The Bulletin of the International Linear Algebra Society

IMAGE

Serving the International Linear Algebra Community
Issue Number 45, pp. 1-48, Fall 2010

Contents continued on page 2

About IMAGE

ISSN 1553-8991. Produced by the International Linear Algebra Society (ILAS). Two issues are published each year, on June 1 and December 1. The editors reserve the right to select and edit material submitted.

Advertisers: Contact ILAS Outreach Director James Weaver (jweaver@uwf.edu).

Subscriptions: To subscribe or send a change of address for IMAGE, contact ILAS Treasurer Leslie Hogben (ilas@iastate.edu). The cost is $40/year for electronic version only, or $50/year if you wish to receive a printed copy of each issue.

For more information about ILAS, visit the home page http://www.ilasic.math.uregina.ca/iic/.

UPCOMING CONFERENCES AND WORKSHOPS

6th Linear Algebra Workshop
Kranjska Gora, Slovenia, May 25-June 1, 2011

View of Kranska Gora

The main theme of this workshop will be the interplay between operator theory and algebra. It will be held at the Hotel Kompas and the Hotel Larix in Kranjska Gora, Slovenia.

It will follow the usual format: a few hours of talks in the morning sessions, with afternoons reserved for work in smaller groups.

The plenary and invited talks will be 40 minutes, and contributed talks will be 20 minutes. Saturday morning will be dedicated to honoring Professor Matjaž Omladič in celebration of his 60th birthday. A conference excursion is planned for Sunday, May 29.

For more information, visit www.law-5.si or contact Heydar Radjavi (hradjavi@waterloo.ca) or Damjana Kokol Bukovšek (damjana.kokol@fmf.uni-lj.si).

You can register online by January 31, 2011. The conference website is http://conferences.imfm.si/conferenceDisplay.py?confId=24.

18th Householder Symposium XVIII
Tahoe City, CA USA, June 12-17, 2011

The 18th Householder Symposium on Numerical Linear Algebra will be held June 12-17, 2011, at the Granlibakken Conference Center & Lodge in Tahoe City, California. This series of conferences is named after its founder, Alston S. Householder, one of the pioneers in numerical linear algebra. This meeting is the eighteenth in a series previously called the Gatlinburg Symposia. The meeting will be very informal, with the intermingling of young and established researchers a priority. Participants are expected to attend the entire meeting. The fourteenth Householder Award for the best thesis in numerical linear algebra since January 1, 2008 will be presented.

Lake Tahoe

Attendance at the meeting is by invitation only. Applications are solicited from researchers in numerical linear algebra, matrix theory, and related areas such as optimization, differential equations, signal processing, and control. Each attendee will be given the opportunity to present a talk. Some will be plenary lectures, while other talks will be shorter presentations arranged in parallel sessions. To apply for an invitation to attend, please go to https://outreach.scidac.gov/HH11/.

For full consideration, conference applications must be received by October 31, 2010. Invitations will be sent in January 2011. It is expected that partial support will be available for some students, early career participants, and participants from countries with limited resources.

MAT-TRIAD 2011 – Conference on Matrix Analysis and its Applications
Tomar, Portugal, July 12-16, 2011

The Conference will be held in Tomar, Portugal on July 12-16, 2011. The aim is to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications in other parts of mathematics and offer them an opportunity to discuss current developments in these subjects.

Researchers and graduate students in linear algebra, statistical models and computation are particularly encouraged to attend. The format will involve plenary talks and sessions with contributed talks.

The workshop will open with talks by the winners of Young Scientists Awards at MAT-TRIAD 2009.

Recognizing the work of young scientists continues to have a special position in the MAT-TRIAD conferences. The best poster as well as the best talk by a graduate student or scientist with a recently completed Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference.

The invited speakers will be Natália Bebiano (Portugal), Jeff Hunter (New Zealand), Charles Johnson (USA) and Juan Manuel Pena (Spain). For details, visit the conference website (http://www.mattriad2011.ipt.pt) or email Francisco Carvalho, Chair of the Conference at mattriad2011@ipt.pt.

ICIAM 2011 Call for Papers
Vancouver, British Columbia, Canada, July 18-22, 2011
Contributed by Dan Cleary

Vancouver

The 7th International Congress on Industrial and Applied Mathematics will be held July 18-22, 2011, in Vancouver, British Columbia, Canada. The sponsoring organizations are CAIMS/SCMAI (Canadian Applied and Industrial Mathematics Society), MITACS (Mathematics of Information Technology and Complex Systems) and SIAM (Society for Industrial and Applied Mathematics). They are hoping to attract over 3000 attendees for the world-class program and to visit beautiful Vancouver.

There will be about 250 minisymposia, several jointly organized by the SIAM Linear Algebra Activity Group and by the Activity Group Applied and Numerical Linear Algebra of GAMM (the German Society for Applied Mathematics and Mechanics). The deadline for minisymposium and contributed paper abstracts is December 15, 2010. For details, visit www.iciam2011.com.

CT11, SIAM Conference on Control and Its Applications
Baltimore, Maryland, USA, July 25-27, 2011

This conference is sponsored by the SIAM Activity Group on Control and Systems Theory and will be held at the Hyatt Regency Baltimore, Baltimore, Maryland, USA. The deadlines are: January 3, 2011 for Minisymposium proposals; January 31, 2011 for abstracts for contributed and minisymposium speakers; and June 27, 2011 (4:00 p.m. EST) for pre-registration. For more information, visit http://www.siam.org/meetings/ct11/ or contact the SIAM Conference Department at meetings@siam.org.

CT11 is a Satellite Meeting of ICIAM 2011, which will be held in Vancouver, Canada on July 18-22, 2011.

17th ILAS Conference “Pure and Applied Linear Algebra: The New Generation”

Call for Abstracts

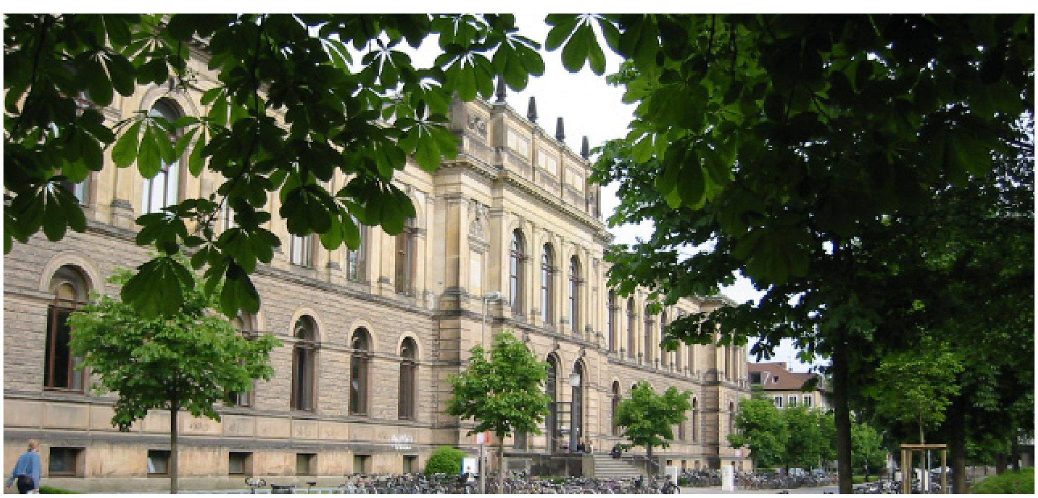
The **2011 ILAS Conference “Pure and Applied Linear Algebra: The New Generation”** will take place in Braunschweig, Germany, at the Technische Universität Braunschweig **August 22-26, 2011.** This conference will have a special emphasis on young researchers, which will be reflected by predominantly young plenary speakers as well as young researchers’ minisymposia. See www.ilas2011.de for more information on invited speakers, invited minisymposia, and local information.

Due to its special focus, at this ILAS conference the only contributed minisymposia will be **Young Researchers’ Minisymposia**. A young researchers’ minisymposium will focus on a specific, timely research subject. It will last two hours with four to six talks. Two organisers from two different institutions can apply for a young researchers’ minisymposium. They should hold a PhD for no longer than 6 years and not yet hold a tenured professor’s position. The speakers in a minisymposium should not all be from the same institution. From the applications received, the scientific committee will select up to five young researchers’ minisymposia. All speakers in and organizers of accepted young researchers’ minisymposia will pay only the reduced student registration fee.

A proposal for a young researchers’ minisymposium consists of a one page abstract, the titles of all lectures and information about the date of the PhD degree and the current position and affiliation of all organizers and speakers Submit proposals by e-mail (plain ASCII) to ilas2011@tu-braunschweig.de. For consideration, the proposal has to be submitted by **December 31, 2010.**

Organizers will be notified by email about accepted proposals by **January 31, 2011.** In case of acceptance, all participants of the young researchers’ minisymposium will be asked to submit an abstract. A LaTeX-template will be provided to the organizers of the young researchers’ minisymposium.

Individuals who are interested in presenting a **contributed talk** at the 2011 ILAS conference are asked to download the LaTeX-template-files from www.ilas2011.de (click on abstract submission) and to follow the guidelines stated in the files. For consideration, the abstract has to be submitted by **February 15, 2011**. Authors will be notified by email about accepted contributed talks by March 30, 2011.



Technische Universität Braunschweig

Other Upcoming Conferences

AMS/MAA Joint Meetings, January 6-9, 2011, New Orleans, LA, USA. MAA is offering a minicourse on Special Relativity Through a Linear Algebraic Lens and a special session for contributed papers on Innovative and Effective Ways to Teach Linear Algebra. One AMS short course is about matroids (http://www.ams.org/meetings/national/jmm/2125_intro.html).

5th International Conference on Mathematics & Statistics, June 13-16, 2011, Athens, Greece (http://www.atiner.gr/mathematics.htm).

2nd SIAM Gene Golub Summer School, Waves and Imaging, July 4-11, 2011, Vancouver, British Columbia, Canada (http://www.siam.org/students/summer.php).

6th International Workshop on the Numerical Solution of Markov Chains (NSMC 2010), September 16-17, 2010, The College of William and Mary, Williamsburg, VA, USA (http://www.cs.bilkent.edu.tr/~nsmc10/).

Triennial SIAM Conference on Applied Linear Algebra, Valencia, Spain, June 18-22, 2012, organized by the SIAM Special Interest Group on Linear Algebra (http://www.math.temple.edu/~siagla/conference.html). Endorsed by ILAS.

REPORTS ON CONFERENCES AND WORKSHOPS

19th International Workshop on Matrices and Statistics, Shanghai, China: June 5-8, 2010

Report by Jeffrey J. Hunter & Yonghui Liu

The 19th International Workshop on Matrices and Statistics (IWMS-2010) was held June 5-8 at the Shanghai Finance University in Shanghai, China. This workshop was co-organized by Shanghai Normal University and the E-institute of Computational Science of Shanghai Universities. The workshop was endorsed by the International Linear Algebra Society and supported by the National Natural Science Foundation of China and SPSS China.

The International Organising Committee comprised George P. H. Styan - Honorary Chair of IOC and Honorary Chair of ISC (Montreal, Canada)*, Zhong-Ci Shi - Honorary Chair of IOC, Chair of ISC (Beijing, China)*, Jeffrey J. Hunter – Chair of the IOC (Auckland, New Zealand)*, Hans Joachim Werner - Vice-Chair of the IOC (Bonn, Germany)*, S. Ejaz Ahmed (Windsor, Canada), Zhi-Dong Bai (Singapore)*, Zhong-Zhi Bai (Beijing, China)*, Guo-Liang Chen (Shanghai, China)*, Ben-Yu Guo (Shanghai, China)*, Er-Xiong Jiang (Shanghai, China)*, Tian-Gang Lei (Beijing, China)*, Augustyn Markiewicz (Poznan, Poland), Simo Puntanen (Tampere, Finland), Götz Trenkler (Dortmund, Germany), Julia Volaufova (New Orleans, USA), Dietrich von Rosen (Uppsala, Sweden), Mu-Sheng Wei (Shanghai, China)*, Xue-Jun Xu (Beijing, China)* where * denotes members of the International Scientific Committee (ISC).

The Local Organising Committee (LOC) was chaired by Yonghui Liu (Shanghai Finance University China) and included Rong-Xian Yue (Co-Chair of LOC, Shanghai Normal University, China), Yongge Tian (Central University of Finance and Economics, Beijing, China) Baoxue Zhang (Northeast Normal University, Changcun, China), Rui Li, Rongqiang Che, Yong Fang, Manrong Wang, Keyan Wang, Lei Fu, (all from Shanghai Finance University).

The invited speakers were:
Owe Axelsson, University of Uppsala, Sweden: "A general framework for the construction and analysis of preconditioners of matrices on two by two block form"

Zhidong Bai, Northeast Normal University, China and National University of Singapore: "Making Markowitz portfolio mean-variance principle practically usuable by RMT"

Ake Bjorck, University of Linkoping, Sweden: “Gram-Schmidt orthogonalization: 100 years and more”

Kai-Tai Fang, BNU-HKBU United International College, Zhuhai, China: “Additive logistic elliptical distributions”

Karl Gustafson, University of Colorado, Boulder, Colorada, USA: “Forty years of antieigenvalue theory and applications”

Stephen Haslett, Massey University, Palmerston North, New Zealand: “Data cloning and design matrices”

Maya Neytcheva, Uppsala University, Sweden: Numerical techniques to enhance the solution of large scale problems in statistics”
Peter Taylor, University of Melbourne, Australia: “Matrix-analytic methods in stochastic models”,

Guofu Zhou, Washington University, St Louis, USA: “Optimal estimation of a matrix toward portfolio choice: A survey”

Lixing Zhu, Hong Kong Baptist University, Hong Kong: “Generalized linear and single-index models re-visited”.

There were a total of 70 contributed talks, spread over sessions on Matrix Analysis, Matrix Computations, Biostatistics, Financial Mathematics and Statistics, Matrix and Statistics, Matrix and Statistics Computations, Statistical Methods, Computational Statistics, Statistics and Design of experiments, and Stochastic Analysis.

This workshop was one of the largest IWMS meetings that has been held, with 186 participants from 23 states and districts from around the world. It began with a formal opening ceremony and photo session. There were several excursions, including a Saturday evening walk along Shanghai’s famous Bund followed by a cruise on the Huang-Pu River. Special events included an evening banquet at the Langyi Fang in Jin Gao Road and a full day excursion on the Monday for all participants to the Expo 2010 concurrently held in Shanghai.

A special issue with selected papers from the workshop, titled “Numerical Linear Algebra with Applications in Statistics”, will be published in Numerical Linear Algebra with Applications. The editors for this issue are Åke Björck, Maya Neytcheva, Musheng Wei and Yonghui Liu. Acta et Commentationes Universitatis Tartuensis de Mathematica will also publish selected papers presented at IWMS 2010. The editors for this issue are Tõnu Kollo and Dietrich von Rosen. See http://math.ut.ee/acta/about.html for details on submission.

The 20th International Workshop on Matrices and Statistics (IWMS-2011) will be held in Tartu, Estonia, June 26 - July 1, 2011 in conjunction with the 9th Tartu Conference on Multivariate Statistics. See http://www.ms.ut.ee/tartu11/ . The Program Committee is chaired by Dietrich von Rosen (Sweden) with George P. H. Styan, Honorary Chair of IWMS (Canada), and Tõnu Kollo, Vice-Chair (Estonia).

16th ILAS Conference, Pisa, Italy, June 21–25, 2010
Submitted by Dario Bini

The 16th ILAS Conference was held in Pisa, Italy, on June 21–25, 2010. About 380 people, from 35 different countries and all over the five continents, participated in this conference. There were 50 students registered. Many relevant topics of great interest in core linear algebra, algorithms and applications were treated.

The 9 plenary speakers were Rajendra Bhatia (India), "Loewner Matrices"; Richard A. Brualdi (USA), "Matrices and Indeterminates"; Pauline van den Driessche (Canada), "Potential Stability and Related Spectral Properties of Sign Patterns"; Nicholas J. Higham (UK), "Computing the Action of Matrix Exponential, with an Application to Exponential Integrators"; Beatrice Meini (Italy), "Nonsymmetric Algebraic Riccata Equations Associated wtih M-matrices: Theoretical Results and Algorithms"; Vadim Olshevsky (USA) – LAMA lecturer, "Potpourri of Quasiseparable Matrices"; Zdenek Strakos (Czech Republic), "Moments, Model Reduction and Nonlinearity in Solving Linear Algebraic Equations"; Daniel B. Szyld (USA), "Modifications to Block Jacobi with Overlap to Accelerate Convergence of Iterative Methods for Banded Matrices"; and Luis Verde Star (Mexico), "Linear Algebraic Foundations of the Operational Calculi". An additional 5 invited lectures were delivered by Oliver Ernst (Germany) – SIAG/LA speaker, "Krylov Subspace Approximations of the Action of Matrix Functions for Large-scale Problems"; Olga Holtz (USA) – Taussky-Todd Lecture " Zeros of Entire Functions: from René Descartes to Mark Krein and Beyond"; Lek-Heng Lim (USA) – LAA Lecture, "Multilinear Algebra and Its Applications"; Cleve Moler (USA) – Hans Schneider Prize lecture, "Evolution of MATLAB"; and Beresford N. Parlett (USA) – Hans Schneider Prize lecture, "Linear Algebra Meets Lie Algebra".

Advanced minisymposia were organized on different important topics where research is particularly active. Some attention was addressed to areas where linear algebra plays a role of growing interests. The scientific committee organised 6 invited mini-symposia and accepted 11 additional contributed mini-symposia. Titles and organizers of the invited mini-symposia were: Combinatorial Linear Algebra (Shaun Fallat, Bryan Shader), Linear Algebra Education (Avi Berman, Steven J. Leon), Markov Chains (Steve Kirkland, Michael Neumann), Matrix Functions and Matrix Equations (Chun-Hua Guo, Valeria Simoncini), Nonnegative Matrices (Judi McDonald, Michael Tsatsomeros), and Structured Matrices (Yuli Eidelman, Lothar Reichel, Marc Van Barel).

Titles and organizers of the contributed mini-symposia were: Application of Linear and Multilinear Algebra in Life Sciences and Engineering (Shmuel Friedland, Amir Niknejad), Generalized Inverses and Applications (Nieves Castro-Gonzalez, Pedro Patricio), Linear Algebra and Inverse Problems (Marco Donatelli, James Nagy), Linear Algebra in Curves and Surfaces Modeling (Costanza Conti, Carla Manni), Linear Algebra in Quantum Information Theory (Vittorio Giovannetti, Simone Severini), Matrix Inequalities - In Memory of Ky Fan (Chi-Kwong Li, Fuzhen Zhang), Matrix Means (Jimmie Lawson, Yongdo Lim), Max Algebras (Peter Butkovic, Hans Schneider), Nonlinear Eigenvalue Problems (Daniel Kressner, Volker Mehrmann), Spectral Graph Theory (Vladimir S. Nikiforov, Dragan Stevanovic), Tensor Computations in Linear and Multilinear Algebra (Lek-Heng Lim, Eugene Tyrtyshnikov). There were about 90 presentations in the sessions of contributed talks. The overall number of talks delivered at mini-symposia and contributed talk sessions was about 290.

On the first day, a box luncheon was offered by ILAS, and the Welcome Reception was sponsored by Elsevier, publishers of Linear Algebra and its Applications. Taylor & Francis, publisher of Linear and Multilinear Algebra, supported the LAMA lecture. The SIAM SIAG/LA group supported the SIAG/LA lecture.

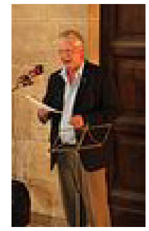
Social events included a guided tour to the historical center of Pisa. This was followed by a wonderful evening at the Church of San Francesco: first a concerto by piano, violin and cello in the church, and then the conference banquet held in the inner garden of the church complex. At the end of the banquet, Richard Brualdi introduced Cleve Moler and Beresford Parlett, who received the Hans Schneider Prize of 2008 but were not able to attend the 2009 ILAS Conference. Harm Bart was the Banquet Speaker.

