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IMAGE

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About IMAGE

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UPCOMING CONFERENCES AND WORKSHOPS

Canadian Mathematical Society Meeting
Special Session on Combinatorial Matrix Theory
Edmonton, Canada
June 3-4, 2011

Edmonton

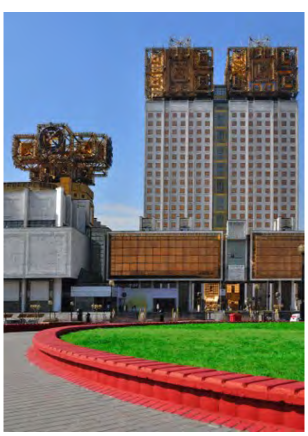
At this summer meeting of the Canadian Mathematical Society, the topics in the Special Session on Combinatorial Matrix Theory will include qualitative matrix theory (allow problems and minimum rank problems) and algebraic graph theory (spectra of graphs and normalized Laplacian).

The speakers will be Wayne Barrett, Michael Cavers, Louis Deaett, Randy Elzinga, Shaun Fallat, Yi Zheng Fan, Chris Godsil, In-Jae Kim, Zhongshan Li, Judi McDonald, Karen Meagher, Vladimir Nikiforov, Dale Olesky, Pauline van den Driessche, and Kevin N. Vander Meulen. For details, visit http://cms.math.ca/Events/summer11/.

3rd International Conference on Matrix Methods in Mathematics and Applications, MMMA-2011
Moscow, Russia, June 21-25, 2011

The previous title of this conference was Matrix Methods and Operator Equations. There will be several plenary talks and three parallel sessions, Matrices and Algebra, Matrices and Algorithms and Matrices and Applications. The conference will be very comprehensive, including essentially all areas of pure and applied linear algebra as well as a session on Application Challenges.

Please visit http://matrix.inm.ras.ru/mmma-2011/ at your earliest convenience for details and to let us know about your intention concerning this conference.



Russian Academy of Sciences

The Organizing Institutions are the Institute of Numerical Mathematics of Russian Academy of Sciences and Lomonosov Moscow State University. The Organizing Committee consists of Dario Bini, Alexander Guterman, Roger Horn, Volker Mehrmann, Maxim Olshanskii, Vadim Olshevsky, Stefano Serra-Capizzano, Hans Schneider, Gilbert Strang, Eugene Tyrtyshnikov (Chair) and Yuri Vassilevski.

The Russian Academy of Sciences (RAS) consists of the national academy of Russia and a network of scientific research institutes from across the Russian Federation, as well as auxiliary scientific and social units like libraries, publishers and hospitals. Headquartered in Moscow, the RAS is incorporated as a civil, self-governed, non-commercial organization chartered by the Russian Government. It consists of nine specialized scientific branches, three territorial branches and 14 regional scientific centres as well as numerous councils, committees and commissions, organized for different purposes.

Topics in Tensors: A Summer School by Shmuel Friedland
Coimbra, Portugal, July 6-8, 2011

This Summer School will be presented by Shmuel Friedland and is supported by the Centre for Mathematics, University of Coimbra. The Organizing Committee is Natália Bebiano and Carlos Fonseca. The daily themes will be Ranks of 3-tensors and a solution of the salmon problem; Nonnegative tensors: eigenvalues, singular values and scaling; and Approximation of tensors by low rank tensors. There will be morning sessions 10:30-12:00 and afternoon sessions 14:00-15:30. For more details, visit http://www.mat.uc.pt/~cmf/SummerSchool/TopicsInTensors.htm.

To register, contact Carlos Fonseca, Department of Mathematics, University of Coimbra, 3001-454, Coimbra, Portugal; email: cmf@mat.uc.pt; phone: +351 239 791 172; fax: +351 239 793 069.

2011 Industrial Math/Stat Modeling Workshop for Graduate Students
Raleigh, North Carolina, July 7-15, 2011

The 17th Industrial Mathematical & Statistical Modeling (IMSM) Workshop for Graduate Students will take place at North Carolina State University, July 7-15, 2011. The workshop is sponsored by the Statistical and Applied Mathematical Science Institute (SAMSI) and the Center for Research in Scientific Computation (CRSC). The organizers are Ilse Ipsen, Pierre Gremaud, and Ralph Smith.

This workshop will expose graduate students in mathematics, engineering, and statistics to exciting real-world problems from industry and government. Besides giving students experience working with a team, the IMSM workshop can help them decide what kind of professional career they want. Local and travel expenses will be covered for students at US institutions. For details, visit http://www.samsi.info/workshop/2011-industrial-mathstat-modeling-workshop-graduate-students-july-7-15-2011.

SAS Hall at NCSU, Math and Statistics

Directions in Matrix Theory 2011
Coimbra, Portugal, July 9-10, 2011

This workshop aims to be a forum for discussion of frontline areas of Matrix Theory and its Applications. It will bring together renowned researchers from diverse fields, fostering the exchange of experiences and insights from different perspectives. This meeting has been endorsed by the International Linear Algebra Society.

The invited plenary speakers are Béla Bollobás (Department of Pure Mathematics and Mathematical Statistics, University of Cambridge, UK); Shmuel Friedland (Department of Mathematics, Statistics and Computer Science, University of Illinois at Chicago, USA); Willem H. Haemers (Department of Econometrics and Operations Research, Tilburg University, The Netherlands); Steve Kirkland (Hamilton Institute, National University of Ireland, Maynooth, Ireland); Volker Mehrmann (Institut für Mathematik Technische Universität, Berlin, Germany); Juan Manuel Peña (Departamento de Matemática Aplicada, Universidad de Zaragoza, Spain); and Natália Bebiano (Departamento de Matemática, Universidade de Coimbra, Portugal).

The Scientific Committee is Carlos Fonseca (chair), Natália Bebiano, Shmuel Friedland, and Steve Kirkland. The Organizing Committee is Carlos Fonseca and Ana Nata.

There will be a peer-refereed special issue of the Electronic Journal of Linear Algebra devoted to papers presented at the workshop. Submitted papers will go through the usual editorial/referee process. The special editors of the issue are Oskar Maria Baksalary, C.M. da Fonseca, and Shmuel Friedland.

Registration will be open through July 9, 2011; the cost is 60 euro. To register and for further information , contact Carlos Fonseca, Department of Mathematics, University of Coimbra 3001-454 Coimbra, Portugal, Email : cmf@mat.uc.pt, Phone +351 239 791 172; or visit http://www.mat.uc.pt/~cmf/Directions2011/DMT2011.htm.

7th International Congress on Industrial and Applied Mathematics, ICIAM 2011
Vancouver, Canada, July 18–22, 2011

This 7th International Congress on Industrial and Applied Mathematics will highlight the most recent advances in the discipline and demonstrate their applicability to science, engineering and industry, providing an extraordinary opportunity for young researchers and graduate students to see much of the vast potential of mathematics. In addition to strong focus on traditional applied mathematics, the Congress will emphasize industrial applications and computational science in new and emerging topic areas, as identified by panels of top international scientists.

Vancouver

There will be 11 invited speakers and 20 thematic minisymposia, each with a Lead Lecture followed by six session speakers. These will cover a wide range of subjects ranging from molecular simulation to materials science. There will also be about 400 contributed minisymposia, several of which specifically emphasize linear algebra, including:

MS6 A View of Spectral Methods for Large-scale Graphs
MS36 Numerical Linear Algebra Beyond Linear Systems and Eigenvalues (2 Sessions)
MS147 Structured Matrices: Applications, Theory and Algorithms (5 Sessions)
MS149 The Linear Algebra of Optimization (2 Sessions)
MS192 Recycling for Sequences of Linear Systems
MS380 Applications of the Gersgorin Theory

The industrial program will include three minisymposia on finance and risk management, imaging and inverse problems, and graduate research internships with industry. The Prize Speakers are Dr. Beatrice Pelloni, University of Reading in Berkshire, UK, Olga Taussky-Todd Prize, Dr. Ingrid Daubechies, Princeton University, John von Neumann Prize, and Dr. Susanne C. Brenner, Louisiana State University, Sonia Kovalevsky Prize.

Other sessions of note include one for posters in areas consistent with the conference theme, and an integrated industry program (http://bit.ly/e4Opbe), which will spotlight the many ways in which applied mathematics is contributing to industry. These should be of broad appeal, providing insights from a wide variety of sectors including industry, government, and not-for-profit.

Students looking to network with professionals from around the world should consider volunteering, from preparing delegate kits and shooting photographs to assisting at events. Some support may be available. For details, visit http://bit.ly/ltP1Hn. Twitter will be used to deliver updates, announcements and enable interaction among delegates (http://twitter.com/#!/iciam2011). If you're tweeting about ICIAM 2011, use the hashtag #ICIAM2011.

A Welcome Reception is scheduled for Sunday, July 17, sponsored by the North American mathematical institutes and societies. Arranged sightseeing trips will include city tours, rainforest walks, and day trips to Whistler Mountain and Victoria. Discounted online registration will close on June 15, 2011 (http://bit.ly/gPYJTN). For the full program, visit http://www.iciam2011.com/.

SIAM Conference on Control and Its Applications, CT11
Baltimore, Maryland, USA, July 25-27, 2011

This conference is sponsored by the SIAM Activity Group on Control and Systems Theory and is a Satellite Meeting of ICIAM 2011. It is a continuation of meetings begun in 1989 in San Francisco. The invited speakers will be:
Alain Bensoussan, University of Texas at Dallas, USA and the Hong Kong Polytechnic University, Hong Kong
Tyrone Duncan, University of Kansas, USA
Birgit Jacob, Universität Wuppertal, Germany
Yannis Kevrekidis, Princeton University, USA
Walter Willinger, AT&T Labs-Research, USA
Enrique Zuazua, Ikerbasque & Basque Center for Applied Mathematics (BCAM), Basque Country-Spain

The preregistration deadline is June 27, 2011. For details, visit http://www.siam.org/meetings/ct11/.

