizer is Ilse Ipsen.

LinStat 2010 - Tomar, Portugal, July 27-31, 2010

LinStat 2010, the 2010 International Conference on Trends and Perspectives in Linear Statistical Inference, will focus on a number of topics: estimation, prediction and testing in linear models, robustness of relevant statistical methods, estimation of variance components, design and analysis of experiments. The proceedings will be published in the Journal of Statistical Computation and Simulation. For more information, visit <http://www.linstat2010.ipt.pt>.



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International Conference on Recent Trends in Graph Theory and Combinatorics
A Satellite Conference of the International Congress of Mathematicians (ICM 2010)
Cochin, India, August 12-15, 2010

This conference will have plenary and invited talks by eminent researchers in graph theory, combinatorics and related topics, contributed presentations and mini symposia on specific themes.

The International Scientific Committee is G.E. Andrews, USA; S. Arumugam, India; R. Balakrishnan, India; R. B. Bapat, India; W. Goddard, USA ; M.C. Golumbic, Israel; G. Gutin, U.K ; P. Hell, Canada; G. O. H. Katona, Hungary; S. Klavzar, Slovenia; J. Nešetřil, Czech Republic; S. B. Rao, India; A. Raspaud, France; A. Rosa, Canada; S. S. Sane, India; V. T. Sos, Hungary; and X. Zhu, Taiwan. For details, visit <http://icrtgc2010.cusat.ac.in/>.

This conference will be held just before ICM-2010, the International Congress of Mathematicians that is to take place at Hyderabad, India on August 19-27, 2010. We extend a cordial invitation to you to come to Cochin for the ICRTGC-2010 and then on to Hyderabad for ICM-2010, which is just ninety minutes by air from Cochin.

2nd Conference on Numerical Linear Algebra and Optimization
University of Birmingham, UK, September 13-15, 2010

This conference is organized by the Institute of Mathematics and Its Applications (IMA) and the University of Birmingham. It is co-sponsored by SIAM, ILAS and the Mathematical Programming Society (MPS), whose members will receive the IMA members' registration rate. Conference topics include Direct and iterative methods for large sparse linear systems, Eigenvalue computation and optimization, Large-scale nonlinear and semidefinite programming, Effect of round-off errors, stopping criteria, Embedded iterative procedures, Optimisation issues for matrix polynomials, Fast matrix computations, Compressed/sparse sensing, PDE-constrained optimization, and Applications and real time optimisation.

The invited speakers are Larry Biegler, Carnegie Mellon Univ.; Nick Higham, Univ. of Manchester; Adrian Lewis, Cornell Univ.; Volker Mehrmann, Technische Univ., Berlin, ILAS Speaker; Mike Saunders, Stanford Univ.; Valeria Simoncini, Univ. di Bologna; Jared Tanner, Univ. of Edinburgh; and Andy Wathen, Univ. of Oxford. For details, visit [//www.ima.org.uk/Conferences/2nd_numerical_linear_algebra.html](http://www.ima.org.uk/Conferences/2nd_numerical_linear_algebra.html).

6th International Workshop on the Numerical Solution of Markov Chains (NSMC 2010)
September 16-17, 2010, Williamsburg, VA, USA

Topics of interest include matrix generation techniques and storage, sparse matrix technologies, matrix geometric solutions and a large number of computational issues and applications. William J. Stewart (North Carolina State University, US) will be the invited speaker. General Conference Chairs are Peter Kemper (College of William and Mary, US) and Evgenia Smirni (College of William and Mary, US); Program Co-chairs are Michele Benzi (Emory University, US) and Tugrul Dayar (Bilkent University, Turkey). For details, visit <http://www.cs.bilkent.edu.tr/~nsmc10>.



Lindos, Island of Rhodes

8th International Conference of Numerical Analysis and Applied Mathematics
Island of Rhodes, Greece, September 19-25, 2010

During this conference, we will celebrate the 65th birthday of Prof. Dr. Peter Deuflhard, President of the Zuse Institute Berlin (ZIP). There will be many distinguished invited speakers and the Proceedings will be published in the American Institute of Physics Conference Proceedings. Papers are invited.

For details, visit <http://www.icnaam.org> or contact Dr. T.E. Simos, Chair and Organiser ICNAAM 2010 (tsimos.conf@gmail.com). Proceedings of previous ICNAAM conferences have been abstracted in: ISI Proceedings, Zentrablatt für Mathematik, MathSciNet, Scopus, INSPEC, Scirus, and Google Scholar.

International Conference on Algebra (ICA2010) Gadjah Mada University, Yogyakarta, Indonesia, October 7-10, 2010



Borobudur Temple

This conference is in honor of the 70th birthday of Professor Shum Kar-Ping at The Faculty of Mathematics and Natural Sciences. It is supported by Faculty of Mathematics and Natural Sciences, Gadjah Mada University, and the Southeast Asian Mathematical Society (SEAMS). Conference activities will be held at the Faculty of Mathematics and Natural Sciences, Gadjah-Mada University, Yogyakarta, Indonesia on October 7-10, 2010. There will be parallel sessions and plenary lectures. For details, visit <http://ica2010.ugm.ac.id/drp/index.php?q=node/4>.

Social activities will include attending the Ramayana Ballet and sightseeing to Borobudur Temple, the world's largest Buddhist temple, Merapi Volcano and the Prambanan Temple. Yogyakarta is located in south-central Java and can be reached by a one hour flight from Jakarta. It has an outstanding historical and cultural heritage. It was the centre of the Mataram Dynasty (1575-1640), and the Kraton (sultan's palace) still exists; it has numerous thousand-year-old temples, beautiful natural panorama and natural beaches to the south. Malioboro, the center of Yogyakarta, is full of artistic handcrafts. For more information, please visit <http://visitingjogja.com/>.

1st Joint SMM-CAIMS-SIAM Meeting Universidad del Mar, Oaxaca, Mexico, December 8-10, 2010

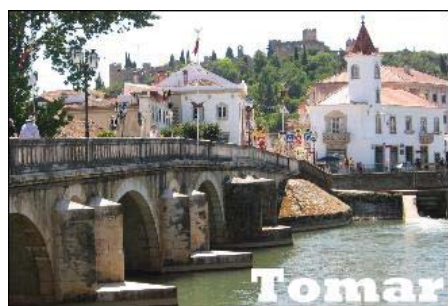
The Sociedad Matemática Mexicana, Canadian Applied and Industrial Mathematics Society, and SIAM are organizing this joint meeting. The main goal is to bring together applied computational scientists, mathematicians, researchers and students with interdisciplinary interests from Mexico, Canada, and the United States.

Themes of the conference include applied probability and statistics; numerical analysis and linear algebra; optimization and operation research; biomathematics; oil, weather and geoscience modeling; financial mathematics and economy; and inverse problems and control. Read more here: <http://www.smm.org.mx/namiam10/>.



Zapotec Temples at
Monte Alban

MAT TRIAD 2011 Tomar, Portugal, July 12-16, 2011



MAT TRIAD 2011 will be held in Tomar, Portugal, on July 12-16, 2011. The purpose of this conference is to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications in other parts of mathematics and offer them a possibility to discuss current developments in these subjects.

Researchers and graduate students in the area of linear algebra, statistical models and computation are particularly encouraged to attend the workshop. The format of this meeting will involve plenary talks and sessions with contributed talks. The list of invited will be opened by winners of Young Scientists Awards of MAT-TRIAD 2009.

We are also planning two short courses delivered by Ali Hadi (The American University in Cairo, Egypt) and Volker Mehrman (Technical University in Berlin, Germany).

The Scientific Committee of Mat-Triad 2011 is chaired by Tomasz Szulc (Poland), and the Organizing Committee by Francisco Carvalho (Portugal). Visit <http://www.mattriad2011.ipt.pt/> for more information.



2011 ILAS Conference “Pure and Applied Linear Algebra: The New Generation”

The **2011 ILAS Conference “Pure and Applied Linear Algebra: The New Generation”** will take place in Braunschweig, Germany, at the Technische Universität Braunschweig **August 22 - 26, 2011**. It will have a special emphasis on young researchers, with predominantly young plenary speakers as well as young researchers’ minisymposia. See www.ilas2011.de for more information.

The Plenary Speakers will be Ioana Dumitriu (University of Washington, USA), Roland Hildebrand (Université Joseph Fourier, Grenoble, France), Didi Hinrichsen (Universität Bremen, Germany), Joseph Landsberg (Texas A&M, USA), Rajesh Pereira (University of Guelph, Canada), Daniel Potts (Technische Universität Chemnitz, Germany). Special Lectures will be announced in the near future.

Five invited minisymposia will highlight the recent developments in specific areas of pure and applied linear algebra:

1. Total positivity: recent advances in theory and applications
2. Quasi- and semiseparable matrices
3. Compressed Sensing and Sparse Approximation Algorithms
4. Tensor Decompositions
5. Matrix Polynomials and their Eigenproblems

Besides these invited minisymposia, the conference will focus on **Young Researchers’ Minisymposia**, each on a timely research subject and lasting two hours with four to six talks. The organisers and speakers in these should hold a PhD for no longer than 6 years and not yet hold a tenured professor’s position. The scientific committee will select up to five young researchers’ minisymposia. More information on the application process will be available soon on www.ilas2011.de.

The Scientific Organising Committee consists of Ravi Bapat, Indian Statistical Institute, India; Albrecht Böttcher, TU Chemnitz, Germany; Angelika Bunse-Gerstner, Universität Bremen, Germany; Tobias Damm, TU Kaiserslautern, Germany; Froilan Dopico, Universidad Carlos III de Madrid, Spain; Shaun Fallat, University of Regina, Canada; Heike Faßbender, TU Braunschweig, Germany, Chair; Steve Kirkland, Hamilton Institute, National University of Ireland, Ireland; Raphael Loewy, Technion - Israel Institute of Technology, Israel; Nil Mackey, Western Michigan University, USA; Bryan Shader, University of Wyoming, USA; and Michael Tsatsomeros, Washington State University, USA.

The local Organising Committee consists of Heike Fassbender, TU Braunschweig, Germany; Matthias Bollhöfer, TU Braunschweig, Germany; and Peter Benner, Max-Planck-Institute for Dynamics of Complex Technical Systems, Germany.

Braunschweig (Brunswick in English), the city of Henry the Lion, is an old medieval city in the north of Germany, located about 200km west of Berlin. It has a population of around 240,000 and is characterised by an appealing and exciting blend of the past and the present, tradition and the future. The city has been given its distinctive profile by the steady development of its cultural, research and technological scenes. The surrounding areas are just as varied as the city, with their cultural, historical and economic richness: a fertile symbiosis, that is unparalleled in Germany. Braunschweig is home to 14 research establishments of international renown. According to a recent study by the European Union (EU), it is the center of the most intensive research region in the whole of the EU. At the heart of this extraordinary research landscape is the oldest technical university in Germany, the Technische Universität Braunschweig, established in 1745. Of its 15,000 students, nearly 1,600 are non-German, from well over 80 different countries, studying here for their first academic degree or within bilateral exchange programmes.

Braunschweig is where Carl Friedrich Gauss was born and raised. His first years of study (1792-1795) were spent at the “Collegium Carolinum”, the predecessor of TU Braunschweig. Here he wrote the *Disquisitiones Arithmeticae* and stayed until he was appointed professor in Göttingen in 1807. His presence can still be felt, and a walk connects several places related to his work and life.

Other Upcoming Conferences of Interest

- * SIAM Annual Meeting, Pittsburgh, PA, July 12-16, 2010. There will be a “Linear Algebra track”.
- * International Conference on Industrial and Applied Mathematics, Vancouver, Canada, July 18-22, 2011. Deadline for minisymposia proposals is August 18, 2010.
- * Triennial SIAM Conference on Applied Linear Algebra, Valencia, Spain, June 18-22, 2012. This will be organized by SIAG/LA, the SIAM Special Interest Group on Linear Algebra (<http://www.math.temple.edu/~siagla/>).

REPORTS ON CONFERENCES AND WORKSHOPS

EPSICON 2010

1st International Conference on Power, Control, Signals, and Computation

Vidya Academy of Science and Technology (VAST)

Thrissur, Kerala, India, January 4-6, 2010

This conference was dedicated to Thomas Kailath, Hitachi Professor Emeritus of Engineering, Stanford University, US, in recognition of his great achievements in science and engineering.

The attendees were welcomed by Professor D. Balakrishnan, Chair of the Organizing Committee and Academic Director of VAST, and Professor Sudha Balagopalan, the Conference Coordinator. The chief guests were Biswa Datta, Thomas Kailath, G.R.C. Reddy, M.C. Chako, Member of Parliament, R. Bindu, Mayor of Thrissur, and Er. K.R. Brahmadathan and Executive Director of VAST. There were welcoming addresses by Biswa Datta, PP Nair, Dean of VAST, K.R. Brahmadathan and S.P. Subramanian, Principal of VAST, and Mr. Chacko. Professor Datta also thanked the new institution VAST for its excellent work organizing the conference.

Professor Kailath gave the opening keynote address, Mathematical Engineering – Its Origin and Impact.

The next day, Prof. D.P. Kothari gave a keynote address, followed by lectures from Narayanan Komerath, Georgia Institute of Technology, US; Amit Bhaya, Federal Univ. of Rio de Janeiro, Brazil; and S. Mani Venkata, Univ. of Washington, US. That evening there was a musical performance by Prof. Bhattacharyy.

Other invited lectures were presented by Biswa Datta, Northern Illinois University, US; James Bunch, Univ. of California at San Diego, US; S.P. Bhattacharyya, Texas A&M Univ., US; V.K. Damodaran, INGCORE, Trivandrum and Hangzhou, China; Karabi Datta, Northern Illinois University, US; Sergej Fatikow, Univ. of Oldenburg, Germany; K. Galkowski, Univ. of Zielona Gora, Poland; B.M. Mohan, IIT-Kharagpur, India; and D.P. Khotari, VIT, India. The topics varied widely, from power systems to many mathematical, computational and other aspects of control and signal processing including automated robot based nanohandling. There were over 100 contributed talks, and about 350 people from at least 10 countries participated in the conference.

Professor Datta received a Gold Medal of Honor during this conference, recognition of his lifetime achievements in various disciplines of science and engineering. (See details on page 19.)

Innovative and Effective Ways to Teach Linear Algebra

Special Session at the Joint AMS/MAA Meetings, San Francisco, US, January 16, 2010

This two-part contributed paper session was organized by David Strong, Pepperdine University; David Lay, University of Maryland; and Gil Strang, MIT. Over 20 talks were given in which a variety of ideas, examples and teaching approaches in linear algebra were presented and discussed.

Topics included ways to help students see what eigenvalues and eigenvectors do and learn linear algebra language; projects and other student-centered activities; unusual topics such as using quaternions for rotations, Laguerre basis, and “Back to Grassmann”; and a variety of ways to use online applets and other technology to enhance students’ understanding of concepts. Also Sang-gu Lee described the Integrated Computer Technology classrooms that his university has implemented, and materials he has developed for the linear algebra courses that he teaches in those. The ICT technology allows students to participate in classes from remote locations in real time, as well as to review lectures later. Links to many of the talks can be found at <http://math.pepperdine.edu/dstrong/LinearAlgebra/2010/>.

This session on Linear Algebra Education has become a regular part of the annual Joint Meetings, and a call for talks at the 2011 Joint Meetings has now appeared in MAA’s Focus Magazine and is also available online. Questions and comments may be directed to the lead organizer, David Strong (David.Strong@pepperdine.edu).

Ed. Note: Slides from history talks about numerical linear algebra are available

There were special sessions on the history of numerical linear algebra at the 10th SIAM Applied Linear Algebra Conference held in Monterey, CA, US, October 16-19, 2009. Slides of presentations given in those sessions can be found at http://www.siam.org/meetings/la09/history_talks.php. There are also audio files for three of these talks. Submitted by Linda Thiel, SIAM Director of Programs and Services (thiel@siam.org). (A report on this conference is in IMAGE issue 43, p. 17.)