The conference was also sponsored by Gruppo Nazionale di Calcolo Scientifico–INdAM; Ministero Università e Ricerca; University of Pisa: Department of Mathematics "Leonida Tonelli", Department of Applied Mathematics "Ulisse Dini", Department of Informatics, and the Interdepartmental Research Center "E. Piaggio". The Scientific Organizing Committee was formed by Michele Benzi (SIAM representative member), Avi Berman, Dario A. Bini, Luca Gemignani, Leslie Hogben, Steve Kirkland (ILAS President), Julio Moro, Ilya Spitkovsky, Francoise Tisseur, and Eugene Tyrtyshnikov. The Local Organizing Committee was formed by: Dario A. Bini (Chair), Gianna Del Corso, Bruno Iannazzo, Beatrice Meini, Ornella Menchi, Federico Poloni, and Sergio Steffé. More information about the conference, along with many more photos and a recording of part of the concerto, can be found at www.dm.unipi.it/~ilas2010.

Concerto

Banquet

Roger & Susan Horn,
Miriam & Raphi Loewy



Banquet Speaker
Harm Bart

Hans Schneider
and Olga Holtz

Leslie Hogben and Dario Bini

A FEW PISA PHOTOS

Martha Takane &
Luis Verde-Star

Peter Lancaster and Bor Plestenjak

Tobias Damm, Volcker Mehrmann
& Peter Benner

Oliver Ernst

Emailers

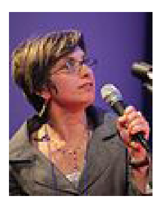
Berkant Savas &
Lek Heng Lim

Rajendra Bhatia

Daniel Szyld

Pauline van den
Driessche

Nick Higham



Beatrice Meini

Vadim Olshevsky

Zdenek Strakos

Coimbra Meeting on 0-1 Matrix Theory and Related Topics
Coimbra, Portugal, June 17-19, 2010
Report by Carlos Fonseca

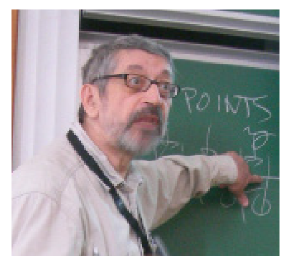
The Coimbra Meeting on 0-1 Matrix Theory and Related Topics was held at the Department of Mathematics of the University of Coimbra, Portugal, June 17-19, 2010. The main aim of this conference was to present recent developments on matrices with entries consisting only of zeros and ones, whose entry sums of rows and columns are constrained, as well as strengthening the interdisciplinary approach to the issue. Several fields in mathematics were involved: Matrix Theory, Combinatorics, Graph Theory, and Combinatorial Optimization.

The scientific committee consisted of Richard A. Brualdi (University of Wisconsin-Madison, USA), Brendan McKay (Australian National University, Australia), Christian Krattenthaler (Universität Wien, Austria), José Dias da Silva (University of Lisbon, Portugal), Natália Bebiano (University of Coimbra, Portugal), and Carlos Fonseca (University of Coimbra, Portugal), Conference Chair. The conference brought together 97 researchers from more than 20 countries, and there were 12 plenary sessions.


Herbert Wilf

Herbert Wilf, Thomas A. Scott Emeritus Professor of Mathematics at the University of Pennsylvania, was the ILAS Lecturer. He gave a very insightful talk on "Some Threads of the Theory of 0-1 Matrices in Graphs and Combinatorics".

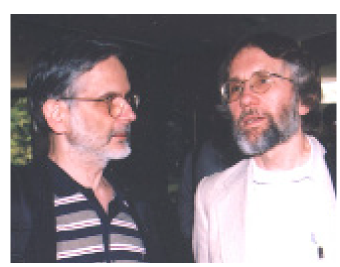
Other invited speakers and their titles included Alexander Barvinok (University of Michigan), "The Number of Matrices and a Random Matrix with Prescribed Row and Column Sums and 0-1 Entries"; Adrian Bondy (Université Claude Bernard Lyon 1, France), "Switching Reconstruction"; Richard A. Brualdi, "Combinatorial Batch Codes, Matroids, and A(R,S)"; Domingos M. Cardoso (University of Aveiro, Portugal), "Graph Eigenvalues in Combinatorial Optimization"; Geir Dahl (University of Oslo, Norway), "Majorization, A(R,S) and Related Matrix Classes"; Michael Drmota (Technische Universität Wien, Austria); "Random Planar Graphs"; Catherine Greenhill (University of New South Wales, Australia), "Asymptotic Enumeration of 0-1 Matrices: the Sparse Case"; Gyula O.H. Katona (Alfréd Rényi Institute of Mathematics, Hungary), "Bounds on the Largest Families of Subsets with Forbidden Subposets"; Hadi Kharaghani (University of Lethbridge, Canada), "Unbiased Hadamard Matrices"; Christian Krattenthaler, "Increasing and Decreasing Chains in 0-1 Matrices and RSK-like Algorithms"; Brendan McKay, "Asymptotic Enumeration of 0-1 Matrices: the Dense Case", and Irene Sciriha (University of Malta), "The Minimal Basis of the Nullspace of Singular Graphs". There was also an interesting special session by Vašek Chvátal, Canada Research Chair in Combinatorial Optimization, Concordia University, Montréal, Québec, Canada, entitled "A de Bruijn-Erdős Theorem in Metric Spaces?".



Vašek Chvátal

Another 33 contributed talks were presented in several afternoon parallel sessions, and they had a high standard. There will be a peer-reviewed special issue of Linear Algebra and its Applications devoted to papers presented at the conference. The guest editors are C.M. da Fonseca, J.A. Dias da Silva, Natália Bebiano, and Geir Dahl.

The banquet was dedicated to Professor José Dias da Silva on the occasion of his retirement from the University of Lisbon, to recognize his many contributions to linear and multilinear algebra and his important role in its development in Portugal. Both Richard A. Brualdi and Melvyn B. Nathanson (City University of New York) spoke about Jose's work and their personal relationship with him and his family. On the last day, the participants took a Mondego River cruise, enjoying a surprising view of the town.


José Dias da Silva
and Richard Brualdi

The conference was impeccably organized thanks to the efforts of the local organizing committee Ricardo Mamede (University of Coimbra, Portugal), Milica Anđelić (University of Aveiro, Portugal), and Ana Nata (Polytechnic Institute of Tomar, Portugal). This meeting was cosponsored by the American Mathematical Society and endorsed by the International Linear Algebra Society. It was also sponsored by Fundação para a Ci□ncia e a Tecnologia (FCT); the Centre for Mathematics of the University of Coimbra (CMUC); the Department of Mathematics, University of Coimbra (DMUC); and the Luso-American Development Foundation (FLAD).

The 10th Workshop on Numerical Ranges and Numerical Radii
Krakow, Poland, June 27-29, 2010
Report by Chi-Kwong Li

The 10th Workshop on Numerical Ranges and Numerical Radii (WONRA 2010) was held at the University Museum in the beautiful old city of Krakow, Poland from June 27-29, 2010, in conjunction with the 2010 ILAS Conference at Pisa, Italy, June 21-25, 2010. The workshop was sponsored by the Instytut Matematyki, Uniwersytet Jagielloński and Instytut Matematyczny, Polska Akademia Nauk, and was endorsed by the International Linear Algebra Society. Just before this workshop, there was also a meeting on Functions and Operators at Krakow, June 21-25, 2010.

Twenty four talks on different aspects of numerical ranges and radii were given. For details of the program and abstracts. see http://www.math.wm.edu/~ckli/10abs.pdf. There will be a special issue of Linear and Multilinear Algebra devoted to the proceedings. The Special editors will be Chi-Kwong Li, College of William and Mary (ckli at math.wm.edu) Yiu-Tung Poon, Iowa State University (ytpoon at iastate.edu) and Tin-Yau Tam, Auburn University (tamtiny at auburn.edu).

A workshop dinner took place on the evening on June 28. Workshop pictures were taken taken at the University Museum (see http://people.wm.edu/~cklixx/Pictures/wonra10a.JPG and http://people.wm.edu/~cklixx/Pictures/wonra10b.JPG).

There were about four dozen participants in the workshop, from more than fifteen countries. A list of participants and more photos can be found at http://www.math.wm.edu/~ckli/wonra10.html. As at previous meetings, participants exchanged ideas, results and problems on the subject in a friendly atmosphere.

University Museum of Collegium Maius (Grand College)
Copernicus was a student here.

Participants, WONRA 2010

The 5th Workshop on Matrices and Operators
Taiyuan City, Shanxi Province, China, July 12-15, 2010
Report by Jinchuan Hou and Chi-Kwong Li

The 5th Workshop on Matrices and Operators was held at the Yingze Hotel in Taiyuan, Shanxi, China, July 12-15, 2010. The workshop was sponsored by the Taiyuan University of Technology, the Shanxi Association for Science and Technology, and Shanxi Normal University, and also endorsed by the International Linear Algebra Society. Over 60 talks on different topics in matrix/operator theory and related subjects were discussed.

There were about 160 participants from more than 8 different countries and regions. Participants exchanged ideas, results and problems on the subject in a friendly atmosphere. A workshop dinner took place on the evening of July 13. Visit the conference webiste (http://www.sxkp.com/mao/eindex.asp) for a more detailed report, including slides of the talks and the group photo below.

Report on the 2010 International Conference on Optimization and Control
Guiyang, China, July 18-23, 2010

The 2010 International Conference on Optimization and Control (ICOCO 2010) was held at the Guizhou Park Hotel in Guiyang, China on July 18-23, 2010. Its purpose was to provide an international forum for scientists, engineers, researchers, and practitioners to exchange ideas and approaches, to present research findings and state-of-the-art solutions, to share experiences on potentials and limits, and to open new avenues of research and developments, on all issues and topics related to optimization and control theory. The conference was dedicated to the 65th birthdays of Professors Kok Lay Teo and Jie Sun who have made fundamental contributions to optimization, optimal control, and their computational methods and applications.

The conference strengthened collaborations and friendship between the scholars in the areas of optimization and control. The program and abstracts of talks can be viewed at http://sci.gzu.edu.cn/icoco/ICOCO-Conference-Programme.pdf.

The Mutually Beneficial Relationship of Matrices and Graphs
NSF-CBMS Regional Conference
Iowa State University, Ames, Iowa USA, July 12-16, 2010

Report by Bryan Shader

Richard Brualdi

This conference discussed the symbiotic relationship between matrices and graphs, and the significant role they jointly play in pure and applied mathematics, science and technology. It was supported by the National Science Foundation and the Institute for Mathematics and its Applications (IMA) through its Participating Institution Program. Highlights of the conference included:

- A series of ten lectures by Professor Richard Brualdi, University of Wisconsin, USA, which provided a contemporary perspective and surveyed new directions for future research on the subject.
- Collaborative research during the afternoon sessions that involved a total 40 people (10 graduate students, 3 postdocs, and 26 faculty members), split among five research groups, each working on problems related to the workshop topics.
- A well-attended poster session featuring the research of nine advanced graduate students.

Leslie Hogben, Iowa State University, and Bryan Shader, University of Wyoming, co-organized the conference. The following experts, in addition to the co-organizers, led research groups: Wayne Barrett, Brigham Young University; Pauline van den Driessche, University of Victoria; H. Tracy Hall, Brigham Young University (visiting); Judith McDonald, Washington State University; and Dale Olesky, University of Victoria. Accessible, yet interesting, problems that were amenable to a collaborative attack were suggested for each research topic. Those topics were:

- Inertially arbitrary patterns
- Sign-pattern problems related to Hopf-bifurcations
- Statistics of random orthogonal representations
- The skew-spectra of a graph
- The relationships between potential stability, nilpotency, and spectrally arbitrary patterns of small order
- Minimum rank problems for structured graphs
- Two tournament problems

ILAS Members who participated were: Wayne Barrett, Elizabeth Bodine, Steven Butler, Minnie Catral, Louis Deaett, Luz DeAlba, Ken Driessel, Craig Erickson, Colin Garnett, Ajith Gunaratne, Syliva Hobart, In-Jae Kim, Judi McDonald, Jillian McLeod, Sivaram Narayan, Maria Ñeco, Dale Olesky, Darren Row, Wasin So, Jeff Stuart, Hector Torres-Aponte, Pauline van den Dreissche, Amy Wehe, and Amy Yielding. The conference website, http://orion.math.iastate.edu/lhogben/CBMS/, has more information and photos.

The NSF-CBMS Regional Research Conference program is now in its 42nd year. The purpose is to stimulate interest and activity in mathematical research, and both established researchers and interested newcomers, including postdoctoral fellows and graduate students, are invited to participate in these conferences. For more information, visit http://www.cbmsweb.org/NSF/2012_call.htm.

International Conference on Trends and Perspectives in Linear Statistical Inference LinStat'2010

Report by Francisco Carvalho and Katarzyna Filipiak

The International Conference on Trends and Perspectives in Linear Statistical Inference was held in Tomar, Portugal, July 27- 31, 2010. The conference was organized by the Polytechnic Institute of Tomar, where the sessions took place, and the Faculty of Sciences and Technology of the New University of Lisbon. The aim of the conference was to bring together researchers who share an interest in a variety of aspects of statistics and its applications, to discuss current developments on these subjects.

During these five days, 167 participants from 23 countries had the chance of taking part in over 55 sessions. The keynote speakers were Somnath Datta, Thomas Mathew and Muni Srivastava. The 15 invited talks were on several topics and delivered by S. Ejaz Ahmed, Carlos Braumann and Sat Gupta on Inference; Anthony Atkinson, Barbara Bogacka and Stanisław Mejza on Experimental Designs; Dinis Pestana on Linear Models; Lynn R. LaMotte, Júlia Volaufová and Roman Zmyślony on Mixed Models, and Charles R. Johnson on Matrix Methods. The winners of the Young Scientists Awards of LinStat'2008 were invited speakers at this conference: Aylin Alin and Thomas Rush spoke on Inference and Klaus Nordhausen and Wojciech Rejchel spoke on Linear Models,

The Scientific Program also comprised Special Sessions devoted to specific subjects such as Robust Analysis of Linear Models, chaired by João Branco; Statistical Methods in Bioinformatics, chaired by Carles Cuadras; and Experimental Designs, chaired by Pierre Druilhet. Special session invited speakers were Rosemary A. Bailey, Financial Mathematics: Models and Statistical Methods chaired by Manuel L. Esquível; On the Methodology of Optimal Design of Nonlinear Models Based on the Functional Approach, chaired by Viatcheslav B. Melas; Nonparametrical Methods, chaired by Hannu Oja; Linear Models chaired by Simo Puntanen with invited speaker Jean-Pierre Masson; and New Ideas in the Analysis of Mixed Linear Models chaired by Dietrich von Rosen with invited speaker Jacek Wesołowski. Among the 94 contributed and special sessions speakers and 35 parallel sessions, 26 were delivered by young scientists. The poster session had 30 posters with 9 presented by young scientists.

The Scientific Committee selected 3 of the talks delivered by Ph.D. students or young scientists, and the best poster, for Young Scientists Awards. The winners were: 1st prize to Olivia Bluder (Austria), 2nd prize to Eero Liski (Finland), 3rd prize to Chengcheng Hao (Sweden), and the prize for best poster went to Paulo C. Rodrigues (Portugal). These winners will be invited speakers at the next LinStat conference, which will be held in Finland and organized by Simo Puntanen in 2012.

The Scientific Committee was constituted by João T. Mexia, Chair (Portugal), Augustyn Markiewicz, Vice Chair (Poland), Simo Puntanen (Finland), Dietrich von Rosen (Sweden), Götz Trenkler (Germany) and Roman Zmyślony (Poland). The Organizing Committee was chaired by Francisco Carvalho (Portugal) and included Katarzyna Filipiak, Vice Chair (Poland), Paulo C. Rodrigues (Portugal), Ricardo Covas (Portugal) and Miguel Fonseca (Portugal).

The proceedings will appear in a special issue of the Journal of Statistical Computation and Simulations, with S. Ejaz Ahmed as Coordinator-Editor and João T. Mexia and Augustyn Markiewicz as Guest Editors and in a special issue of Communications in Statistics – Theory and Methods, with N. Balakrishnan as Coordinator-Editor and Dietrich von Rosen and Katarzyna Filipiak as Guest Editors. These special issues will also include selected papers strongly correlated to the talks and posters presented during the LinStat'2010 conference.

Participants agreed that the conference was extremely fruitful and well organized with a friendly and warm atmosphere. A list of participants and abstracts can be found at http://www.linstat2010.ipt.pt.

ARTICLES

Some Recent Interesting Results Related to Linear Algebra

Thomas J. Laffey, University College Dublin, Ireland

1. Sums and Products of Sets

Tom Laffey

Let R be a ring and S, T finite nonempty subsets of R. Let $S+T := \{s + t : s \in S, t \in T\}$ and $ST := \{st : s \in S, t \in T\}$.

A famous result first found by Cauchy and rediscovered by Davenport states that if p is a prime, then for $R = Z_p$, the ring of integers modulo p, the cardinality $|S+T| \geq \min\{p, |S| + |T| - 1\}$. While the early proofs of this result were combinatorial, a very nice approach was found by Dias da Silva and Hamidoune (LAA 141 (1990), 283-287). Letting D_S and D_T denote the diagonal matrices with the elements of S, resp. T, along the diagonal, they observe that the cardinality of $S+T$ is the number of distinct eigenvalues of the Kronecker product matrix $M := D_S \otimes I + I \otimes D_T$, where I is the identity matrix of appropriate size. This cardinality is also the degree of the minimal polynomial of M since M is diagonalizable, as well as the maximum of the dimensions of span$\{w, Mw, M^2w, ...\}$ as w ranges over all column vectors of appropriate size. So for $S+T \neq Z_p$, it suffices to exhibit a w for which this dimension is at least $|S| + |T| - 1$, and they do this.

They and their collaborators developed this idea into a general method by means of which Grassmann algebraic results were used to prove additive number theory results and resolve a number of conjectures. Nathanson's book "Additive Number Theory: Inverse Problems and the Geometry of Sumsets", Springer-Verlag GTM volume 165, contains a good account of this area.

The Cauchy-Davenport result holds for subsets of integers (replacing p by ∞) and it is much easier in this situation, as one can use the natural ordering of the integers. In this situation with $S = T$, one gets equality when the elements of S form an arithmetic progression. By choosing S to have most of its elements in arithmetic progression, one gets $|S+S|$ close to $2|S|$, in contrast to close to the value $|S|(|S| + 1)/2$ expected when S is randomly chosen. Notionally, taking logs, one expects and indeed gets the same Cauchy-Davenport inequality for the cardinality of the product SS of a finite set of nonzero integers, with equality occurring for S a geometric progression.

Since arithmetic progressions and geometric progressions have very different profiles, one expects that for no such set S is the cardinality of both $S+S$ and SS close to the minimum. This property that one does not expect the cardinalities of the sum set $S+S$ and the product set SS to be both of the order of $|S|$, even for Z_p, was observed earlier by Elekes and Erdős, and a conjecture known as the Erdős ring problem was formulated. Over the last 15 years, questions of this form have become a central theme in number theory and combinatorics. If S is a finite subgroup of an additive group, then $S+S = S$ implies that S is a subgroup. So one can say that S is an approximate group if $|S+S| \leq k|S|$, for some given constant k close to 1. A similar definition can be used in the multiplicative context. In a ring situation, if both $|S+S|$ and $|SS|$ are small in this sense, we may think of S as an approximate ring. Using Gowers' quantitative version of the Balog-Szemerédi theorem (Geom. Funct. Anal. 8 (1998), 529-551) and the methods of solution of the Erdős ring problem by Edgar and Miller (Proc. AMS 131 (2003), 1121-1129), Bourgain, Katz and Tao (Geom. Funct. Anal. 14 (2004), 27-57) prove in colloquial terms that there are no "approximate subrings" of the finite field Z_p.