ILAS 2011
Pure and Applied Linear Algebra: The New Generation
Braunschweig, Germany, August 22 - 26, 2011

This 17th ILAS Conference will have a special emphasis on young researchers, with predominantly young plenary speakers as well as seven special minisymposia featuring talks by young researchers. We are excited about many promising young researchers in our field and hope that you will share this excitement with us. This conference will provide an excellent setting for fruitful interaction between them and more senior ones. The special minisymposia for young speakers and their organizers are:

1. "Max-plus Linearity and its Applications in Computer Science and Scheduling", Rob M.P. Goverde and Sergei Sergeev
2. "The Theory of Orbits in Numerical Linear Algebra and Control Theory", Fernando De Terán and Marta Peña
3. "Matrix Means: Theory and Computation", Miklos Palfia and Bruno Iannazzo
4. "Modern Methods for PDE Eigenvalue Problems", Joscha Gedicke and Agnieszka Miedlar
5. "Combinatorial Matrix Theory", Minerva Catral and Amy Wangsness Wehe
6. "Numerical Methods for the Solution of Algebraic Riccati Equations", Frederico Poloni and Timo Reis
7. "Parallel Computing in Numerical Linear Algebra", Alfredo Remon and Jens Saak

Forty talks will be selected for these special minismposia, from proposals received. There will also be regular sessions for other invited minisymposia and contributed talks.

Technische Universität Braunschweig

The conference will be held on the main campus of the Technische Universität Braunschweig, which is close to the city center. The old and new integrated buildings provide excellent facilities including lecture rooms, poster areas and coffee break areas. Braunschweig is located in the north of Germany, 200 km west of Berlin and 200 km south of Hamburg. It is an important research region with a population of about 300,000, including 20,000 students.

The Scientific Committee is Ravi Bapat (Indian Statistical Institute, New Delhi, India), Angelika Bunse-Gerstner (Universität Bremen, Bremen, Germany), Albrecht Böttcher (TU Chemnitz, Chemnitz, Germany), Tobias Damm (TU Kaiserslautern, Kaiserslautern, Germany), Froilán Dopico (Universidad Carlos III de Madrid, Madrid, Spain), Shaun Fallat (University of Regina, Saskatchewan, Canada), Heike Faßbender, Chair (TU Braunschweig), Steve Kirkland (Hamilton Institute, National University of Ireland, Ireland), Raphael Loewy (Technion - Israel Institute of Technology, Haifa, Israel), Niloufer Mackey (Western Michigan University, Kalamazoo, USA), Bryan Shader (University of Wyoming, USA) and Michael Tsatsomeros (Washington State University, Pullman, USA).

The Local Organizing Committee is Heike Faßbender (TU Braunschweig, Carl-Friedrich-Gauß-Fakultät, Institut Computational Mathematics), Matthias Bollhöfer (TU Braunschweig, Carl-Friedrich-Gauß-Fakultät, Institut Computational Mathematics) and Peter Benner (Institute of Complex Technical Systems, Magdeburg and TU Chemnitz, Fakultät für Mathematik, Mathematik in Industrie und Technik).

The proceedings will appear as a volume of Linear Algebra and its Applications, with special editors Ravi Bapat, Matthias Bollhöfer, Froilán Dopico, and Heike Faßbender. Peter Šemrl will be the Editor in Chief responsible for this volume.

Online registration will be available through July 31, 2011. The registration fee for students is 130 Euro and 265 Euro for all other participants. Registration includes coffee breaks, the book of abstracts, the conference dinner and an excursion to Goslar on Wednesday afternoon. Visit www.ilas2011.de to register and find details about the program as well as local information. Send email to ilas2011@tu-braunschweig.de.

Medieval City of Goslar, a World Heritage Site (http://www.goslar.de/english.html)

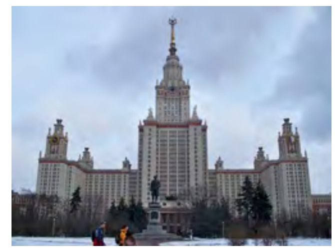
ICERM Workshops in 2011

Providence, Rhode Island, USA, August-November, 2011

The National Science Foundation established a new Institute for Computational and Experimental Research in Mathematics (ICERM) at Brown University in August 2010, the 8th NSF-funded mathematics institute in the United States. It intends to support and broaden the relationship between mathematics and computation: to expand the use of computational and experimental methods in mathematics, support theoretical advances related to computation, and use mathematical tools, research and innovation to address problems posed by the use of computers. The 2011 workshops will be Mathematical Aspects of P versus NP and its Variants (August 1-5, 2011), Cluster Algebras and Statistical Physics (August 15-19, 2011), Vlasov Models in Kinetic Theory (September 19-23, 2011), Applications of Kinetic Theory and Computation (October 17-21, 2011), and Boltzmann Models in Kinetic Theory (November 7-11, 2011. To apply, visit http://icerm.brown.edu/participation.php?show=forms. Direct questions to Lauren Barrows, lauren_barrows@icerm.brown.edu.

2nd Rome-Moscow School on Matrix Methods and Applied Linear Algebra

September 3-17, 2011 and September 18-October 2, 2011



Lomonosov University

This second Rome-Moscow School intends to encourage the exchange of ideas and scientific collaborations between Italian and Russian universities and institutions, in matrix methods and applied linear algebra. It will take place for 2 weeks in Moscow and then 2 in Rome, offering young students a long time for learning advanced scientific topics and for direct contact with people and institutions of excellence in the field. For details,visit www.mat.uniroma2.it/~tvmsscho, 2011 Edition.

The Lecturers in 2011 will include the main Lecturers of the 2010 School, Professors Eugene E. Tyrtyshnikov (INM-RAS-LMSU), Carmine Di Fiore (University of Tor Vergata), Khakim Ikramov (Lomonosov Moscow State University), and Daniele Bertaccini and Zellini (University of Rome Tor Vergata). Other Lecturers will be Professors Yuri Vassilevski (INM-RAS Moscow), Maxim Olshanskii (Lomonosov University Moscow), Dario Fasino (University of Udine, both in Moscow and Rome), S. A. Goreinov (INM-RAS, both in Moscow and Rome), and B. Khoromskij (Leipzig, Germany and MIS MPG, Rome). The school is approved by the Faculty of Sciences MM.FF.NN. of Tor Vergata University, and by the Faculty of Computational Mathematics and Cybernetics and the Faculty of Mechanics and Mathematics of Lomonosov University. The organizers of the School are Professors Carmine Di Fiore and Eugene E. Tyrtyshnikov.

Attending the school is equivalent to credits of extracurriculum activity. Interested students are encouraged to also attend the 3rd International Conference on Matrix Methods in Mathematics and Applications, Moscow, June 20-24, 2011. See http://matrix.inm.ras.ru/.

Workshop and Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices

January 2-7, 2012 and January 10-11, 2012, Manipal, India

The Workshop and Conference will both be held at Manipal University (www.manipal.edu), Manipal, India. The Workshop topics are Combinatorial Matrix Theory and Applications, Generalized Inverses of Matrices and Applications and Linear Model and Error Analysis, with special lectures on advanced topics. The conference will be a continuation of the workshop. For details and registration, please visit (www.cmtgim2012.org) or contact K. Manjunatha Prasad (cmtgim@gmail.com or km.prasad@manipal.edu).

18th International Symposium on Mathematical Methods Applied to Sciences
San Jose, Costa Rica, February 21-24, 2012

This symposium SIMMAC VIII is the most important event in applied mathematics in Central America. It is organized by the Center for Research in Pure and Applied Mathematics (CIMPA) of the University of Costa Rica every two years, with the support of the School of Mathematics. The topics will include Multivariate Statistics, Numerical Analysis, Modeling, Optimal Control, and many other application areas. For details, visit http://www.cimpa.ucr.ac.cr/simmac/.

Other Upcoming Conferences

18th Householder Symposium, Tahoe City, California, USA, June 12-17, 2011
5th Intern. Conf. on Mathematics & Statistics, Athens, Greece, June 13-16, 2011
2nd SIAM Gene Golub Summer School, Waves and Imaging, Vancouver, British Columbia, Canada, July 4-11, 2011
Mat-Triad 2011, Tomar, Portugal, July 12-16, 2011
ICNAAM 2011, International Conference of Numerical Analysis and Applied Mathematics, Halkidiki, Greece, Sept. 9-25, 2011
11th SIAM Conference on Applied Linear Algebra, Valencia, Spain, June 18-22, 2012. Jointly sponsored by SIAM and ILAS

ARTICLES

Linear Algebra Research in Portugal

By João Filipe Queiró, University of Coimbra, jfqueiro@mat.uc.pt

In the second issue of IMAGE, Graciano de Oliveira published a personal, non-mathematical account of the beginnings of the Portuguese school in linear algebra research [17]. While he acknowledges earlier influences, the main story begins with Graciano de Oliveira's interest in matrices in the 1960's, around the time of his two-year stay in England. There he met Hazel Perfect in Sheffield and he became acquainted with Leon Mirsky's papers, some of which (especially [2]) inspired some of his later research.

In 1969 Graciano de Oliveira obtained his doctorate in Coimbra with the thesis "On stochastic and doubly stochastic matrices", where he displayed his interest in inverse problems for nonnegative matrices, later to be extended to other kinds of inverse problems and existence theorems for matrices, including the classical additive and multiplicative inverse eigenvalue problems. An example of a result on this theme concerns the existence of matrices with prescribed characteristic polynomial and top left submatrix. He started the tradition of study of this kind of problems (sometimes also called completion problems) in Portugal, pursued to this day by many people, in several countries.

Another early interest of Oliveira was multilinear algebra. A series of lectures by him in Coimbra in 1971-72 on generalized matrix functions marked the beginning of research on this topic in Portugal. The course notes were published in book form [3].

The audience for these lectures (held outside the University) was a very small group of interested students, which included Jośe Dias da Silva and Eduardo Marques de Sá. Oliveira attracted them both to linear algebra problems, and they have been leaders in the field ever since. One publication from this period is a paper by all three [5], one of the first to consider similarity invariants of sums of matrices in prescribed similarity classes, a notoriously difficult question.

Between 1972 and 1976, Graciano de Oliveira was a researcher at the University of Lisbon. A very influential paper from this period was [4], which started a line of enquiry that eventually turned into a long-term program of research.

In 1976 he became a professor at the University of Coimbra. There he immediately started a regular program of seminars, attracting new people and forming a research group. He invited many well-known mathematicians to visit Coimbra. R. Merris, D. Carlson, H. Minc, B. Cain, M. Fiedler, H. Wimmer, T. Laffey, S. Friedland and R. Loewy were some of the early visitors in the 1970's and 80's. Two international meetings were organized in Coimbra, in 1982 and a much larger one in 1984. Between the two, a linear algebra meeting was held in 1983 in Vitória, Spain, following the increasing contacts between Coimbra and linear algebraists from the Basque Country (see [17] and [19]).