OBITUARIES

Ky Fan: September 19, 1914 – March 22, 2010

By Fuzhen Zhang, Nova Southeastern University, Fort Lauderdale, FL 33314, US,
with the help of the Mathematics Department, of UC Santa Barbara



Ky Fan

Ky Fan, Emeritus Professor of the University of California at Santa Barbara (UCSB), died in Santa Barbara on March 22, 2010 at age 95.

Ky Fan was born in Hangzhou, China on September 19, 1914. He enrolled in Peking University in 1932, and received his B.S. degree from Peking University in 1936. Initially Fan wanted to study engineering, but shifted to mathematics, largely because of the influence of his uncle who was a renowned mathematician in China and the Chair of the Department of Mathematics of Peking University.

Fan went to France in 1939 and received his D.Sc. under Frechet (a student of J. Hadamard) from the University of Paris in 1941. A member of the Institute for Advanced Study in Princeton from 1945 to 1947, he then joined the faculty of the University of Notre Dame, eventually becoming full professor. In 1965, Fan became professor of mathematics at UC Santa Barbara, retiring in 1985.

In addition to Frechet, Fan was also influenced by John von Neumann and Hermann Weyl. Fan made fundamental contributions to operator and matrix theory, convex analysis and inequalities, linear and nonlinear programming, topology, and topological groups. His work in fixed point theory, in addition to influencing nonlinear functional analysis, has found wide application in mathematical economics and game theory, potential theory, calculus of variations, and differential equations.

There are many stories about Ky Fan, but the most famous one is about the graduate student who went to Fan for advice: "I don't seem to be making any progress on my thesis, even though I'm working eight hours a day." Fan replied "eight hours? Eight Hours? You must do math EVERY WAKING MOMENT!" The graduate students subsequently had T-shirts made with Fan's picture and this slogan.

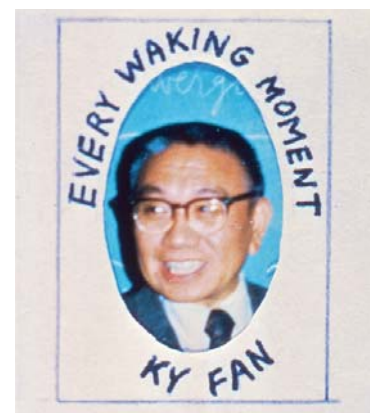
Fan had 22 Ph.D. students and 89 mathematical descendants. His 140 papers and books have been cited over 4000 times. Fan was elected to the Academia Sinica in 1964, and served as the Director of its Institute of Mathematics in Taiwan from 1978 to 1984.

In 1989, having been away from China for 50 years, Ky Fan returned to his motherland in June. He was very excited from the first day he stepped on his motherland and was welcomed warmly everywhere he went. He received Honorary Professorships from Peking University and Beijing Normal University.

In 1999, Ky Fan and his wife made a gift of \$1M to the American Mathematical Society to establish the Ky and Yu-Fen Fan Endowment, to foster collaborations between Chinese mathematicians and those in other parts of the world. The Ky Fan Visiting Assistant Professorship in the Mathematics Department at UCSB is also due to his generosity.

A memorial service for Ky Fan was held in Santa Barbara on April 24, 2010. Attendees included UCSB Chancellor, Mathematics Department Chair, friends and students.

Ky Fan will be remembered as a great mathematician. His elegant and profound mathematical results will be studied and used forever.



The Famous T-shirt

Textbook

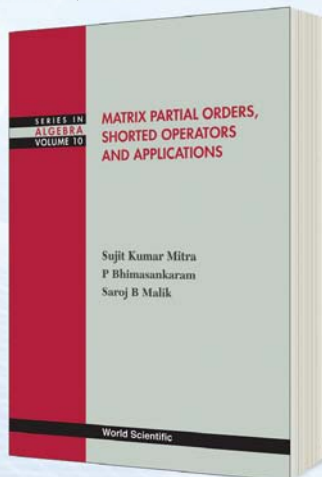
Series in Algebra - Vol. 10

MATRIX PARTIAL ORDERS, SHORTED OPERATORS AND APPLICATIONS

by **Sujit Kumar Mitra** (Indian Statistical Institute, India),
P Bhimasankaram (University of Hyderabad, India), &
Saroj B Malik (Hindu College, University of Delhi, India)

Contents: Introduction; Decompositions and Generalized Inverses; Minus Order; Sharp Order; Star Order; One-Sided Orders; Löwner Order and Majorization; Unified Theory of Matrix Partial Orders through Generalized Inverses; Parallel Sums; Schur Complements and Shorted Operators; Shorted Operators II; Supremum and Infimum for a Pair of Matrices; Partial Orders for Modified Matrices; Statistics; Electrical Network Theory.

Readership: Graduate students in mathematics; researchers in mathematics, statistics and electrical engineering.



480pp
978-981-283-844-5
Mar 2010
US\$88

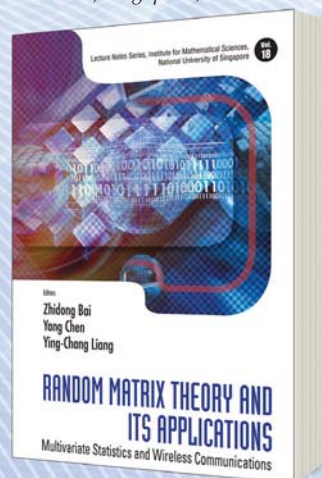
Lecture Notes Series Institute for Mathematical Sciences,
National University of Singapore - Vol. 18

RANDOM MATRIX THEORY AND ITS APPLICATIONS

Multivariate Statistics and Wireless Communications

edited by **Zhidong Bai** (National University of Singapore, Singapore & Northeast Normal University, P R China),
Yang Chen (Imperial College London, UK), &
Ying-Chang Liang (Institute for Infocomm Research, Singapore)

Random matrix theory has a long history, beginning in the first instance in multivariate statistics. It was used by Wigner to supply explanations for the important regularity features of the apparently random dispositions of the energy levels of heavy nuclei. The subject was further deeply developed under the important leadership of Dyson, Gaudin and Mehta, and other mathematical physicists.



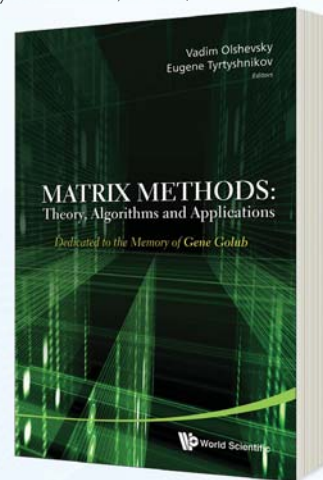
176pp
978-981-4273-11-4
Jul 2009
US\$68

MATRIX METHODS: THEORY, ALGORITHMS AND APPLICATIONS

Dedicated to the Memory of Gene Golub

edited by **Vadim Olshevsky** (University of Connecticut, USA) &
Eugene Tyrtyshnikov (Russian Academy of Sciences, Russia)

Matrix methods provide the key to many problems in pure and applied mathematics. However, linear algebra theory, numerical algorithms and matrices in FEM/BEM applications usually live as if in three separate worlds. In this volume, maybe for the first time ever, they are compiled together as one entity as it was at the Moscow meeting, where the algebraic part was impersonated by Hans Schneider, algorithms by Gene Golub, and applications by Guri Marchuk. All topics intervened in plenary sessions are specially categorized into three sections of this volume.



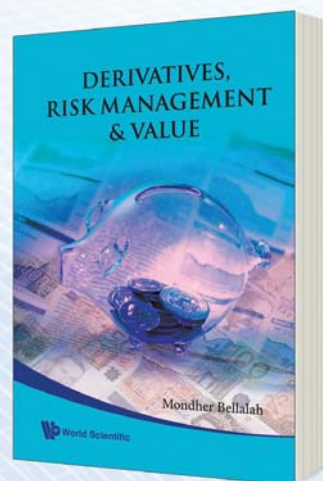
Readership: Advanced graduate students in mathematics and professionals. Besides the academia and industry, also those who consider using matrix methods in their work or who major in other fields of mathematics, engineering and sciences.

600pp
978-981-283-601-4
May 2010
US\$115

DERIVATIVES, RISK MANAGEMENT & VALUE

by **Mondher Bellalah** (Université de Cergy-Pontoise, France)

This book covers fundamental concepts in financial markets and asset pricing such as hedging, arbitrage, speculation in different markets, classical models for pricing of simple and complex derivatives, mathematical foundations, managing and monitoring portfolios of derivatives in real time, etc. It explains different applications of these concepts using real world examples. The book also covers topics like financial markets and instruments, option pricing models, option pricing theory, exotic derivatives, second generation options, etc. Written in a simple manner and amply supported by real world examples, questions and exercises, the book will be of interest to students, academics and practitioners alike.



Readership: Undergraduate and graduate students, academics and professionals interested in options.

996pp
978-981-283-862-9
Oct 2009
US\$135

More Memories and Special Events Planned to Honor Ky Fan

“I met Ky Fan several times. I especially remember having dinner with him at an ILAS meeting. He was a lot of fun, and had a lot of stories. He was full of life as well as mathematics.” Roger Horn (University of Utah)

“I first met Ky Fan in the Spring of 1986 when I came to US at the Matrix Conference at Auburn; though it was the first time I met Ky Fan, he was very kind to me and gave me a lot of advice and encouragement. I remember very well that he told me about some of his views about mathematics: A good mathematical result should have a simple hypothesis and a beautiful-deep-elegant conclusion. I miss him much!” Chi-Kwong Li (College of William and Mary)

“I am saddened by the news. Ky Fan was a kind elder and surely a world class scholar in many different math areas.” Tin-Yau Tam (Auburn University)

“Professor Fan’s results will last forever.” Boying Wang (Beijing Normal University)

“I first met Ky in Beijing in 1989, the day before the Tiananmen Square Shooting. At that time I had been admitted to UCSB. He was very happy that another Chinese student was coming to the department. On the first day of my adventure in the US, I had a conversation with him over the phone in Bob Thompson’s office that I will remember forever. I (and other Chinese students) were invited to Fan’s home for dinner many times. Fan’s results on matrices are among the most elegant ones. I wish a collection of his papers could be published.” Fuzhen Zhang (Nova Southeastern University)

* “He was an inspiration to all of his students and one of the most memorable people I’ve ever met. His dedication to mathematics was unmatched, and though he was demanding as a teacher, he never asked anything of anyone that he wouldn’t have expected of himself. As many people know, his advisor was the noted mathematician Maurice Frechet, and Frechet’s advisor was the even more famous Jacques Hadamard. Frechet died while I was working on my thesis. Shortly thereafter, someone point out to Fan that Hadamard had lived until 97, Frechet until 94, and concluded that Fan would live to 91. Now, for most people in their late 50’s (as he was at the time), that would be good news. But Dr. Fan resented any limitation on his opportunity to do mathematics, so he replied testily, “How do you know it’s a straight line?!? Maybe it’s a parabola!” Dennis Wldfogel, Fan’s Ph.D. student at UCSB, 1974.

There is another biography of Ky Fan, written by R. Ye, at <http://www.math.ucsb.edu/~yer/KyFan-biography.pdf>.

An article “In Memoriam Ky Fan” is being written for LAA by Bor-Luh Lin of the University of Iowa, who was a PhD student of Fan. Professor Lin reports that Ky Fan will be buried in Hangchow (Hangzhou), China, next to his parents, and he also reported about the following special events being organized:

* Kung-Ching Chang of Peking University is organizing a conference in Beijing, to be held in August, 2010, in memory of Professor Fan.

* The 7th International Conference of Nonlinear Convex Analysis, which will be held in 2011 in Japan, will be dedicated to Professor Fan.

* The Journal of Nonlinear Convex Analysis and the Bulletin of the Institute in Mathematics, Academia Sinica, Taiwan, will publish special issues in Ky Fan’s honor. His friends and colleagues are invited to attend and/or contribute papers to these conferences and special issues.



Family Photographs of Ky Fan

Obituary: M.J.C. Gover

by A.M. Byrne

It is with great sadness that I inform you of the death of M.J.C. Gover. He died peacefully on January 19, 2010. He was a Professor in the Applied Mathematics Research Group at the University of Bradford, UK, a former member of ILAS, and a valued colleague. Professor Gover worked on structured matrices, and co-edited with S. Barnett the proceedings of a conference held at the University of Bradford, published as Applications of Matrix Theory by Clarendon Press, Oxford, 1989.

HONORS AND AWARDS

Thomas Kailath Wins BBVA Foundation Frontiers of Knowledge Award



Thomas Kailath

The 2009 BBVA Foundation Frontiers of Knowledge Award in the Information and Communication Technologies category has been awarded to engineer and mathematician, Thomas Kailath, for creating knowledge with transformative impact on the information and communication technologies that permeate everyday life. These pioneering developments laid the mathematical foundations enabling solutions to some of the challenging problems in this area and have also served to break through the barrier of chip miniaturization.

The BBVA Foundation is the corporate social responsibility arm of the BBVA Group, a multinational financial services corporation based in Spain. The Foundation collaborated with the Spanish National Research Council in the awards process. The prize, which carries a purse of 400,000 Euros, will be awarded at ceremonies in Madrid in June 2010.

Kailath, the Hitachi America Professor Emeritus in the School of Engineering at Stanford University, was cited as “that rare combination: a scientist with the ability to solve profound mathematical problems and translate them into practical applications, generating new technologies and transferring them to industry.”

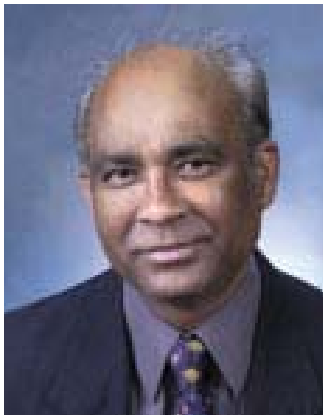
In the course of his teaching career, Professor Kailath has mentored over a hundred doctoral and postdoctoral students. “I was able to see the opportunities and enter new fields because I learned to use my students as intelligence amplifiers,” says Kailath. “So, I regard this prize as a tribute also to them, to their brilliance and dedication.” Together with some of his students, he has founded four engineering companies.

He was nominated by Professor Jose M. F. Moura of the Department of Electrical and Computer Engineering at Carnegie Mellon University and President of the IEEE Signal Processing Society. Supporting letters came from individuals at the Spanish Royal Academy of Engineering; the Institute of Advanced Studies of the Technical University of Munich; the School of Engineering of Stanford University; the Department of Mathematics of the Massachusetts Institute of Technology; the Princeton School of Engineering and Applied Science; the Israel Institute of Technology; and the Imperial College, London.

Professor Kailath has made major contributions to engineering, applied mathematics, and to the teaching of these fields. Here are a few of the great number of awards and honors he has received. The IEEE gave him their Medal of Honor in 2007 for “exceptional development of powerful algorithms in the fields of communications, computing, control and signal processing,” their Millenium Award in 2000, Jack S. Kilby Signal Processing Medal in 2006 and Education Medal in 1995. He belongs to numerous professional societies, has given many keynote addresses at their conferences, and has been elected to the National Academy of Engineering, National Academy of Sciences, American Academy of Arts and Sciences, and the Silicon Valley Engineering Hall of Fame. He is also a Foreign Member of the Royal Society of London, England, the Academy of Sciences of the Developing World, the Indian Academy of Engineering, and the Royal Spanish Academy of Engineering.