A concrete version of the sum-product bound is the following result of Katz and Shen (Proc. AMS 136 (2008), 2499-2504): Let p be a prime and S a nonempty subset of the field of p elements with $|S| \leq \sqrt{p}$. Then at least one of $S+S$ and SS has cardinality at least $|S|^{14/13-\varepsilon}$. This work improved an earlier paper of Garaev (arXiv math/07027808) in which the fraction $15/14$ appeared in place of $14/13$.

The paper "Approximation subgroups for linear groups" by Breuillard, Green and Tao (arXiv 1005.1881v1 (2010)) contains information on approximate subgroups of $GL_n(\boldsymbol{C})$ and other matrix groups. A very diverse set of techniques coming from algebra, analysis, geometry and combinatorics are used in the proofs. While the sum-product phenomenon does not extend to matrices without certain restrictions, Solymosi (Bull. LMS 41 (2009), 817-822) obtains an interesting result of this type for complex $d \times d$ matrices.

2. Octonions

Shortly after Hamilton discovered the division ring of real quaternions, H, in 1843, Graves discovered the division ring of real octonions Φ. While H is four dimensional and associative, Φ is eight dimensional and non-associative. However, Φ is alternative, that is, every *2*-generated subalgebra of Φ is associative. A famous theorem of Milnor and Kervaire states that a finite dimensional (not necessarily associative) real division algebra has dimension *1, 2, 4* or *8*. A *4*-dimensional one must be isomorphic to *H*, but there are *8*-dimensional ones not isomorphic to Φ. While the quaternions have played a quite visible role in mathematics since their discovery, octonions have not been so prominent. However, in recent times, octonions have gained prominence. Gauss defined his composition of binary quadratic forms almost two centuries ago and it has played a significant role in number theory since then. The paper "Composition of binary quadratic forms" by Pall (Acta Arith. 24 (1973), 401-409) gives a nice account of this. However, the composition did not have a "natural" interpretation. Bhargava has now found a new way of approaching it, within the context of octonions. Weissman ("Octonions, cubes, embeddings", "What is ... G_2?", University of California Sage talks) gives a nice exposition of these ideas. The automorphism group of Φ is the compact Lie group G_2.

One can perform the standard construction of the quaternions and octonions over any field, but in this case the resulting objects need not be division algebras. A simple example is to use complex instead of real coefficients, and notice that if *j* has the standard meaning as a quaternion or octonian, $(j-\sqrt{-1})(j+\sqrt{-1}) = 0$, where $\sqrt{-1}$ is chosen from the complex numbers. (This does not happen with the quaternion or octonion *i* since *i*, *j* do not commute.) Algebras of quaternions or octonions over a field that are not division algebras are called split. Such algebras are always split over finite fields. The automorphism group of the split quaternions over a field *K* is the split Lie group $G_2(K)$.

By examining the action of the automorphism group on the subspace of pure octonions, Gow (arXiv 0811.1291v1 (2008)) has constructed for every field *K* of characteristic $\neq 2$, a remarkable seven dimensional subspace of 7×7 skew-symmetric matrices in which all non-zero elements have rank *6* in the non-split case and rank *4* or *6* in the split case. Landsberg and Manivel (arXiv: math. RT/0402/157) identified a distinguished six dimensional subalgebra (which they call the algebra of sextonions) of the algebra of split octonions containing the quaternions, and applying their triality construction they obtain an exceptional Lie algebra that they call $\mathfrak{e}_{7\frac{1}{2}}$ since it lies between $\mathfrak{e}_7$ and $\mathfrak{e}_8$.

Sextonions are to be distinguished from sedenions. Algebras of sedenions are generalizations of the octonions. They have dimensions 2^n, $(n = 1,2,3,...)$,with $n = 2$ being a quaternion algebra and $n = 3$ being an algebra of octonions. For n > *3*, sedenion algebras are power associative (each *1*-generated subalgebra is associative but not alternative) but not division algebras. However, Moreno (Soc. Mat. Mex. Bol.Teicera Serie 4 (1), 13-28) has related their zero divisors to G_2.

3. Simultaneous Similarity of Matrices

Given two *k*-tuples $(A_1, A_2, ..., A_k)$ and $(B_1, B_2, ..., B_k)$ of $n \times n$ matrices over a field *K*, one says that they are simultaneously similar over *K* if there exists a nonsingular matrix *T* with entries in *K* satisfying $T^{-1}A_iT = B_i$, for all *i*. See Friedland (Adv. in Math. 50 (1983), 189-265) for a comprehensive treatment of this topic. If $k = 1$, then it follows from the theory of rational canonical forms that if A_1 and B_1 are similar over an extension field *E* of *K* (that is, *T* is allowed to have entries in the bigger field *E*), then they are similar over *K*. The question of whether for $k > 1$, simultaneous similarity over an extension field of *K* implies simultaneous similarity over *K* had not been answered until recently. If *K* has *n* or more elements, the result follows from a specialization argument (essentially, the fact that a nonzero polynomial of degree at most *n* cannot vanish identically on *K*), but for very small fields, there seemed to be a general expectation that the answer would be no. An approach via canonical forms, as in the case $k = 1$, is not available. However, a positive solution, showing that simultaneous similarity is field independent, has been found by de Seguins Pazzis (LAA 433 (2010), 618-624). His solution uses the Kronecker theory of pencils.

Finally, a nice expository account, with a strong linear algebraic flavour, of the ideas and context underlying the Fundamental Lemma of the Langlands programme is given in the lecture of Ben Zvi (University of Texas at Austin GRASP lecture series). Ngô

Note about Tom Laffey

By Bryan Shader

Tom Laffey is a Professor in the Department of Mathematics at the University College Dublin, where he has served two terms as department head.

He received his PhD from Sussex University in 1968 under the supervision of Walter Ledermann. His research interests are broad and include Group Theory, Ring Theory and Matrix Theory. He credits Fergus Gaines, a PhD student of Olga Taussky Todd, for drawing him into linear algebra, and Bob Thompson, Helene Shapiro, Charlie Johnson, Frank Uhlig, Richard Brualdi, Hans Schneider, Trevor West and Jaroslav Zemánek with greatly influencing his research.

His current research concerns nonnegative matrices, Liapunov stability theory, and similarity of matrices. He has made major contributions to the nonnegative inverse eigenvalue problem (NIEP) (which asks for necessary and sufficient conditions on a given list L of complex numbers to be the list of eigenvalues of a nonnegative matrix), on the problem of finding necessary and sufficient conditions for the existence of a common Lyapunov quadratic function (CQLF) for a pair of systems, and on the problem of deciding whether two given integer matrices are integrally similar and whether two given pairs of matrices over a field are simultaneously similar.

For over 30 years, Tom has supported the development of young mathematicians in Irish schools, through the organization of the American High School Mathematics Exams and by coaching the Irish International Mathematics Olympiad team.

In addition to his great breadth of knowledge about contemporary mathematics, Tom is very knowledgeable about world politics, history, art and music, and it is always an educational treat to join Tom for a tour of a museum or chat with him during a conference. A very nice interview of Tom conducted by Joao Queiro is available at http://at.yorku.ca/i/a/a/h/45.htm.

Addendum to "The History of Linear Algebra in Israel - A Personal View"

By Abraham Berman

In "The History of Linear Algebra in Israel – A Personal View" (Image issue 44, Spring 2010), I referred to the Amitsur-Levitzki Theorem as the "first Israeli" linear algebraic result. I want to thank Uriel Rothblum for pointing out that this title should be given to the fundamental work of Theodore Motzkin about linear inequalities. Motzkin wrote a thesis on linear inequalities, which was submitted to the University of Basel under Ostrowski in 1934. He was on the faculty of the Hebrew University from 1935 through 1948 where he continued to work on linear inequalities. The publication of his thesis came out in Jerusalem as well as in Basel [1].

Quoting a biographical note from an article of Adi Ben-Israel on Motzkin's work, [2]: "Theodore S. Motzkin (1908-1970) made important contributions to linear inequalities and polyhedral combinatorics. His name is associated with the Motzkin transportation theorem [3], Motzkin numbers, Motzkin paths, Fourier-Motzkin elimination method and its dual, the double description method. His father, Leo Motzkin (1867-1933), who studied mathematics and sociology at the University of Berlin, was a Zionist politician and cultural leader. The Israeli city of Kiryat Motzkin, a suburb of Haifa, is named after Leo Motzkin." Adi refers readers to the online biography [4] for further details.

To this I want to add that Motzkin gave a necessary and sufficient condition for copositivity [5] and to mention that his son, Gabriel Motzkin, is the Director of the Van Leer Jerusalem Institute (http://www.vanleer.org.il/eng/), which is a center for the interdisciplinary study and discussion of issues related to philosophy, society, culture and education.

References

[1] T.S. Motzkin, Beiträge zur Theorie der linearen Ungleichungen, Inaugural Dissertation, Basel, Jerusalem, 1936.

[2] A. Ben-Israel, "Motzkin's Transposition Theorem and the Related Theorems of Farkas, Gordan and Stiemke", *Encyclopaedia of Mathematics*, Supplement III, Kluwer Academic Publishers, Dordrecht 2001, ISBN 1-4020-0198-3 and http://benisrael.net/MOTZKIN.pdf.

[3] T.S. Motzkin, Two consequences of the transposition theorem on linear inequalities, *Econometrica* 19 (1951), 184-185.

[4] J.J. O'Connor and E.F. Robertson, Theodore Samuel Motzkin, http://www-history.mcs.st-andrews.ac.uk/Biographies/Motzkin.html.

[5] T.S Motzkin, Copositive quadratic forms, National Bureau of Standards Report 1818 (1952), 11-12.

Linear Algebra in Mexico
By Luis Verde-Star


Luis Verde-Star

In this note we present a brief and sketchy description of the linear algebra activity in Mexico during the last forty years. We begin with some comments about the history of the academic and scientific activity previous to the time when the first mathematical research institutions were created in Mexico.

The Royal School of Mining (Real Seminario de Mineria) was created in 1790. The curriculum had two years of Mathematics, which included algebra, geometry, trigonometry, analytic geometry and applied geometry, which contained mechanics and fluid dynamics. Later the infinitesimal calculus was incorporated. By 1843 it had become a school of Mining Engineering and in 1867 it was transformed into the National School of Engineers and included civil, mining, mechanical, topographic, geographic, and hydraulic engineering. When the National University of Mexico was created in 1910, the School of Engineers became an important part of the University.

Other schools of the University that would play an important role in the development of higher studies and scientific research were the National Preparatory School (Escuela Nacional Preparatoria) and the School of Higher Studies (Escuela de Altos Estudios).

An engineer who was a teacher of mathematics in those schools, Prof. Sotero Prieto (1884-1935) had a profound impact on the future of science and mathematics in Mexico. He convinced many of his students to pursue advanced studies in science, mathematics, and engineering. One of those was Manuel Sandoval-Vallarta (1899-1977), who became the most famous Mexican physicist. He went to MIT in 1917, where he studied Electrical Engineering, and later obtained a PhD in Physical Sciences in 1924. Then he took courses in Germany from Einstein, Planck, and Schrodinger.

Sandoval-Vallarta was on the faculty of MIT from 1923 until 1943 and was the tutor of R. Feynman and J. Stratton. In 1923 he wrote a book about Heaviside's operational calculus, published by the Electrical Engineering Department of MIT, and in 1926 wrote the paper [62] about Heaviside's expansion theorem. Since operational calculus is now considered part of the theory of linear functional equations, these works are probably the first ones written by a Mexican researcher on a subject related to linear algebra.

Sandoval-Vallarta visited Mexico frequently during his time at MIT and participated, together with other physicists, engineers, and former students of Sotero Prieto, in the creation of the School of Sciences (Facultad de Ciencias) in 1936 and the Mathematics Institute in 1942, both in the National University (UNAM). In 1943 Sandoval-Vallarta returned to Mexico where he participated in founding the Mexican Mathematical Society and also contributed greatly to the development of the physics research community.

We present next some information about the period after 1943 until around 1970, paying special attention to the events related to algebra, and in particular, to the parts of algebra that had later some connection with linear algebra.

During the early years of the School of Sciences and the Mathematics Institute, several teachers and students went to the United States to take graduate level courses and some of them obtained a PhD. Some visiting professors had a strong influence in the evolution of the Mexican mathematical institutions. For example, G.D. Birkhoff from Harvard, Dirk Struik from MIT, and Solomon Lefschetz from Princeton visited Mexico frequently from 1945 until 1966.

Among the first Mexican research mathematicians we must mention the following. Félix Recillas obtained a PhD from Princeton in 1948, with a thesis on functional analysis directed by Chevalley and Bochner. José Adem (1921-1991) got his PhD from Princeton in 1949 under the direction of N. E. Steenrod, with a thesis on algebraic topology. Emilio Lluis, who was born in Spain in 1925 and lived in Russia, studied mathematics at the Lomonosov University of Moscow during three semesters before coming to Mexico. Lluis worked with Lefschetz in Mexico, took some graduate courses in Paris with Pierre Samuel, and got his PhD from UNAM in 1954. His research subjects were group theory and algebraic geometry. Humberto Cárdenas also studied in Princeton, where he obtained his PhD in 1965 working on group theory and cohomology. Other important pioneers of mathematics were Alberto Barajas, Guillermo Torres, Francisco Zubieta, and Francisco Tomás. They all played an important role in the formation of the Mexican mathematical community, mainly through their teaching activities.

Raymundo Bautista got his PhD from UNAM in 1965. His first research papers were on cohomology of rings, group rings, and finite algebras, but then he worked on representation theory of groups and algebras and became the leader of an important research group on representation theory of algebras. Other members of this group that have played a leading role are Roberto Martínez and José Antonio de la Peña.

Another important algebra research group that started in the late 1960's in UNAM is the ring theory research group. Some of its members are F. Raggi, José Ríos, A.G. Raggi, C. Signoret, R. Fernández-Alonso, and M.J. Arroyo.

Lluis and Cárdenas are considered the founders of the Mexican algebra school. They are now Emeriti professors of UNAM, but still active and publishing papers on discrete geometry, in collaboration with some younger colleagues.

The paper [1] by J. Adem is one of the first papers related to linear algebra and written by a Mexican mathematician. Adem considers bilinear maps Φ: $R^p \times R^q \rightarrow R^r$ that satisfy $n(\Phi(x,y)) = n(x)n(y)$ where n is the Euclidean norm. Such maps are called normed. The usual multiplications in the complex numbers, the quaternions, and the Cayley numbers are normed maps. Adem found a systematic way to construct a family of normed maps, different from the family of Hurwitz-Radon maps. This paper had a large impact and motivated other Mexican mathematicians to work on similar problems and on the general theory of Cayley-Dickson algebras. See [3], [14], [43], [51], [53], and [54]. Adem's paper [2] is also of a linear algebra nature.

Another early linear algebra related paper is Bautista's [9]. Bautista has also worked on linear algebra related to mathematical physics [10] and category theory [65]. Some of his students and collaborators have done work in linear algebra. José Antonio de la Peña has a relatively large number of linear algebra related publications on several subjects, for example, quadratic forms [7], [19], spectral radius of graph cleavages [20], Coxeter matrices [21], and Galois coverings of graphs [22]. In [6], M.Takane collaborated with Hans Schneider and other well-known linear algebraists. Rafael H. Villarreal, from CINVESTAV, obtained a PhD from Rutgers under the direction of W. Vasconcelos. He works in several areas of algebra and combinatorics and has published several papers about linear algebra, some with his students. See [26] and [64].

There are several fields of mathematics in which groups of people in Mexico publish papers related to linear algebra. Some of these include people from different institutions and some groups are very small. We will mention next such groups in no particular order.

Probability and Statistics

Professor José A. Díaz-García, from Universidad Autónoma Agraria Antonio Narro, has a very large number of publications on several aspects of linear algebra in statistics and probability, with several collaborators, both in Mexico and other countries. See, for example, [23] and [24]. Víctor Pérez-Abreu, from CIMAT, Rolando Cavazos-Cadena, from Universidad Autónoma Agraria Antonio Narro, and Daniel Hernández, also from CIMAT, work on random matrices. See [46] and [17]. Cavazos-Cadena has also some core linear algebra papers, such as [16]. Some Mexican physicists work with random matrices in statistical physics.

Control Theory

There is a group of researchers, most of them in the Department of Automatic Control of CINVESTAV, who work on linear algebraic problems related to control. They collaborate with many colleagues from other countries. See, for example, [12], [29], [36], and [37].

Clifford Algebra and Geometric Algebra

There is a relatively large number of people from different institutions and backgrounds that publish in this field, especially in the journal Adv. Appl. Clifford Algebr. See, for example, [4], [25], and [34]. Some members of this group have organized several international conferences on Clifford algebras and related topics.

Here we should mention Garret Sobczyk (Universidad de las Américas, Cholula, Puebla), who got his PhD from Arizona State in 1971, working under David Hestenes, and has worked on geometric algebra for a long time. He also works on several topics of linear algebra. He retired recently but is still active in research. See [33], [56] and [57].

Numerical Analysis

The Mexican numerical analysis community is quite large and diverse. They work very actively in areas such as fluid dynamics, PDE, optimization, simulation in finance, biology, and other fields, approximation theory, computational physics, etc. Regarding numerical linear algebra, we should mention Jesús López-Estrada, Susana Gómez, Humberto Madrid, Pedro González-Casanova, and José Luis Morales. See [32], [38], and [50].

Toeplitz matrices and operators

This group includes S. M. Grudsky, N. Vasilevski, E. Ramírez de Arellano, R. Quiroga-Barranco, M. Shapiro and some of their students. They collaborate with a large number of well-known researchers from other countries, such as A. Böttcher and I. Spitkovsky. See [13], [48], and [49].

Operator theory

Here we have C. Hernández,, C. Bosch, R. del Río, R. Weder, J.H. Arredondo, S. Pérez-Esteva, A. Fraguela, J. Cruz, S. V. Djordjević, R. Martínez-Avendaño, and E. Roldán. Some recent works are [41], [42], and [52].

Crystallography

There is a group of physicists who work on theoretical crystallography, using group theory, linear algebra, and Clifford algebra. Among them we have D. Romeu, A. Gómez, J.L. Aragón, G. Roldán, and M. A. Rodríguez. See, for example, [30], [4], and [5].

Linear Algebra Education

The main researchers in Linear Algebra Education in Mexico are Asuman Oktaç (CINVESTAV) and María Trigueros (ITAM) and their students. They organized the minisymposium Linear Algebra Education at the ILAS Conference held in Cancún in 2008. See [45] and [47]. We should also mention Humberto Madrid, who has written several notes on the teaching of linear algebra. Regarding textbooks, Jorge Ludlow (UAM) wrote a linear algebra textbook [39] in 1981. Emilio Lluis-Puebla is the author of a textbook on linear and multilinear algebra available at the web page of the Sociedad Matemática Mexicana. The textbook that has been used by the largest number of Mexican students is [58], because the linear algebra course offered by the School of Engineering (Facultad de Ingeniería) of UNAM is taken by more than 300 students each year. Professor Leda Speziale and some of her colleagues who teach that course have done a lot of work on linear algebra education.

Other miscellaneous topics

There are other colleagues who work on very diverse areas who occasionally publish papers related to linear algebra. For example, from mathematical physics some examples are [28], [61], and [60]. From mathematical biology we have [27]. From approximation theory see [15]. See also [8], [11], [31], [35], [55], [59], and [63].

Final remarks

In the book [18], several chapters describe different aspects of the history of mathematics in Mexico.

Reference [44] is one of the first papers related to linear algebra published in Mexico. It is based on a series of lectures presented by Oldenburger at the School of Engineering of UNAM in 1940.

The 15th Conference of the International Linear Algebra Society was held in Cancún, Mexico, in June 2008. The local organizers were María José Arroyo, Martha Takane, Rubén Martínez-Avendaño, and Luis Verde-Star. The author of this article was invited to deliver a plenary lecture at the 16th Conference of ILAS, held in Pisa, Italy, in June 2010.