Graciano de Oliveira's enthusiasm led and inspired many people in Portugal and other countries. Problems he has proposed through the years continue to be analyzed to this day. One famous example is a conjecture, also formulated independently by M. Marcus, on the determinant of the sum of two normal matrices with prescribed eigenvalues [9]: the conjecture, still open, simply states that such a determinant should lie in the convex hull of the n! points obtained by taking diagonal matrices with the given eigenvalues.

*

Both Jośe Dias da Silva and Eduardo Marques de Sá obtained their PhD's at the University of Coimbra in the late 1970's. Shortly after that, Dias da Silva settled at the University of Lisbon, whereas Marques de Sá, a few years later, came to the University of Coimbra to stay.

Dias da Silva's thesis centered around his work on decomposable symmetric tensors. In [8] he presented conditions for equality of such tensors, a result that R. Merris qualified as "dramatic" in his MR review of the paper.

Dias da Silva proceeded to build an impressive body of work while at the same time organizing a research group in Lisbon, now the Center for Linear and Combinatorial Structures. In the mid 1980's he studied groups of matrices preserving generalized matrix functions, starting with the paper [10] with Graciano de Oliveira. While maintaining his earlier interests, in the following two decades he branched out to other subjects like combinatorics and matroids. An interesting paper on nonnegative matrices is [14].

In the 1990's Dias da Silva became interested in additive number theory. In collaboration with Y. Hamidoune [18], he gave a new proof of the Cauchy-Davenport theorem on the cardinality of the sum of two subsets of Z_p by using a lower bound for the degree of the minimal polynomial of the Kronecker sum of two linear operators. Later, in the remarkable paper [22], Dias da Silva and

Hamidoune used linear algebra techniques to prove a stronger version of a conjecture by Erdös and Heilbronn on the cardinality of the set of sums of the 2-subsets of a subset of Z_p.

Among many other papers by Dias da Silva in more recent years, with several coauthors, it is worthwhile to mention a study with T. Laffey of an equivalence for polynomial matrices [26], which they apply to the problem of simultaneous similarity of matrices.

Recently, Dias da Silva wrote, in collaboration with the Lisbon geometer Armando Machado, the chapter on multilinear algebra in the *Handbook of Linear Algebra* [28].

*

One of Marques de Sá's first papers [7] contained a deep result describing the complete relations between the similarity invariants of a square matrix (over an arbitrary field) and those of a principal submatrix. This paper was the backbone of his PhD thesis, for which he was awarded the Householder Prize in 1981. The result was found independently, with a very different proof, by R. C. Thompson. Connections of this type of problem to questions in control theory were explored by Ion Zaballa in the 1980's, starting with his important theorem on the completion of matrices with prescribed rows and invariant factors [12].

Sá's later research, apart from a continued interest on invariant factor problems, focused, among other topics, on norms, convexity, inertia results for Hermitian matrices and combinatorial matrix theory. The four-author paper [11], on conditions for a partially given matrix with a given graph to have a positive definite completion, has been very influential. Sá remains active at the Algebra and Combinatorics research group in Coimbra.

Concerning Sá's research, a remark is in order: he has left some interesting results unpublished, after presenting them in talks. I give just one example. In a December 1990 meeting commemorating the 50th anniversary of the Portuguese Mathematical Society, Sá gave a talk where he presented a solution to the perturbation problem for matrix pencils. A manuscript circulated in some circles. When a different solution was published in 1997 by Edelman, Elmroth and Kågström [23], Sá included his manuscript in the Coimbra preprint collection [24] and left it there. Another example concerns a more obscure problem: find the (1,▮)-approximation numbers of I_n; call them a_k. For each $n \geq 2$, it is easy to see that $a_1 = 1$, $a_2 = 1/2$, $a_n = 1/n$, and $a_k \geq 1/k$ for all k. Around 1988 Sá presented in a Coimbra talk a proof that

$$a_3 = \frac{\cos(\frac{\pi}{n})}{1 + \cos(\frac{\pi}{n})}$$

for all $n \geq 3$, which appears quite surprising. I advertised this more than once ([20], [27]), but Sá hasn't yet published the proof.

The strongest linear algebraist in Portugal after the three mathematicians mentioned above is Fernando Silva, a former PhD student of J. A. Dias da Silva and a professor at the University of Lisbon. In a career spanning two and a half decades, he has produced a stream of solid papers, many with co-authors, about similarity invariants of sums and products, completion problems, control theory and other subjects. He gave full solutions to difficult and fundamental problems, sometimes with long and technical proofs. See e.g. [13], [15], [21].

Like Oliveira, Dias da Silva and Marques de Sá, Fernando Silva has had many PhD students: the total for the four is around 40, from Portugal and abroad. Many are now professors in several Portuguese universities.

*

Among the many mathematical descendants of Graciano de Oliveira, I mention the following:

-Natália Bebiano wrote several papers on the Oliveira determinantal conjecture, and from there she went on to study variations of the numerical range concept. More recently, she analyzed the same kind of problems in the presence of an indefinite inner product.

-Amélia Fonseca has been and continues to be one of the main collaborators of Dias da Silva in his work in multilinear algebra and combinatorics.

-Antonió Leal Duarte has been interested in spectral properties of acyclic matrices, starting with [16]. He has developed a fruitful collaboration with C. Johnson on this topic. A survey can be found in chapter 34 of the Handbook of Linear Algebra [26].

-Olga Azenhas went from the problem of the invariant factors of the product of two matrices over a principal ideal domain to a longstanding study of Young tableaux and Littlewood-Richardson coefficients.

-Susana Furtado has published on similarity invariants, congruence of matrices, canonical forms, and other topics.

I could mention other names. Mathematically speaking, there is a certain continuity in research themes, but also some diversification as well as connections with people working in other areas, such as graphs, operator theory, differential equations, control, and representation theory. An example involving the latter is the paper [25], by A.P. Santana, myself and E. Marques de Sá, dealing with the Hermitian sum eigenvalue problem (fully solved shortly afterwards), incidentally one of the topics Graciano de Oliveira lectured on at the Coimbra seminar in the late 1970s, based on Alfred Horn's celebrated paper [1].

*

The evolution of the linear algebra research community in Portugal may be viewed as a success story. There are many international contacts and collaborations. Some people from Portugal have served on the boards of the International Linear Algebra Society. Two ILAS Conferences have been held in Portugal: the 2nd (Lisbon, 1992) and the 11th (Coimbra, 2004).

The community is large and diverse, less concentrated than before both geographically and scientifically. The challenges for the future lie in improving mathematical standards, resisting the too common tendency to evaluate a person's research by counting publications instead of emphasizing quality.

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The Gatlinburg Symposia and Their Influence on the Development of Numerical Linear Algebra

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Abstract

About 1960, Alton S. Householder initiated the idea of "Gatlinburg Symposia on Numerical Linear Algebra". They were named after the resort of Gatlinburg, Tennessee, where the early meetings took place; later on they shifted to various other locations in North America and also in Europe. Responsible for their program was the "Gatlinburg Committee"; the first consisted of A.S. Householder, J.H. Wilkinson, W. Givens, G.E. Forsythe, P. Henrici, and F.L. Bauer. There were only invited lectures and there were, at least initially, no parallel sessions. The list of participants contains many well known names and continues to read like a "Who's Who" in the area of Numerical Linear Algebra, so that it is not surprising that the Symposia have had a tremendous influence on its development, both with respect to theory and the design of reliable and efficient algorithms. Such well-known software packages as LINPACK, LAPACK, EISPACK, and SPARSEPACK had their root in the collection of ALGOL programs in the book of Wilkinson and Reinsch, *Linear Algebra*, in many critical discussions during Gatlinburg Symposia, and in the close cooperation of scientists attending these meetings.

1 Background

In 1961, the proposal for a series of Gatlinburg Symposia was initiated by Alton S. Householder (1904-1993). He was then Director of the Mathematics Division of Oak Ridge National Laboratory and Ford Professor at the University of Tennessee.

Alston S. Householder

The background of his initiative was the new situation in numerical analysis, which had changed radically with the advent of digital computers and their growing availability at universities. Between 1950 and 1960 and before, only a few institutions associated with universities and government (like the National Bureau of Standards, the Oak Ridge National Laboratory and the Rand Corporation) had access to digital computers. In particular, many of the interested scientists concentrated at the National Bureau of Standards near Washington D.C. were eager to face the challenges of the new situation, and they knew the many results of classical mathematics that still could be used.

Using a digital computer was then tedious: programs had to be written in machine code or later in assembler language, as command driven languages like Fortran were still in their infancy. It was quickly seen that traditional numerical methods were inadequate when trying to realize them on digital computers: not only was it difficult to program them, but many of them turned out to be numerically unstable. A new foundation of numerical analysis and in particular of numerical linear algebra was necessary. This inspired the development of new methods and concepts in the years 1950-1960.

In 1950, D. Young proposed the SOR Algorithm, leading to new iterative methods for solving linear equations. This was systematically enhanced by the book of R. S. Varga, Matrix Iterative Analysis (1962). About the same time C. Lanczos (1950) and W. E. Arnoldi (1952) proposed Krylov-subspace methods. In 1952, M. Hestenes and E. Stiefel excited the community by their Conjugate Gradient Algorithm, because their method combined features of an iterative and a finite algorithm for solving linear equations.

In 1958, H. Rutishauser proposed the radically new LR Algorithm for computing the eigenvalues of a matrix, which was quickly extended by J.G. Francis (1961/62) to the now famous backward stable QR method. The basic idea, but without essential practical modifications, was also proposed by V.N. Kublanowskaya (1961) in the USSR.

The influence of round-off errors was analyzed by J. H. Wilkinson (1960, 1963) in a novel way, leading to the distinction between the condition number of a mathematical problem and the numerical stability properties of an algorithm to solve it. He developed so-called backward error analysis to judge the stability of an algorithm. The first formal backward error analysis was presented by W. Givens in 1954 in a technical report put out by the Oak Ridge National Laboratory.

In 1958 W. Givens also introduced the tool now called Givens rotations to numerical linear algebra; it was supplemented by the equally important tool introduced by Householder (1958), now called Householder matrices, which are reflectors, not rotators.