He received the third highest civilian honor of the Government of India, the Padma Bhushan Award. He has received honorary doctorates from Universities in France, Spain, Sweden, Scotland and India, many prestigious fellowships, and scores of other awards, prizes and recognitions. He has received several Outstanding Paper awards from IEEE and other engineering and mathematics journals. He has served on the editorial boards of over 20 professional journals, and continues that work for the India Institute of Electronics and Telecommunication Engineers Journal, Integral Equations and Operator Theory, Linear Algebra and Its Applications, and the Asian Journal of Mathematics. He has been Editor of the Prentice-Hall Series on Information and System Sciences since 1963.

Medal of Honor to Biswa Datta Presented at EPSICON 2010



Biswa Datta

At the First International Conference on Power, Control, Signals and Computation (EPSCICON 2010), a Gold Medal of Honor was presented to Professor Biswa Datta from the Vidya Academy of Science and Technology. Professor Datta is a Distinguished Research Professor at Northern Illinois University, US, and this award was in recognition of his lifetime achievements in various disciplines of science and engineering. The EPSICON conference was sponsored by IEEE, several Government of India Organizations and the Vidya Academy of Science and Engineering. It was held in Thrissur, Kerala, India, January 4-6, 2010.

Professor Datta is a Fellow of IEEE, an IEEE Distinguished Lecturer, an Academician of the Academy of Nonlinear Sciences, and recipient of two Senior Fulbright Research Awards, to support visits to Mongolia and to Egypt. Other Plaques of Honor have been awarded to him by the IIT Kharagpur IEEE Section and Northern Illinois University. He was also honored at the International Workshop on Signals, Systems and Control, IIT-Kharagpur, India in 2007, and more recently at the IMA Conference, Linear and Numerical Linear Algebra: Theory, Methods and Applications, held at Northern Illinois University, US, in 2009.

In a Special Honoring Ceremony during EPSICON, Professor Thomas Kailath of Stanford University, US, presented the award. He remarked on Datta's ability to bring diverse groups of researchers together, his extensive contributions to interdisciplinary research that blends mathematics and computation with several areas of engineering, and his leadership role in organizing several very successful interdisciplinary conferences sponsored by many professional societies, including the American Mathematical Society, Society for Industrial and Applied Mathematics and Mathematical Theory of Networks and Systems. He also made some kind remarks about the contributions of his wife, Karabi Datta, to his success.



Other speakers in the ceremony were Professors Shankar Bhattacharayya of Texas A&M University; Amit Bhaya of Federal University of Rio de Janeiro; James Bunch of University of California, San Diego; K. Datta of Northern Illinois University; K. Galkowski of University of Zielona Gora, Poland; P.P. Nair, Dean, Vidya Academy Of Science and Technology; Sudha Balagopalan, Head, Department of Electrical Engineering, Vidya Academy of Science and Technology; Prof. V.K. Damodaran, Managing Director of International Centre on Small Hydro Power, Kerala and China; Prof. Balasubrahmaian, IIT Delhi, Prof. Ashwani Kumar, NIT Kurukshethra, and Dr. D.P. Kothari, Vice-Chancellor of VIT University, Tamil Nadu, India. These speakers echoed remarks made by Professor Kailath and additionally commented on the impact made by Datta's two acclaimed textbooks, *Numerical Linear Algebra and Applications* and *Numerical Methods for Control Systems Design and Analysis*, the educational and industrial software packages, MATCOM, MATCONTROL and Advanced Numerical Methods for Control Systems Design and Analysis, and how his work has influenced the work of numerous researchers around the world.

Speakers from India were appreciative about the close scientific contact that Professor Datta has maintained with India, and his contributions to the development of education and research in India. Professors Balagopalan and Prof. Nair gratefully acknowledged his contributions to the success of EPSCICON 2010.

Datta's wife Karabi Datta, who is also a mathematician, humorously remarked that he is only a part-time husband while passionately working on his research and books almost full-time.

Datta responded that he was very proud of the mathematical heritage of India, the country that invented the fundamental concepts of zero and infinity and of this honor given to him in the state of Kerala. Kerala is the state with the best literacy rate and highest number of people with higher education in India, as well as a rich culture, and is very special to him.

ARTICLES

Eduard Weyr and Linear Algebra

By Jindřich Bečvář, Prague



Eduard Weyr

Czech mathematician Eduard Weyr was born in Prague on June 22, 1852, the fourth child of Franz Weyr (1820-1889), a famous professor of mathematics at the Realschule (secondary school) in Mikulandská Street in Prague, and his wife Marie born Rumplová (1825-1889). He had three brothers and six sisters. His older brother Emil (1848-1894) was an outstanding mathematician and professor at the University of Vienna. His younger brother Bedřich (1853-1908) was a chemist. For more information about the Weyr family, see [1], [2], [3].

From 1858 to 1862, Eduard attended elementary school and grammar school in Prague and later he studied at the Realschule where his father taught.

Beginning in the academic year 1868/69, Eduard was a regular student at the Prague Polytechnic. He spent the academic year 1872/73 as a state scholarship holder at the University in Göttingen, where he attended lectures of A. Clebsch, E.Ch.J. Schering, and E.F.W. Klinkerfues. On May 28, 1873 he obtained a doctor's degree at Göttingen based on his thesis *Ueber algebraische Raumcurven*. The following year he was awarded a new state scholarship to study in Paris at the Faculté des Sciences and at the Collège de France. He attended lectures of Ch. Hermite and J.A. Serret. In the academic year 1874/75, he habilitated at the Czech Polytechnic thanks to his research in geometry; the next year, he was named a salaried docent. In 1876, he was appointed an extraordinary professor at the Czech Polytechnic and he taught there until his death. In the same year, Ed. Weyr became a private docent at the University in Prague and in 1881, he was named an ordinary professor at the Czech Polytechnic. In the academic year 1884/85, he was elected rector of the Czech Polytechnic, a post that he held again in the academic year 1890/91.

Eduard Weyr spent the winter semester 1885/86 in Berlin where he attended the lectures and seminars of L. Kronecker and I.L. Fuchs.

In 1891, Eduard Weyr obtained his second academic post. He became a substitute professor at the Czech University in Prague; his lectures on geometry supplemented the basic mathematical courses of F.J. Studnička (1836-1903). In the academic year 1902/03, Eduard became ill and on July 23, 1903 he passed away.

The Weyr brothers were deeply involved in the Czech mathematical community. Eduard often published his results in the publications printed by the Union of Czech Mathematicians. In 1875, after Emil moved to Vienna, Eduard became permanent secretary of the Union and held this position until his death. From 1882 to 1883 he was the Chief Editor of the *Journal for Cultivation of Mathematics and Physics*, which was founded by the Union in 1872.

Eduard Weyr started his mathematical career quite early. At the age of sixteen he published his first paper, *Erweiterung des Satzes von Désargues nebst Anwendungen* (1868). He and Emil wrote a three-volume textbook *The Elements of Higher Geometry* (1871, 1874, 1878), which became the first Czech textbook dealing with projective geometry. In addition to many articles, Eduard wrote two monographs, *On the Theory of Bilinear Forms* (1889) and *On the Theory of Surfaces* (1891), and two textbooks, *Projective Geometry of Elementary Formations of the First Order* (1st edition 1898, 2nd edition 1911) and *Differential Calculus* (1902).

During his life, Eduard was regarded as a geometer; in fact, most of his publications are geometric (conic sections and quadrics, projective geometry, synthetic geometry, geometrical affinities, differential geometry of curves and surfaces, algebraic curves). The thematic scope of his works is impressive; some of them have a very high standard, others contain only methodical notes and comments. The books of Emil and Eduard and later Eduard's textbooks played a very important role in the development of Czech geometrical terminology as well as of Czech geometrical studies. The Weyrs influenced many of their Czech contemporaries, younger colleagues, and successors. Some of Eduard's works dealt with analysis (series, infinite products, elliptic functions, integration of linear differential equations, expressions of roots by continued fractions), and often followed some of the ideas of Hermite. Nowadays, his most valued results deal with linear algebra, matrix theory, and theory of hypercomplex systems.

In the 1880's, Eduard Weyr was one of few European mathematicians acquainted with matrix theory and working in it. He was one of the first mathematicians who gave a general proof of the Cayley-Hamilton theorem; J.J. Sylvester knew about Weyr's proof in 1884 (see *Coll. Papers IV*, p. 202).

Eduard Weyr developed some of J.J. Sylvester's ideas in an 1884 paper (*Sur la théorie des quaternions*), which solved some problems of algebraic equations in the field of quaternions and studied the exponential and logarithmic functions of matrix argument. However, C.C. MacDuffee failed to mention Weyr's work on functions of matrices in his book *The Theory of Matrices* (1933).

In an 1885 paper (On the solving of linear equations), Eduard studied questions about systems of linear equations. This paper was addressed to the whole Czech mathematical community. Let us point out that an effective formulation of the necessary and sufficient condition of the solvability of the system of linear equations (often connected with the names of Frobenius, Kronecker, Cappelì) came into existence only at the start of the 20th century.

Weyr's papers *Sur la théorie des matrices* and *Répartition des matrices en especes et formation de toutes les especes* (*C. R. Acad. Sci. Paris*, 1885) contain the basic ideas of his important results in matrix theory. In them, he presented his theory of "characteristic numbers" and "typical matrices", which today are known as the *Weyr characteristic* and the *Weyr canonical form*. He used the concept of nullity introduced by J.J. Sylvester. He described a complete system of invariants for matrix similarity. His 1885 papers contains a result about nullity of the product of two matrices, equivalent conditions for diagonalization of a matrix, conditions for existence of a minimal polynomial of degree smaller than the order of matrix, etc. His approach, different from the classical theory of elementary divisors, was close to modern algebra and functional analysis.

Natural concepts in mathematics are often developed independently by various authors who are unaware of the work of their contemporaries. The "Jordan" canonical form was introduced by Weierstrass and independently by Jordan. The Weyr canonical form introduced in Eduard's 1885 paper is permutation similar to the Jordan canonical form, so one can be derived from the other. It is interesting that Jordan's canonical form is not mentioned in Weyr's papers and on the other hand, that Weyr's canonical form is almost forgotten nowadays. A partial explanation may be that Weyr's theory was too modern for many of his contemporary readers; it used matrix language that became standard on the continent only after the turn of the 20th century (in historical considerations there is always some portion of speculation and thus a portion of uncertainty).

In 1887, Eduard published a Czech monograph (On Binary Matrices) in which he developed the theory of matrices of second order and explained their connection with linear transformations. This monograph was addressed to the Czech readers and its main aim was to introduce and disseminate a new theory, a new mathematical language, and connections between matrices and transformations and quaternions.

In two 1887 papers, Eduard Weyr studied various problems involving associative algebras. In 1889, he published a 111-page Czech paper (On the theory of bilinear forms), in which he explained the fundamental parts of matrix theory and connected them with the theory of bilinear and quadratic forms. He clarified in detail results presented in his previous papers and analyzed the convergence of power series in matrices. He re-proved (in matrix language) the classification of equivalence of certain pairs of bilinear forms that had been studied by K. Weierstrass and L. Kronecker, presented modern proofs of some other outstanding results, and gave some applications.

In 1890, in the first volume of the journal *Monatshefte für Mathematik und Physik* (founded by Gustav von Escherich and Emil Weyr), Eduard Weyr published a German version of his long 1889 paper (Zur Theorie der bilinearen Formen). It is not just a literal translation; some parts were reduced and others expanded. One notable omission was the chapter on scalar functions of matrices, which contained Weyr's important result on the convergence of power series with the matrix argument. Necessary and sufficient conditions for convergence of power series in matrices are presented only in Weyr's 1889 Czech paper.

Eduard Weyr was one of the first mathematicians on the European continent who accepted the language of matrices, developed ideas of matrix theory, and contributed to its connections with the theory of bilinear and quadratic forms. Today, his understanding of similarity of matrices, via his complete system of invariants of similarity (the *Weyr characteristic*), can be acknowledged as quite modern.

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Ed. note: For photo and other information about Eduard Weyr, visit <http://turnbull.mcs.st-and.ac.uk/history/Biographies/> .

THE HISTORY OF LINEAR ALGEBRA IN ISRAEL – A PERSONAL VIEW

By Abraham Berman, The Technion, Haifa, Israel

The first “Israeli” linear algebra result that I can think of is the classical Amitsur-Levitzki theorem [3]:

Theorem 1. For every $2n$ matrices of size $n \times n$, $A_1, A_2, \dots, A_{2n}$, $\sum \text{sgn}(\sigma) \prod_{i=1}^{2n} A_{\sigma(i)} = 0$ where the sum is taken over all $(2n)!$ permutations of $1, 2, \dots, 2n$ and sgn denotes the sign of σ .

A lovely description of the theorem is given in Gil Kalai’s blog [18]. Yaacov Levitzki was one of the founders of the Department of Mathematics in the Hebrew University, the first mathematics department in Israel. Shimshon Amitsur was, in 1950, a student of Levitzki and then became one of the world’s leading ring theorists. There are several proofs of the Amitsur-Levitzki identity. Of them I want to mention a short (and also “Israeli”) proof given in 1976 by Shmuel Rosset of Tel-Aviv University [23].

A major part of the linear algebra activity in Israel takes place at the Technion and most of this article will be Technion-centered. The founder of the Technion matrix group is Binyamin (Benny) Schwarz; see the tribute to him on the occasion of his 70th birthday [12]. He received his Master of Science in 1942 at the Hebrew University and his Ph.D at Washington University under the direction of Zeev Nehari (after serving in the British Army during the world war and in the newly created Israeli Defense Force at the birth of the state of Israel). He joined the Technion in 1955 and was one of the founders of its Mathematics Department. I had the great pleasure of taking his matrix theory course as an undergraduate student and of returning to the department as a faculty member in 1972, when he was its chair. Schwarz’s research activity included complex functions, differential equations, calculus of variations and matrix theory. One of his first matrix results is the following beautiful theorem on rearrangements [25]:

Theorem 2. Let S be the set of all $n \times n$ nonnegative square matrices whose entries form a given set of n^2 nonnegative numbers. Then the maximal value of the spectral radius, where the maximum is taken over all the matrices in S , is achieved for a matrix A such that the entries in each of its rows and columns form a decreasing sequence.

Schwarz’s interest in nonnegative matrices was influenced by his visit to Madison during 1961-1962. Similar Madisonian influence was exerted later on the work of Shmuel Friedland, Raphi Loewy, Uri Rothblum, Allan Pinkus and Danny Hershkowitz.

Benny had many graduate students including three Ph.D students in matrix theory: David London, Moshe Katz and Shmuel Friedland, to whom a recent issue of LAA was dedicated [4]. Shmuel also won the Hans Schneider Prize at the Pensacola meeting of ILAS in 1993. David London was the supervisor of the masters theses of Raphi Loewy, Danny Hershkowitz and myself. Raphi got his Ph.D in Caltech under the supervision of Olga Taussky, I got my Ph.D at Northwestern under Adi Ben-Israel and was the Ph.D adviser of Danny. I had students in matrix theory and in math education. My matrix Ph.D students were, in addition to Danny: Ron Aharoni, Dan Shemesh, Dafna Shasha (co-supervised by Danny), Naomi Shaked-Monderer, Natali Kogan (supervised also by Danny) and Mark Krupnik. Another doctoral student, about to graduate, is Felix Goldberg. Ron and Danny served as Chairmen of our department. Danny was the President of ILAS and is now the Minister of Science and Technology of Israel.