There are a few conclusions that we can obtain from the information presented above. The first one is that research activity in linear algebra in Mexico began around 1975. The second one is that we don't have yet a "Mexican school of linear algebra". Only a few people publish regularly on linear algebra and most of them started, and continue, working in other branches of mathematics. Nor is there a clearly defined "family tree" of Mexican mathematicians that work on linear algebra.

We can also observe that foreign mathematicians who moved to Mexico have influenced the development of mathematics in Mexico in a very strong way.

Finally, it is clear that the information presented in this article is far from being complete. There is a lot of historic information missing and there are certainly many colleagues that have done some linear algebra work that we didn't mention here.

References

[1] Adem, José, Construction of some normed maps, Bol. Soc. Mat. Mexicana (2) 20 (1975) 59-75.
[2] Adem, José, On maximal sets of anticommuting matrices, Bol. Soc. Mat. Mexicana (2) 23 (1978) 61-67.
[3] Adem, José, On the Hurwitz problem over an arbitrary field. II, Bol. Soc. Mat. Mexicana (2) 26 (1981) 29-41.

[4] Aragón-Gonzalez, G., J.L. Aragón, M.A. Rodríguez-Andrade, L. Verde-Star, Reflections, rotations and Pythagorean numbers, Adv. Appl. Clifford Alg. 19 (2009) 1-14.
[5] Aragón-Gonzalez, G., J.L. Aragón, M.A. Rodríguez-Andrade, The decomposition of an orthogonal transformation as a product of reflections, J. Math. Phys. 47 (2006) 013509, 10 pp.
[6] Barker, George P., Max Neumann-Coto, Hans Schneider, Martha Takane, Bit-Shun Tam, Equilibria of pairs of nonlinear maps associated with cones, Integral Equations Operator Theory 51 (2005) 357-373.
[7] Barot, M., J.A. de la Peña, Root-induced integral quadratic forms, Linear Algebra Appl. 412 (2006) 291-302.
[8] Barradas, Ignacio, Francisco J. Solis, Nonnegative rank, Int. Math. J. 1 (2002) 601-610.
[9] Bautista, Raymundo, On the problem of *n* matrices, (Spanish) An. Inst. Mat. Univ. Nac. Autónoma México 14 (1974) 15-43.
[10] Bautista, R., A. Criscuolo, M. Durdević, M. Rosenbaum, J.D. Vergara, Quantum Clifford algebras from spinor representations, J. Math. Phys. 37 (1996) 5747-5775.
[11] Bengochea, Gabriel, Luis Verde-Star, Linear algebraic foundations of the operational calculi, Adv. Appl. Math. (in press) doi:10.1016/j.aam.2010.08.001.
[12] Bonilla Estrada, Moisés, Maricela Figueroa Garcia, Michel Malabre, Juan Carlos Martínez García, Left invertibility and duality for linear systems, Linear Algebra Appl. 425 (2007) 345-373.
[13] Böttcher, A., S. Grudsky, E. Ramírez de Arellano, On the asymptotic behavior of the eigenvectors of large banded Toeplitz matrices, Math. Nachr. 279 (2006) 121-129.
[14] Calvillo, Gilberto, Isidoro Gitler, José Martínez-Bernal, Intercalate matrices: II, A characterization of Hurwitz-Radon formulas and an infinite family of forbidden matrices, Bol. Soc. Mat. Mexicana (3) 3 (1997) 207- 219.
[15] Campos, Rafael G., Claudio Meneses, Differentiation matrices for meromorphic functions, Bol. Soc. Mat. Mexicana (3) 12 (2006) 121-131.
[16] Cavazos-Cadena, Rolando, An alternative derivation of Birkhoff's formula for the contraction coefficient of a positive matrix, Linear Algebra Appl. 375 (2003) 291-297.
[17] Cavazos-Cadena, Rolando, Daniel Hernández-Hernández, A central limit theorem for normalized products of random matrices, Period. Math. Hungar. 56 (2008) 183-211.
[18] Cosmos: Enciclopedia de las ciencias y la tecnología en México, Tomo V, Universidad Autónoma Metropolitana, México D.F., 2009.
[19] Dean, A., J. A. de la Peña, Algorithms for weakly nonnegative quadratic forms, Linear Algebra Appl. 235 (1996) 35-46.
[20] de la Peña, José A., Cleavages of graphs: the spectral radius, Linear Multilinear Algebra 57 (2009) 641-649
[21] de la Peña, José A., Periodic Coxeter matrices, Special issue on linear algebra methods in representation theory, Linear Algebra Appl. 365 (2003) 135-142.
[22] de la Peña, J. A., M. Takane, The spectral radius of the Galois covering of a finite graph, Linear Algebra Appl. 160 (1992) 175-188.
[23] Díaz-García, José A., Graciela González-Farías, Singular random matrix decompositions: Jacobians, J. Multivariate Anal. 93 (2005) 296-312.
[24] Díaz-García, José A., Ramón Gutiérrez-Jáimez, Distribution of the generalised inverse of a random matrix and its applications, J. Statistical Planning and Inference 136 (2006) 183-192.
[25] Durdević, Mićo, Zbigniew Oziewicz, Clifford algebras and spinors in braided geometry. Differential geometric methods in theoretical physics (Ixtapa-Zihuatanejo, 1993). Adv. Appl. Clifford Algebras 4 (1994) Suppl. 1, 461-467.
[26] Escobar, César A., José Martínez-Bernal, Rafael H. Villarreal, Relative volumes and minors in monomial subrings, Linear Algebra Appl. 374 (2003) 275-290.
[27] Esteva-Peralta, L., J.X. Velasco-Hernández,, *M*-matrices and local stability in epidemic models, Math. Comput. Modeling 36 (2002) 491-501.
[28] Felipe, Raúl, Nancy López-Reyes, Fausto Ongay, Raúl Velásquez, Yang-Baxter equations on matrix dialgebras with a bar unit, Linear Algebra Appl. 403 (2005) 31-44.
[29] Fernández-Anaya, G., J.C. Martínez-García, V. Kučera, Characterizing families of positive real matrices by matrix substitutions on scalar rational functions, Systems Control Lett. 55 (2006) 871-878.
[30] Gómez, A., J.L. Aragón, O. Caballero, F. Dávila, The applications of Clifford algebras to crystallography using Mathematica, Clifford Algebras with Numeric and Symbolic Computations, 251-266, Birkhauser Boston, Boston, MA, 1996.
[31] Gómez, Ricardo, José Miguel Salazar, Spanning tree invariants, loop systems and doubly stochastic matrices, Linear Algebra Appl. 432 (2010) 556-565.
[32] Gómez, Susana, José Luis Morales, Performance of the Chebyshev iterative method, GMRES and ORTHOMIN on a set of oil-reservoir simulation problems, Mathematics for Large Scale Computing, 265-295, Lecture Notes in Pure and Appl. Math., 120, Dekker, New York, 1989.

[33] Hestenes, David, Garret Sobczyk, Clifford algebra to geometric calculus: A unified language for mathematics and physics, Fundamental Theories of Physics, D. Reidel Publishing Co., Dordrecht, 1984.
[34] Keller, Jaime, Suemi Rodríguez-Romo, Multivectorial representation of Lie groups, Int. J. Theoret. Phys. 30 (1991) 185-196.
[35] Königsberg, Zvi Retchkiman, A generalized eigenmode algorithm for reducible regular matrices over the max-plus algebra, Int. Math. Forum 4 (2009) 1157-1171.
[36] Leyva-Ramos, J., A new look at partial fraction expansion of transfer function matrices from a computational viewpoint, Comput. Math. Appl. 26 (1993) 27-35.
[37] Loiseau, Jean Jacques, Sabine Mondié, Ion Zaballa, Petr Zagalak, Assigning the Kronecker invariants of a matrix pencil by row or column completions, Linear Algebra Appl. 278 (1998) 327-336.
[38] López Estrada, Jesús, Humberto Madrid de la Vega, Valia Guerra Ones, Numerical calculation of the Moore-Penrose pseudoinverse of a matrix, Investigación Oper. 17 (1996) 117-132.
[39] Ludlow, Jorge A., Álgebra Lineal, Limusa, Mexico City, 1981.
[40] Martínez Adame, Carmen, On the spectral behaviour of a non-selfadjoint operator with complex potential, Math. Phys. Anal. Geom. 10 (2007) 81-95.
[41] Martínez-Avendaño, Rubén A., Higher-rank numerical range in infinite dimensional Hilbert space, Oper. Matrices 2 (2008) 249-264.
[42] Martínez-Avendaño, Rubén A., Peter Rosenthal, An introduction to operators on the Hardy-Hilbert space, Graduate Texts in Mathematics, 237, Springer, New York, 2007.
[43] Moreno, Guillermo, The zero divisors of the Cayley-Dickson algebras over the real numbers, Bol. Soc. Mat. Mexicana (3) 4 (1998) 13-28.
[44] Oldenburger, Rufus, The theory of polynomials of higher order, (Spanish) Ingeniería (Mexico City) 14 (1940). 192-201, 239-244.
[45] Parraguez, Marcela, Asuman Oktaç, Construction of the vector space concept from the viewpoint of APOS theory, Linear Algebra and its Appl. 432 (2010) 2112-2124.
[46] Pérez-Abreu, Víctor, A random matrix model for the Gaussian distribution, Periodica Math. Hungarica 52 (2006) 47-65.
[47] Possani, E., M. Trigueros, J.G. Preciado, M.D. Lozano, Use of models in the teaching of linear algebra, Linear Algebra Appl. 432 (2010) 2125-2140.
[48] Quiroga-Barranco, Raúl, Nikolai Vasilevski, Commutative C*-algebras of Toeplitz operators on the unit ball, II, Geometry of the level sets of symbols, Integral Equations Operator Theory 60 (2008) 89-132.
[49] Ramírez de Arellano, Enrique, Michael V. Shapiro, Nikolai L. Vasilevski, Two types of analysis associated to the notion of Hurwitz pairs, Differential geometric methods in theoretical physics (Ixtapa-Zihuatanejo, 1993), Adv. Appl. Clifford Algebras 4 (1994) Suppl. 1, 413-422.
[50] Rodríguez-Gómez, Gustavo, Pedro González-Casanova, Jorge Martínez-Carballido, Computing general companion matrices and stability regions of multirate methods, Int. J. Numer. Methods Eng. 61 (2004) 255-273.
[51] Rodríguez-Ordóñez, Hugo, Homotopy classification of bilinear maps related to octonion polynomial multiplications, Linear Algebra Appl. 432 (2010) 3117-3131.
[52] Roldán-Pensado, Edgardo, Infinite numerical range is convex, Linear Multilinear Algebra 56 (2008) 731-733.
[53] Romero, David, New constructions for integral sums of squares formulae, Bol. Soc. Mat. Mexicana (3) 1 (1995), 87-90.
[54] Sánchez-Flores, Adolfo, A method to generate upper bounds for the sums of squares formulae problem, Bol. Soc. Mat. Mexicana (3) 2 (1996) 79-92.
[55] Sánchez-Valenzuela, Oscar Adolfo, Matrix computations in linear superalgebra, Linear Algebra Appl. 111 (1988) 151-181.
[56] Sobczyk, Garret, Geometric matrix algebra, Linear Algebra Appl. 429 (2008) 1163-1173.
[57] Sobczyk, Garret, The missing spectral basis in algebra and number theory, Amer. Math. Monthly 108 (2001) 336-346.
[58] Solar, Eduardo, Leda Speziale, Apuntes de Álgebra Lineal, 3a. Ed., Limusa-Facultad de Ingeniería, UNAM, Mexico DF, 1996.
[59] Torres-Cházaro, Adolfo, Ernesto Vallejo, Sets of uniqueness and minimal matrices, J. Algebra 208 (1998) 444-451.
[60] Torres del Castillo, G. F., Spinors in three dimensions, III, Rev. Mexicana Fís. 40 (1994) 713-729.
[61] Urrutia, L. F., N. Morales, An extension of the Cayley-Hamilton theorem to the case of supermatrices, Lett. Math. Phys. 32 (1994) 211-219.
[62] Vallarta, M. S., Heaviside's proof of his expansion theorem, J. Am. Inst. Elect. Eng. 45 (1926) 383-387.
[63] Verde-Star, Luis, Functions of matrices, Linear Algebra Appl. 406 (2005) 285-300.
[64] Villarreal, Rafael H., Rees cones and monomial rings of matroids, Linear Algebra Appl. 428 (2008) 2933-2940.
[65] Zhang, Yingbo, Tiangang Lei, Raymundo Bautista, A matrix description of a wild category, Sci. China, Ser. A, 41 (1998) 461-475.

Department of Mathematics, Universidad Autónoma Metropolitana, Mexico City. verde@xanum.uam.mx

2008 Hans Schneider Award Winners Cleve Moler and Beresford Parlett Celebrated

Remarks by Richard Brualdi at the ILAS Conference Banquet, in Pisa, Italy, June 23, 2010

It is a great honor and pleasure for ILAS to celebrate in person with Cleve Moler and Beresford Parlett, the winners of the Hans Schneider Prize in 2008. Neither was able to attend the 2008 ILAS Conference in Cancun, but we are happy that they are here in Pisa and will present their Hans Schneider lectures at this conference. They have both made major contributions to linear algebra and it is appropriate to share here some of the details in the letters nominating them for the Hans Schneider Prize.

It was Cleve Moler's dream, devotion, and vision that created MATLAB. He invented this software for his students at the University of New Mexico in the 1970's, in order to provide easy access to the LINPACK and EISPACK libraries of algorithms that he had helped create. Written in FORTRAN and first for mainframes, then for PC's and Apples, MATLAB included simplicity of use and extensibility from the beginning. It kept evolving and in the early 1980's Cleve encouraged John Little when Little said he would like to update and develop it commercially. Interest in the software grew quickly and in 1984, together they founded MathWorks. Cleve's involvement was mainly advisory until he joined the firm full time in 1989, as Chairman and Chief Scientist. Hundreds of people have contributed to keep the language, algorithms and implementations of MATLAB up to date, and to supply new features and numerous special toolboxes.

The MATLAB User's Guide of 1982 explained that the name MATLAB came from "MATrix LABoratory". It could perform all standard matrix computations numerically to the highest accuracy and stability standards of the numerical analysis community, and this quality standard has been maintained. MATLAB is now in its 7th major version, containing over 8000 command functions and over 25 special toolboxes for applications such as statistics, simulation, control, wavelets, image processing, neural networks, signal processing, PDEs, optimization, and the list goes on. It truly is a matrix based laboratory for doing all kinds of numerical calculations. In order to serve students well, MathWorks has always provided a relatively inexpensive MATLAB license for educational institutions, and a somewhat smaller student version of the software, which is still quite powerful, for about the price of a technical textbook.

Cleve received his PhD from Stanford University in 1965 with George Forsythe, during the time Forsythe was creating a computer science program and department at Stanford. For almost 20 years, Cleve was a Professor of Mathematics and Computer Science at the University of Michigan, Stanford University, and then the University of New Mexico. In 1984 he left his tenured position at New Mexico to work for 5 years at Intel and Ardent on developing software for parallel systems.

The Math Genealogy Project lists 15 PhD students and 81 PhD descendants for Cleve Moler. While this number of descendants is phenomenal, what is even more phenomenal is the vast number of researchers -- mathematicians, scientists, engineers, and technicians -- who have made and are making substantial use of MATLAB throughout the world. Electrical, mechanical, aerospace, chemical, computer, and biological engineers use it on a daily basis. There are over 500 books that use MATLAB codes, both student textbooks and treatises for engineers and scientists. In addition to his own publications, there are many papers in which Cleve is a shadow coauthor. MATLAB is now an important tool for researchers in linear algebra and matrix theory, and an important teaching and learning tool that emphasizes the natural advantages of viewing modeling and numerical analysis from the principal ingredients of matrices and linear algebra.

Cleve has received many honors. He was elected to the National Academy of Engineering in 1997 "for conceiving and developing widely used mathematical software." He has served SIAM in many capacities, including as President in 2007-09. In 2009 he received the fourth SIAM/ACM Prize in Computational Science and Engineering, a prize that recognizes "outstanding contributions to the development and use of mathematical and computational tools and methods for the solution of science and engineering problems." To that we can add theoretical matrix theory and linear algebra, for almost all of us have used MATLAB to make and test conjectures. SIAM commissioned a long interview with Moler about his early work and development of MATLAB, which can be read at http://history.siam.org/pdfs2/Moler_final.pdf.

Cleve has been instrumental in getting linear algebra and matrix theory to attain its rightful place in the modern mathematical, scientific, and engineering world. Just as I find it difficult now to imagine the mathematical enterprise without the resource of MathSciNet, I am sure most of us would find it hard to imagine our community without MATLAB and Cleve Moler. Through MATLAB, he has changed the way most of us do research in matrix theory and linear algebra. He fully deserves the title "Father of MATLAB."

Beresford Parlet has been a major figure in the field of Numerical Analysis beginning in the 1960s when the subject started to flourish, all the way to the present day. MathSciNet lists over 100 publications for him, of which 27 were published from 2000 on. Beresford's main speciality is the eigenvalue problem, and he is the unchallenged leading world expert on the symmetric eigenvalue problem. He is a master expositor and speaker, with a flair for words and ideas. For example, his most recent publication (coauthored with A.A. Dubrulle) has the title "Revelations of a transposition matrix". I am proud to say that he is one of the Distinguished Editors of LAA.

Beresford received the PhD from Stanford University in 1962 with George Forsythe as advisor, and settled down at Berkeley in 1965 where he is now Professor Emeritus of Mathematics and Computer Science. According to the Math Genealogy Project, he has 26 PhD students and 54 PhD descendants. Beresford has made major contributions across the entire spectrum of linear algebra, ranging from deep theoretical results to the design and development of algorithms and software. The letter nominating him for the Hans Schneider Prize highlighted six major aspects of his work.

1. After the invention in the early 1960s of the QR algorithm for solving the symmetric and nonsymmetric eigenvalue problems, the race was on to derive convergence results to explain the observed excellent behavior of the algorithm and to guide the choice of shifts. Beresford made many fundamental and lasting contributions. He gave necessary and sufficient conditions for the convergence of the basic algorithm, an elegant proof of global convergence when Wilkinson's shift is used, and necessary and sufficient conditions for forward instability of the QR algorithm for symmetric tridiagonal matrices. With C. Reinsch he developed an algorithm for balancing the row and column sum norms of a matrix, an algorithm used in EISPACK, LAPACK, and MATLAB. The QR algorithm was named in 2000 as one of the "top 10 algorithms of the century" in science and engineering by the journal *Computing in Science and Engineering*, and Beresford was the person who was asked to, and did, write about it for that journal.

2. His book "The Symmetric Eigenvalue Problem," first published in 1980 and republished in 1998 as a SIAM Classic, is indeed a classic and has been very influential. It is a masterpiece of exposition.

3. The mathematical foundation for the resurgence of the Lanczos algorithm was laid by Chris Paige, but it was Beresford Parlett and his students D.S. Scott and H. Simon who are responsible for making it an indispensable tool. A paper with D.R. Taylor and Z.A. Liu ushered in the (now standard) look ahead device for dealing with most instabilities in the nonsymmetric Lanczos algorithm.

4. Among its symmetric eigenvalue SVD solvers, LAPACK includes codes based on the work of Parlett and coauthors since the early
1990s. With K.V. Fernando, he discovered the shifted differential quotient-difference algorithm (the qd-algorithms), which are simultaneously the fastest and most accurate known algorithms for finding the eigenvalues or singular values of a symmetric tridiagonal (or bidiagonal) matrix.

5. Beresford's most significant recent contribution, with his student I.S. Dhillon, is the discovery of a new algorithm that lowers the complexity of finding both the eigenvalues and eigenvectors of a symmetric tridiagonal matrix, from $O(n^3)$ to $O(n^2)$, an unimprovable complexity bound since the eigenvector matrix consists of n^2 numbers. In fact, the algorithm finds k accurate orthogonal eigenvectors of length n in $O(kn)$ time in an "embarrassingly parallel fashion". This dual optimality of minimal complexity and embarrassing parallelism has led this algorithm to be dubbed the "Holy Grail." The final version of this algorithm is employed in version 4 of LAPACK. Given the wide usage of LAPACK, this will have an enormous impact on users of linear algebra tools worldwide.