In 1947 G. B. Dantzig proposed the Simplex Method (see Dantzig (1963)) for solving a linear program (optimizing a system of linear inequalities subject to linear constraints), thereby starting the development of optimization methods which play a crucial role in many branches of engineering and other fields.

The ensuing rapid development of numerical analysis gave rise to new journals, such as Mathematics of Computation in 1954 (formerly Mathematical Tables and other Aids to Computation, 1943) and Numerische Mathematik in 1959 (see Figure 1 in the Appendix).

2 The Gatlinburg Symposia and Their Style

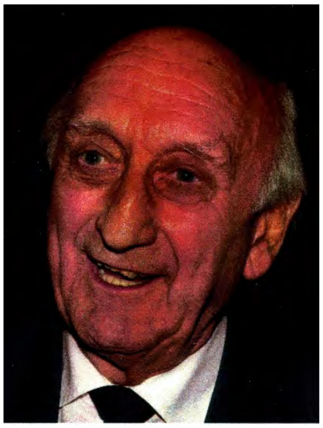
The Gatlinburg Symposia, officially started by Alston S. Householder in 1961, were influenced in part by the existence of two earlier and similar symposia, the first organized by Olga Taussky in 1951 in Los Angeles and the second by Wallace Givens in 1957 in Detroit. Householder had a special gift for persuading other top scientists to cooperate with him in launching a new conference series to foster a new foundation for numerical analysis, in particular for numerical linear algebra, made essential by the advent of digital computing. Due to his reputation and influence, Householder obtained the financial support of the National Science Foundation, allowing all participants of the early symposia to be fully paid.

The style of the Symposia was unusual. They were originally designed to be "closed" meetings (which, later on, led to growing concern and dislike by many about this particular aspect of the symposia). The reason for closed meetings was simply to limit the number of participants to about 100, so as to ensure that participants really could interact with one another.

Accordingly, the Symposia resembled more forums for discussion among scientists interested in computing, rather than traditional conferences to present results. There were no conference proceedings and no obligation for the participants to give a lecture. This loose structure was controlled by a Committee that assisted Householder to prepare and run the symposium. This Committee issued the invitations and, using a nice formulation of Richard Varga (1990), helped to "prompt" people to offer to give "spontaneous" lectures. There was always a full, but not overloaded, regular program with lectures of variable length and ample time for intensive discussions between them. Also, there were no parallel sessions in the early Symposia, and the official program was always supplemented by evening lectures organized on the spot.

The Committee of Gatlinburg III, shown on the left, is well known as this photo appears in the MATLAB Users Guide. The members, left to right, are James H. Wilkinson, Wallace Givens, George E. Forsythe, Alston S. Householder (Chair), Peter Henrici, and Friedrich L. Bauer.

The membership of the Committee could change with each successive meeting. For instance, the Gatlinburg IV Committee in 1969 consisted of F.L. Bauer, Miroslav Fiedler, A.S. Householder (Chair), A.M. Ostrowski, John Todd, Richard S. Varga, and J.H. Wilkinson. By 2005 the Householder XVI Committee was Angelika Bunse-Gerstner, Tony F. Chan, Alan Edelman, Nicholas J. Higham, Roy Mathias, Dianne P. O'Leary, Michael L. Overton, Henk A. van der Vorst, Paul Van Dooren, and Charles F. Van Loan (Chair).



F.L. Bauer

J.H. Wilkinson

The Committees were always careful to invite not only well known scientists but also promising young postgraduates, and to include all areas having both algebraic aspects and numeric content or implications. All this is illustrated by the program of Gatlinburg IV and the (partial) copy of its list of participants in Figures 3, 4, and 5 in the Appendix. In the early meetings an effort was made to interest experts in pure matrix theory but that did not succeed and an unfortunate separation emerged between these two related areas.

The attendance at the Symposia was international, in particular, there were always participants (already in the early Symposia) from behind the Iron Curtain. The author of this paper still remembers the sad discussion in April 1969 among the Czech attendants of Gatlinburg IV (I. Babuska, M. Fiedler, V. Pták and I. Marek), when they learned during the Symposium that the reform government of their country had been crushed by a Soviet invasion.

The first four Symposia took place in Gatlinburg, a resort village in Tennessee. Later on, the workshop moved to other places in North America and Europe. After Gatlinburg X, the first to return to Tennessee, they were renamed Householder Symposia on Numerical Linear Algebra (see Table 1 below).

Also Gatlinburg X was the first "open" meeting. Each person wishing to participate was requested to submit a short abstract of his or her research interest, and the Committee decided on whom to invite, based on this information.

Since 1970, a Householder prize for the best Ph.D. thesis has been awarded during each Symposium. Its subject should be (broadly) related to Numerical Linear Algebra. Applications for the prize, accompanied by an appraisal by the thesis advisor, are invited. The Committee decides on the recipient and announces the winner during the Symposium, who is expected give a lecture. The prize is entirely financed by voluntary contributions of the Conference participants. So far, the Householder Award has been granted to the following:

1971: F. Robert (Grenoble)
1974: Ole Hald (New York U.)
1977: Daniel D. Warner (UCSD)
1981: E. Marques de Sá (Coimbra) and Paul Van Dooren (K. U. Leuven)
1984: Ralph Byers (Cornell U.) and James M. Demmel (UC Berkeley)
1987: Nicholas J. Higham (U. of Manchester)
1990: Alan Edelman (MIT), Maria Beth Ong (U. of Washington)
1993: Hong-Guo Xu (Fudan U.) and Barry Smith (New York U.)
1996: Ming Gu (Yale U.)
1999: Jörg Liesen (U. Bielefeld)
2002: Jing-Rebecca Li (MIT)
2005: Jasper van den Eshof (U. Utrecht)
2008: David Bindel (UC Berkeley)

Table 1

Gatlinburg Symposia

No.	Year	Place (Organizers)
I	1961	Gatlinburg, USA (A.S. Householder)
II	1963	Gatlinburg, USA (A.S. Householder, F.W.J. Olver)
III	1964	Gatlinburg, USA (A.S. Householder)
IV	1969	Gatlinburg, USA (A.S. Householder)
V	1972	Los Alamos, USA (R.S. Varga)
VI	1974	Hopfen am See, Germany (F.L. Bauer)
VII	1977	Asilomar, USA (G.H. Golub)
VIII	1981	Oxford, England (L. Fox, J.H. Wilkinson)
IX	1984	Waterloo, Canada (J.A. George)
X	1987	Fairfield Glades, USA (R.C. Ward, G.W. Stewart)

Householder Symposia

No.	Year	Place (Organizers)
XI	1990	Tylosand, Sweden (Å. Björck)
XII	1993	Lake Arrowhead, USA (T.F. Chan, G.H. Golub)
XIII	1996	Pontresina, Switzerland (W. Gander, M.H. Gutknecht, D.P. O'Leary)
XIV	1999	Chateau Whistler, Canada (J.M. Varah, G.W. Stewart)
XV	2002	Peebles, Scotland (P. Knight, A. Ramage, A. Wathen, N.J. Higham)
XVI	2005	Campion, USA (J. Barlow, D. Szyld, H. Zha)
XVII	2008	Zeuthen, Germany (L. Liesen, V. Mehrmann, R. Nabben)

Householder VIII, Pontresina, Switzerland

3 Main Topics of the Symposia and Their Influence

The Symposia served as a forum for discussion among scientists about all aspects of using digital computers as a tool for the design and realization of reliable mathematical algorithms. This influenced and started developments not only in numerical mathematics but also in areas later belonging to computer science. For example, the design of the new well defined procedural programming language ALGOL was supported, in order to have a clean shareable tool for programming numerical software. This led to the so-called "handbook project" established in 1965 under the roof of Springer Verlag, in cooperation with Numerische Mathematik. It was essentially J.H. Wilkinson and his co-workers, mainly C. Reinsch, who established a fixed pattern to describe numerical software. Each program was accompanied by a description of the theoretical background of the underlying algorithm, of its applicability, and of its formal parameter list that precisely defined the meaning of its input and output parameters. Then followed a well-structured and syntactically correct ALGOL program, the description of its organisational and notational details, a discussion of numerical properties of the algorithm, typical examples of its use, and finally test results, illustrating the behaviour of the program in critical situations.

The handbook project started with prepublications in Numerische Mathematik in 1965, then collected in the famous *Handbook of Linear Algebra* ("HALA") of J.H. Wilkinson and C. Reinsch, which appeared in 1971 as Volume II of the *Handbook for Automatic Computation* published by Springer Verlag (see Figure 2 in the Appendix). Volume I, part a, on ALGOL itself, was written by H. Rutishauser in 1967. Although he had a profound influence on numerical linear algebra, poor health prevented him from attending more than one Gatlinburg meeting. Many attendants of the Symposia contributed to Volume II and to other related software packages that appeared later on.

As only a few others, Wilkinson (1965) strongly influenced the Gatlinburg Symposia and the contributions to "HALA" by his monograph *The Algebraic Eigenvalue Problem*. Using many classical results for his analysis, he described numerous new numerically stable algorithms for eigenvalue problems and improved many old methods.

Another new journal, Linear Algebra and Its Applications, was founded in 1968 with several attendants of the Symposia on its board of editors.

To the main subjects of the earlier Symposia belonged the QR algorithm and its properties (convergence, shift techniques, implicit shifts). The book of Wilkinson (1965) had stimulated the development of related algorithms. It led to the generalization of the QR method for computing the SVD (singular value decomposition) of matrices (Golub, Reinsch 1971).

Several results of classical mathematics also became of renewed interest in the context of numerical linear algebra, for instance, the moment problem, orthogonal polynomials, Gaussian inegration, summation methods, and approximation theory. Bauer and Householder (1960) estimated the eigenvalues of matrices in terms of moments. Other estimates were tied to classical results of Gerschgorin (see Varga (1962). A systematic study is found in Varga (2004). All these estimates turned out to be valuable tools for

G.H. Golub and R.S. Varga

G.W. (Pete) Stewart and W. Gautschi

investigating the sensitivity of the eigenvalue problem and the numerical stability of corresponding algorithms.
Orthogonal polynomials were used by Hestenes and Stiefel (1952) to estimate the approximation error of the iterates of their cg-algorithm (conjugate gradient). The classical role of these polynomials in weighted Gaussian integration was greatly enhanced by new results. The construction of such quadrature rules was considered by Gautschi (1968) and Golub and Welsch (1969), and is tied to the construction of positive definite tridiagonal matrices and the computation of their eigenvectors. The qd-algorithm of Rutishauser (1954) is closely related to his LR-algorithm, but also gave rise to new summation methods to speed up the convergence of sequences and series. In particular, this created new interest in Richardson-extrapolation techniques and their application to ordinary and partial differential equations.