I also supervised several masters students. With Raphi Loewy I supervised Ron Adin who continued his studies at the Hebrew University and is now the chairman of the Department of Mathematics in Bar-Ilan University. In his thesis Ron solved a problem on extreme positive operators on minimal and almost minimal cones and published it in LAA [2]. We thought that the paper sufficed as a masters thesis but the graduate school wanted the paper to be translated to Hebrew (now theses can be written in English but this was not allowed in 1982). Luckily Bryan Cain was a visiting professor at the Technion at that time so we asked him to be on Ron’s committee and got a special permission to publish the thesis in English. Another masters student of mine was Marina Arav, who later completed her doctorate under the supervision of Danny and is now a professor in Atlanta. Here we can extend, via marriage, the Israeli linear algebra community by adding Hein van der Holst, Marina’s husband. The connection is, of course, deeper. Hein cooperated with Raphi on problems of minimum rank and gave good advice to Felix on his thesis. Other “matrix descendants” of Benny are Nader Agah who was a student of Danny, and Nizar Radwan who was a student of Raphi. Moshe Baruch wrote his masters thesis with Raphi (and his doctorate with Piatetski-Shapiro) and is now a professor in the department.

One cannot talk about linear algebra in Israel without mentioning the very strong operator theory and linear algebra school of Israel Gohberg. He won, among many other honors, the Hans Schneider prize. The impact and production of this school is so extensive that a special article ought to be devoted to it, that will hopefully appear in this series on the history of linear algebra. The representative of the Gohberg group at the Technion is Leonid Lerer. He immigrated to Israel and joined the Technion in 1974. His doctoral students include Miron Tismeneski (Timor), Ben-Zion Kon, Irina Karelin, Grasiela Gomez, Julian Haimovici and Igor Margulis.

An important feature of the Israeli linear algebra scene is the Haifa matrix conference series. The series started in 1984 and one of the conferences coincided with the 8th ILAS conference. The 15th conference, the most recent one so far, (see [1]) took place in 2009. Most of the world's leading matrix theorists have participated in these very pleasant, if I may say so, meetings. Some are frequent participants. Two even celebrated their honeymoon at one of the meetings. Very prominent Israeli mathematicians participate in these meetings, and for many of them, their major area of research is not linear algebra. They work in operator theory, functional analysis, combinatorics and graph theory, group theory, system theory, computer science and signal processing, but nevertheless they are also actively involved in linear algebra. Such is the power and strength of linear algebra that people come to it in a natural way in the course of the intrinsic development of their area and become interested in linear algebra and its applications. These leading mathematicians include Gil Kalai, Nati Linial and Alex Lubotzky from the Hebrew University, Noga Alon (who won the Israel Prize in Mathematics), Israel Gohberg, Assaf Goldberger and Yuli Eidelman from Tel-Aviv University, Harry Dym from Weizmann (President of the Israeli Math Union), Ron Adin, Chair of the Mathematics Department of Bar-Ilan University, Daniel Alpay, Genrich Belitskii, Abe Feintuch, Paul Fuhrmann, Itzhak Lewkowicz, Alexander Markus and Victor Vinnikov from Ben-Gurion University; Yonathan Arazi and Yair Censor from the University of Haifa; and Freddy Bruckstein, Israel Cedrbaum, David Chillag, Miki Elad, Moshe Goldberg, Shaul Gutman, Yuri Lubich, Roy Meshulam, Allan Pinkus, Uriel Rothblum, Avram Sidi and Baruch Solel from the Technion. Mathematicians from industry, for example Achya Dax and Tamir Shalom, also participate in the Haifa matrix meetings.

In the next section I want to recall several important past members and visitors to the Technion. Mordechai Lewin worked on matrices related to graphs. He had Yehoshua Vitek as a doctoral student and Nati Linial and Noga Alon as masters students. Between Northwestern and Delaware and Rutgers, Adi Ben-Israel was the head of the Department of Applied Mathematics. One of his students at the Technion was Yair Censor. Henryk Minc was a visiting professor for many years and this is probably one of the reasons for the interest that people in the department had concerning the van der Waerden conjecture. David London and Shmuel Friedland made important advances in attacking the conjecture that was later proved independently by Egorichev and Falikman. When Falikman immigrated to Israel he completed his doctorate at the Technion under the supervision of David London and Raphi Loewy. A few years after that, Leonid Gurvitz spent some months at the Technion and gave a series of lectures on generalized matrix functions.

The Technion has a special position of permanent visiting professor. Two such permanent visiting professors were Alan Hoffman and Paul Erdos. A theorem of Erdos and Minc that resulted from their simultaneous stay at the Technion is a necessary and sufficient condition for the existence of a nonnegative matrix with a given diagonal and given row sums and column sums [9]. Other important active visitors were Leiba Rodman and M.A. Kaashoek, who mainly worked with Lerer at the Technion and with Gohberg in Tel-Aviv, and Wayne Barrett who visited Raphi.

Important matrix theory is also done at the Technion in other departments. Prominent examples are Uriel Rothblum in Industrial Engineering and Alfred Bruckstein, Miki Elad and Avram Sidi in Computer Science. Miki Neumann started his linear algebra career in the Technion Computer Science Department.

This article is a very personal (and Technion-centered) look at Linear Algebra in Israel and I want to apologize to those whom I forgot to mention and to the many that I only mentioned without describing their important work. Ben Gurion University has a strong group of linear algebraists and people who use linear algebra, that were mentioned above. Here I want to mention, very briefly, the linear algebra activities in Tel-Aviv University and the Weizmann Institute.

Harley Flanders was a professor at Tel Aviv University between 1970 and 1977. Israel Gohberg immigrated to Israel in 1974 and joined the Department of Mathematics at Tel-Aviv University. Using the words of Leiba Rodman [22] "he left an enormous, lasting legacy". He authored and edited 90 books and supervised 40 Ph.D students including Leonid Lerer, Leiba Rodman, Tamir Shalom, Asher Ben-Artzi, Yuli Eidelman, Nir Cohen, Israel Koltracht, Sonia Levin and I. Zuker. Gohberg received six honorary doctorates, including one from the Technion. From the early seventies up to 1996 (when Gohberg became seriously ill) about half of the talks in his seminar were related to linear algebra.

Gohberg also had a position in Weizmann and a very productive cooperation with Harry Dym. Talking about Weizmann one should mention Sam Karlin who between 1970 and 1976 divided his time between Stanford and the Weizmann Institute in Rehovot in which he became Head of the Applied Mathematics Department. One of his doctoral students was Allan Pinkus who later joined the Technion. One of his postdocs was Shmuel Friedland who later joined the Hebrew University before leaving to go to the University of Illinois at Chicago.

I want to conclude this article by observing that Israelis play an important role in the Linear Algebra community. Many are or were Editors of *Linear Algebra and Its Applications*, *Linear and Multilinear Algebra*, *The Electronic Journal of Linear Algebra* (ELA),

the SIAM *Journal of Matrix Analysis and Operators and Matrices*. Quite a few of them have published books. As examples I want to mention a book on general matrix theory by Miron Tismenetsky and Peter Lancaster [19], two books by Gohberg, Lancaster and Rodman on polynomials [14] (republished in [15]), the book [13] by Paul Fuhrmann, several books on nonnegative matrices [6] (republished in [7]), the books [5] and [8] that I wrote with Bob Plemmons, Miki Neumann, Ron Stern and Naomi Shaked-Monderer, and the book [21] by Allan Pinkus.

Finally, an important recent publication in linear algebra is the *Handbook of Linear Algebra* edited by Leslie Hogben. The Israeli linear algebraists contributed their share. Miki Neumann coauthored the chapter on inner product spaces [16]. Uri Rothblum contributed the chapter on nonnegative and stochastic matrices [24]. Raphi Loewy wrote the chapter on quadratic, bilinear and sesquilinear forms [20]. Danny Hershkowitz wrote the chapter on matrix stability and inertia [17] and Shmuel Friedland wrote the chapters on matrices over integral domains and similarity of families of matrices [10, 11].

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BOOK REVIEWS

Linear Algebra in Action, by Harry Dym
AMS Graduate Studies in Mathematics, Vol. 78
American Mathematical Society, 2006. 541 pages. ISBN-13: 978-0-8218-3813-6

Reviewed by Douglas Farenick

A few years ago I noted the release of a new book in linear algebra, authored by a distinguished linear algebraist, Harry Dym. I had always found Professor Dym's conference lectures to be informative and interesting, and so without hesitation or much foreknowledge of the contents of the book, I placed my order and in a short time the book found its way into my personal library. *Linear Algebra in Action* now occupies a position on my primary bookshelf - the place where I keep the books I will want to have handy when I am working on mathematics.

Dym's book, 541 pages consisting of 23 chapters and two appendices, is directed toward graduate students. As its focus is on matrix analysis rather than linear algebra (in the "pure algebra" sense), the book is in direct competition with other established graduate-level books. I am thinking of, for example, *Matrix Analysis* and *Topics in Matrix Analysis*, both by Horn and Johnson, and *The Theory of Matrices*, 2nd Edition, by Lancaster and Tismenetsky. (This latter text was a major source of inspiration for Dym, according to the Author's Preface.) Whereas most books in mathematics do not have much personality, Dym's book does: it is enlivened with quotes (many of them from baseball, some witty, some off the wall, all worthy) and is written in a manner that seems almost to have been transcribed word for word from an oral lecture. The voice of the "teacher" is present throughout the text.

The first few chapters have material that is covered in generic undergraduate texts. One could question the value of including these chapters, as I first did when I opened the book, but the treatment is in no way stale and it provides Dym with the opportunity to impart some important principles to students. As an illustration, after proving the Fredholm alternative he describes the result in - as he puts it - its most provocative form: "if $Ax = b$ has at most one solution, then it has exactly one." Nice!

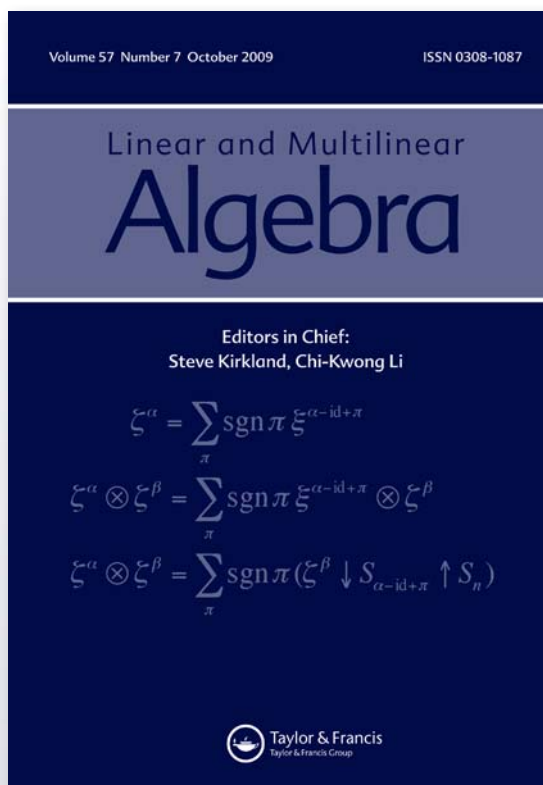
I cannot mention all the features of these early chapters that I like, but here are a few. The chapter on applications of Gaussian elimination concludes with a good set of exercises. Dym takes a determinant-free approach to eigenvalues, which I personally favour, but he does not do so in a condescending way. (Where, for example, would the theory of Lie groups be without determinants?) Determinants are indeed studied in a subsequent chapter; the approach is axiomatic, and there are some useful results such as the Binet-Cauchy Formula included. In the treatment of Jordan canonical form, effort is made to convey the conceptual meaning of the data encoded by the form, and the issue of determining the Jordan form is faced head on, with examples that students will find of great value.

The study of matrix analysis begins with a discussion of normed vector spaces, Banach spaces, and linear functionals. The proof that the general linear group $GL_n(C)$ is an open set in $M_n(C)$ is achieved by analytic methods rather than through the continuity of the determinant. This is just one example of where Dym aims to provide readers with general mathematical methods that apply equally well to functional analysis. Another example is the spectral radius formula of Gelfand, which is proved in manner that is particularly transparent and instructive. Again, a lot of the material could be considered standard, but one finds interesting (and, for me, often new) facts here and there: for example, Takagi's Theorem on the diagonalisation of symmetric complex matrices.

As one gets deeper into the book, the material becomes more specialised and sophisticated, and it is here that the book's distinctive value is most evident. Dym gives an expert treatment of a wide selection of topics that includes differential and difference equations, extremal problems, positive-definite Toeplitz matrices, Riccati equations, holomorphic and rational matrix functions, applications to control theory, eigenvalue inclusion regions, Bezoutians and resultants, convexity, nonnegative matrices, and majorisation. A lot of the advanced and specialised topics in Dym's book I had heard about, but had never before studied myself. Each topic is treated in enough detail to permit a reader or student to acquire the basic background that one might need to read a paper on an unfamiliar subject or simply to widen one's knowledge of matrix theory.

Dym's book is a worthy competitor to the books I mentioned earlier. Similar in spirit to the book of Lancaster and Tismenetsky and some of the monographs of Gohberg, Lancaster, and Rodman (such as *Invariant Subspaces of Matrices*), Dym's book is nevertheless a singular and significant addition to the literature, with many important topics treated here in one volume. The emphasis on examples will be appreciated by many.

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It is gratifying to see the inclusion of *Linear Algebra in Action* in the high quality AMS Graduate Studies in Mathematics series, as it is an indirect affirmation of a subject that is both important and vibrant. Dym's book should go far in bringing serious matrix analysis to the next generation of mathematicians.

Department of Mathematics and Statistics, University of Regina, Regina, Saskatchewan S4S 0A2, Canada
Doug.Farenick@uregina.ca

Introductory Combinatorics, 5th edition, by Richard Brualdi
Pearson Prentice Hall, 2010. 648 pages. ISBN-13: 978-0-13-602040-0

Reviewed by Jeffrey L. Stuart

How many ways can you partition thirty-three years into five editions and fourteen chapters? This year marks the arrival of the fifth edition of Richard Brualdi's *Introductory Combinatorics*, a textbook whose first edition appeared in 1977. For those acquainted with other works by Professor Brualdi, or who have had the pleasure of taking a course from him, the style of this work will be familiar. The book is built around a wealth of varied and informative examples that are revisited as new ideas are introduced. The examples punctuate a steady progression of carefully stated definitions and theorems. The exposition is clear and directed, but deliberately eschews prolixity; the proofs are insightful and provide enough detail that a moderately strong undergraduate should be able to read them without (much) assistance. The intended audience is a reader with the mathematical maturity gained from successfully completing a calculus sequence and a first course in linear algebra (although neither calculus nor linear algebra is significantly employed here). Given the level of the presentation and the accessibility of the material, this text is well suited for a bridge course designed to develop the skills needed for senior level courses in abstract algebra and real analysis – working with examples and counter-examples, reading and writing proofs. To support its use as a textbook, the author includes roughly 700 exercises of varying levels of difficulty. Exercises appear at the end of each chapter rather than at the end of each section; short answers and hints to selected problems are included at the end of the book.

What should be in a combinatorics text? An author should be guided by common practice, personal preferences, and, of course, the desire that the book be both useful and successful. Some authors have written books organized around a single topic such as enumeration, generating functions or graph theory. This is *not* such a text. As an overarching organizational theme, Professor Brualdi uses the idea of combinatorics as the study of arrangements of objects from a set into patterns that obey specified rules. In this context, he focuses on four questions:

- Does an arrangement exist that satisfies the rules?
- Is there a classification or enumeration of such arrangements?
- What properties does such an arrangement have?
- Is there an attractive optimality criterion that leads to an optimal arrangement?

The opening chapter offers six extended examples spread across twenty pages: perfect covers of chessboards, magic squares, the four-color problem, the problem of 36 officers, a shortest route problem, mutually overlapping circles, and the game of Nim. Already, coverage of some of the standard topics of combinatorics should be evident. There are three chapters on graph theory (including trees, Eulerian and Hamiltonian graphs, bipartite graphs, planarity, chromatic numbers, clique numbers, matchings, digraphs and networks); two chapters on permutations and combinations; two chapters on systems of distinct representatives and combinatorial designs; as well as single chapters on the pigeonhole principle, inclusion exclusion (including forbidden position problems and Möbius inversion), recurrence relations and generating functions, binomial coefficients and their generalizations, special counting sequences, and Pólya enumeration. Recognizing that there is at least two semesters' worth of content here, the author provides extensive suggestions about how to organize several different one semester courses from this wealth.