6. While the king of rounding error analysis was Jim Wilkinson, Beresford Parlett is a prince, and he has done as much as anyone to advance the subject, with many of his papers containing penetrating rounding error analyses.

As the nomination letter for the ILAS Hans Schneider Prize summarized so well, "all the best methods in use today for dense and sparse symmetric eigenvalue problems can be traced to the work of Parlett and his students."

It's hard to imagine Numerical Linear Algebra without Beresford Parlett, and without him it would be a lot poorer. It's also hard to imagine the linear algebra community without his grace, skill, knowledge, and goodness.

Ed. Note: The 2008 Hans Schneider Awards were previously announced in IMAGE issue 40, Spring 2008.

Aspects of Projective Geometry

Richard William Farebrother

1. **Introduction**

Consider an artist painting a picture in the Early Renaissance. He examines the scene with one eye closed in order to maintain perfect monocular vision. In this context, any point on the line joining his eye to a point in the scene will be represented in the picture surface by a single point, and any point on the plane de.ned by his eye and two specified points in the scene will be represented in the picture surface by a point on the straight line joining the points in this surface corresponding to the two selected points, and so on. This practical technique in which objects with real dimensions one, two, and three are regarded as points, lines and planes respectively was subsequently formalised under the title of projective geometry, see Cameron [3] or Hirschfeld [11] for details. Moreover, Farebrother [7, 8] has shown that the concept of projective geometry duality plays a natural role in linear statistical estimation.

2. **Dürer's Practical Procedure**

Albrecht Dürer (1471-1528) has described a practical procedure for obtaining correct perspective in art by means of a picture surface ruled with a rectangular grid of crossed lines together with a corresponding rectangular grid of crossed wires in a frame installed at a suitable distance between the artist and the scene he is painting. The artist then copies what he sees in each rectangle into the corresponding rectangle on the picture surface. In this context, standard perspectives are obtained when the angle between the plane of the grid of wires and the line joining the artist's eye to the centre of the scene is close to a right angle whilst nonstandard perspectives are obtained if the plane of the grid of wires makes a much smaller angle with this line. The use of unfamiliar perspectives had a considerable vogue in the middle of the sixteenth century. Perhaps the most familiar example is the mystery object in the foreground of the portrait "The Ambassadors", by Hans Holbein the Younger (1497-1543).

Readers may be interested to learn in passing that I originally subscribed to the albums prepared by the Living Paintings Trust (www.LivingPaintings.org.uk) with a view to discovering how the concept of linear perspective could be explained to people who had been blind from birth. Almost four hundred years after Dürer described his practical technique, Walter Richard Sickart (1860-1942) employed a variant of this procedure to transfer his small studies to surfaces of larger size by ruling corresponding rectangular grids of crossed lines on both surfaces. (Sickart achieved considerable notoriety in 2002 when Patricia Cornwell [5] identified him as her prime candidate in the role of Jack the Ripper, who perpetrated a series of notorious murders in the East End of London in 1888-90.)

3. **Mathematisation of Projective Geometry**

Although Piero della Francesca (1420.1492) and Albrecht Dürer (1471-1528) had each described a method for representing three-dimensional figures by means of orthogonal projections (plan and elevation) onto two planes set at right angles to one another, it seems appropriate to regard their work as early contributions to the subject area of descriptive geometry whilst that of Girard Desargues (1591-1661) and Blaise Pascal (1623-1662) provided the impetus that lead to the formalisation of projective geometry. See [1, 10, 12] for details.

4. Nonlinear Projective Geometry

In her analysis of the mathematical symbolism of Lewis Carroll's Alice in Wonderland [4], Melanie Bayley [2] suggests that Carroll's transformation of the Duchess' baby into a pig may be regarded as a parody of Jean-Victor Poncelet's (1788-1867) modern projective geometry. Keith Devlin [6] is content to report Bayley's views without further comment, but I prefer to regard Carroll's baby to pig transformation as a precursor of the type of nonlinear transformations from one nonrectangular grid to another employed with considerable success some fifty years later by the zoologist D'Arcy Wentworth Thompson (1860-1948).

References

[1] Percy Anderson and Ivor Grattan-Guinness (1994). Descriptive Geometry, in [9, 887-896].

[2] Melanie Bayley (2009). Alice's adventures in algebra: Wonderland solved, New Scientist Issue 2739, 16 December 2009: online open access.

[3] Peter J. Cameron (1991). Projective and Polar Spaces, QMW Maths Notes 13, Queen Mary and Westfield College, London: available from http://www.maths.qmul.ac.uk/ pjc/pps/.

[4] Lewis Carroll (1865). Alice's Adventures in Wonderland. Clarendon Press, Oxford.

[5] Patricia Cornwell (2002). Portrait of a Killer: Jack the Ripper Case Closed, Little, Brown and Company, Boston.

[6] Keith Devlin (2010). The hidden math behind Alice in Wonderland, Devlin's Angle. http://www.maa.org/devlin/devlin0310:html
[7] Richard William Farebrother (1989). Some early work on the duality between points and lines, Communications in Statistics (Series B: Computation and Simulation), 18, 719-727.
[8] Richard William Farebrother (2002). Visualizing Statistical Models and Concepts, Marcel Dekker, New York: online access via taylorandfrancis.com.
[9] Ivor Gratten-Guinness (Ed.) (1994). Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences, Vol. II. Routledge, London.
[10] J. J. Gray (1994). Projective Geometry, in [9, 897-907].
[11] J.W. P. Hirschfeld (1985). Finite Projective Spaces of Three Dimensions, Oxford University Press, Oxford.
[12] Peter Scheider (1994). Art and Architecture, in [9, 1593.1611]. Richard William Farebrother R.W.Farebrother@Manchester.ac.uk

11 Castle Road, Bayston Hill, Shrewsbury, England SY3 0NF.

Mihaly Bakonyi: January 6, 1962 – August 7, 2010

By Frank Hall

Mihaly Bakonyi was born in Arad Romania on January 6, 1962. He graduated from the University of Timisoara with a B.S. degree in Mathematics in 1984, obtained his M.S. degree in Mathematics from the University of Bucharest in 1985, and then his Ph.D. from the College of William and Mary in 1992, under the guidance of Charles Johnson.

Mihaly joined the Department of Mathematics and Statistics at Georgia State University in 1992 and was a highly respected colleague, teacher, and researcher. He frequently gave his time and expertise to other faculty members. His research was in functional analysis/operator theory, matrix analysis (completions and factorizations), and moment problems. With his superb network of co-authors, he produced many excellent journal articles and two outstanding books. He visited the Mathematical Institute of the Romanian Academy during the fall of 2006 and the Center for Linear and Combinatorial Structures of the University of Lisbon from 2008 to 2009.

The Department of Mathematics and Statistics of Georgia State University is working with his wife Gina to set up a Memorial Fund to honor Mihaly. For information, contact Frank Hall (matfjh@langate.gsu.edu).

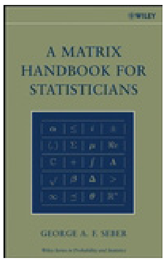
Additional Memories of Ky Fan

An obituary of Ky Fan appeared in IMAGE issue 44. Since then, Steven Johnson reported that Dennis Wildfogel has created a blog (http://drfantales.blogspot.com/) where more memories about Professor Fan have been recorded.

BOOKS

A Matrix Handbook for Statisticians
by G.E.F Seber
Wiley, 2010. 559 pages. ISBN: 978-0-471-74869-4

Reviewed by Dietrich von Rosen



This is a very special book which has arisen from the author's collection of useful matrix results, assembled during more than 20 years. I can definitely recommend the book to anyone who is working with matrices in statistics. However, one should be aware of what the author notes in the preface, that a number of problems arise when writing a book of this kind, for example: what material should be included; how to handle proofs; which order the material should be presented in; and what level of generality should be used. Moreover, the author never meant to provide a complete work that covers the majority of matrix applications in statistics. It is a personal selection. The mathematical sophistication corresponds probably to what is satisfactory to most researchers in statistics. There exist, however, several statistical schools that use more general mathematics than that presented here.

The material is divided into 24 chapters. The sections within chapters start with definitions, then results are presented and thereafter references to proofs and related material. This structure makes the book easy to follow. It includes a vast variety of material, among others, subspace relations, rank relations, results on patterned matrices, matrix functions such as the determinant, trace, inverse, g-inverse, norms, eigenvectors and eigenvalues.

Different classes of matrices are considered, for example, Hermitian, symmetric, skew symmetric, normal, diagonally dominant, Toeplitz, circulant, Hankel, Hadamard, Vandermonde, Fourier and others. Usually the underlying field is real or complex valued but some results for quaternion matrices are also presented. Moreover, results on matrices with nonnegative elements have been collected, including irreducible, stochastic and doubly stochastic matrices.

In the book one finds results on linear equations, factorizations and inequalities. Differentiation and Jacobians are covered. The book ends with 5 chapters on statistical applications, including random vectors and matrices, the least squares and maximum likelihood methods as well as applications of different types of majorizations.

The book is an impressive one-man project but it might have benefited from engaging a couple of coauthors. There are parts which are not up-to-date and the reference list could have been longer. It could have been of interest to briefly indicate what kind of generalizations exist. Basic information about tensor products is one example of an area which I am missing. Much of the material can be traced back to classical statistical books on linear models and multivariate analyses, and people from these areas will feel comfortable with the notation and the way results are presented here.

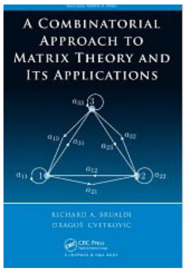
A similar reference book has been written recently by Dennis Bernstein, *Matrix Mathematics:Theory, Facts and Formulas*, 2nd ed. Princeton University Press, Princeton, 2009 (first edition 2005). The content of that certainly overlaps with the book discussed here. The main difference is that the book under review includes material that is closer to various statistical applications than Bernstein's. The latter includes more results (facts) and references as well as covers different areas. Even together these two books definitely do not cover the whole matrix world.

The material in the book under review is very clearly presented. In general, however, it is difficult to learn from the presentation since very few ideas and details are given as to how to prove results. Also, it would perhaps be advantageous to mention more results from journals, since nowadays such items can often be obtained from some electronic data base.

Department of Energy and Technology, Swedish University of Agricultural Sciences, Sweden. dietrich.von.rosen@et.slu.se

A Combinatorial Approach to Matrix Theory and its Applications
by Richard A. Brualdi and Dragos Cvetkovic
CRC Press, 2009. 267 pages. ISBN 978-1-4200-8223-4

Reviewed by Wasin So



The mutually beneficial relation between combinatorics (in particular, graph theory) and matrix theory is well known and much appreciated. Unfortunately, this relation is often not considered to be fundamental, but of auxiliary nature only. In the book under review here, the authors argue the other way around.

Are combinatorics (in particular, graph theory) and matrix theory really dissimilar subjects? This question was asked by the first author almost 20 years ago in the article ”The Symbiotic Relationship of Combinatorics and Matrix Theory”, *Linear Algebra and its Applications* 162-164 (1992) 65-105. In the book discussed here, one finds that the answer to this question is absolutely NO. In fact, the authors proclaim in the preface that a matrix and a (weighted) graph can be regarded as different models of the same mathematical concept.

To achieve their goal, the authors introduce three weighted digraphs associated with a matrix A. When A is m×m, the digraph D(A) is the complete graph on m vertices {1,...,m} and for each entry a_{ij} in A there is an edge $i \rightarrow j$ with weight a_{ij}. The Coates digraph D*(A) of a square matrix A is defined to be $D(A^T)$. When A is m×n, the König digraph G(A) is defined to be the complete bipartite graph having m black vertices, n white vertices, and an edge $i \rightarrow j$ from each black vertex i to each white vertex j, with weight a_{ij}. The authors use these graphs to explain and illuminate matrix operations (chapter 2), matrix power (chapter 3), determinant (chapter 4), matrix inverse (chapter 5), Cramer’s formula (chapter 6), Cayley-Hamilton Theorem and Jordan Canonical Form (chapter 7), Perron-Frobenius Theorem (chapter 8), eigenvalue inclusion regions and permanent (chapter 9), and applications in electrical engineering, physics, and chemistry (chapter 10).

Even though nice exercises are included in each chapter, the book may not be suitable as an introductory textbook. However, it can be a good supplementary source of alternative proofs for standard results in matrix theory. A favorite one of the reviewer is the graph theoretic proof of the Cayley-Hamilton Theorem. As in every book written by humans, there are typos, but very few were spotted by the reviewer.

Although the book is about matrices, one has a different feeling when reading it because of its combinatorial approach. Indeed this reviewer finds it refreshing and revealing that the tie between matrix theory and graph theory is more fundamental than he thought.

Department of Mathematics, San Jose State University, San Jose, CA 95192, USA. so@math.sjsu.edu

---

New Book
Matrices: Theory and Applications, 2nd edition, by Denis Serre
Springer-Verlag Graduate Text in Mathematics, Vol. 216, 2010

This is based on a course the author taught at the École Normale Supérieure de Lyon. The new edition is 40 percent longer than the first, and material in the first edition has been reorganized, with new topics, exercises and applications. The author has always been concerned with matrix theory because of its importance in every area of mathematics as well as science and engineering.

List of Books of Interest to Linear Algebraists

Oskar Baksalary, Book Consulting Editor for IMAGE, maintains a list of books about linear algebra and its applications on the ILAS website (http://www.ilasic.math.uregina.ca/iic/). He includes references to reviews of selected titles. Please tell him when you find additional titles or informative reviews that should be included (baxx@amu.edu.pl).

CALL FOR PAPERS FROM JOURNALS

Special Issue of LAMA in Memory of Ky Fan

There will be a special issue of Linear and Multilinear Algebra dedicated to the memory of Ky Fan, who passed away in March 2010. Professor Fan made significant contributions to many areas of mathematics. His work in linear algebra, particularly in matrix inequalities, positive definite matrices, nonnegative and M-matrices, inequalities for eigenvalues, majorization, matrix norms, location of eigenvalues and linear programming has had a lasting influence.

The journal invites papers related to his work, particularly in the areas of linear algebra, matrix theory and operator theory. The special issue editors are Ravindra Bapat, Indian Statistical Institute, India; Raphael Loewy, Technion-Israel Institute of Technology, Israel; and Fuzhen Zhang, Nova Southeastern University, USA. The Editor in Chief responsible for this special issue is Steve Kirkland. The deadline for submissions is July 31, 2011 with target date May 15, 2012 for publication.

Special Issue of LAA on Matrix Patterns
Deadline extended to Dec. 31, 2010

Papers can be submitted by December 31, 2010 for the special issue of Linear Algebra and Its Applications on the occasion of the Workshop "Theory and Applications of Matrices described by Patterns", held at the Banff International Research Station in Alberta, Canada on January 31 - February 5, 2010.

Papers within the scope of the workshop are solicited from all interested, whether or not a participant in the workshop. Papers should be submitted to one of the special editors: Shaun Fallat <sfallat@math.uregina.ca>, Leslie Hogben <lhogben@iastate.edu>, Bryan Shader <bshader@uwyo.edu>, and Pauline van den Driessche <pvdd@math.uvic.ca>. The responsible Editor-in-Chief is Richard A. Brualdi <brualdi@math,wisc.edu>.

Special Issue of LAA in honor of Abraham Berman, Moshe Goldberg and Raphael Loewy
Submitted by Richard Brualdi

Linear Algebra and its Applications is pleased to announce a special issue in honor of Professors Abraham Berman, Moshe Goldberg, and Raphael Loewy in recognition of their retirements from the Technion within the next two years, their many important contributions to linear algebra and other topics in mathematics, and their important roles in the organization of the successful Haifa Matrix Meetings.

We solicit papers for the special issue within the entire scope of LAA or the research interests of the three honorees. The deadline for submission of papers is March 1, 2011. Submissions will be subject to normal refereeing procedures and usual standards of LAA. They should be sent as pdf attachments in email, to one of the following special editors: Wayne Barrett, Brigham Young University, USA (wayne@math.byu.edu); Michael Neumann, University of Connecticut, USA (neumann@math.uconn.edu); Naomi Shaked-Monderer, Emek Yezreel College, Israel (nomi@tx.technion.ac.il); and Eitan Tadmor, University of Maryland, USA (tadmor@cscamm.umd.edu).

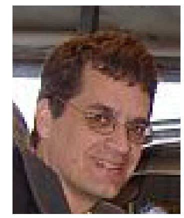
Special Issue of NLAA on Inverse Problems in Science and Industry, In Honor of Biswa Datta

Numerical Linear Algebra with Applications is pleased to announce a special issue dedicated to Professor Biswa Datta, on the topic of Inverse Problems in Science and Industry. The Guest Editors are Eric King-wah Chu, Monash University (eric.chu@monash.edu), Wen-Wei Lin, National Taiwan University (wwlin@math.nctu.edu.tw), and Lothar Reichel, Kent State University (reichel@math.kent.edu). The responsible Associate Editor of the special issue is Maya Neytcheva, Uppsala University (Maya.Neytcheva@it.uu.se). Submit your paper to one of the guest editors at http://mc.manuscriptcentral.com/nla, by February 28, 2011.

ILAS NEWS

Editorial Leadership of ELA Changes
From Steve Kirkland, ILAS President

After fifteen years of tireless efforts on behalf of ILAS, Danny Hershkowitz has retired as an Editor in Chief of the ILAS Electronic Journal of Linear Algebra (ELA). In recognition of Danny's longstanding service, he has been conferred the new title of Founding Editor of ELA, and appears now on the ELA masthead under that title. The ILAS Board extends its appreciation for his long efforts as Editor in Chief of ELA, which have brought it from only an idea to its current stature as a successful and respected journal.



I am pleased to announce that Bryan Shader has been appointed as an Editor in Chief of ELA, effective July 15, 2010, and that Ludwig Elsner will continue in his current capacity as an Editor in Chief. On behalf of the ILAS Board and membership, I wish Bryan and Ludwig every success with the journal.

Please note that submissions to the journal should be sent to elamath@uwyo.edu.

Upcoming 2010 ILAS Elections
Submitted by Steve Kirkland, ILAS President

The Nominating Committee for the 2010 ILAS elections has completed its work, and ballots will be sent out soon.

Nominated for a three year term, beginning March 1, 2011, as ILAS President is Steve Kirkland. Nominated for the two open three-year terms, beginning March 1, 2011, as "at-large" members of the ILAS Board of Directors are Robert Guralnick, Los Angeles USA; Francoise Tisseur, Manchester UK; David Watkins, Pullman USA; and Henry Wolkowicz, Waterloo Canada.

The Nominating Committee is Ravi Bapat, Avi Berman (Chair), Leiba Rodman, Valeria Simoncini and Hugo Woerdeman. Many thanks for their important service to ILAS. Thanks also to the nominees for agreeing to stand for election.

Jim Weaver Retiring as Outreach Director of ILAS
Interview by Jane Day

Judy and Jim Weaver

Jim Weaver is retiring as Outreach Director of ILAS as of February 2011. He says he needs a break, but he will be sorely missed. His nonstop involvement and contributions to ILAS began before the organization existed. Its predecessor the International Matrix Group (IMG) was formed in 1987. The initial leaders, Hans Schneider, Bob Thompson and Danny Hershkowitz were exploring how to incorporate this organization and make it legally a nonprofit. Jim had spent a sabbatical at the University of Wisconsin in 1984, where he met Hans (and also Danny and Volker Mehrmann). In 1987 Hans contacted Jim to ask if he would be interested in being treasurer of a newly formed linear algebra organization and help with the legal matters. Jim agreed and by 1989 ILAS was founded, incorporated and legally became a Corporation Not For Profit.