To the highlights of Gatlinburg V (1972) in Los Alamos belong the presentation by Pete Stewart of the QZ algorithm for solving $Ax = \lambda Bx$ (C. Moler, G.W. Stewart (1971)), evoking the admiration (coupled with admitted envy) of Jim Wilkinson. This QZ paper launched the algorithmic study of generalized eigenvalue problems and matrix pencils, and the development of a perturbation theory for them (Stewart (1978), Sun (1977)).

The ALGOL program collection in HALA stimulated enlarged software packages of FORTRAN programs written in the style of HALA, EISPACK (Smith et al. 1976, 1977) and LINPACK (Dongarra et al. 1979).

A major theme since Gatlinburg VII has been the exploitation of sparsity in direct methods for solving linear equations. Preserving sparsity and numerical stability by proper pivoting led to the program package SPARSEPACK (George, Liu, Ng (1980)) and to the books of George, Liu (1981), and Duff, Erisman and Reid (1986). To enhance the use of vector machines in numerical linear algebra, the development of BLAS (basic linear algebra subroutines) was started, and they were incorporated into the software package LAPACK (Anderson et al. (1992)).

For distributed memory computers, a suitable program package is ScaLAPACK (L.S. Blackford et al. (1997)), which is based on LAPACK. The importance of all these packages is also seen from the fact that they form an essential part of current test-beds to assess the speed of today’s supercomputers and the quality of their operating systems. Many of these routines were incorporated into MATLAB, MATHEMATICA and MAPLE. At several Symposia, during the coffee breaks between lectures, Cleve Moler demonstrated the easy use of MATLAB to solve problems in numerical linear algebra, and impressed an ever growing audience.

To the regular visitors of the Symposia belonged W. (Velvel) Kahan. His critical remarks were feared by all, but helped to improve many results; in particular his repeated and insistent critique of the low quality of the arithmetic operations of the then available computers finally led to the IEEE standard for computer arithmetic, adopted in 1985 and still in current use.

Major themes of the Symposia originated from a fresh view of the cg algorithm as an iterative method, thanks to J. Reid (1971), and then to Krylov spaces. This led to investigations of a whole nest of problems:

– the relation of the cg-algorithm to the algorithms of Lanczos (1950) and Arnoldi (1951)
– the solution of the symmetric eigenvalue problem (Parlett, 1980)
– the Kaniel-Paige theory (Kaniel 1966, Paige 1971)
– solving linear equations with a symmetric but indefinite matrix (Paige, Saunders (1975))
– the GMRES method of Saad and Schultz (1986)
– the fast but unreliable BI-CG method (Lanczos (1950), Fletcher (1976)) and stabilizations of it
– the Bi-CGSTAB-method (van der Vorst 1992) presented at Gatlinburg XI
– the QMR-method (Freund, Nachtigal (1991)), also presented at Gatlinburg XI

The convergence of all these iterative methods can be sped up by preconditioning techniques, a recurrent topic of the Symposia. With very large problems coming from partial differentiation in important applications, in order to be able to solve such problems at all, it became necessary to adapt algorithms for finite element methods, multigrid algorithms and domain decomposition in order to take into account the special structure of matrices, and this became one of the regular topics of the Symposia.

Filtering and control were increasingly addressed at the Symposia. The use of the the Kronecker normal form in relevant algorithms was studied by P. van Dooren (1979), who won the Householder prize of Gatlinburg VIII in 1981. The late R. Byers (1983) won the Householder prize of Gatlinburg IX in 1984 for the solution of the algebraic Riccati equation by exploiting the structure of Hamiltonian and symplectic matrices. Statistical calculations profit very much from the increasingly refined arsenal of SVD related algorithms.

Early contributions of numerical linear algebra to the broad area of optimization should also be noted. This was to be expected as the solution of positive definite linear systems is equivalent to the minimization of a strictly convex quadratic function. It motivated new algorithms for solving linearly constrained quadratic programs and, more generally, to solve nonlinearly constrained nonlinear programs (by so-called sequential quadratic programming (SQP) methods). Even though these methods for nonlinear problems are the proper subject of other societies like the Mathematical Programming Society, their realization led to interesting specially structured problems of numerical linear algebra also considered during the Gatlinburg/Householder Symposia. An example is the MINOS program package started by Murtagh and Saunders in 1977 (see Murtagh and Saunders (1978)), with later additions by P.E. Gill, W. Murrray and M.H. Wright. There are relations to the interior point methods of mathematical programming since all these methods have to repeatedly solve large sets of linear equations with a similar structure.

These remarks also apply to other modern optimization problems such as semidefinite programming. Here one considers linear programs in the Hilbert space (with respect to the Frobenius norm) of symmetric matrices, ordered by the cone of positive definite matrices. These programs lead to intricate problems of numerical linear algebra that have been addressed during the Symposia, e.g. by M. Overton during the Symposium of 1996. Also see the later paper by Alizadeh, Haeberly and Overton (1998).

All this shows that there are fruitful relations between numerical linear algebra and various areas of applications. These areas typically lead to consideration of new problems with an interesting mathematical structure, which require efficient and stable algorithms of numerical linear algebra to solve them. This interplay is fostered by the Gatlinburg/Householder Symposia. From their beginning, they have served as meetings of mathematicians who are willing to face the problems connected with the development of such methods.

Acknowledgement. The author thanks Beresford Parlett for the careful reading and correction of a draft of this paper, and Walter Gander and Martin Gutknecht for providing photos from the Householder Conference XIII in Pontresina. Also the author gratefully admits that he had access to the reminiscences of Richard Varga (1990) on the Gatlinburg Symposia.

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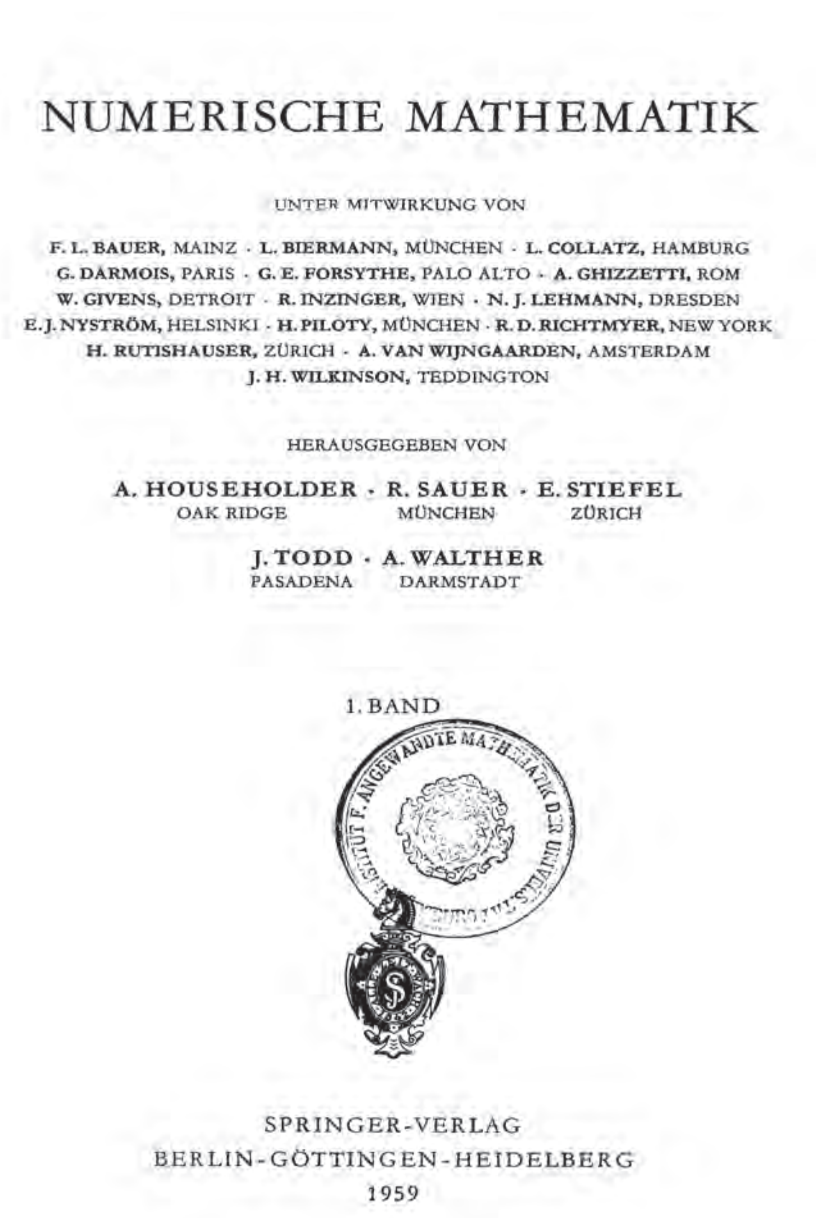
Ed. Note: A brief history of the Householder Symposia is at http://www3.math.tu-berlin.de/householder_2008/history.php. An interested reader might also look at http://www.mathworks.com/company/newsletters/news_notes/clevescorner/dec04.html (where Cleve Moler reminisces about the Gatlinburg meetings and shows more photos of the early organizers), as well as articles about numerical linear algebra that have appeared in IMAGE: "The ILAS 2008 Hans Schneider Prize in Linear Algebra Goes to Beresford Parlett and Cleve Moler", issue 40; "Finding John Francis Who Found QR Fifty Years Ago", issue 43; and "Hans Schneider Award Winners Celebrated", issue 45. All issues of IMAGE may be viewed at http://www.ilasic.math.uregina.