This review ends with a few final observations. One feature of this book that distinguishes it from other introductory combinatorics texts is that there is an entire chapter devoted to algorithms for systematically creating and modifying permutations and combinations. One minor quibble is with the typesetting: the large fonts coupled with the narrowest of margins can make the text appear to spill off of the page, and may inhibit annotation. Finally, many students today are cynical about new editions of established texts, viewing them as consisting of modest alterations designed primarily to annihilate the market in used copies. In the introduction, Professor Brualdi is careful to justify this new edition, detailing how (and sometimes why) he has moved, expanded, and occasionally deleted material in the new edition. This reviewer believes that this is an improved version of an already excellent book.

Pacific Lutheran University, Tacoma, Washington, 98447, USA. jeffrey.stuart@plu.edu

NEW BOOKS

Numerical Linear Algebra and Applications, 2nd ed., Biswa Nath Datta, 2010

This new edition covers the major topics of computational linear algebra. Important features of the original edition have been updated and improved, including a variety of motivating applications. When a physical problem is posed, the scientific and engineering significance of the solution is discussed. Each chapter has a summary of its important concepts, suggestions for further reading, and numerous exercises, both theoretical and MATLAB® and MATCOM based. There is a list of key words for quick reference.

This book is intended for undergraduate and graduate students in applied and computational mathematics, scientific computing, computer science, financial mathematics, actuarial sciences, and electrical and mechanical engineering. It can also be useful for researchers in mathematics, computer science, physics, chemistry, biology, economics, statistics and engineering, practicing engineers and industrial mathematicians.

The chapter titles are: 1: Linear Algebra Problems, Their Importance, and Computational Difficulties; 2: A Review of Some Required Concepts from Core Linear Algebra; 3: Floating Point Numbers and Errors in Computations; 4: Stability of Algorithms and Conditioning of Problems; 5: Gaussian Elimination and LU Factorization; 6: Numerical Solutions of Linear Systems; 7: QR Factorization, Singular Value Decomposition, and Projections; 8: Least-Squares Solutions to Linear Systems; 9: Numerical Matrix Eigenvalue Problems; 10: Numerical Symmetric Eigenvalue Problem and Singular Value Decomposition; 11: Generalized and Quadratic Eigenvalue Problems; 12: Iterative Methods for Large and Sparse Problems: An Overview; 13: Key Terms in Numerical Linear Algebra; Bibliography; Index. Online materials include Chapter 14 on special topics, some software, a brief introduction to MATLAB, and partial solutions and answers to selected problems.

Matrices, Moments and Quadrature with Applications, Gene H. Golub & Gérard Meurant Princeton Series in Applied Mathematics, Princeton University Press, 2009

This computationally oriented book describes and explains the mathematical relationships among matrices, moments, orthogonal polynomials, quadrature rules, and the Lanczos and conjugate gradient algorithms. It bridges different mathematical areas to obtain algorithms to estimate bilinear forms involving two vectors and a function of the matrix. The first part provides the necessary mathematical background and explains the theory. The second part describes applications and gives numerical examples of the algorithms and techniques developed in the first part. Applications here include computing elements of functions of matrices; obtaining estimates of the error norm in iterative methods for solving linear systems and computing parameters in least squares and total least squares; and solving ill-posed problems using Tikhonov regularization.

This book will interest researchers in numerical linear algebra and matrix computations, as well as scientists and engineers working on problems involving computation of bilinear forms. For more information and Matlab codes used for the numerical experiments, visit <http://pagesperso-orange.fr/gerard.meurant/>.

A State Space Approach to Canonical Factorization With Applications H. Bart, I. Gohberg, M.A. Kaashoek, A.C.M. Ran OT 200, Birkhäuser Verlag, 2010

This book deals with factorization of matrix and operator functions that appear in state space form or that can be transformed into such a form. The emphasis is on canonical factorization problems, including spectral and J-spectral factorization problems and related Riccati equations. A unified geometric approach is used. The main results are all expressed explicitly in terms of matrices or operators, which are parameters of the state space representation. Applications concern elements of H-infinity control theory and the corresponding Nehari approximation problem, problems in mathematical analysis (inversion problems for singular integral equations and Wiener-Hopf integral equations, related Riemann Hilbert problems), and problems from mathematical physics (linear transport theory). A large part of the book deals with rational matrix functions only.

Matrix Partial Orders, Shorted Operators and Applications, by S. K. Mitra, P. Bhimasankaram, and S. B. Malik. World Scientific, 2010.

This monograph is the first on its topic, and makes a unique presentation of many partial orders on matrices. Except for the Löwner order, the partial orders considered are relatively new and came into being in the late 1970s. After a detailed introduction to generalized inverses and decompositions, the three basic partial orders - namely, the minus, the sharp and the star - and the corresponding one-sided orders are presented using various generalized inverses. The authors then give a unified theory of all these partial orders as well as study the parallel sums and shorted matrices, the latter being studied at great length. Partial orders of modified matrices are a new addition. Finally, applications are given in statistics and electrical network theory. The first author is now deceased.

Ed. Note: List of books of interest to linear algebraists is available on the ILAS website

There exist many fine books about linear algebra and its applications. Our Book Editor Oskar Baksalary maintains a list of such on the ILAS website (<http://www.ilasic.math.uregina.ca/iic/>). He also posts references to reviews of selected titles. Please have a look at this resource and tell him when you find other titles or informative reviews that should be included (baxx@amu.edu.pl).

NEWS ABOUT JOURNALS

The Electronic Journal of Linear Algebra (ELA)

ELA is a refereed all-electronic journal published by ILAS, which welcomes mathematical articles of high standards that contribute new information and new insights to matrix analysis and the various aspects of linear algebra and its applications. Accepted papers are posted online immediately.

Editor Daniel Hershkowitz reports that ELA grew impressively in 2009. In 2008, there were 698 pages in 45 papers published, whereas in 2009 there were 918 pages in 61 papers. This includes the regular Volume 18 and special Volume 19 on the occasion of the workshop “Nonnegative Matrix Theory: Generalizations and Applications”, held at the American Institute of Mathematics, Palo Alto, CA, US.

Special Issue of Linear and Multilinear Algebra on Quantum Information Science

Original results, surveys, research problems, etc. are welcome for this special issue on quantum information science. Linear and multilinear techniques are useful in the study of quantum information science. Most researchers of quantum information science have published results and problems in specialized journals of the subject, which may fail to get the attention of researchers on linear and multilinear algebra. In view of this, Linear and Multilinear Algebra would like to publish a special issue on this topic so as to provide a platform for mathematicians and physicists to exchange problems, ideas, and results. In particular, it is desirable to have papers that are accessible to a general mathematical audience. Appropriate topics include, but are not limited to: quantum operations, quantum error correction, quantum tomography, quantum state discrimination, entanglement theory, channel capacities, mathematical foundations of quantum information.

The deadline for submissions for this issue is August 31, 2010 and targeted date of publication is September, 2011. Submit papers online at <http://mc.manuscriptcentral.com/glma>. Accepted papers will be posted online immediately.

The Special Editors for this issue are:

- * Koenraad Audenaert <Koenraad.Audenaert at rhul.ac.uk>, University of London, UK.
- * David Kribs <dkribs at uoguelph.ca>, University of Guelph, Canada.
- * Chi-Kwong Li <ckli at math.wm.edu>, College of William and Mary, USA.

Special Issue of Linear Algebra and its Applications On Linear and Multilinear Algebra, In Honor of Jose Dias da Silva

Linear Algebra and Its Applications (LAA) is pleased to announce a special issue in honor of Professor Jose Dias da Silva on the occasion of his retirement from the University of Lisbon and in recognition of his many important contributions to linear and multilinear algebra and other topics in mathematics. The responsible editor-in-chief is Richard A. Brualdi.

Papers are solicited for the special issue within the entire scope of LAA or the research interests of Jose Dias da Silva. The deadline for submission of papers is November 1, 2010. Submitted papers will be subject to normal refereeing procedures according to the usual standards of LAA. They should be sent by November 1, 2010, preferably as pdf attachments in email, to one of the following four special editors:

Thomas J. Laffey (Thomas.Laffey@ucd.ie)
 Thomas H. Pate (patetho@auburn.edu)
 Joao F. Queiro (jfqueiro@mat.uc.pt)
 Edward Marques de Sa (emsa@mat.uc.pt).

International Journal of Combinatorics

This journal provides a rapid forum for the dissemination of original research articles as well as review articles in all areas of combinatorics. It is published using an open access publication model, meaning that all interested readers are able to freely access the journal online, without the need for a subscription, at <http://www.hindawi.com/journals/ijct/contents.html>.

The journal has a distinguished editorial board with extensive academic qualifications, ensuring that the journal maintains high scientific standards and has a broad international coverage. A current list of the journal's editors can be found at <http://www.hindawi.com/journals/ijct/editors.html>.

Manuscripts should be submitted to the journal online at <http://www.hindawi.com/journals/ijct/>. Once a manuscript has been accepted for publication, it will undergo language copyediting, typesetting, and reference validation in order to provide the highest publication quality possible.

Oriental Journal of Mathematics (OJM)

The Oriental Journal of Mathematics invites original research papers and critical survey articles covering all aspects of mathematics and mathematical sciences and recent developments in these areas. This is a new bimonthly peer-reviewed international journal. For more information, visit our website <http://oapsci.org/> to see the OJM 1(1), 2009.

Papers may be submitted through e-mail directly to the Editorial Head Office on harringtonhouse.math@rediffmail.com or oapsci@yahoo.com or niraj@oapsci.org. They should be .pdf or .doc files, and should be accompanied by a covering letter of submission. Special attention should be taken that papers are original and that they add new ideas and innovations in the existing literature of their respective areas. The reviewing of a paper generally takes approximately one month and if the paper is found suitable, it will be published within two months from the date of submission.

We are also going to publish a new bimonthly peer-reviewed international journal in 2010, covering all aspects of theory and applications in statistics and statistical sciences – the Oriental Journal of Statistical Methods, Theory and Applications. The first issue of this journal is expected to appear by mid February 2010. Other relevant information can be seen on our website.

Journal of Algebra, Number Theory: Advances and Applications (JANTAA)

The Scientific Advances Publisher invites original research papers and critical survey articles for consideration of possible publication in its new journal, named the Journal of Algebra, Number Theory: Advances and Applications. The JANTAA is being published in two volume annually and each volume consisting of two issues being released in March, June and September, December. It is my pleasure to inform you that No. 2 of Vol. 2 (2009) December issue of the JANTAA has been published. To see the contents of this issue please visit the following link: <http://scientificadvances.org/journals4P5.htm>. One volume of this journal is expected to comprise of 300 pages and consist of duly refereed original research papers and survey articles in all areas of Algebra, Number Theory and their Applications. We have approached Mathematical Reviews of the American Mathematical Society and Zentralblatt für Mathematik of the European Mathematical Society (EMS) to seek their consent for Indexing and Reviewing.

Please submit manuscripts electronically as .tex, .dvi, .pdf, .ps or .doc file attachments to pkagrawal@scientificadvances.org or atul@scientificadvances.org or jantaa@scientificadvances.org. Authors with no access to internet two hard copies of papers with a letter of submission should mail to: Journal of Algebra, Number Theory: Advances and Applications, Scientific Advances Publishers, 71/52 Bhusoli Tola, Khuldabad, Allahabad 211 006 (INDIA).

For details please visit our websites: Editorial board at <http://scientificadvances.org/journals4P2.htm>, Authors Guidelines at <http://scientificadvances.org/journals4P3.htm>, and Subscription information at website: <http://scientificadvances.org/journals4P4.htm>.

JP Journal of Algebra, Number Theory and Applications (JPANTA)

This journal provides a common forum for significant research both on theoretical and on applied aspects of current interest in algebra and number theory and innovative links with various applications. It is reviewed in Mathematical Reviews, MathSciNet, Zentralblatt für Mathematik, Scopus, Ei databases index, EMBASE, and EMCare. It is published in four volumes annually, each comprised of two issues. Number 1 of Volume 17 was released in April 2010. For information about submission, visit <http://www.pphmj.com/journals/jpanta.htm>.

ISST Journal of Mathematics and Computing Systems (IJMCS)

IJMCS is a new peer reviewed bi-annual journal that aims to publish high quality original research papers about applied problems of mathematics, operations research, engineering and statistics, without compromising mathematical precision. It will be reviewed in Zentralblatt für Mathematik and welcomes any topics related to computer science, including linear and nonlinear algebra, matrix theory, functional analysis and operator theory. Articles that address both the theoretical and application aspects are particularly encouraged, for example computational techniques in operations research or statistics.

This is one of 6 new journals launched by the Intellectuals Society for Socio-Techno Welfare (ISST) in 2010. These journals will be published in June and December each year. For more information, visit www.isst.org.in.

ILAS NEWS

Report on the ILAS Program Review Submitted by Steve Kirkland, ILAS President

Since its founding in 1989, ILAS has established and maintained a number of activities and programs that further the Society's principal goal of supporting research and education in linear algebra. In June of 2008, the ILAS Executive decided to undertake a comprehensive review of ILAS's suite of activities in order to determine which ones were working well, and which ones could be improved. Consequently, the ILAS Program Review Committee (PRC) was appointed to undertake that review.

The PRC was comprised of Harm Bart (chair), Shaun Fallat, Steve Kirkland (ex-officio), Francoise Tisseur, and Xingzhi Zhan. The members of the Committee were asked to review the existing ILAS programs, and make recommendations for their improvement, or expansion, or possible elimination. The PRC was also asked to recommend new activities or directions that ILAS might undertake in the course of furthering its aims. A call for input from the ILAS membership was issued in July of 2008, and the PRC consulted extensively with key people in the linear algebra community. After extensive discussions, the PRC submitted its report and recommendations to the ILAS Board of Directors in February 2009. The Board gave the report careful consideration over several months. What follows is a summary, organised by topic, of some of the main recommendations arising from the PRC report that were adopted by the Board.

Conferences:

- * In order to facilitate communication between the Board and organisers of ILAS conferences, a member of the Board will be appointed to the Organising Committee of each ILAS conference. Shaun Fallat has now been appointed in that capacity.

- * An ILAS Conference Best Practices Committee shall be formed in order to make an inventory of best practices at recent ILAS meetings. This committee, chaired by Shaun Fallat, has been formed and is expected to have completed its work by the end of this calendar year.

ELA:

- * The journal has grown substantially since its launch in 1996, with an accompanying increase in workload on both the editorial and production sides. ILAS should look carefully at the possibility of providing paid part-time administrative support for ELA's technical editing and production needs. This issue is currently being considered by the ILAS Journals Committee.

IMAGE:

- * Members should be strongly encouraged to select the online reading option for IMAGE, in order that the Society may save on printing and postage costs. The most recent round of membership renewals indicates that many members have chosen to read IMAGE online.

- * Searchable back issues of IMAGE should be made available. This is now the case, as all issues of IMAGE are available for download as PDF files from the ILAS web site.

ILAS Homepage:

- * The 'Research in Linear Algebra' section of the ILAS web page should be redeveloped in order to better reflect the depth and diversity of the discipline. A committee chaired by Chi-Kwong Li is in the process of updating the research section of the ILAS homepage.

Cooperation with other organisations:

- * ILAS should seek out opportunities to cooperate with other groups that are engaged with linear algebra. To that end, ILAS has supported the efforts of the GAMM Activity Group on Applied and Numerical Linear Algebra in securing funding for speakers; ILAS has also made a special effort to encourage students attending the 2010 International Summer School on Numerical Linear Algebra to also participate in the ILAS meeting in Pisa.