Jim served as Treasurer from 1989-1994, as Secretary-Treasurer from 1995-2000, and then as Outreach Director from 2001-2010. He has also been a Consulting Editor for IMAGE since 2007. As Outreach Director, he found corporate sponsors and obtained advertisements from them for IMAGE, solicited materials from publishers and displayed those at ILAS meetings, chaired the ILAS Institutional Membership Committee, and archived ILAS' official documents. He has also staffed a membership table at each ILAS meeting, and his wife Judy has been a constant and delightful helper with that and with the book displays.

Jim started college with a President's scholarship to Kent State, where he earned a math degree with a minor in physical education and also a teaching certificate. But his math profs encouraged him to try graduate school. Michigan State offered a teaching assistantship and he says he liked the math department as well as the lively campus culture (he actually said "lots of sports, music and pretty girls"). He met Judy right away and they married two years later when she finished her BA. She began teaching second grade while Jim finished his PhD. His thesis was in group theory, with Joseph Adney as advisor. He was hired at the University of West Florida in Pensacola and made his career there. He and Judy have two children and now four grandchildren.

In the fall of 1991 Jim visited the Institute for Mathematics and its Applications in Minneapolis, to participate in the Institute's year of emphasis on applied linear algebra. He says the intense creative atmosphere at IMA was very exciting, and there were great bonuses –a beautiful 3 foot snowfall in late October, and the Minnesota Twins were battling the Atlanta Braves in the World Series that fall. Jim loves baseball and got to see the 6^{th} game of the Series, which the Twins won against all predictions. (That forced a 7^{th} game, which they also won, making the Twins the 1991 World Champions!)

In 1993, Jim was local organizer for the third ILAS meeting, held in Pensacola. He says ILAS has had a major impact on his life and he is most appreciative of the generous contributions to this grass roots organization by so many people for so many years. He loves linear algebra and it's been fun to work with many coauthors.

To learn more about the history of ILAS, see the article by Jim Weaver in IMAGE issue 40, Spring 2008 (http://www.ilasic.math.uregina.ca/iic/IMAGE/).

Linear Algebra Education Corner

Submitted by Steve Leon

One of the many highlights of the ILAS conference in Pisa, Italy was the invited minisymposium on linear algebra education. The opening speaker was Guershon Harel (UC San Diego USA). He discussed research about the algebraic ways of thinking necessary for success in beginning mathematics courses, including linear algebra, how well high school textbooks prepare students for these ways of thinking, how well high school teachers employ teaching practices that promote these, and how can teachers be trained to advance these ways of thinking among students.

Other speakers included Boris Koichu (Technion, Israel), who discussed strategies, cost and their overall effect of using clickers in teaching linear algebra. Clickers allow students to give immediate feedback during the lecture presentation. Edgar Possani (Inst. Tecnológico Autótonomo, México) outlined an approach to teaching linear algebra using a model from economics and a method for analyzing students' learning processes called APOS (constructing mental *actions*, *processes*, and *objects* and organizing them in *schemas* in order to understand how to solve problems). Jane Day (San Jose State Univ., USA) summarized insights about how people learn that she had acquired over time, including the writings of Maria Montessori, Jean Piaget, and Guershon Harel, plus recommendations of the Linear Algebra Curriculum Study Group (LACSG) and feedback from her own students.

Frank Uhlig (Auburn Univ., USA) posed a number of thought provoking questions that could lead us to rethink much of what we do in our courses. Sang-Gu Lee (Sungkyunkwan Univ., Korea) talked about the linear algebra course that he teaches using Sage-Math software. Sage-Math provides a very new mobile, multimedia and flexible learning environment for students. It utilizes tehnology and the internet in innovative ways and is ideal for teaching linear algebra. Avi Berman spoke on "Principles and Tools in Teaching Linear Algebra". He discussed the objectives of the course in terms of student understanding and skills, ways of achieving those objectives through motivation, visualization, interaction, and challenge, and examples of how this is being done at the Technion.

Steve Leon emphasized the importance of requiring a second course in linear algebra for all mathematics majors, as recommended by the LACSG in 1990, and outlined a number of alternative courses that could be offered. He has taught one in which students work in teams on projects, applying linear algebra to areas such as digital imaging, computer animation, and coordinate metrology. Many of the projects involved original undergraduate level research. The final speaker was Kyung-Won Kim (Sungkyunkwan Univ., Korea), who spoke about "The Development of Excel and Sage Math Tools for Linear Algebra".

For pdf files with more details of the talks, visit the ILAS Education Web page http://galois09.skku.ac.kr/newilas/.

ILAS 2009-2010 Treasurer's Report
March 1, 2009 – March 31, 2010

Net Account Balances on February 28,2009		
Vanguard (ST Fed Bond Fund 6893.256 Shares)	$ 74,653.96	
Checking Account	$ 24,186.85	
Pending Credit Card Amounts	$ 5,200.00	
Pending Deposits	$ 2,365.00	
		$106,405.81
		===========
General Fund	$ 43,464.18	
Conference Fund	$ 11,956.01	
ILAS/LAA Fund	$ 12,840.57	
Olga Taussky Todd/John Todd Fund	$ 10,592.14	
Frank Uhlig Education Fund	$ 4,844.34	
Hans Schneider Prize Fund	$ 22,708.57	
		$ 106,405.81

Income:	First Federal	First Nat'l
Dues	$ 12,170.00	
Corporate Dues	$ -	
General Fund	$ 3,210.00	
Conference Fund	$ 140.00	
Taussky - Todd Fund	$ 240.00	
Uhlig Education Fund	$ 70.00	
Schneider Prize Fund	$ 200.00	
Interest – First Federal S&L Account	$ 53.32	$ 10.03
	$ 16,083.32	$ 10.03

Expenses:	First Federal	First Nat'l
ILAS Speakers (2)	$ 1,316.91	
IMAGE (2 issues)	$ 1,623.97	
Supplies (checks)	$ 19.50	
License Fees	$ 61.25	
Dues Mailing	$ 375.36	
Credit Card & Bank Fees	$ 522.51	$ 224.38
Labor-Conference	$ 230.89	
	$ 4,150.39	$ 224.38

Net Account Balances on March 31, 2010		
Vanguard (ST Fed Bond Fund 7196.162 Shares)	$ 77,358.74	
Checking Account - First Federal	$ 18,782.93	
Certificate of Deposit	$ 20,000.00	
Checking Account - First National	$ 1,600.00	
Variances for Currency Transfers	$ 382.72	
		$ 118,124.39
		===========
General Fund	$ 54,532.76	
Conference Fund	$ 12,096.01	
ILAS/LAA Fund	$ 12,840.57	
Olga Taussky Todd/John Todd Fund	$ 10,832.14	
Frank Uhlig Education Fund	$ 4,914.34	
Hans Schneider Prize Fund	$ 22,908.57	
		$ 118,124.39
		===========

Other Recent ILAS Business

The following official reports were published in 2010 on ILAS-NET (http://www.ilasic.math.uregina.ca/iic/ilas-net/): Minutes of the ILAS Board Meetings at Cancun in 2008 and in Pisa in 2010 were published as Messages 1772 (Cancun) and 1765 (Pisa); and the President/Vice President Annual Report is Message 1763.

New SIAM Fellows

We congratulate members of ILAS recently inducted as Fellows of SIAM: Iain S. Duff, Rutherford Appleton Laboratory, UK; Carl D. Meyer, North Carolina State University, USA; and Olof B. Widlund, Courant Institute of Mathematical Sciences, USA. For details, visit http://fellows.siam.org/.

Send News for IMAGE Issue 46 by April 1, 2011

All items of interest to the linear algebra community are welcome, including photos and longer articles as well as short notes like historical tidbits. Suggestions are also welcome, such as ideas for survey articles, history topics, books to review, people to interview, etc.

Announcements and reports of conferences and workshops
News about striking developments in linear algebra and its applications
Honors and awards
Articles on history
Survey articles
Books, websites, funding sources
News about journals
Employment and other funding opportunities
Problems and solutions
Transitions: new appointments, responsibilities, deaths
Linear algebra education
Possible new corporate sponsors
Letters to the editor

News should be sent by April 1, 2011 (but see note below):

Problems and Solutions to Fuzhen Zhang (zhang@nova.edu).
Book news and reviews to Oskar Baksalary (baxx@amu.edu.pl).
History of linear algebra to Peter Semrl (peter.semrl@fmf.uni-lj.si).
Linear algebra education to Steve Leon (sleon@umassd.edu).
Advertisements to Jim Weaver (jweaver@uwf.edu).
All other material and ideas to Editor Jane Day (day@math.sjsu.edu).

If you wish to submit an article after April 1, let the appropriate editor know as soon as possible when to expect your item, and it may be possible to include it.

Send material in plain text, Word, or both Latex and PDF (Article.sty with no manual formatting is preferred). Send photos in JPG format; higher resolution for best reproduction. Issue 45 will be published on June 1, 2011.

OPPORTUNITIES

Assistant Professorship Open at the University of Connecticut

The Department of Mathematics at the University of Connecticut invites applicants for a tenure-track position at the Assistant Professor level starting in Fall 2011. Highly qualified candidates in all mathematical disciplines are encouraged to apply, but logic, geometry and topology, and numerical linear algebra and numerical analysis are areas of particular, but not exclusive, focus of the search.

Minimum Qualifications: A completed Ph.D. in Mathematics by August 23, 2011, and demonstrated evidence of excellent teaching ability and outstanding research potential.

Preferred Qualifications: Research focus of logic, geometry and topology, and numerical linear algebra and numerical analysis; and the ability to contribute through research, teaching and/or public engagement to the diversity and excellence of the learning experience.

This position is at the Storrs campus. Candidates may have the opportunity to work at the campuses at Avery Point, Hartford, Stamford, Torrington, Waterbury, and West Hartford.

Review of applications will begin on November 15, 2010, and continue until the position is filled. Applications and at least 3 letters of reference should be submitted online at http://www.mathjobs.org/jobs. Questions or requests for further information should be sent to the Hiring Committee at mathhiring@uconn.edu.

The University of Connecticut is an Equal Opportunity and Affirmative Action Employer. We enthusiastically encourage applications from underrepresented groups, including minorities, women, and people with disabilities.

SIAM Travel Grants for Students to Attend ICIAM 2011

Thanks to a grant from the National Science Foundation, SIAM can provide travel support to selected students and early career participants, such as postdoctoral candidates, to attend ICIAM 2011. The nominal limit of support is $1500. Applicants must be affiliated with a U.S. institution on the start date of the meeting, which is July 18, 2011. Early career applicants must be within five years of their PhD. Preference will be given to candidates presenting their work at the conference.

For details, visit http://www.iciam2011.com/index.php?option=com_content&view=article&id=34&Itemid=36.

Invitation for Nominations for the 14th Householder Award

The Alston S. Householder Award is given every three years for the best PhD dissertation in numerical linear algebra. It is presented at the triennial Householder Symposium on Numerical Linear Algebra, which will be held next on June 12-17, 2011 at Tahoe City, CA, USA. The dissertation must have been submitted between Jan 1, 2008 and Dec 31, 2010. Nominations should be made by the student's advisor and the deadline for submission is Feb 1, 2011.

Entries will be assessed by an international committee consisting of Michele Benzi (Emory U.), James Demmel (UC Berkeley, Chair), Howard Elman (U. Maryland), Volker Mehrmann (TU Berlin), Sabine Van Huffel (Catholic U. Leuven) and Stephen Vavasis (U. Waterloo). Details may be found at https://outreach.scidac.gov/HH11/index.html.

IMAGE Problem Corner: Old Problems With Solutions

We present solutions to Problems 44-1 through 44-7. Seven new problems are on the back cover; solutions are invited.

Problem 44-1: Constrained Characterization of Hermitianness
Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

It is known that every Hermitian matrix is necessarily EP, i.e., the following implication holds for every $A \in \mathbb{C}_{n,n}$

$$A^* = A \;\Rightarrow\; \mathcal{R}(A^*) = \mathcal{R}(A), \tag{1}$$

where $\mathcal{R}(\cdot)$ stands for the column space of a matrix argument and A^* denotes the conjugate transpose of A. Suppose now that A is EP. Find a nontrivial condition involving A and A^* which ensures that A is Hermitian.

Solution 44-1.1 by the proposers Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

A solution is established in the following proposition.

PROPOSITION. Let $A \in \mathbb{C}_{n,n}$. Then

$$A^* = A \;\Leftrightarrow\; \mathcal{R}(A^*) = \mathcal{R}(A),\; AAA^* = AA^*A^*.$$

PROOF. From Corollary 6 in Hartwig & Spindelböck (1984) it is known that any matrix $A \in \mathbb{C}_{n,n}$ of rank r can be represented as

$$A = U \begin{pmatrix} \Sigma K & \Sigma L \\ 0 & 0 \end{pmatrix} U^*, \tag{2}$$

where $U \in \mathbb{C}_{n,n}$ is unitary, $\Sigma = \mathrm{diag}(\sigma_1 I_{r_1}, ..., \sigma_t I_{r_t})$ is the diagonal matrix of singular values of A, $\sigma_1 > \sigma_2 > \cdots > \sigma_t > 0$, $r_1 + r_2 + \cdots + r_t = r$, and $K \in \mathbb{C}_{r,r}$, $L \in \mathbb{C}_{r,n-r}$ satisfy $KK^* + LL^* = I_r$.

It was pointed out by Baksalary and Trenkler (2008) that A of the form (2) is Hermitian if and only if $L = 0$ and $\Sigma K = K^*\Sigma$, whereas is EP if and only if $L = 0$ (which confirms the validity of implication (1)). Direct calculations show that $AAA^* = AA^*A^*$ is equivalent to $K\Sigma = \Sigma K^*$. Taking into account that $L = 0$ ensures that K is nonsingular and satisfies $K^* = K^{-1}$, we arrive at the conclusion that within the class of EP matrices $A^* = A$ holds if and only if $AAA^* = AA^*A^*$. □

Reference

[1] O.M. Baksalary & G. Trenkler (2008). Characterizations of EP, normal, and Hermitian matrices. *Linear and Multilinear Algebra* **56**, 299-304.
[2] R.E. Hartwig & K. Spindelböck (1984). Matrices for which A^* and $A^\dagger$ commute. *Linear and Multilinear Algebra* **14**, 241-256.

Solution 44-1.2 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

We first show that $A = A^*$ *is equivalent to* $AA^* = A^2$. Evidently, $A = A^* \Rightarrow AA^* = A^2$. Conversely, let A be such that $AA^* = A^2$. Then, $AA^* = A^2 = (AA^*)^* = (A^*)^2$. Hence, $\mathcal{R}(A) = \mathcal{R}(AA^*) = \mathcal{R}((A^*)^2) \subseteq \mathcal{R}(A^*)$, and so, in virtue of $\mathrm{rank}(A) = \mathrm{rank}(A^*)$, $\mathcal{R}(A) = \mathcal{R}(A^*)$, i.e., A is EP. Consequently, $\mathcal{R}(A^* - A) \subseteq \mathcal{R}(A^*)$. It is a well-known fact that the range of A^* has only the origin in common with the nullspace of A, i.e., $\mathcal{R}(A^*) \cap \mathcal{N}(A) = \{0\}$. Because $AA^* = A^2$ is equivalent to $A(A^* - A) = 0$, it is now clear that $AA^* = A^2$ implies that $A^* - A = 0$, i.e., A is Hermitian.

It's worth mentioning that $A = A^*$ can actually be characterized in myriads of different manners such as the following one: *A is Hermitian if and only if A is* EP *and A^2A^* is Hermitian.* The *'only if'* part is straightforward. To prove the *'if'* part, let the EP-matrix A be such that $AAA^* = AA^*A^*$. Multiplying both sides of this equation on the left by the Moore-Penrose inverse $A^\dagger$ of A and observing that $A^\dagger A$ is the orthogonal projector onto $\mathcal{R}(A^*)$ produces $AA^* = A^*A^*$ or, equivalently, $AA^* = A^2$, so that $A = A^*$ follows according to our first characterization of $A = A^*$.

Problem 44-2: Sum of Entries of Inverses of Submatrices
Proposed by Ravindra Bapat, *Indian Statistical Institute, New Delhi, India,* rbbapat@rediffmail.com

Let A be an $n \times n$, nonsingular, doubly stochastic matrix and let $B = A^{-1}$. Let S and T be nonempty, proper subsets of $\{1, \ldots, n\}$ of equal cardinality. Let X be the submatrix of A formed by taking the rows indexed by S and the columns indexed by T, and let Y be the submatrix of B formed by excluding the rows indexed by T and the columns indexed by S. Suppose X and Y are nonsingular. Find the sum of all the entries of X^{-1} and Y^{-1}.

Solution 44-2.1 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

The answer is n. Let P and Q be $n \times n$ permutation matrices such that the matrix PAQ has the rows of A indexed by S as its initial rows and the columns of A indexed by T has its initial columns. Then the matrix

$$(PAQ)^{-1} = Q^{-1}A^{-1}P^{-1} = Q^{\mathsf{T}}BP^{\mathsf{T}}$$

has the rows of B indexed by $\{1, \ldots, n\} - T$ as its final rows and the columns of B indexed by $\{1, \ldots, n\} - S$ as its final columns. Furthermore, PAQ is also nonsingular and doubly stochastic. We may therefore simply use the names A and B to refer to PAQ and $(PAQ)^{-1}$, respectively. Partition A and B in accordance with their submatrices X and Y:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

where $A_{11} = X$ is $k \times k$ and $B_{22} = Y$ is $n-k \times n-k$. Let $\mathbf{z}_m$ denote the column m-vector of ones. The sum of the entries of X^{-1} plus those of Y^{-1} may then be written as

$$\mathbf{z}_k^{\mathsf{T}} X^{-1} \mathbf{z}_k + \mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1} \mathbf{z}_{n-k} \tag{3}$$

Also, the statement that A is doubly stochastic may be written as $A\mathbf{z}_n = \mathbf{z}_n$ and $\mathbf{z}_n^{\mathsf{T}} A = \mathbf{z}_n^{\mathsf{T}}$. Using the partitioned form of A, these two equations become

$$A_{11}\mathbf{z}_k + A_{12}\mathbf{z}_{n-k} = \mathbf{z}_k \qquad A_{21}\mathbf{z}_k + A_{22}\mathbf{z}_{n-k} = \mathbf{z}_{n-k} \tag{4}$$

$$\mathbf{z}_k^{\mathsf{T}} A_{11} + \mathbf{z}_{n-k}^{\mathsf{T}} A_{21} = \mathbf{z}_k^{\mathsf{T}} \qquad \mathbf{z}_k^{\mathsf{T}} A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} A_{22} = \mathbf{z}_{n-k}^{\mathsf{T}} \tag{5}$$

Similarly, the single equation $BA = I_n$ becomes

$$B_{11}A_{11} + B_{12}A_{21} = I_k \qquad B_{11}A_{12} + B_{12}A_{22} = O_{k,n-k} \tag{6}$$

$$B_{21}A_{11} + B_{22}A_{21} = O_{n-k,k} \qquad B_{21}A_{12} + B_{22}A_{22} = I_{n-k} \tag{7}$$

To find the value of (3), we multiply the second of equations (7) on the left by $\mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1}$, then apply the second of equations (5), then use the first of equations (7) in the form $Y^{-1}B_{21} = -A_{21}X^{-1}$, and then apply the first of equations (5) in the form $-\mathbf{z}_{n-k}^{\mathsf{T}} A_{21} X^{-1} = \mathbf{z}_k^{\mathsf{T}} - \mathbf{z}_k^{\mathsf{T}} X^{-1}$:

$$\begin{aligned} \mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1} &= \mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1} B_{21} A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} A_{22} \\ &= \mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1} B_{21} A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} - \mathbf{z}_k^{\mathsf{T}} A_{12} \\ &= -\mathbf{z}_{n-k}^{\mathsf{T}} A_{21} X^{-1} A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} - \mathbf{z}_k^{\mathsf{T}} A_{12} \\ &= \left(\mathbf{z}_k^{\mathsf{T}} - \mathbf{z}_k^{\mathsf{T}} X^{-1}\right) A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} - \mathbf{z}_k^{\mathsf{T}} A_{12} \\ &= -\mathbf{z}_k^{\mathsf{T}} X^{-1} A_{12} + \mathbf{z}_{n-k}^{\mathsf{T}} \end{aligned}$$