A Appendix

Figure 1: Numerische Mathematik



NUMERISCHE MATHEMATIK

UNTER MITWIRKUNG VON

F. L. BAUER, MAINZ · L. BIERMANN, MÜNCHEN · L. COLLATZ, HAMBURG
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J. TODD · A. WALTHER
PASADENA DARMSTADT

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BERLIN-GÖTTINGEN-HEIDELBERG
1959

Figure 2: Handbook

Handbook for Automatic Computation

Edited by

F. L. Bauer · A. S. Householder · F. W. J. Olver

H. Rutishauser † · K. Samelson · E. Stiefel

Volume II

J. H. Wilkinson · C. Reinsch

Linear Algebra

Chief editor

F. L. Bauer

Springer-Verlag Berlin Heidelberg New York 1971

Figure 3: Program of Gatlinburg IV

PROGRAM

FOURTH GATLINBURG SYMPOSIUM ON NUMERICAL ALGEBRA

Mountain View Hotel April 14-19, 1969

Monday, April 14, 1969

9:30	Welcome	A. S. Householder
9:45	Some Basic Results in Automatic Solution of Polynomial Equations	A. M. Ostrowski
11:15	Convergence Theorems for Secant Methods in n Dimensions	J. M. Ortega
2:00	Alternating Direction Methods for Galerkin Approximations for Parabolic Problems	J. Douglas
3:30	The Computational Aspects of Applying Variational Techniques to Boundary Value Problems	R. S. Varga

Tuesday, April 15, 1969

9:30	On the Numerical Improvement of Optimizing Algorithms	J. Stoer
11:00	Another Algorithm for Minimizing a Sum of Squares of Nonlinear Functions	M. J. D. Powell
2:00	Bounds for the Second Eigenvalue	F. L. Bauer
3:30	Techniques for the Eigenvalue Problems	N. A. Gastinel

Wednesday, April 16, 1969

9:30	Natural Norms in Algebraic Processes	V. N. Faddeeva
11:00	Generalized Eigenvalue Problem	V. N. Kublanovskaja

- Afternoon free -

Thursday, April 17, 1969

9:30	On Some Classes of Matrices	M. Fiedler
11:00	The Critical Exponent and the Spectral Radius of Matrices	V. Pták
2:00	The $Ax = \lambda Bx$ and Related Problems	J. H. Wilkinson
3:30	Some New Results on the QR and LR Algorithms	J. F. Traub

Friday, April 18, 1969

9:30	Matrix Bound Norms	H. Schneider
11:00	Bounds on the Eigenvalues of a Matrix	A. J. Hoffman
2:00	Bounds for the Abscissa of Stability of a Stable Polynomial	P. Henrici
3:30	The Method of Odd/Even Reduction and Factorization with Application to Poisson's Equation	G. H. Golub

Saturday, April 19, 1969

9:00	Sectionally Linear Functions	H. Schwerdtfeger
10:30	On Some Finite Element Procedures	M. Zlámal

Figure 4: Participants of Gatlinburg IV

PARTICIPANTS

FOURTH GATLINBURG SYMPOSIUM ON NUMERICAL ALGEBRA

Mountain View Hotel April 14-19, 1969

D. E. Arnurius
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Oak Ridge, Tennessee

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College Park, Maryland

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University of Texas
Austin, Texas

Robert C. F. Bartels
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Leningrad, USSR

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Stanford, California

Leslie Fox
Oxford University
Oxford, England

Figure 5: Participants of Gatlinburg IV, ctd.

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Christian Reinsch
Case Western Reserve University
Cleveland, Ohio

John R. Rice
Purdue University
Lafayette, Indiana

Francois Robert
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Maxine Rockoff
Washington University
St. Louis, Missouri

Hans Schneider
University of Wisconsin
Mathematics Research Center
Madison, Wisconsin

Johann Schröder
Boeing Laboratories
Seattle, Washington

Hans Schwerdtfeger
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G. W. Stewart, III
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Austin, Texas

Josef Stoer
University of California, San Diego
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V. R. R. Uppuluri
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Technischen Hochschule
Munchen, Germany

Katharina Zimmermann
Zurich, Switzerland

Milos Zlamal
Technical University
Brno, Czechoslovakia

Fred Dorr
Los Alamos Scientific Laboratory
Los Alamos, New Mexico

OBITUARIES

Michael Neumann, 1946-2011

By Stephen Kirkland

Michael Neumann

Michael Neumann died suddenly on April 21, 2011, at the age of 64. Miki, as he is known to his many friends and loved ones, was a devoted husband and father, a faithful friend, a valuable colleague, and a dedicated scholar. He is survived by his wife Helen and his children Joseph and Rachel.

Miki earned his B.Sc. from Tel Aviv University in 1970 and Ph.D. in computer science from London University in 1972. He held teaching positions at the University of Reading, the Technion, Nottingham University, UC Riverside, and the University of South Carolina before moving to the University of Connecticut as a full professor in 1985. He supervised nine Ph.D. students and served for many years on the editorial boards of the Electronic Journal of Linear Algebra, Linear Algebra and its Applications, and Operators and Matrices. Over the course of his career, he published more than 150 papers, primarily in pure and applied matrix theory, numerical analysis, and numerical linear algebra. He also coauthored a book *Nonnegative Matrices in Dynamic Systems* with Abraham Berman and Ronald J. Stern. Miki was a strong supporter of ILAS, serving on several of its committees and on the ILAS Board of Directors.

He has received multiple honors at the University of Connecticut. He has been Head of the Department of Mathematics since 2003; he received the Provost's Research Excellence Award for 2004/05, and in 2007 was designated a Board of Trustees Distinguished Professor and elected to the Connecticut Academy of Arts and Sciences. In 2010 he was awarded the Stuart and Joan Sidney Professorship of Mathematics (http://today.uconn.edu/?p=23927).

In addition to his many mathematical contributions, Miki will be remembered for his warmth, generosity, and sense of humour. In considering Miki's life and work, I am reminded of the words of the poet Emerson: "The purpose of life is not to be happy. It is to be useful, to be honorable, to be compassionate, to have it make some difference that you have lived and lived well." Michael Neumann did indeed live well, did indeed make a difference. He is greatly missed.

The funeral was held Friday, April 22 at Beth David Synagogue in West Hartford. The formal Shiva began on April 26, at the end of Passover, and concluded on May 2.

Index of Obituaries in IMAGE Issues 1-46

A reader has suggested that an index be maintained of obituaries that have appeared in IMAGE. Here is the list to date, with the issues and the names. All issues of IMAGE can be read at http://www.ilasic.math.uregina.ca/iic/IMAGE/.

15, Fall 1995	Arthur Asquith Rayner, Richard D. Sinkhorn, Albert William Tucker
16, Spring 1996	Olga Taussky Todd, Robert Charles Thompson, Bill Larry Neal
18, Spring 1997	John Maybee (brief announcement)
19, Fall 1997	John Maybee, Kermit Sigmon
21, Fall 1998	Patricia James Eberlein
22, Spring 1999	Dennis Ray Estes
23, Fall 1999	Vlastimil Ptak, Norman J. Pullman
25, Fall 2000	Sally Rear
38, Spring 2007	Morris Newman
39, Fall 2007	John Todd, Victor Klee, Ralph DeMarr, Gene Golub
43, Fall 2009	Israel Gohberg
44, Spring 2010	Ky Fan, M.J.C. Gover
45, Fall 2010	Mihaly Bakonyi
46, Spring 2011	Michael Neumann

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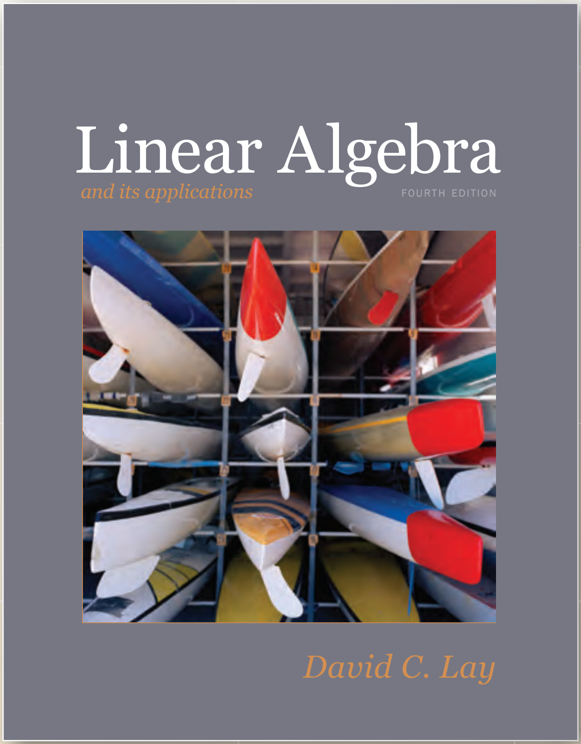

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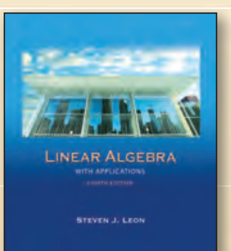

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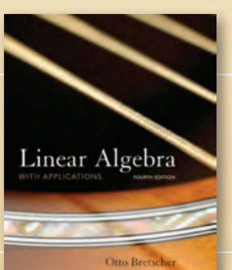

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BOOKS

Functions of Matrices, Theory and Computation by Nicholas J. Higham

SIAM, 2008. 425 pages. ISBN: 978-0-898716-46-7

Reviewed by Dario A. Bini
University of Pisa, bini@dm.unipi.it

General overview.
Since the time of Cayley and Sylvester, there has been great interest in the concept of matrix function. This interest, mainly motivated by the beauty and elegance of the underlying theory, has increased in the years due to the many applications that this concept has had in scientific computing and engineering. Nowadays, the interest is mainly addressed to the design and analysis of algorithms for computing matrix functions. The current research is very active and is continuously fed by the increasing demand from applications, to provide highly effective methods for solving diffcult computational problems.

The research monograph under review is the first treatment devoted to matrix functions and to algorithms for their computation. It constitutes a comprehensive, detailed and self-contained treatise on this subject. The author has collected an impressive amount of material, including theory, algorithms, applications, references and historical notes, reorganized and presented in an extraordinarily clear, neat and rigorous way.

The state of the art in research is reported with the description and analysis of algorithms for computing the most important matrix functions. Besides the matrix exponential, which is by far the most studied matrix function, the book deals with logarithm, matrix sign, square root, p^{th} root, polar decomposition, matrix cosine and matrix sine. Additional algorithms not treated in the book are addressed and commented with references to the current literature.