Membership:

- * In order that ILAS can continue to support a broad range of activities, the PRC recommended, and the Board approved, an increase in the membership dues.

* A recommendation was made to make it easier for people to join ILAS and renew their membership in the Society. In response to that, memberships can now be renewed online via the ILAS web site.

It is expected that all of the items noted above will serve to strengthen ILAS as it continues in its mission to support and promote the discipline of linear algebra. On behalf of the Society, I would like to extend sincere thanks to the members of the Program Review

New ILAS Officers **Submitted by Steve Kirkland, ILAS President**

Chi-Kwong Li was elected to the position of Vice-President for a three-year term, beginning on March 1, 2010; and Christian Mehl and T.Y. Tam were elected to three-year terms as members of the ILAS Board, beginning on March 1, 2010.

On behalf of ILAS, I take this opportunity to thank the members of the Nominating Committee - Rajendra Bhatia (Chair), Dario Bini, Roger Horn, Tom Laffey and Francoise Tisseur - for their efforts on behalf of ILAS, and also to thank all of the nominees for their participation in the election. Finally, I extend thanks to Ravi Bapat and Isha Dewan, who helped to count the ballots.

Call for Proposals for ILAS Lecturers at Non-ILAS Meetings **Submitted by Steve Kirkland, ILAS President**

As part of ILAS's commitment to supporting activities in Linear Algebra, the Society maintains a program of ILAS Lectureships at non-ILAS conferences. In a typical year, ILAS supports two such Lectureships, and in an exceptional year, ILAS may support as many as four Lectureships.

This announcement is a call for proposals for ILAS Lectureships to take place in 2011. The deadline for proposals is September 30, 2010. Details about the format of proposals and the submission process can be found at: http://www.ilasic.math.uregina.ca/iic/misc/non_ilas_guidelines.html.

ILAS Members Are SIAM Officers and New SIAM Fellows

ILAS members Nicholas Higham and Michele Benzi have been elected to be officers of SIAM: Higham as Vice President-at-Large of SIAM, and Benzi as Chair of SIAM's special interest group SIAG/LA, for 2010-2012.

Also, three members of ILAS are among the 34 new Fellows of SIAM: Ian S. Duff, Rutherford Appleton Laboratory, UK; John R. Gilbert, Univ. of California Santa Barbara, US; and Olaf B. Widlund, Courant Institute, US. This honor is conferred on SIAM members who have made outstanding contributions to the fields of applied mathematics and computational science. All of SIAM's new Fellows and prize winners will be honored at a Prizes and Awards Luncheon during the SIAM Annual Meeting in Pittsburg, PA, US, July 12-16, 2010.

Send News for IMAGE Issue 45 by October 1, 2010

Announcements and reports of conferences and workshops
 News about striking developments in linear algebra and its applications
 Honors and awards
 Articles on history
 Survey articles
 Books, websites, funding sources
 News about journals
 Employment and other funding opportunities
 Problems and solutions
 Transitions: new appointments, responsibilities, deaths
 Linear algebra education
 Possible new corporate sponsors
 Letters to the editor

All items of interest to the linear algebra community are welcome, longer articles as well as short notes like historical tidbits. Photos are always of interest. Suggestions are welcome, such as ideas for new topics, survey articles, history topics, books to review, etc.

Send all material to the appropriate editor by October 1, 2010 (or as soon as possible – see note below):

Problems and solutions to Fuzhen Zhang (zhang@nova.edu)
 Book news and reviews to Oskar Baksalary (baxx@amu.edu.pl).
 History of linear algebra to Peter Semrl (peter.semrl@fmf.uni-lj.si).
 Linear algebra education to Steve Leon (sleon@umassd.edu).
 Advertisements to Jim Weaver (jweaver@uwf.edu).
 All other material to Jane Day (day@math.sjsu.edu).

If you wish to submit an article after October 1, let the appropriate editor know as soon as possible when to expect your item, and it may be possible to include it.

Send material in plain text, Word or both Latex and PDF (Article.sty with no manual formatting is preferred). Send photos in JPG format. Issue 45 will be published on December 1, 2010.

OPEN POSITIONS

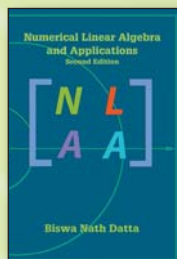
Tenure Track Positions Available at Umea University, Sweden

Outstanding candidates are being sought for the following new tenure track positions at Umea University, Sweden:

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Numerical Linear Algebra and Applications, Second Edition

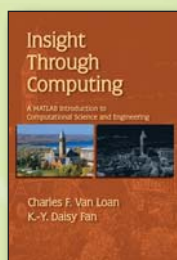
Biswa Nath Datta

This second edition of the author's acclaimed textbook covers the major topics of computational linear algebra, including solution of a system of linear equations, least-squares solutions of linear systems, computation of eigenvalues, eigenvectors,

and singular value problems. It is intended for undergraduate and graduate students and will also appeal to researchers.

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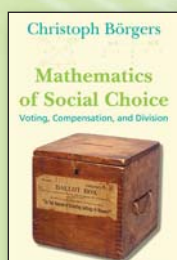
Charles F. Van Loan and K.-Y. Daisy Fan

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Christoph Börgers

How do you select a winner from a field of candidates? How do you rank a field of candidates? How do you share a divisible resource like a cake, or an indivisible one like a pet or a house? These are the questions addressed in this fun and accessible

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Jan Modersitzki

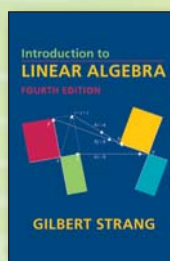
Fundamentals of Algorithms 6

This book provides an overview of state-of-the-art registration techniques from theory to practice. It also provides, via a supplementary Web page, free access to FAIR.m, a package that is based on the MATLAB® software

environment, which enables readers to experiment with the proposed algorithms and explore the presented examples in more depth.

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Introduction to Linear Algebra, Fourth Edition

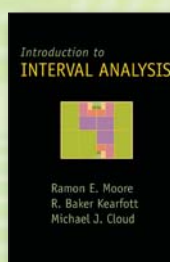
Gilbert Strang

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challenge problems to complement the review problems that have been highly praised in previous editions.

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Introduction to Interval Analysis

Ramon E. Moore, R. Baker Kearfott, and Michael J. Cloud

This unique book provides an introduction to a subject whose use has steadily increased over the past 40 years. An update of Ramon Moore's previous books on the topic, it provides broad coverage of the subject as well as the historical perspective

of one of the originators of modern interval analysis. The authors provide a hands-on introduction to INTLAB, a high-quality, comprehensive MATLAB® toolbox for interval computations.

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IMAGE Problem Corner: Old Problems With Solutions

We present solutions to Problems 43-1 through 43-7. Seven new problems are on the back cover; solutions are invited.

Problem 43-1: Characterization of EP Matrices

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland*, baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany*, trenkler@statistik.uni-dortmund.de

Let A be an $n \times n$ complex matrix. Show that A is EP (i.e., $AA^\dagger = A^\dagger A$, where $A^\dagger$ is the Moore-Penrose inverse of A) if and only if the column space of A^2 coincides with the column space of the conjugate transpose of A .

Solution 43-1.1 by Ravindra Bapat, *Indian Statistical Institute, New Delhi, India*, rbbapat@rediffmail.com

Let A^* denote the conjugate transpose of A . If A is EP (i.e., $AA^\dagger = A^\dagger A$) then $A^2 = AA^\dagger A^2 = (A^\dagger A)A^2 = (A^\dagger A)^* A^2 = A^*(A^\dagger)^* A^2$, and hence the column space of A^2 is contained in that of A^* . Similarly, $A^* = A^*(A^\dagger)^* A^*(A^\dagger)^* A^* = (A^\dagger A)^*(A^\dagger A)^* A^* = (A^\dagger A)(A^\dagger A)A^* = A^2(A^\dagger)^2 A^*$ and the column space of A^* is contained in that of A^2 . Thus we have shown that if A is EP, then the column spaces of A^2 and A^* coincide.

Conversely, suppose the column space of A^* equals the column space of A^2 . Since the column space of A^2 is clearly contained in the column space of A , we see that the column space of A^* is contained in that of A . However the dimensions of the column spaces of A^* and A are equal (to the rank of A) and hence the two spaces must coincide. This is a well-known characterization of an EP matrix.

Solution 43-1.2 by Johanns de Andrade Bezerra, *Campina Grande, PB, Brazil*, pav.animal@hotmail.com

Since $Im A^2 \subset Im A$, it follows that $Im A^* \subset Im A$, but $rank A = rank A^*$, hence $Im A = Im A^*$, that is, A is EP. Conversely, if A is EP, then $Im A = Im A^* \Rightarrow (Im A)^\perp = (Im A^*)^\perp \Rightarrow Ker A = Ker A^*$. Clearly $Ker(A^*)^2 \supset Ker A^*$, and so $Ker(A^*)^2 \supset Ker A$. $B^\#$ denotes the group inverse of an $n \times n$ complex matrix B , that is, $B^\#$ satisfies $BB^\#B = B$, $B^\#BB^\# = B^\#$ and $BB^\# = B^\#B$. It is well known that $B^\# = B^\dagger$ if and only if B is EP-matrix, B is EP-matrix if and only if B^* is EP-matrix, and that a group inverse of B exists if and only if $rank(B) = rank(B^2)$. Hence $rank(A^*)^2 = rank(A^*)$, and so $dim Ker(A^*)^2 = dim Ker(A^*) = dim Ker A$. Thus, $Ker(A^*)^2 = Ker A$, which implies $(Ker(A^*)^2)^\perp = (Ker A)^\perp$, and therefore $Im(A^2) = Im(A^*)$.

Solution 43-1.3 by Sachindranath Jayaraman, *Indian Institute of Science Education and Research - Kolkata, West Bengal, India*, sachindranath@iiserkol.ac.in

We start by recalling the following well known result:

For a square matrix A with complex entries, the following three statements are equivalent.

- (1) A is an EP matrix.
- (2) $R(A) = R(A^*)$.
- (3) $A^\dagger = A^\#$, where $A^\#$ is the unique group inverse of A .

Suppose A is an EP matrix. Then, $R(A^*) = R(A)$ (by the equivalence of (1) and (2) in the above result). Since the group inverse of A , $A^\#$ exists, we have that $R(A) = R(A^2)$. Therefore, $R(A^2) = R(A^*)$.

Conversely, assume that $R(A^*) = R(A^2)$ and let $y = Ax \in R(A)$. Then, $x = A^\dagger y + z$ for some $z \in N(A)$. Now $A^\dagger y \in R(A^\dagger) = R(A^*) = R(A^2)$. Therefore, $x - z = A^\dagger y = A^2 u$ for some u and Consequently, $y = Ax = A(x - z) = A^3 u \in R(A^3) \subseteq R(A^2)$. Thus, $R(A) \subseteq R(A^2)$. The inclusion $R(A^2) \subseteq R(A)$ trivially holds good and we conclude that $R(A) = R(A^2) = R(A^*)$. It now follows from the equivalence of (1) and (2) in the above result that A is EP.

Solution 43-1.4 by Hans Joachim Werner, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

First, recall that $AA^\dagger$ and $A^\dagger A$ are the orthogonal projectors onto $\mathcal{R}(A)$ and $\mathcal{R}(A^*)$, respectively, and so $AA^\dagger = A^\dagger A$ if and only if $\mathcal{R}(A) = \mathcal{R}(A^*)$. It therefore suffices to show that $\mathcal{R}(A) = \mathcal{R}(A^*)$ is equivalent to $\mathcal{R}(A^2) = \mathcal{R}(A^*)$. Clearly, $\mathcal{R}(A^2) = \mathcal{R}(A^*)$ can happen only if $\mathcal{R}(A^*) \subseteq \mathcal{R}(A^2) \subseteq \mathcal{R}(A)$ or, equivalently, in view of $rank(A) = rank(A^*)$, only if $\mathcal{R}(A^*) = \mathcal{R}(A)$. Conversely, if $\mathcal{R}(A) = \mathcal{R}(A^*)$, then $\mathcal{R}(A^2) = A\mathcal{R}(A^*) = \mathcal{R}(AA^*) = \mathcal{R}(A)$.

Note: In the above solutions, $\mathcal{R}(X)$, $R(X)$, and $Im(X)$ all represent the column space of matrix X .

Also solved by Mao-Ting Chien, Richard Farebrother, Eugene A. Herman, Nestor Thome, Hongxing Wang and Jin Zhong, and the proposers.

Problem 43-2: Square Roots of (Skew-)Involutory Matrices

Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@Manchester.ac.uk

Identify a family of square matrices with integral elements (few of them zeros) whose squares are nonzero multiples of involutory or skew-involutory matrices. [Recollect that an $n \times n$ matrix A is said to be involutory if it satisfies $A^2 = I_n$ and skew-involutory if it satisfies $A^2 = -I_n$.]

Solution 43-2 by proposer Richard William Farebrother, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@Manchester.ac.uk

In connection with his work on the stability of difference equations, Farebrother (1974, 2000) has defined the n th member of a family of matrices in the following terms. Let $m = n - 1$ then, for $i, j = 0, 1, \dots, m$, the $(i + 1, j + 1)$ th element of the $n \times n$ Samuelson transformation matrix S_n is given by the coefficient of $x^i y^{m-i}$ in the binomial expansion of $(x + y)^{m-j}(x - y)^j$.

The first five members of this class are thus

$$S_1 = [1], \quad S_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad S_3 = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix},$$

$$S_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 1 & -1 & -3 \\ 3 & -1 & -1 & 3 \\ 1 & -1 & 1 & -1 \end{bmatrix}, \quad S_5 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 4 & 2 & 0 & -2 & -4 \\ 6 & 0 & -2 & 0 & 6 \\ 4 & -2 & 0 & 2 & -4 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix}.$$

Direct multiplication establishes that $Q_n = 2^{-(n-1)/2} S_n$ is an $n \times n$ involutory matrix satisfying $Q_n^2 = I_n$. Further, since reversing the order of the columns of S_n is equivalent to negating its odd-numbered rows, we find that $T_n = H_n Q_n = Q_n J_n$ satisfies $T_n^2 = H_n Q_n Q_n J_n = H_n J_n$ where H_n is an $n \times n$ matrix with alternate plus ones and minus ones on its principal primary diagonal, $h_{fg} = (-1)^{f+1}$ if $f = g$ and zeros elsewhere; J_n is an $n \times n$ matrix with plus ones on its principal secondary diagonal, $j_{fg} = 1$ if $f + g = n + 1$ and zeros elsewhere; and $K_n = H_n J_n$ is an $n \times n$ matrix with alternate plus ones and minus ones on its principal secondary diagonal, $k_{fg} = (-1)^{f+1}$ if $f + g = n + 1$ and zeros elsewhere.

Clearly $T_n^2 = K_n$ is an $n \times n$ skew-involutory matrix satisfying $K_n^2 = -I_n$ if n is even and an $n \times n$ involutory matrix satisfying $K_n^2 = I_n$ if n is odd. And, for all values of n , we find that the $n \times n$ matrix $H_n S_n = S_n J_n$ satisfies the requirements of the problem.

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Problem 43-3: Reconstructing a Positive Semidefinite Matrix

Proposed by Bryan T. Kelly, *NYU Stern School of Business, New York, USA*

and Chi-Kwong Li, *College of William and Mary, Williamsburg, USA*, ckli@wm.edu

Suppose $A = (A_{rs})_{1 \leq r, s \leq k}$ is a block complex Hermitian (or real symmetric) positive semidefinite matrix such that A_{rr} is n_r -by- n_r . Show that $\tilde{A}$ is positive semidefinite if $\tilde{A}$ is obtained from A as follows:

- (a) For $r \neq s$, replace each entry of A_{rs} by the average of its entries, and
- (b) Replace each diagonal entry of each diagonal block A_{rr} by the average of its diagonal entries, and replace each off-diagonal entry of A_{rr} by the average of its off-diagonal entries.