Finally, multiply this result on the right by $\mathbf{z}_{n-k}$ and add $\mathbf{z}_k^{\mathsf{T}} X^{-1} \mathbf{z}_k$, and then apply the first of equations (4) in the form $-X^{-1}A_{12}\mathbf{z}_{n-k} = \mathbf{z}_k - X^{-1}\mathbf{z}_k$:

$$\begin{aligned} \mathbf{z}_k^{\mathsf{T}} X^{-1} \mathbf{z}_k + \mathbf{z}_{n-k}^{\mathsf{T}} Y^{-1} \mathbf{z}_{n-k} &= \mathbf{z}_k^{\mathsf{T}} X^{-1} \mathbf{z}_k - \mathbf{z}_k^{\mathsf{T}} X^{-1} A_{12} \mathbf{z}_{n-k} + \mathbf{z}_{n-k}^{\mathsf{T}} \mathbf{z}_{n-k} \\ &= \mathbf{z}_k^{\mathsf{T}} X^{-1} \mathbf{z}_k + \mathbf{z}_k^{\mathsf{T}} \left(\mathbf{z}_k - X^{-1}\mathbf{z}_k\right) + n - k \\ &= \mathbf{z}_k^{\mathsf{T}} \mathbf{z}_k + n - k = k + n - k = n. \end{aligned}$$

Solution 44-2.2 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

Permuting the rows and/or columns of a doubly stochastic nonsingular $n \times n$ matrix always produces a matrix of the same kind. Therefore, without loss of generality, let $S = T = \{1, \ldots, n_1\}$ be a proper subset of $\{1, \ldots, n\}$ such that X and Y are nonsingular. Consider the block partitioning

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where A_{11} is the submatrix of A formed by taking the rows and columns indexed by S, i.e., $X = A_{11}$. Put

$$Q := \begin{pmatrix} I_{n_1} & 0 \\ -A_{21}A_{11}^{-1} & I_{n_2} \end{pmatrix} \quad \text{and} \quad P := \begin{pmatrix} I_{n_1} & -A_{11}^{-1}A_{12} \\ 0 & I_{n_2} \end{pmatrix},$$

with I_{n_1} and I_{n_2} denoting the identity matrix of order n_1 and $n_2 := n - n_1$, respectively. Observe that

$$QAP = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} - A_{21}A_{11}^{-1}A_{12} \end{pmatrix}.$$

Then

$$\begin{aligned} A^{-1} &= P \begin{pmatrix} A_{11}^{-1} & 0 \\ 0 & (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1} \end{pmatrix} Q \\ &= \begin{pmatrix} A_{11}^{-1} + A_{11}^{-1}A_{12}(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}A_{21}A_{11}^{-1} & -A_{11}^{-1}A_{12}(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1} \\ -(A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}A_{21}A_{11}^{-1} & (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1} \end{pmatrix}, \end{aligned}$$

thus showing that the submatrix Y of A^{-1} formed by excluding the rows and columns indexed by $S(= T)$ is given by

$$Y = (A_{22} - A_{21}A_{11}^{-1}A_{12})^{-1}.$$

For solving this problem it thus suffices to determine

$$\iota_1' A_{11}^{-1} \iota_1 + \iota_2'(A_{22} - A_{21}A_{11}^{-1}A_{12})\iota_2,$$

where ι_1 and ι_2 are suitable vectors with all entries equal to unity. Because A is a doubly stochastic matrix, i.e., $A\iota = \iota$ and $\iota' A = \iota'$ (here ι is the vector from $\mathbb{R}^n$ whose entries are all equal to unity), clearly $A_{12}\iota_2 = (I_{n_1} - A_{11})\iota_1$, $\iota_2' A_{21} = \iota_1'(I_{n_1} - A_{11})$, and $\iota_1' A_{12}\iota_2 + \iota_2' A_{22}\iota_2 = (\iota_1' A_{12} + \iota_2' A_{22})\iota_2 = \iota_2'\iota_2$. Therefore,

$$\begin{aligned} \iota_1' A_{11}^{-1}\iota_1 + \iota_2'(A_{22} - A_{21}A_{11}^{-1}A_{12})\iota_2 &= \iota_1' A_{11}^{-1}\iota_1 + \iota_2' A_{22}\iota_2 - \iota_2' A_{21}A_{11}^{-1}A_{12}\iota_2 \\ &= \iota_1' A_{11}^{-1}\iota_1 + \iota_2' A_{22}\iota_2 - \iota_1'(I_{n_1} - A_{11})A_{11}^{-1}(I_{n_1} - A_{11})\iota_1 \\ &= \iota_1' A_{11}^{-1}\iota_1 + \iota_2' A_{22}\iota_2 + \iota_1'(2I_{n_1} - A_{11} - A_{11}^{-1})\iota_1 \\ &= \iota_2' A_{22}\iota_2 + 2n_1 - \iota_1' A_{11}\iota_1 \\ &= n_1 + \iota_1'(A_{11}, A_{12}) \begin{pmatrix} \iota_1 \\ \iota_2 \end{pmatrix} - \iota_1' A_{11}\iota_1 + \iota_2' A_{22}\iota_2 \\ &= n_1 + \iota_2'\iota_2 = n_1 + n_2 = n. \end{aligned}$$

In other words, the sum of all entries of X^{-1} and Y^{-1} always coincides with the order of the given doubly stochastic nonsingular matrix A.

Also solved by the proposer.

Problem 44-3: Curiously Commuting Vectors

Proposed by Adam J. Brzezinski, *University of Michigan, Ann Arbor, USA*
Eva Wu, *State University of New York, Binghamton, USA*
and Dennis S. Bernstein, *University of Michigan, Ann Arbor, USA*, dsbaero@umich.edu

Let $a_1, a_2, \ldots, a_n \in \mathbb{C}$, let $b,\ c \in \mathbb{C}^n$, and let $A = \begin{bmatrix} 0 & a_1 \\ I_{n-1} & \alpha \end{bmatrix}$, where $\alpha = (a_2, \ldots, a_n)^t$ (here ^{t} is for transpose). Show that

$$\begin{bmatrix} b & Ab & \cdots & A^{n-1}b \end{bmatrix} c = \begin{bmatrix} c & Ac & \cdots & A^{n-1}c \end{bmatrix} b.$$

Solution 44-3.1 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

Denoting the standard unit basis vectors in $\mathbb{C}^n$ by $e_1, e_2, \ldots, e_n$, we see that $Ae_j = e_{j+1}$ for $j = 1, 2, \ldots, n-1$. Hence

$$A^k e_j = e_{j+k} \quad \text{whenever } j + k \le n \tag{8}$$

We write the desired identity in the equivalent form

$$\sum_{k=1}^{n} A^{k-1}(c_k b - b_k c) = 0, \text{ where } (b_1, \ldots, b_n)^{\mathrm{T}} = b,\ (c_1, \ldots, c_n)^{\mathrm{T}} = c \tag{9}$$

Since the left side of (9) is linear in both b and c, it suffices to prove (9) when $c = e_j, b = e_m, 1 \le j, m \le n$. We may assume $j < m$ by symmetry and since (9) is clearly true whenever $b = c$. Thus,

$$c_k b - b_k c = e_m \text{ if } k = j,\ \ -e_j \text{ if } k = m, \text{ and } 0 \text{ otherwise}$$

Therefore, by using equations (8), we have

$$\sum_{k=1}^{n} A^{k-1}(c_k b - b_k c) = A^{j-1}e_m - A^{m-1}e_j = A^{j-1}e_m - A^{j-1}A^{m-j}e_j = A^{j-1}e_m - A^{j-1}e_m = 0.$$

Solution 44-3.2 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Let $c = (c_1, c_2, \cdots, c_n)^t$. Then the left hand side becomes $c_1 b + c_2 Ab + \cdots + c_n A^{n-1} b = (c_1 I + c_2 A + \cdots + c_n A^{n-1})b$. So it suffices to show that the kth ($1 \le k \le n$) column of $\sum_{i=1}^{n} c_i A^{i-1}$ is just $A^{k-1}c$, or

$$\sum_{i=1}^{n} c_i A^{i-1} e_k = A^{k-1}c, \tag{10}$$

where e_k denotes the kth column of the identity matrix of size n. We shall use induction to prove (10). It ia easy to see $Ae_m = e_{m+1}$ for $1 \le m \le n-1$. So when $k = 1$, (10) is true. Suppose (10) is true for $k = m$ ($1 \le m \le n-1$), i.e.,

$$\sum_{i=1}^{n} c_i A^{i-1} e_m = A^{m-1}c,$$

then pre-multiplying it by A, we obtain

$$\sum_{i=1}^{n} c_i A^{i-1} e_{m+1} = A^{m}c,$$

which means (10) is true for all $1 \le k \le n$. This completes the proof.

Also solved by the proposers.

Problem 44-4: Square-nilpotent Matrix

Proposed by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

Let A be a square complex matrix. If $A^2 = 0$ and A has rank r, show that $A + A^*$ has rank $2r$ and that half of its nonzero eigenvalues are positive and the other half are negative.

Solution 44-4.1 by Johanns de Andrade Bezerra, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let $A + A^* = X$. Then $(A + A^*)(A + A^*) = X^2$, and so $AA^* + A^*A = X^2$. Since $AA^*A^*A = A^*AAA^* = 0$, it follows that there exists $U \in \mathbf{C}^{n\times n}$ such that $UU^* = I$, $UAA^*U^* = diag(A_1, A_2)$ and $UA^*AU^* = diag(B_1, B_2)$, where $A_1, B_1 \in \mathbf{C}^{r\times r}$, A_1, A_2, B_1 and B_2 are diagonal matrices, with A_1 nonsingular and $A_2 = 0$, since if $Y \in \mathbf{C}^{n\times n}$, then $rankY = rankYY^*$. Suppose that $A_1B_1 \ne 0$, then $UAA^*U^*UA^*AU^* \ne 0$, but this is a contradiction, so $B_1 = 0$ and $rankB_2 = r$. Hence, $rankUX^2U^* = rankX^2 = rankXX^* = rankX = 2r$. Moreover, let $A_1 = diag(\lambda_1, \lambda_2, \ldots, \lambda_r)$, then $B_2 = diag(0, \ldots, 0, \lambda_1, \lambda_2, \ldots, \lambda_r, 0, \ldots, 0)$ since AA^* and A^*A have the same spectrum. Let $\alpha_1, \ldots, \alpha_r, \beta_1, \ldots, \beta_r$ be the nonzero eigenvalues of the Hermitian matrix X, then $\alpha_1^2, \ldots, \alpha_r^2, \beta_1^2, \ldots, \beta_r^2$ are the nonzero eigenvalues of X^2, and so $\alpha_1^2 = \lambda_1, \ldots, \alpha_r^2 = \lambda_r, \beta_1^2 = \lambda_1, \ldots, \beta_r^2 = \lambda_r$, which implies $\pm\alpha_i = \beta_i$, $1 \le i \le r$. Since $TrX = TrA + TrA^* = 0$, it follows that $\alpha_1 + \cdots + \alpha_r + \beta_1 + \cdots + \beta_r = 0$, hence $\beta_i = -\alpha_i$.

Solution 44-4.2 by Jan Hauke, *Adam Mickiewicz University, Poznań, Poland,* jhauke@amu.edu.pl

Let $\mathbf{A}$ be a square $n\times n$ complex matrix of rank r. For SVD of $\mathbf{A} = \mathbf{UDV}^*$, with unitary matrices $\mathbf{U}$ and $\mathbf{V}$ partitioned as follows: $\mathbf{U} = [\mathbf{U}_1, \mathbf{U}_2]$, $\mathbf{V} = [\mathbf{V}_1, \mathbf{V}_2]$, where matrices $\mathbf{U}_1$ and $\mathbf{V}_1$ are $n \times r$ matrices, we obtain

$$\mathbf{A} = \mathbf{UDV}^* = \mathbf{U}_1\mathbf{D}_r\mathbf{V}_1^* = \mathbf{U}\begin{pmatrix} \mathbf{D}_r\mathbf{W}_1 & \mathbf{D}_r\mathbf{W}_2 \\ \mathbf{0} & \mathbf{0} \end{pmatrix}\mathbf{U}^*, \tag{11}$$

where

$$\mathbf{D} = \begin{pmatrix} \mathbf{D}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad \mathbf{V}^*\mathbf{U} = \begin{pmatrix} \mathbf{W}_1 & \mathbf{W}_2 \\ \mathbf{W}_3 & \mathbf{W}_4 \end{pmatrix},$$

with diagonal $r \times r$ matrix $\mathbf{D}_r$ having on diagonal square roots of nonzero eigenvalues of $\mathbf{AA}^*$, $\mathbf{W}_1 = \mathbf{V}_1^*\mathbf{U}_1$, and $\mathbf{W}_2 = \mathbf{V}_1^*\mathbf{U}_2$.
OBSERVATION 1. Let $\mathbf{A}^2 = \mathbf{0}$. Then

$$\mathbf{U}_1\mathbf{D}_r\mathbf{V}_1^*\mathbf{U}_1\mathbf{D}_r\mathbf{V}_1^* = \mathbf{0} \Longleftrightarrow \mathbf{V}_1^*\mathbf{U}_1 = \mathbf{W}_1 = \mathbf{0},$$

which [by (11)] implies

$$\mathbf{A} + \mathbf{A}^* = \mathbf{U}\begin{pmatrix} \mathbf{0} & \mathbf{D}_r\mathbf{W}_2 \\ \mathbf{W}_2^*\mathbf{D}_r & \mathbf{0} \end{pmatrix}\mathbf{U}^*. \tag{12}$$

OBSERVATION 2. [see Fact 5.8.20 of Bernstein (2010)] If rank of an $n \times n$ matrix $\mathbf{B} = \begin{pmatrix} \mathbf{0} & \mathbf{C} \\ \mathbf{C}^* & \mathbf{0} \end{pmatrix}$ is equal to r, then the matrix $\mathbf{B}$ has r positive eigenvalues and r negative eigenvalues.

REMARK 1. If rank of an $n \times n$ matrix $\mathbf{A}$ is equal to r and $\mathbf{A}^2 = \mathbf{0}$, then [by (12)] the matrix $\mathbf{A} + \mathbf{A}^*$ has r positive eigenvalues and r negative eigenvalues. Moreover, $rank(\mathbf{A} + \mathbf{A}^*) = 2r$, $rank(\mathbf{A}) \leq \frac{1}{2}n$ for even n, and $rank(\mathbf{A}) < \frac{1}{2}n$ for odd n.

REMARK 2. Remark 1 is not true if $\mathbf{A}^k = \mathbf{0}$ for $k > 2$. The matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ shows that $\mathbf{A}^3 = \mathbf{0}$, but the matrix $\mathbf{A} + \mathbf{A}^*$ has one negative and two positive eigenvalues [$\lambda_1 = -2,\ \lambda_2 = \lambda_3 = 1, rank(\mathbf{A}) = 3$].

Reference D.S. Bernstein (2010). Matrix Mathematics. Theory, Facts and Formulas. Princeton University Press, Princeton, USA.

Solution 44-4.3 by Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu

Let A be an n-by-n complex matrix and suppose that $\operatorname{rank} A = r$. If A^2 is Hermitian, then A is unitarily similar to a direct sum of blocks, each of which is [1, Theorem 8.12]

$$[\lambda],\ [i\lambda],\ \text{or}\ \tau\begin{bmatrix} 0 & 1 \\ \mu & 0 \end{bmatrix},\ \lambda, \mu, \tau \in \mathbb{R}, \tau > 0,\ -1 < \mu < 1.$$

If the square of each of these blocks is zero, then $\lambda = \mu = 0$, so A is unitarily similar to

$$0_{n-r} \oplus \tau_1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \oplus \cdots \oplus \tau_r\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \tau_1, \ldots, \tau_r > 0$$

and $A + A^*$ is unitarily similar to

$$0_{n-r} \oplus \tau_1\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \oplus \cdots \oplus \tau_r\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tau_1, \ldots, \tau_r > 0,$$

which has rank $2r$ and eigenvalues $\pm\tau_1, \ldots, \pm\tau_r$.

Reference

[1] R. A. Horn and V. V. Sergeichuk, Canonical forms for unitary congruence and *congruence, *Linear Multilinear Algebra* 57 (2009) 777-815.

Solution 44-4.4 by Jakub Kierzkowski, *Warsaw University of Technology, Poland*, J.Kierzkowski@mini.pw.edu.pl

Let $A \in \mathcal{M}_n$ be nilpotent of rank r. Define hermitian matrix $H = A + A^*$. It is known [1, Theorem 1.2] that A can be written in the Schur form

$$A = QRQ^*, \;\; R = \begin{bmatrix} 0_k & B \\ 0_{(n-k)\times k} & 0_{(n-k)} \end{bmatrix}, \tag{13}$$

where $Q \in \mathcal{M}_n$ is unitary and B is $k \times (n-k)$.
Since $rank(A) = rank(R) = rank(B)$ we have $r \le \min(k, n-k)$.

From (13) we obtain $H = Q(R + R^*)Q^* = QSQ^*$, where $S = \begin{bmatrix} 0 & B \\ B^* & 0 \end{bmatrix}$. Notice that

$$S = S_1 + S_2, \;\; S_1 = \begin{bmatrix} 0 & B \\ 0 & 0 \end{bmatrix}, \;\; S_2 = \begin{bmatrix} 0 & 0 \\ B^* & 0 \end{bmatrix},$$

hence $rank(H) = rank(S) \le rank(S_1) + rank(S_2) = 2r$.

Let $B = U\Sigma V^*$ be the SVD of B, where $U \in \mathcal{M}_k$ and $V \in \mathcal{M}_{n-k}$ are unitary and Σ is a k-by-$n-k$ real "diagonal" matrix, whose diagonal elements are the singular values of B: $\sigma_1 \ge \sigma_2 \ge \ldots \ge \sigma_r > 0$.
Let $U = [u_1, \ldots, u_r, \ldots, u_k]$ and $V = [v_1, \ldots, v_r, \ldots, v_{n-k}]$. The columns u_i of U are called the left singular vectors and the columns v_i of V the right singular vectors of B. They satisfy the relations $Bv_i = \sigma_i\, u_i$, $B^*u_i = \sigma_i\, v_i$ for $i = 1, \ldots, r$, hence for all $i = 1, \ldots, r$ we get

$$\begin{bmatrix} 0 & B \\ B^* & 0 \end{bmatrix} \cdot \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} Bv_i \\ B^*u_i \end{bmatrix} = \sigma_i \begin{bmatrix} u_i \\ v_i \end{bmatrix}$$

and

$$\begin{bmatrix} 0 & B \\ B^* & 0 \end{bmatrix} \cdot \begin{bmatrix} u_i \\ -v_i \end{bmatrix} = \begin{bmatrix} -Bv_i \\ B^*u_i \end{bmatrix} = -\sigma_i \begin{bmatrix} u_i \\ -v_i \end{bmatrix}.$$

This means, that the $2r$ eigenvalues of H are $\pm\sigma_i$ with corresponding unit eigenvectors $\frac{\sqrt{2}}{2}\begin{bmatrix} u_i \\ \pm v_i \end{bmatrix}$, $i = 1, \ldots, r$. Hence $rank(H) = 2r$ and the remaining eigenvalues of H are 0.

Remark. In numerical analysis the matrix S is called Golub's matrix and it can be used in computing singular values of matrices, see [2, p. 86].

Reference

[1] M. Aleksiejczyk, A. Smoktunowicz, *On properties of quadratic matrices*, Mathematica Pannonica 11/2 (2000), 239-248.
[2] Å. Björck, Numerical methods for least squares problems, SIAM Philadelphia 1996.