The reader is guided on a fascinating trip which touches nice pieces of elegant mathematics, including matrix theory and approximation theory, and where the analysis of numerical algorithms has a prominent role. Concepts and results are presented with great clarity and through illuminating examples. Throughout the book, intuition always has the priority with respect to technicalities in a clean mathematical framework. Plenty of historical notes enrich the presentation and give the subject great appeal.

Different aspects, like abstract theoretical properties and more concrete issues related to numerical analysis and algorithms, have a nice and effective interplay. For any function of interest, the fundamental steps of analyzing the conditioning of the problem and the algorithms for its solution, including accuracy of the approximation, convergence speed and the stability of the proposed iterations, together with the complexity of the related computations, are systematically pursued in a formally rigorous way. Besides the specific results of interest, the book implicitly teaches about the methodological approach that any numerical analyst should autonomously follow.

A reader interested in matrix analysis, numerical algorithms, and computational issues may find a lot of useful and amusing material whose interest goes beyond the subject of matrix functions. Here are just a few examples to give the flavor: the evaluation of a matrix polynomial by means of four different algorithms, including the Paterson-Stockmeyer method; the role played by the Fréchet derivative in the analysis of the sensitivity and conditioning of matrix functions and in the analysis of the stability of matrix iterations; the analysis of Newton's iteration, Padé approximation, and the role of scaling applied in different forms for different computational problems.

Each chapter contains a section with historical notes, references and other details concerning subjects of related interest not covered in the book. These notes are not limited just to simple citations but contain precise comments and descriptions of the different approaches encountered in the literature, including historical comments and the chronology of discoveries. Many of these comments have general interest and go beyond the specificity of the subject. Each chapter has also a list of exercises which range from simple problems addressed to graduate students, to more difficult exercises and advanced research problems which should stimulate scholars to work on this topic. At the end of the book, the solutions to all the exercises are reported.

The bibliography with its 625 references spans the period going from the early 1800's to nowadays. Honestly, it was a great pleasure bumping into quotations from Peano's work of the last decades of the 19^{th} century and ones from the two Italian mathematicians, Michele Cipolla and Giovanni Giorgi, who greatly contributed to the subject in the 1920's and 30's.

Outline of the chapters.

Chapter one contains a concise treatment of the theory of matrix functions, including definitions and properties valid for general functions. Three definitions of primary matrix function are given, via Jordan canonical form, Hermite interpolation and the Cauchy integral theorem. Their equivalence is proved. The concept of nonprimary matrix function is introduced.

Chapter 2 is addressed to applications, touching an unexpectedly large spectrum of cases including differential equations, Markov models, control theory, theoretical particle physics, nonlinear matrix equations, and matrix geometric means.

Chapter 3 concerns sensitivity analysis of matrix functions with respect to perturbations of the data, i.e., the analysis of conditioning. Properties of the Fréchet derivative are proved and used for providing estimates of the condition number. Several algorithms for computing the condition number are described and discussed.

Chapter 4 surveys the main techniques for computing or approximating a general matrix function. The main tools of this chapter are algorithms for matrix polynomial evaluation, properties of matrix Taylor series, rational approximation including Padé approximation and continuous actions, and diagonalization and triangularization techniques, including Schur decomposition and the Parlett recurrence. Functional iterations are analyzed in detail, including the related problem of convergence, termination criteria and stability. Preprocessing techniques and upper bounds to the norm of a function of a matrix are the last two topics of the chapter.

The following Chapters 5-8 are focused on the algorithmic core of the book. They deal with specific matrix functions, namely, matrix sign, matrix square root, matrix p^{th} root, and polar decomposition, with their properties and algorithms for their computation. Each chapter contains the fundamental steps of sensitivity analysis, and, concerning iterative algorithms, the analysis of convergence and stability of the iteration, together with terminating conditions. Here the major part is played by the Schur method, Newton's iteration and its variants, the Padé family of iterations, and the matrix sign iteration. The interplay between these matrix functions is pointed out.

Chapter 9 deals with a general purpose algorithm for computing a matrix function. The algorithm is based on the reordered Schur decomposition of the matrix, followed by the application of the Parlett recurrence in the block form.

Chapters 10, 11 and 12 focus on properties and algorithms of matrix exponential, matrix logarithm and matrix cosine and sine, respectively. Once again, the design and analysis of algorithms for their computation are the main issues of these chapters together with sensitivity analysis and results for estimating the condition numbers.

Computing the product of a matrix function and a vector is the core of Chapter 13. This subject is a very hot topic of increasing interest because of the need of dealing with very large and sparse input matrices. The approaches based on Krylov subspaces, quadrature and differential equations are outlined.

Chapter 14, the final and the shortest, considers the case where the input matrix has some specific structure. A list of relevant structured matrices associated with some scalar product is just reported, including Hamiltonian and symplectic matrices. The exponential decay properties of functions of band matrices are analyzed.

The Appendix is divided into five parts. Part A contains the notation list. Part B reports the main basic tools of matrix analysis and numerical analysis used throughout the book. Useful tables with the arithmetic cost of the main matrix computations are reported in Part C, while Part D contains the list of functions in the Matrix Function Toolbox of MATLAB, which contains MATLAB implementations of many of the algorithms described in the book. Part E offers solutions for the problems of each chapter.

Conclusions.

I enjoyed reading this book a lot and I am sure that anyone fond of matrices and linear algebra will be greatly delighted by it. Certainly, students and scholars interested in computational issues will find the book even more useful and stimulating.

The book is recommended to any scholar who has a theoretical or computational interest on functions of matrices, and to anyone playfully devoted to matrix theory, computations and algorithms.

The book is fundamental for a graduate course in functions of matrices, but it will also be of great support and interest in any course touching topics in computational linear algebra.

Matrix Partial Orders, Shorted Operators and Applications
By S.K. Mitra, P. Bhimasankaram and S.B. Malik

World Scientific, New Jersey, 2010. 464 pages. ISBN-13 978-981-283-844-5, ISBN-10 981-283-844-9

Reviewed by Götz Trenkler
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The number of articles dealing with matrix orders and the related concepts of parallel sums and shorted operators for nonnegative definite matrices has distinctly increased over the recent years. The central objective of this monograph is to present these results and their applications, enriched with hitherto unpublished research. Parallel sum and shorted operator have their origin in the study of impedance matrices of electrical networks.

Being a generalization of the harmonic mean, the parallel sum of two matrices is well defined and known in the literature. The shorted operator, which is not so well known, can be introduced in a number of ways. In its most common form, for example in Statistics, the shorted operator is the maximal element with respect to the Löwner order in a certain class of nonnegative definite matrices. More generally, the shorted operator can be defined as a limit of a sequence of parallel sums. Various other definitions are considered in the book with a detailed investigation of their relationships.


S.K. Mitra

The main tools used in the book consist of various matrix decompositions such as simultaneous normal form, simultaneous singular value decomposition and the generalized singular value decomposition. The matrix orders under study relate to matrices over an arbitrary field, but fairly often the complex case is considered.

Obviously the monograph is based mainly on results of the first author, Sujit Kumar Mitra, who was a leading expert in the theory of matrix partial orders and also well known in the Statistics community, passed away in 2004.

After a short introduction, Chapter 2 presents the fundamentals of matrix decompositions and generalized inverses, as far as they are needed for the further reading of the book. Some new results and their proofs are included.

Chapter 3 is devoted to the minus order with some introductory considerations of the space pre-order, "the big daddy of (the most basic among) all order relations". It is seen that the rank partial order is a space pre-order along with some additional conditions. Subsequently, a large number of equivalent statements is given for the rank partial order of two rectangular matrices with entries from an arbitrary field. Some attention is paid to the problem of finding all matrices lying above or below a given matrix. The chapter ends with a study of the minus order for idempotent and complex matrices.

In Chapter 4 the sharp order, originally introduced by Mitra, is considered. Several characterizations are given by mainly using a special decomposition of square matrices whose index of nilpotence is one. The minus order is defined independently of the sharp order, and the authors give some conditions to assure their coincidence. Special attention is paid again to idempotent matrices. Finally, the sharp order is generalized to matrices that are not necessarily of index one. This so-called Drazin order relies on the Core-Nilpotent Decomposition of a square matrix.

The star order of complex matrices is considered in Chapter 5. It originates from the research work of Drazin in the framework of star semi-groups with involution. Several characterizations of the star order are given, as previously done with the other matrix orders. Here the so-called normal form and the reduced form turn out to be very useful. A detailed study is performed for the subclasses of range-Hermitian, normal, Hermitian and nonnegative definite matrices. Special sections deal with the star order of idempotents and Fisher-Cochran type theorems.

Each of the previously discussed orders is characterized by two corresponding conditions. In Chapter 6 the question is investigated whether one still gets a partial order when one of the conditions is dropped or modified.

Partial orders defined by generalized inverses are explored in Chapter 7. The new orders are mainly characterized by the inclusion of certain classes of generalized inverses in others. Also some order relations defined by outer inverses are considered. Special focus is on the so-called G-based order (left and right).

Chapter 8 is dedicated to the most widely used order, the Löwner order, which is defined for the class of Hermitian matrices. However, its main application is found for the subclass of nonnegative definite matrices. Several properties are collected here, which have previously been dispersed over the literature: for example Albert's Theorem, relationships with the inertia, Löwner order for powers and generalized inverses, interplay with other partial orders, and generalizations for rectangular matrices.

In Chapter 9 parallel sums of matrices are studied, even for rectangular matrices, using generalized inverses. Some links to various partial orders are provided. It is shown that the set of nonnegative definite matrices constitutes a partially ordered abelian group with respect to parallel addition. Also some results for the Drazin inverse of parallel sums are presented.

Many facets of Schur complements and shorted operators are treated in Chapter 10. Formally the Schur complement is introduced as the shorted operator of a nonnegative definite matrix. Among other results, the Schur complement is shown to be the supremum of a certain class of matrices under the minus order. Most of the considerations are extended to matrices over arbitrary fields, serving as a basis for the next chapter.

How to characterize the shorted operator further is the topic of Chapter 11. Moreover, some hints for its computation are given.

Chapter 12 investigates the problem of identifying under which conditions a class of matrices, equipped with a certain partial order, becomes a lattice. This means, in other words, finding the infimum or supremum of a given pair of matrices. The problem is tackled for the minus, the star and the sharp orders.