Solution 43-3.1 by Ravindra Bapat, *Indian Statistical Institute, New Delhi, India*, rbbapat@rediffmail.com

For a positive integer m , let S^m be the set of permutations of $\{1, \dots, m\}$. If $\sigma \in S^m$, then let P^σ be the $m \times m$ permutation matrix corresponding to σ . We denote the transpose of A by A^t . We first state two results which are easily verified.

Lemma 1. Let B be an $m \times m$ matrix. Then the matrix

$$\frac{1}{m!} \sum_{\sigma \in S^m} P^\sigma B (P^\sigma)^t$$

equals the $m \times m$ matrix with each diagonal entry equal to the average of the diagonal entries of B , and each off-diagonal entry equal to the average of the off-diagonal entries of B .

Lemma 2. Let B be an $m \times p$ matrix. Then the matrix

$$\frac{1}{m!p!} \sum_{\sigma \in S^m} \sum_{\tau \in S^p} P^\sigma B P^\tau$$

equals the $m \times p$ matrix with each entry equal to the average of the entries of B .

Now if $\sigma_r \in S^{n_r}$, $r = 1, \dots, k$, then let $Q(\sigma_1, \dots, \sigma_k)$ be the direct sum of $P^{\sigma_1}, \dots, P^{\sigma_k}$. Using Lemmas 1 and 2 it can be seen that the matrix $\hat{A}$ constructed in the problem equals

$$\frac{1}{n_1! \cdots n_k!} \sum_{\sigma_1 \in S^{n_1}} \cdots \sum_{\sigma_k \in S^{n_k}} Q(\sigma_1, \dots, \sigma_k) A Q(\sigma_1, \dots, \sigma_k)^t.$$

This being a sum of positive semidefinite matrices, is positive semidefinite.

Solution 43-3.2 by the proposers Bryan T. Kelly, *NYU Stern School of Business, New York, USA* and Chi-Kwong Li, *College of William and Mary, Williamsburg, USA*, ckli@wm.edu

Let $M_{m,n}$ (M_n) be the set of $m \times n$ ($n \times n$) complex (or real) matrices, and let S_n be the set of $n \times n$ permutation matrices. Denote by $J_{m,n}$ the $m \times n$ matrix with all entries equal to one. If $m = n$, write $J_{m,n}$ as J_n . We have the following observations.

(1). A matrix $X \in M_n$ satisfies $PXP^t = X$ for all $P \in S_n$ if and only if $X = aI_n + bJ_n$.

(2). A matrix Y satisfies $PYQ = Y$ for all permutation matrices P and Q if and only if $Y = J_{m,n}$.

We can now prove the assertion when $k = 2$. The general case can be done similarly. In fact, we prove the following.

Suppose $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{12}^* & A_{22} \end{pmatrix}$ is positive semidefinite, where $A_{11} \in M_m$ and $A_{22} \in M_n$. If

$$\tilde{A} = [(m!)(n!)]^{-1} \sum_{P \in S_m, Q \in S_n} \begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix} A \begin{pmatrix} P & 0 \\ 0 & Q \end{pmatrix}^t,$$

then

$$\tilde{A} = \begin{pmatrix} d_1 I_m + a_1 J_m & b J_{m,n} \\ \bar{b} J_{n,m} & d_2 I_n + a_2 J_n \end{pmatrix}$$

is positive semidefinite, where b is the average of the entry of A_{12} , d_j (resp. a_j) is the average of the diagonal (resp. off-diagonal) entries of A_{jj} for $j = 1, 2$.

Proof. Since $\tilde{A}$ is a nonnegative combination of positive semidefinite matrices, it is positive semidefinite. Note that the $(1, 1)$ block $X \in M_m$ of $\tilde{A}$ satisfies $PXP^t = X$ for all $P \in S_m$, so it has the asserted form by the lemma. Similarly, the $(1, 2)$ and $(2, 2)$ blocks have the asserted forms.

Solution 43-3.3 by Edward Poon, *Embry-Riddle Aeronautical University, Prescott, USA*, poon3de@erau.edu

Let A be as given in the problem. Say a matrix P is a *block diagonal permutation* if it is block diagonal and its block diagonal entries, P_r , are n_r -by- n_r permutation matrices. In this case PAP^{-1} is positive semidefinite since P is orthogonal. Thus the average of

$$\{P^{-1}AP : P \text{ is a block diagonal permutation}\}$$

(which we denote by A_{avg}) is positive semidefinite. It suffices to show that $\tilde{A}$ equals this average.

Note the (r, s) -block entry of $P^{-1}AP$ is $(P_r)^{-1}A_{rs}P_s$. Since all the P_r are independent of one other, the (r, s) -block entry of A_{avg} is

$$(A_{avg})_{rs} = \frac{1}{n_r!n_s!} \sum_{Q_1 \in S_{n_r}, Q_2 \in S_{n_s}} Q_1^{-1} A_{rs} Q_2, \quad \text{if } r \neq s.$$

If we identify a permutation σ on n letters with the matrix Q defined by $Qe_i = e_{\sigma(i)}$ (that is, $Q_{ij} = \delta_{i\sigma(j)}$), the (i, j) -entry of $(A_{avg})_{rs}$ is

$$\frac{1}{n_r!n_s!} \sum_{\rho \in S_{n_r}, \sigma \in S_{n_s}} (A_{rs})_{\rho(i)\sigma(j)} = \frac{1}{n_r n_s} \sum_{u, v} (A_{rs})_{uv} = (\tilde{A}_{rs})_{ij},$$

since there are $(n_r - 1)!$ permutations $\rho \in S_{n_r}$ mapping i to u ; $(n_s - 1)!$ permutations $\sigma \in S_{n_s}$ mapping j to v . So $(A_{avg})_{rs} = \tilde{A}_{rs}$ when $r \neq s$.

On the other hand, the (r, r) -block entry of A_{avg} is

$$(A_{avg})_{rr} = \frac{1}{n_r!} \sum_{Q \in S_{n_r}} Q^{-1} A_{rr} Q.$$

Its i th diagonal entry is

$$\frac{1}{n_r!} \sum_{\rho \in S_{n_r}} (A_{rr})_{\rho(i)\rho(i)} = \frac{1}{n_r} \sum_u (A_{rr})_{uu} = (\tilde{A}_{rr})_{ii}$$

by the same reasoning as before, and its (i, j) th entry ($i \neq j$) is

$$\frac{1}{n_r!} \sum_{\rho \in S_{n_r}} (A_{rr})_{\rho(i)\rho(j)} = \frac{1}{(n_r)(n_r - 1)} \sum_{u \neq v} (A_{rr})_{uv} = (\tilde{A}_{rr})_{ij}$$

since there are $(n_r - 2)!$ permutations $\rho \in S_{n_r}$ mapping (i, j) to (u, v) for $u \neq v$. Thus $(A_{avg})_{rr} = \tilde{A}_{rr}$ as well.

Solution 43-3.4 by Hans Joachim Werner, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

Our solution is based on the following four well-known elementary facts.

FACT 1. If M and N are complex Hermitian positive semidefinite matrices of the same order, then $M + N$ is positive semidefinite.

FACT 2. For given $a, b \in \mathbb{R}$ with $a \neq 0$, let $T := aI_n + bE_n$, where I_n and E_n denote the identity matrix of order n and the $n \times n$ matrix of ones, respectively. Then T is a real symmetric matrix with a and $a + nb$ as its only eigenvalues. If b is nonzero, then the multiplicity of eigenvalue a is $n - 1$, whereas the multiplicity of eigenvalue $a + nb$ is 1. If $b = 0$, then the multiplicity of eigenvalue a is n . Therefore, $aI_n + bE_n$ is positive semidefinite if and only if $a \geq 0$ and $a + nb \geq 0$.

FACT 3. If M and N are Hermitian positive semidefinite matrices, then the block-diagonal matrix $\text{diag}(M, N)$ is positive semidefinite.

FACT 4. If M is a Hermitian positive semidefinite matrix and matrix L is such that ML is defined, then matrix L^*ML is positive semidefinite.

In what follows, let $A = (A_{rs})_{1 \leq r, s \leq k}$ be a given symmetrically block-partitioned Hermitian positive semidefinite matrix. For convenience, for each $n_r \times n_r$ diagonal block A_{rr} , let $\bar{d}_r$, $\bar{a}_r$, and $\bar{b}_r$ denote the average of its diagonal entries, the average of its entries, and the average of its off-diagonal entries, respectively. Of course, we always interpret $\bar{b}_r$ as absent whenever $n_r = 1$. Observe that $\bar{d}_r \geq 0$, $\bar{a}_r \in \mathbb{R}$ and $\bar{b}_r \in \mathbb{R}$. Since A_{rr} is positive semidefinite, clearly $\iota' A_{rr} \iota \geq 0$, where $\iota \in \mathbb{R}^{n_r}$ is the vector whose entries are all equal to 1. Needless to say, $n_r^2 \bar{a}_r = \iota' A_{rr} \iota \geq 0$, and

$$\bar{b}_r = \frac{\iota' A_{rr} \iota - n_r \bar{d}_r}{n_r(n_r - 1)} = \frac{n_r \bar{a}_r - \bar{d}_r}{n_r - 1}.$$

Now, let $e_i \in \mathbb{R}^{n_r}$ denote the vector with 1 at the i th position and zeros elsewhere. By letting a_{ij} denote the entry in the (i, j) -position of A_{rr} , we obtain $0 \leq (e_i - e_j)' A_{rr} (e_i - e_j) = a_{ii} + a_{jj} - a_{ij} - a_{ji}$ irrespective of the choice of $1 \leq i, j \leq n_r$. Consequently, for each pair (i, j) , $a_{ii} + a_{jj} \geq a_{ij} + a_{ji}$, and so, summing these inequalities over all pairs (i, j) with $i < j$,

$$(n_r - 1) \sum_{i=1}^{n_r} a_{ii} \geq \iota' A_{rr} \iota - \sum_{i=1}^{n_r} a_{ii}.$$

This inequality implies

$$n_r \sum_{i=1}^{n_r} a_{ii} \geq \iota' A_{rr} \iota \text{ or, equivalently, } \bar{d}_r \geq \bar{a}_r.$$

Theorem. Let A_{rr} be a diagonal block in the symmetrically block-partitioned Hermitian positive semidefinite matrix $A = (A_{rs})_{1 \leq r, s \leq k}$, and let $\bar{a}_r$, $\bar{b}_r$ and $\bar{d}_r$ be defined as before. Then the matrix

$$V_r := (\bar{d}_r - \bar{b}_r)I_{n_r} + (\bar{b}_r - \bar{a}_r)E_{n_r}$$

is positive semidefinite.

Proof. The result follows by means of Fact 2. First note that

$$\bar{d}_r - \bar{b}_r = \bar{d}_r - \frac{n_r \bar{a}_r - \bar{d}_r}{n_r - 1} = \frac{n_r}{n_r - 1} (\bar{d}_r - \bar{a}_r) \geq 0$$

whenever $\bar{d}_r \geq \bar{a}_r$. Next, observe that $n_r(\bar{b}_r - \bar{a}_r) = \frac{n_r}{n_r - 1}(\bar{a}_r - \bar{d}_r) = -(\bar{d}_r - \bar{b}_r)$. Consequently,

$$\bar{d}_r - \bar{b}_r + n_r(\bar{b}_r - \bar{a}_r) = 0. \quad \square$$

Fact 3 along with our Theorem now tells us that the block-diagonal matrix

$$V := \text{diag}(V_1, V_2, \dots, V_k)$$

is positive semidefinite. Letting

$$D := \text{diag}\left(\frac{1}{n_1} E_{n_1}, \frac{1}{n_2} E_{n_2}, \dots, \frac{1}{n_k} E_{n_k}\right),$$

it follows from Fact 4 that the matrix

$$W := D'AD$$

is positive semidefinite. This matrix W is obtained from the symmetrically block-partitioned matrix $A = (A_{rs})_{1 \leq r, s \leq k}$ by replacing each entry in every block A_{rs} by the average of its entries. In virtue of Fact 1, it finally follows that $V + W$ is positive semidefinite. This matrix has exactly the desired structure.

Problem 43-4: A Trace Inequality for Positive Semidefinite Matrices

Proposed by Minghua Lin, *University of Regina, Saskatchewan, Canada*, lin243@uregina.ca

Let A be positive semidefinite and B be positive definite. Show that $\text{tr}(A^{p+1}B^{-p}) \geq (\text{tr}A)^{p+1}(\text{tr}B)^{-p}$ for any positive integer p .

Solution 43-4.1 by Koenraad M.R. Audenaert, *Royal Holloway, University of London, United Kingdom*, Koenraad.Audenaert@rhul.ac.uk

We will assume A to be positive definite; for $p > -1$, using a familiar continuity argument the statement can be shown to hold for singular A , too.

Under this extra condition $A > 0$, we can prove a little bit more, namely that the stated inequality is true for all real p outside the interval $[-1, 0]$, while the inequality is reversed for p within that interval.

Without loss of generality, we can consider A and B which both have trace equal to 1: then the statement is $\text{Tr}(A^{p+1}B^{-p}) \geq 1$ for p outside the interval $[-1, 0]$. By homogeneity of the left-hand side, the general statement indeed follows directly from this.

Consider the function $p \mapsto f(p) = x^{p+1}y^{-p}$, for $x, y > 0$. The main point of the proof is that this function is convex. As this function is analytic, its convexity is easily confirmed by calculating its second derivative: $f''(p) = x^{p+1}y^{-p}(\log(x) - \log(y))^2$, which is non-negative.

Consider now a basis in which A is diagonal: $A = \text{Diag}(a_1, a_2, \dots, a_n)$. In that basis, B has eigenvalue decomposition $B = U \text{Diag}(b_1, b_2, \dots, b_n) U^*$, where U is unitary. Then $\text{Tr}(A^{p+1}B^{-p}) = \sum_{i,j=1}^n a_i^{p+1} b_j^{-p} |U_{ij}|^2$. This is a sum with non-negative weights of terms that are convex in p , and, therefore, the whole sum is convex in p .

For $p = 0$, $\text{Tr}(A^{p+1}B^{-p}) = \text{Tr}(A) = 1$, and for $p = -1$, $\text{Tr}(A^{p+1}B^{-p}) = \text{Tr}(B) = 1$, by assumption. Convexity in p then implies that $\text{Tr}(A^{p+1}B^{-p}) \leq 1$ for $p \in [-1, 0]$ and $\text{Tr}(A^{p+1}B^{-p}) \geq 1$ otherwise.

Solution 43-4.2 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

We assume more generally that p is any positive real number. We may assume that $A \neq O$ and $n \geq 2$, where $n \times n$ is the size of A and B , since otherwise the result is trivial. Let

$$M = \frac{1}{(\text{tr} A)^{p+1}} A^{p+1}, \quad N = \frac{1}{(\text{tr} B)^{-p}} B^{-p}$$

Thus, it suffices to show $\text{tr}(MN) \geq 1$. Let $\mu_1, \dots, \mu_n$ and $\lambda_1, \dots, \lambda_n$ denote the eigenvalues of A and B , respectively. The eigenvalues of M and N are therefore

$$\alpha_j = \frac{\mu_j^{p+1}}{(\sum_{k=1}^n \mu_k)^{p+1}} \quad \text{and} \quad \beta_j = \frac{\lambda_j^{-p}}{(\sum_{k=1}^n \lambda_k)^{-p}}, \quad \text{for } j = 1, \dots, n$$

respectively. Hence

$$\alpha_1, \dots, \alpha_n \in [0, 1], \quad \sum_{k=1}^n \alpha_k^{1/(p+1)} = 1, \quad \beta_1, \dots, \beta_n \in (1, \infty), \quad \sum_{k=1}^n \beta_k^{-1/p} = 1 \quad (1)$$

Consider first the special case when M and N are diagonal. Then $\text{tr}(MN) = \sum_{j=1}^n \alpha_j \beta_j$. Using the conditions (1) and letting $c_j = \beta_j^{-1/(p+1)}$, $d_j = (\alpha_j \beta_j)^{1/(p+1)}$ for $j = 1, \dots, n$, we apply Hölder's inequality:

$$\left(\sum_{j=1}^n \alpha_j \beta_j \right)^{1/(p+1)} = \left(\sum_{j=1}^n c_j^{(p+1)/p} \right)^{p/(p+1)} \left(\sum_{j=1}^n d_j^{p+1} \right)^{1/(p+1)} \geq \sum_{j=1}^n c_j d_j = \sum_{j=1}^n \alpha_j^{1/(p+1)} = 1$$

Therefore $\text{tr}(MN) \geq 1$ when M and N are diagonal.