Solution 44-4.5 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Since $A^2 = 0$, we have $range(A) \subseteq ker(A)$. So $range(A) \bigcap (ker(A))^\perp = \{0\}$, i.e., $range(A) \bigcap range(A^*) = \{0\}$. Therefore, $rank(A + A^*) = rankA + rankA^* = 2\, rankA = 2r$.
Let λ be a nonzero eigenvalue of $A + A^*$ with eigenvector x. Then

$$(A + A^*)x = \lambda x. \tag{14}$$

Pre-multiplying (14) by A and A^*, respectively, we have $AA^* = \lambda Ax$ and $A^*A = \lambda A^*x$ (Note that $(A^*)^2 = 0$). It follows that $(A + A^*)(Ax - A^*x) = -AA^*x + A^*Ax = -\lambda(Ax - A^*x)$. If $Ax - A^*x = 0$, then $2Ax = \lambda x$, which is impossible since the eigenvalues of A are zero. Therefore $-\lambda$ is also an eigenvalue of $A + A^*$.

Also solved by Oskar Maria Baksalary, Götz Trenkler, Hans Joachim Werner and the proposer.

Problem 44-5: Matrix Congruence

Proposed by Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu

Let A be a given square complex matrix. Show that A is congruent (respectively, *-congruent) to $\bar{A}$ if and only if A is congruent (respectively, *-congruent) to a real matrix.

Solution 44-5 by the proposer Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu

Let $A \in M_n(\mathbb{C})$. Let $J_k(\mu) \in M_k(\mathbb{C})$ denote a Jordan block with eigenvalue μ and define $H_{2k}(\mu) = \begin{bmatrix} 0_k & I_k \\ J_k(\mu) & 0_k \end{bmatrix}$. The matrix A is congruent to a *congruence canonical form*, which may be expressed as a direct sum of blocks of the three types

$$J_k(0),\ \Gamma_k,\ \text{and } H_{2k}(\mu)$$

in which μ is complex and $0 \neq \mu \neq (-1)^{k+1}$; the blocks $\Gamma_k \in M_k(\mathbb{R})$ are certain *real* matrices whose structure is not relevant to the problem at hand. This congruence canonical form is unique up to permutation of its direct summands and replacement of μ by μ^{-1}. [1, Theorem 1.1(a)] If A and B are nonsingular, then A is congruent to B if and only if $A^{-T}A$ is similar to $B^{-T}B$. [1, Lemma 2.1]

If $A = SRS^T$ for some nonsingular S and some real R, then $\bar{A} = \bar{S}R\bar{S}^T$ and $A = (S\bar{S}^{-1})\bar{A}(S\bar{S}^{-1})^T$ is evidently congruent to its conjugate. A similar calculation shows that if A is *congruent to a real matrix, then it is *congruent to its conjugate.

To prove that A is congruent to a real matrix if it is congruent to its conjugate, one can proceed as follows:

1. Let $S = \frac{e^{i\pi/4}}{\sqrt{2}} \begin{bmatrix} 0_k & I_k \\ -iI_k & 0_k \end{bmatrix}$, let $B \in M_k(\mathbb{C})$, and let $C = B \oplus \bar{B}$. Compute SCS^T and SCS^*; both are real.
2. Show that $H_{2k}(\mu)$ is congruent to its conjugate if and only if either μ is real or $|\mu| = 1$. *Hint:* $H_{2k}(\mu)^{-T}H_{2k}(\mu)$ is similar to $J_k(\mu) \oplus J_k(\mu^{-1})$.
3. If A is congruent to $\bar{A}$, then the congruence canonical form of A is identical to its conjugate, after permuting direct summands and replacement of some summands $H_{2k}(\mu)$ with $|\mu| = 1$ by $H_{2k}(\mu^{-1})$.
4. If A is congruent to $\bar{A}$ then the congruence canonical form of A is a direct sum of (a) real blocks of the form $J_k(0)$, Γ_k, or $H_{2k}(r)$ in which r is real and either $r = (-1)^k$ or $|r| > 1$; (b) blocks of the form $H_{2k}(\mu)$ in which $|\mu| = 1$, $\mu \neq \pm 1$, and μ is determined up to replacement by $\bar{\mu}$; and (c) pairs of blocks of the form $H_{2k}(\mu) \oplus H_{2k}(\bar{\mu})$ in which μ is not real and $|\mu| > 1$. Uniqueness of the congruence canonical form is essential to this argument.
5. Combine (4) with (1) and conclude that A is congruent to a real matrix.

The matrix A is *congruent to a **congruence canonical form*, which may be expressed as a direct sum of blocks of the three types

$$J_k(0),\ \lambda\Gamma_k,\ \text{and } H_{2k}(\mu)$$

in which λ and μ are complex, $|\lambda| = 1$, and $|\mu| > 1$. This *congruence canonical form is unique up to permutation of its direct summands. [1, Theorem 1.1(b)]

If A is *congruent to $\bar{A}$, one can use uniqueness of the *congruence canonical form to show that the *congruence canonical form of A is a direct sum of (a) real blocks of the form $J_k(0)$, $\pm\Gamma_k$, or $H_{2k}(r)$ in which r is real and $|r| > 1$; (b) pairs of blocks of the form $\lambda\Gamma_k \oplus \bar{\lambda}\Gamma_k$ in which $|\lambda| = 1$ and $\lambda \neq \pm 1$, or of the form $H_{2k}(\mu) \oplus H_{2k}(\bar{\mu})$ in which μ is not real and $|\mu| > 1$. Invoke (1) to conclude that A is *congruent to a real matrix.

Reference

[1]. R. A. Horn and V. V. Sergeichuk, Canonical forms for complex matrix congruence and *congruence, *Linear Algebra Appl.* 416 (2006) 1010-1032.

Note: This problem is taken from Section 4.4 of the second edition of R. A. Horn and C. R. Johnson, Matrix Analysis.

Problem 44-6: Unitary Matrix Sum

Proposed by Dennis Merino, *Southeastern Louisiana University, Hammond, USA,* dmerino@selu.edu

Let $\alpha \in \mathbb{C}$ and a unitary $U \in M_n$ be given. Find the minimum positive integer k (depending on α) so that there exist unitary matrices $U_1, \ldots, U_k$ such that $U_1 + \cdots + U_k = \alpha U$. For instance, if $\alpha = 0$, then $k = 2$.

Solution 44-6.1 by by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

We will show that

$$k(\alpha) = 1 \text{ if } |\alpha| = 1;\quad k(\alpha) = 2 \text{ if } |\alpha| < 1;\quad k(\alpha) = n \text{ if } n-1 < |\alpha| \leq n \text{ for } n = 2, 3, 4, \ldots$$

Note that $k(\alpha) = k(|\alpha|)$, since $\alpha U = |\alpha|e^{i\arg(\alpha)}U$ and $e^{i\arg(\alpha)}U$ is unitary. So, from now on, we may assume $\alpha \geq 0$. Under that restriction, $k(\alpha) = 1$ if and only $\alpha = 1$, so we also assume $\alpha \neq 1$. Let $n = \lceil \alpha \rceil$. We claim that $k(\alpha) \geq n$. For, if $k(\alpha) \leq n-1$, we have the following contradiction, based on the fact that the L_2-norm of a unitary matrix is 1:

$$n - 1 < \alpha = \|\alpha U\| = \|U_1 + \cdots + U_{k(\alpha)}\| \leq \|U_1\| + \cdots + \|U_{k(\alpha)}\| = k(\alpha) \leq n - 1$$

Since any unitary matrix is unitarily similar to a diagonal matrix, we may assume that $U = \text{diag}(e^{i\theta_1}, \ldots, e^{i\theta_m})$, where U is $m \times m$. The key case is when $\alpha \in [0,1) \cup (1,2]$. Then $\alpha U = U_1 + U_2$, where $U_1 = [a_{jk}]_{j,k=1}^m$ and

$$a_{jk} = \begin{cases} e^{i\theta_k}\frac{1}{2}(\alpha - (m-2)z) & \text{when } j = k \\ e^{i\theta_k} z & \text{when } j \neq k \end{cases}, \quad z = \frac{i}{m}\sqrt{4-\alpha^2}$$

It is straightforward to check that $U_1U_1^* = I_m$ and $(\alpha U - U_1)(\alpha U - U_1)^* = I_m$, and so $k(\alpha) = 2$. The remaining case is when $n = \lceil \alpha \rceil \geq 3$. Then $\alpha U = (n-2)U + (\alpha - n + 2)U$ and $1 < \alpha - n + 2 \leq 2$. So we can apply the case $\alpha \in (1,2]$ to $(\alpha - n + 2)U$, which is therefore a sum of two unitary matrices. Since $(n-2)U$ is a sum of $n-2$ unitary matrices (all equal to U), we have shown that $k(\alpha) = n$.

Solution 44-6.2 by the proposer Dennis Merino, *Southeastern Louisiana University, Hammond, USA,* dmerino@selu.edu

First, notice that $U_1 + \cdots + U_k = \alpha U$ if and only if $W_1 + \cdots + W_k = \alpha I$, where each $W_i = U_i U^*$ is unitary. We show (i) $k \geq |\alpha|$; and (ii) if $t \geq 2$ is an integer such that $|\alpha| \leq t$, then αI can be written as a sum of t unitary matrices.

Suppose that $W_1 + \cdots + W_k = \alpha I$, where each W_i is unitary. Write $W_i = \left[w_{lm}^{(i)}\right]$, and notice that $w_{11}^{(1)} + \cdots + w_{11}^{(k)} = \alpha$, so that $|\alpha| = \left|w_{11}^{(1)} + \cdots + w_{11}^{(k)}\right| \leq \left|w_{11}^{(1)}\right| + \cdots + \left|w_{11}^{(k)}\right| \leq k$, as each $\left|w_{11}^{(i)}\right| \leq 1$ because W_i is unitary.

Now, notice that for $\theta \in \mathbb{R}$, the function $f(\theta) = e^{i\theta}$ has the unit circle $\{z \in \mathbb{C} : |z| = 1\}$ for its range. Let $m \geq 2$ be an integer. Set $\mathcal{A}_m \equiv \{z \in \mathbb{C} : |z| \leq m\}$. For $\theta_1, ..., \theta_m \in \mathbb{R}$, we define

$$f_m(\theta_1, ..., \theta_m) \equiv e^{i\theta_1} + \cdots + e^{i\theta_m}.$$

We show that the range of f_m is $\mathcal{A}_m$. If $z = re^{i\beta}$, with $r, \beta \in \mathbb{R}$ and $r \geq 0$, then

$$e^{i\theta_1} + \cdots + e^{i\theta_m} = re^{i\beta} \text{ if and only if } e^{i(\theta_1 - \beta)} + \cdots + e^{i(\theta_m - \beta)} = r.$$

Hence, $z = re^{i\beta}$ is the range of f_k if and only if r is in the range of f_k.

Now, let $m = 2$ so that $f_2(\theta_1, \theta_2) = e^{i\theta_1} + e^{i\theta_2}$. Let $0 \leq r \leq 2$ be given. Then, $0 \leq \frac{1}{2}r \leq 1$ and there exists $\alpha \in \mathbb{R}$ so that $\cos\alpha = \frac{1}{2}r$. Choose $\theta_1 = \alpha$ and $\theta_2 = -\alpha$, and notice that $f_2(\theta_1, \theta_2) = r$. Therefore, the range of f_2 is $\mathcal{A}_2$.

Let $m = 3$, and suppose $0 \leq r \leq 3$. Set $\theta_3 = 0$ and set $\theta_1 = \theta = -\theta_2$. Then, $f_3(\theta_1, \theta_2, \theta_3) = 1 + 2\cos\theta$, and θ may be chosen so that $0 \leq r \equiv 1 + 2\cos\theta \leq 3$.

We use mathematical induction to show the general case. The base cases $m = 2$ and $m = 3$ have already been shown. Assume that $m > 3$ and suppose that the range of $f_m(\theta_1, ..., \theta_m)$ is $\mathcal{A}_m$.

Consider $f_{m+1}(\theta_1, ..., \theta_m, \theta_{m+1}) \equiv e^{i\theta_1} + \cdots + e^{i\theta_m} + e^{i\theta_{m+1}}$. Let $z = re^{i\beta}$ be given with $0 \leq r \leq m+1$. We show that r is in the range of f_{m+1}.

First, we show that $\mathcal{A}_2 \subset \text{range}(f_{m+1})$. If m is even, choose $\theta_3 = ... = \theta_{m+1} = 0$ and $\theta_4 = ... = \theta_m = \pi$. Then $f_{m+1}(\theta_1, ..., \theta_m, \theta_{m+1}) = e^{i\theta_1} + e^{i\theta_2}$. If m is odd, choose $\theta_4 = ... = \theta_{m+1} = 0$ and $\theta_5 = ... = \theta_m = \pi$. Then $f_{m+1}(\theta_1, ..., \theta_m, \theta_{m+1}) \equiv e^{i\theta_1} + e^{i\theta_2} + e^{i\theta_3}$. In both cases, notice that $\mathcal{A}_2 \subset \text{range}(f_{m+1})$. Hence, we may assume further that $r \geq 1$; that is we need to show that $r \in \text{range}(f_{m+1})$, for $1 \leq r \leq m+1$.

Choose $\theta_{m+1} = 0$, so that $f_{m+1}(\theta_1, ..., \theta_m, \theta_{m+1}) = f_m(\theta_1, ..., \theta_m) + 1$. Now, the equation $f_m(\theta_1, ..., \theta_m) + 1 = r$ has a solution since $0 \leq r - 1 \leq m$.

Notice now that for integers $p \geq m \geq 2$, we have $\mathcal{A}_m \subset \mathcal{A}_p$. Hence, if $\alpha = f_m(\theta_1, ..., \theta_m)$ has a solution, then for each integer $p \geq m$, the equation $\alpha = f_p(\theta_1, ..., \theta_p)$ has a solution.

Now, we are ready to find k. Suppose that $|\alpha| \leq 2$. Then, there exist $\theta_1, \theta_2 \in \mathbb{R}$ so that $\alpha = e^{i\theta_1} + e^{i\theta_2}$. Notice now that $W_j = e^{i\theta_j}I$ is unitary and $W_1 + W_2 = \alpha I$. Hence, in this case, $k \leq 2$. If $|\alpha| = 1$, then αU is unitary and only 1 unitary matrix is necessary. If $|\alpha| \neq 1$, then αU is not unitary, hence $k \neq 1$ and thus $k = 2$. If $m \geq 2$ is an integer so that $m < |\alpha| \leq m+1$, then $k \geq |\alpha| > m$. Notice now that $\alpha \in \mathcal{A}_{m+1}$, hence $k \leq m+1$, and hence, $k = m+1$.

Problem 44-7: Real Diagonal Entries

Proposed by Edward Poon, *Embry-Riddle Aeronautical University, Prescott, USA,* poon3de@erau.edu

Suppose A is an $n \times n$ (complex) matrix such that its off-diagonal entries are $A_{ij} = (b_i - b_j)^{-1}$, where $b_1, \ldots, b_n$ are distinct real numbers. Suppose all the eigenvalues of A are real. Show that the diagonal entries of A must be real.

Solution 44-7 by the proposer Edward Poon, *Embry-Riddle Aeronautical University, Prescott, USA,* poon3de@erau.edu

This result is precisely Theorem 2 of "Frontiers of reality in Schubert calculus" by Frank Sottile (see *Bulletin of the American Mathematical Society*, Vol 47 No. 1 (Jan 2010), page 48). Details of the proof may be found in the same location.

IMAGE Problem Corner: Problems and Solutions

Problems: We introduce 7 new problems in this issue and invite readers to submit solutions for publication in IMAGE. A problem marked with (°) indicates that no complete solution to the problem is currently available yet. Solutions: We present solutions to all problems in the previous issue [IMAGE 43 (Fall 2009), p. 43]. Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to IMAGE Problem Corner editor Fuzhen Zhang (zhang@nova.edu). The working team of the Problem Corner consists of Dennis S. Bernstein, Nir Cohen, Shaun Fallat, Dennis Merino, Edward Poon, Peter Šemrl, Wasin So, Nung-Sing Sze, and Xingzhi Zhan.

NEW PROBLEMS:

Problem 45-1: Column Space Counterparts Of The Known Conditions For Orthogonal Projectors

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.uni-dortmund.de

Let $\mathbb{C}_n^{\mathsf{OP}}$ denote the set of orthogonal projectors in $\mathbb{C}_{n,1}$ (Hermitian idempotent matrices of order n), i.e.,

$$\mathbb{C}_n^{\mathsf{OP}} = \{K \in \mathbb{C}_{n,n} :\ K^2 = K = K^*\},$$

where K^* means the conjugate transpose of the complex matrix K. It is known that $P, Q \in \mathbb{C}_n^{\mathsf{OP}}$ satisfy:

(i) $PQ \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = QP$,

(ii) $P + Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = 0$,

(iii) $P - Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = Q$.

Provide other forms of the equalities on the right-hand sides of these equivalences, now as subspace inclusions of the form $\mathcal{R}(X) \subseteq \mathcal{R}(Y)$, where $\mathcal{R}(\cdot)$ is a column space of a matrix argument, and X and Y are functions of P and Q.

Problem 45-2: Nilpotent Matrix Commuting With A Nilpotent Matrix And With Its Conjugate Transpose

Proposed Johanns de Andrade Bezerra, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let N_0 and N in $\mathbf{C}^{n\times n}$ be nilpotent matrices, where N is nonderogatory, i.e., $dimKerN = 1$. Let N^* be the conjugate transpose of N. If $N_0 N = N N_0$ and $N_0 N^* = N^* N_0$, show that $N_0 = 0$.

Problem 45-3: PSD Solution To A Matrix Equation

Proposed by Christopher Hillar, *Mathematical Sciences Research Institute, Berkeley, California, USA* chillar@msri.org

Fix an $n \times n$ positive definite matrix A. Determine all positive semidefinite solutions X to the equation $XAX^3AX = 0$.

Problem 45-4: Nilpotent 0-1 Matrix

Proposed by Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu
and Fuzhen Zhang, *Nova Southeastern University, Ft. Lauderdale, USA,* zhang@nova.edu

Show that every nilpotent 0-1 matrix is permutation-similar to a strict upper-triangular 0-1 matrix.

Problem 45-5: Comparison Of The Trace Of Two Inverse Matrices

Proposed by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Let A and B be $n \times n$ positive definite matrices. It is easy to see $Tr(A^2 + AB^2A) = Tr(A^2 + BA^2B)$. Show a less trivial inequality

$$Tr[(A^2 + AB^2A)^{-1}] \geq Tr[(A^2 + BA^2B)^{-1}].$$

Problem 45-6: Matrix Unitary Decomposition

Proposed by Dennis Merino, *Southeastern Louisiana University, Hammond, USA,* dmerino@selu.edu

Let U and Q be given $n \times n$ complex matrices.

(i) Suppose that U is unitary. Show that for every integer $k \geq 2$, U can be written as a sum of k unitary matrices.

(ii) Suppose that Q is orthogonal. When do there exist orthogonal Q_1 and Q_2 such that $Q = Q_1 + Q_2$?

Problem 45-7°: A Generalization of Roth's Theorem

Proposed by Harald K. Wimmer, *Universität Würzburg, 97074 Würzburg, Germany,* wimmer@mathematik.uni-wuerzburg.de

Let K be a field and let $A \in K^{r\times r}$, $B \in K^{s\times s}$, $C_1, C_2 \in K^{r\times s}$. Prove or disprove that the following statements are equivalent:

(i) The matrices $M_1 = \left(\begin{smallmatrix} A & C_1 \\ 0 & B \end{smallmatrix}\right)$ and $M_2 = \left(\begin{smallmatrix} A & C_2 \\ 0 & B \end{smallmatrix}\right)$ are similar.

(ii) There exist $X \in K^{r\times s}$ and nonsingular P and Q with $AP = PA$ and $QB = BQ$ such that $AX - XB = C_2 - PC_1Q$.

Solutions to Problems 44-1 through 44-7 are on page 40.