For the general case, let U and V be unitary matrices such that

$$U^* M U = \text{diag}(\alpha_1, \dots, \alpha_n) = D_M \quad \text{and} \quad V^* N V = \text{diag}(\beta_1, \dots, \beta_n) = D_N$$

Then, with $W = U^* V$, we have

$$\text{tr}(MN) = \text{tr}(V^* M V V^* N V) = \text{tr}(V^* U D_M U^* V D_N) = \text{tr}(W^* D_M W D_N)$$

Since the (i, k) entry of $W^* D_M W D_N$ is $\sum_{j=1}^n \overline{w_{ji}} \alpha_j w_{jk} \beta_k$ (where $W = [w_{jk}]$), we have

$$\text{tr}(MN) = \sum_{k=1}^n \sum_{j=1}^n \alpha_j \beta_k |w_{jk}|^2 = \beta^T X \alpha$$

where $\alpha = (\alpha_1, \dots, \alpha_n)^T$, $\beta = (\beta_1, \dots, \beta_n)^T$, and $X = [x_{jk}] = [|w_{jk}|^2]$. In the first part of the proof, we showed that $\beta^T \alpha \geq 1$ whenever α, β satisfy the conditions (1). Therefore, to complete the proof for the general case, we need only show that $\gamma = X \alpha$ satisfies the same conditions as α in (1). Note that X is doubly stochastic, since W is unitary. Hence $\gamma_j = \sum_{k=1}^n x_{jk} \alpha_k \in [0, 1]$.

We need only show that $\sum_{j=1}^n \gamma_j^{1/(p+1)} \geq 1$, since that was all we needed for the α_j in the proof for the special case. To that end, let $c_k = x_{jk}^{p/(p+1)}$, $d_k = (x_{jk} \alpha_k)^{1/(p+1)}$ for $k = 1, \dots, n$ and any fixed value of j . By Hölder's inequality,

$$\left(\sum_{k=1}^n x_{jk} \alpha_k \right)^{1/(p+1)} = \left(\sum_{k=1}^n c_k^{(p+1)/p} \right)^{p/(p+1)} \left(\sum_{k=1}^n d_k^{p+1} \right)^{1/(p+1)} \geq \sum_{k=1}^n c_k d_k = \sum_{k=1}^n x_{jk} \alpha_k^{1/(p+1)}$$

Therefore

$$\begin{aligned} \sum_{j=1}^n \gamma_j^{1/(p+1)} &= \sum_{j=1}^n \left(\sum_{k=1}^n x_{jk} \alpha_k \right)^{1/(p+1)} \geq \sum_{j=1}^n \sum_{k=1}^n x_{jk} \alpha_k^{1/(p+1)} \\ &= \sum_{k=1}^n \left(\sum_{j=1}^n x_{jk} \right) \alpha_k^{1/(p+1)} = \sum_{k=1}^n \alpha_k^{1/(p+1)} = 1 \end{aligned}$$

Solution 43-4.3 by the proposer Minghua Lin, *University of Regina, Saskatchewan, Canada*, lin243@uregina.ca

Lemma 1. (Marcus) Let $X \geq 0$, $A \geq 0$, $p, q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$. Then

$$\operatorname{tr} XA \leq (\operatorname{tr} X^p)^{\frac{1}{p}} (\operatorname{tr} A^q)^{\frac{1}{q}}.$$

Lemma 2. (Lieb-Thirring) Let $X \geq 0$, $A \geq 0$ and m be any positive integer. Then

$$\operatorname{tr}\{(XA)^m\} \leq \operatorname{tr}\{X^m A^m\}.$$

Using the above known results we can prove the trace inequality in Problem 43-4: Let $X = \operatorname{diag}(x_1, x_2, \dots, x_n)$, $A = \operatorname{diag}(a_1, a_2, \dots, a_n)$, where $x_k \geq 0$ and $a_k > 0$, $k \in \{1, 2, \dots, n\}$. Then the trace inequality reduces to the following classical Radon's inequality.

Radon's inequality. If $x_k \geq 0$ and $a_k > 0$, $k \in \{1, 2, \dots, n\}$, then for any positive integer p ,

$$\sum_{k=1}^n \frac{x_k^{p+1}}{a_k^p} \geq \frac{(\sum_{k=1}^n x_k)^{p+1}}{(\sum_{k=1}^n a_k)^p}.$$

Problem 43-5: The Matrix Equation $YTY = Y$

Proposed by Heinz Neudecker, *University of Amsterdam, Amsterdam, The Netherlands*, ericaengels173@live.nl

Let Y and T be $n \times n$ real matrices and let $\mathbf{1}$ be the n -column vector of ones. Assume that Y is positive semidefinite, of rank $n - 1$, $Y\mathbf{1} = 0$, $YTY = Y$, and that T is symmetric and has full rank. Find a relationship between $\mathbf{1}'T^{-1}\mathbf{1}$ and $\det T$ that shows they have the same sign. As usual, $\det T$ is the determinant of the matrix T .

Solution 43-5 by Ravindra Bapat, *Indian Statistical Institute, New Delhi, India*, rbbapat@rediffmail.com

The hypotheses imply that the cofactors of Y are equal. We denote the common value by α , which must be positive, since Y is positive semidefinite and has rank $n - 1$. Thus $\operatorname{adj} Y = \alpha \mathbf{1}\mathbf{1}'$, where adj denotes adjoint and $\mathbf{1}$ is the column vector of all ones. From $YTY = Y$, we get $(\operatorname{adj} Y)(\operatorname{adj} T)(\operatorname{adj} Y) = \operatorname{adj} Y$. Thus $\alpha^2 \mathbf{1}\mathbf{1}'(\operatorname{adj} T)\mathbf{1}\mathbf{1}' = \alpha \mathbf{1}\mathbf{1}'$. It follows that $\mathbf{1}'(\operatorname{adj} T)\mathbf{1} = \alpha^{-1}$. Thus $(\det T)\mathbf{1}'T^{-1}\mathbf{1}$, which is the sum of the cofactors of T , equals $\alpha^{-1} > 0$. It follows that $\mathbf{1}'T^{-1}\mathbf{1}$ and $\det T$ have the same sign.

Also solved by Eugene A. Herman, Minghua Lin, Hans Joachim Werner and the proposer.

Problem 43-6: Matrix Similarity

Proposed by Xingzhi Zhan, *East China Normal University, Shanghai, China*, zhan@math.ecnu.edu.cn

Let A and B be complex matrices of the same order. If A and B are similar and $*$ -congruent and have the same value of Frobenius norm, then does it follow that A and B are unitarily similar?

Solution 43-6.1 by Roger Horn, *University of Utah, Salt Lake City, USA*, rhorn@math.utah.edu

No, it does not follow that A and B are unitarily similar. Any real matrix and its transpose are similar (via a real symmetric matrix) and $*$ -congruent (via a real involutory matrix); they also have the same singular values (and hence have the same norm for any unitarily invariant norm, not just the Frobenius norm). Consider

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} \quad (2)$$

and $B = A^T$. If there is a unitary U such that $A^T = UAU^*$, then $A = (A^T)^* = UA^T U^*$ and $A + A^T = U(A + A^T)U^*$, that is, $(A + A^T)U = U(A + A^T)$. Since U commutes with $A + A^T = \operatorname{diag}(2, 0, -2)$, U must itself be diagonal, say $U = \operatorname{diag}(e^{i\theta}, e^{i\phi}, e^{i\psi})$. Examination of the entries of $AU = UA^T$ in positions 1, 2, 1, 3, and 3, 2 leads to a contradiction: $e^{i\phi} = -e^{i\theta} = e^{i\psi}$, and $e^{i\phi} = -e^{i\psi}$.

The example (2) is due to H. Vermeer [3], who discovered two general methods to construct real matrices that are not real orthogonally similar to their transposes. Similarity of a matrix and its transpose is discussed in [2]. Congruences of a matrix and its transpose are discussed in [1]. For both similarity and congruence, one may consider matrices over general fields, with or without an involution.

References

- [1] R. Horn and V. Sergeichuk, Congruences of a square matrix and its transpose, *Linear Algebra Appl.* 389 (2004) 347-353.
- [2] O. Taussky and H. Zassenhaus, On the similarity transformation between a matrix and its transpose, *Pacific J. Math.* 9 (1959) 893-896.
- [3] H. Vermeer, Orthogonal similarity of a real matrix and its transpose, *Linear Algebra Appl.* 428 (2008) 382-392.

Solution 43-6.2 by Minghua Lin, *University of Regina, Saskatchewan, Canada*, lin243@uregina.ca

It is not true generally. Here is a counterexample. Let $A = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{pmatrix}$, $B = A^T$, the transpose of A . It is well known that A

and its transpose are similar, $*$ -congruent (see [1] Theorem 3.), and have the same Frobenius norm. But A and B are not unitarily similar by Specht's theorem [2, p.76, Theorem 2.2.6], since

$$\text{tr}(A^2(A^*)^2AA^*) = 44101 \neq \text{tr}(B^2(B^*)^2BB^*) = 47701.$$

References

- [1] R. Horn and V. Sergeichuk, Congruences of a square matrix and its transpose, *Linear Algebra Appl.* 389 (2004) 347-353.
- [2] R. Horn and C. Johnson, *Matrix Analysis*, Cambridge, University Press, 1985.

Problem 43-7: Three Positive Definite Matrices

Proposed by Fuzhen Zhang, *Nova Southeastern University, Fort Lauderdale, USA*, zhang@nova.edu

It is known that any two positive semidefinite matrices of the same size are simultaneously $*$ -congruent to diagonal matrices. Can this be generalized to three positive semidefinite matrices? That is, if A , B , and C are positive semidefinite matrices, does there always exist an invertible matrix P such that P^*AP , P^*BP , and P^*CP are all diagonal?

Solution 43-7 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

There is no such matrix P for the positive semidefinite matrices

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Let $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be any 2×2 invertible matrix. If P^*AP , P^*BP , and P^*CP are all diagonal, then the $(2, 1)$ entry of each equals zero. That is,

$$a\bar{b} + c\bar{d} = 0, \quad a\bar{b} = 0, \quad (a + c)(\bar{b} + \bar{d}) = 0$$

From the first two of these equations, we have $a\bar{b} = 0$ and $c\bar{d} = 0$. Since P is nonsingular, it follows that $a = d = 0$ or $b = c = 0$. Inserting either of these results into the third equation shows that P must be singular, which is a contradiction.

Also solved by Minghua Lin.

IMAGE Problem Corner: Problems and Solutions

Problems: We introduce 7 new problems in this issue and invite readers to submit solutions for publication in IMAGE. **Solutions:** We present solutions to all problems in the previous issue [IMAGE 43 (Fall 2009), p. 43]. **Submissions:** Please submit proposed problems and solutions in macro-free L^AT_EX along with the PDF file by e-mail to IMAGE Problem Corner editor Fuzhen Zhang (zhang@nova.edu). The working team of the Problem Corner consists of Dennis S. Bernstein, Nir Cohen, Shaun Fallat, Dennis Merino, Edward Poon, Peter Šemrl, Wasin So, Nung-Sing Sze, and Xingzhi Zhan.

NEW PROBLEMS:

Problem 44-1: Constrained Characterization of Hermitianness

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland*, baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany*, trenkler@statistik.uni-dortmund.de

It is known that every Hermitian matrix is necessarily EP, i.e., the following implication holds for every $A \in \mathbb{C}_{n,n}$

$$A^* = A \Rightarrow \mathcal{R}(A^*) = \mathcal{R}(A),$$

where $\mathcal{R}(\cdot)$ stands for the column space of a matrix argument and A^* denotes the conjugate transpose of A . Suppose now that A is EP. Find a nontrivial condition involving A and A^* which ensures that A is Hermitian.

Problem 44-2: Sum of Entries of Inverses of Submatrices

Proposed by Ravindra Bapat, *Indian Statistical Institute, New Delhi, India*, rbbapat@rediffmail.com

Let A be an $n \times n$, nonsingular, doubly stochastic matrix and let $B = A^{-1}$. Let S and T be nonempty, proper subsets of $\{1, \dots, n\}$ of equal cardinality. Let X be the submatrix of A formed by taking the rows indexed by S and the columns indexed by T , and let Y be the submatrix of B formed by excluding the rows indexed by T and the columns indexed by S . Suppose X and Y are nonsingular. Find the sum of all the entries of X^{-1} and Y^{-1} .

Problem 44-3: Curiously Commuting Vectors

Proposed by Adam J. Brzezinski, *University of Michigan, Ann Arbor, USA*

Eva Wu, *State University of New York, Binghamton, USA*

and Dennis S. Bernstein, *University of Michigan, Ann Arbor, USA*, dsbaero@umich.edu

Let $a_1, a_2, \dots, a_n \in \mathbb{C}$, let $b, c \in \mathbb{C}^n$, and let $A = \begin{bmatrix} 0 & a_1 \\ I_{n-1} & \alpha \end{bmatrix}$, where $\alpha = (a_2, \dots, a_n)^t$ (here t is for transpose). Show that

$$\begin{bmatrix} b & Ab & \dots & A^{n-1}b \end{bmatrix} c = \begin{bmatrix} c & Ac & \dots & A^{n-1}c \end{bmatrix} b.$$

Problem 44-4: Square-nilpotent Matrix

Proposed by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Let A be a square complex matrix. If $A^2 = 0$ and A has rank r , show that $A + A^*$ has rank $2r$ and that half of its nonzero eigenvalues are positive and the other half are negative.

Problem 44-5: Matrix Congruence

Proposed by Roger Horn, *University of Utah, Salt Lake City, USA*, rhorn@math.utah.edu

Let A be a given square complex matrix. Show that A is congruent (respectively, $*$ -congruent) to $\bar{A}$ if and only if A is congruent (respectively, $*$ -congruent) to a real matrix.

Problem 44-6: Unitary Matrix Sum

Proposed by Dennis Merino, *Southeastern Louisiana University, Hammond, USA*, dmerino@selu.edu

Let $\alpha \in \mathbb{C}$ and a unitary $U \in M_n$ be given. Find the minimum positive integer k (depending on α) so that there exist unitary matrices $U_1, \dots, U_k$ such that $U_1 + \dots + U_k = \alpha U$. For instance, if $\alpha = 0$, then $k = 2$.

Problem 44-7: Real Diagonal Entries

Proposed by Edward Poon, *Embry-Riddle Aeronautical University, Prescott, USA*, poon3de@erau.edu

Suppose A is an $n \times n$ (complex) matrix such that its off-diagonal entries are $A_{ij} = (b_i - b_j)^{-1}$, where $b_1, \dots, b_n$ are distinct real numbers. Suppose all the eigenvalues of A are real. Show that the diagonal entries of A must be real.

Solutions to Problems 43-1 through 43-7 are on page 36.