In Chapter 13 partial orders of modified matrices are considered. Here modification of a matrix means appending or deleting rows/columns or adding a matrix of rank one. It is also investigated whether the main partial orders remain valid under these modifications.

In Chapter 14 equivalence relations on generalized and outer inverses are studied by using the main orders. Thus more structure in these classes is identified.

Chapter 15 shows how the matrix partial orders and the shorted operators can be applied in statistics or in electrical networks. Special emphasis is on the BLUE (Best Linear Unbiased Estimator) in the linear regression model, incomplete block designs and testing. Modifications of shorting mechanisms in n-part reciprocal resistive networks is also considered.

This unique book can be recommended to anybody interested in matrix orders. It contains a lot of results hitherto dispersed in research papers or unpublished work of the authors. Although written a bit from the perspective of statistics, from which many applications come, it should be consulted by students and researchers in both matrix theory and practice.

The book has few typographical errors. Its printing is excellent, making very pleasant reading. Containing a number of exercises, it may also serve as a textbook for an advanced matrix course. Written in the Definition-Theorem-Lemma-Proof-Remark style, this monograph might not necessarily be to everybody's taste, but I liked its precise and succinct presentation. Occasionally references to original work are missing. Nevertheless, this is a very remarkable book.

List of Books of Interest to Linear Algebraists

Oskar Baksalary, Consulting Editor for IMAGE, maintains a list of books about linear algebra and its applications on the ILAS website (http://www.ilasic.math.regina.ca/iic/). He includes references to reviews of selected titles. Please tell him when you find additional titles or informative reviews that should be included (baxx@amu.edu.pl).

NEWS ABOUT JOURNALS AND WEB PAGES

Report on Linear Algebra and its Applications

By Richard Brualdi, Volker Mehrmann, and Hans Schneider, Editors-in-Chief

Linear Algebra and its Applications is finishing a banner year with more than 1000 submissions in 2010 and the year is not quite over yet. This speaks well for the breadth and depth of our field - linear algebra and matrix theory - and the substantial interest in it by a great number of mathematicians from all over the world, and not just those who consider themselves linear algebraists or matrix theorists. With so many papers we have to be very selective in what we publish. At the same time we want to ensure the continued development and application of linear algebra and matrix theory, to promote new directions as well as new insights in established topics, and to encourage young people to enter the field.

The year 2011 will see some changes developing in LAA. First, Peter Šemrl will join us as an Editor-in-Chief, and we welcome him very warmly and with great enthusiasm. We look forword to working together with Peter to promote linear algebra and LAA. Second, sometime during 2011 we plan to switch over to web submission making use of Elsevier's electronic editorial system. Third, Hans, after 40 years as an editor-in-chief of LAA, will start to reduce his presence in LAA and will step down as an Editor-in-Chief of LAA at the end of 2012.

We extend our thanks to all who contributed to LAA in 2010 with papers and refereeing work.

Editorial Changes for the Electronic Journal of Linear Algebra

Contributed by Bryan Shader, ELA Editor-in-Chief

2010 was a good year for The Electronic Journal of Linear Algebra (ELA). The journal published 78 papers in two issues, and received 168 submissions. Many thanks go to the editorial board and the referees for evaluating the papers in a thoughtful and timely fashion, and to the authors for carefully preparing their submissions and revisions.

The new year brings many exciting developments for ELA. Ludwig Elsner's term as Editor-in-Chief of ELA is complete, but his efforts and care for ELA will be long lasting. Joining the editorial board as Associate Editors are:

Oskar Baksalary (http://www.staff.amu.edu.pl/~baxx)
T.Y. Tam (http://www.auburn.edu/~tamtiny)
Christian Mehl (http://www.math.tu-berlin.de/~mehl)
Xingzhi Zhan (http://math.ecnu.edu.cn/~zhan/intro.html)
Jim Nagy (http://www.mathcs.emory.edu/~nagy)
Leslie Hogben (http://www.public.iastate.edu/~lhogben/homepage.html)

Panagiotis (Panos) Psarrakos (http://www.math.ntua.gr/~ppsarr) has taken over the duties of Associate Managing Editor. Many thanks go to former Associate Managing Editor Michael Tsatsomeros for his years of effort in this role and for his help in making a smooth transition. Michael Cavers (http://www.michaelcavers.com) and In-Jae Kim (http://cset.mnsu.edu/mathstat/dir/kim.html) join the board as Assistant Managing Editors. The board members' expertise and renown, as well as high-quality submissions from the ILAS community, position ELA to continue to develop into a leading mathematical journal.

Change in Submission Procedure for the Electronic Journal of Linear Algebra

Contributed by Bryan L. Shader, Editor-in-Chief

The submission procedure for ELA papers has changed. The change is designed to align ELA with open-publication standards, to allow ELA authors to more easily disseminate their results, and to streamline part of the editorial process. Logistically, the main change is that the corresponding author will now complete and submit an Author Declaration and Consent to Publish form at the time of submission. Legally, the main changes are that authors will henceforth maintain the copyrights, agree to allow ELA to publish the paper (if the paper is accepted) and to reference ELA in any future publication of material from the ELA paper. The instructions and form can be found at the ELA website http://www.math.technion.ac.il/iic/ela/

Thanks go to the ELA Editorial Board, the ILAS Publications Committee, and in particular Steve Kirkland and Roger Horn for their advice and help in making this change.

Call for Papers:
Special Issue of Linear and Multilinear Algebra in Memory of Ky Fan

A special issue of Linear and Multilinear Algebra will be dedicated to the memory of Ky Fan, who passed away in March 2010. The journal invites papers related to his work, particularly in the areas of linear algebra, matrix theory and operator theory.

The special issue editors are Ravindra Bapat, Indian Statistical Institute, India (rbbapat@rediffmail.com), Raphael Loewy, Technion-Israel Institute of Technology, Israel (loewy@techunix.technion.ac.il), and Fuzhen Zhang, Nova Southeastern University, USA (zhang@nova.edu). The Editor in Chief for this special issue is Steve Kirkland (stephen.kirkland@nuim.ie). The deadline for submissions is July 31, 2011 with target date May 15, 2012 for publication.

Call for Papers:
Special Issue of Reliable Computing on the Use of Bernstein Polynomials
In Honor of the 100th Anniversary of the Introduction of Bernstein Polynomials

This issue is to celebrate the 100th anniversary of the introduction of the Bernstein polynomials by S.N. Bernstein in 1912.

The shape preserving properties of Bernstein polynomials are closely related to totally positive matrices. During the last decades, on account of their polynomial range enclosing property, Bernstein polynomials have become a useful tool in many fields, such as robust control, automatic theorem proving, global optimization, and computer aided geometric design. Other fields where Bernstein polynomials have recently been applied include image processing, pattern recognition, and ordinary differential equations, with potential for verified computations as well. We feel that these developments have matured sufficently such that the time has come for a special issue devoted to this approach, and we invite submission of papers to it.

If you are interested in submitting a paper for this special issue, please send to us the tentative title and an abstract as soon as possible. Jürgen Garloff (garloff@htwg-konstanz.de) and Andrew P. Smith (smith@htwg-konstanz.de) are the Guest Editors for this issue.

Papers should be submitted electronically via http://www.cs.utep.edu/interval-comp/rcjournal.html. Read the Information and Instructions for Authors and follow the link to the online paper submission site (with EasyChair). To assign your submitted paper to the special issue it is important that you begin the title of your paper with "Bernstein Issue:". Deadline for paper submissions is July 31, 2011.

New Web Page for Spectral Graph Theory
Submitted by Nair Maria Maia de Abreu, Federal University of Rio de Janeiro, Brazil

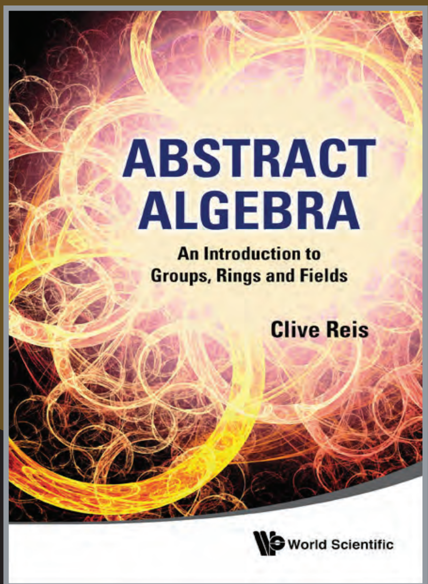
There is a Spectral Graph Theory Home Page (http://www.sgt.pep.ufrj.br/) that is updated regularly and already contains over two thousand references of papers. This web page is hosted by the Program of Production Engineering at COPPE/UFRJ, because of the many applications of graph theory and linear algebra to engineering.

The page includes the main references and results of the field as well as links to events. If you are a researcher in spectral graph theory and you feel something or someone is missing from our lists, please contact us via the web page. The page also includes information about COPPE (Alberto Luiz Coimbra Institute Graduate School and Research in Engineering, Rio de Janeiro, Brazil).

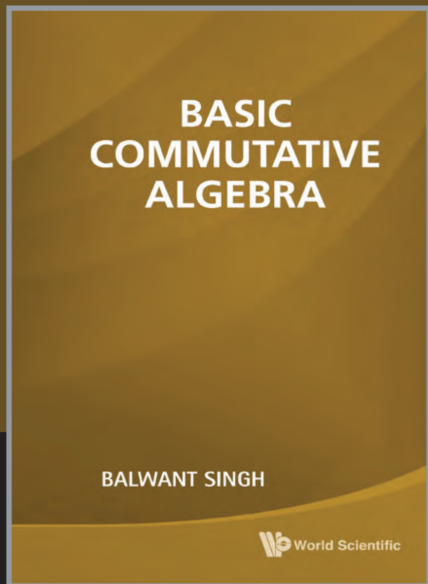
The Editorial Board for this web page is Chris Godsil - University of Waterloo, Dragan Stevanović - University of Nis, Dragos Cvetković - University of Belgrade - (Chairman), Domingos Cardoso - University of Aveiro, Ivan Gutman - University of Kragujevac, Nair Maria Maia de Abreu - Federal University of Rio de Janeiro, Peter Rowlinson - University of Stirling (Co-Chairman), Pierre Hansen - University of Montreal, Steve Kirkland - Hamilton Institute, National University of Ireland, and Vladimir Nikiforov - University of Memphis.

Connecting Great Minds

New



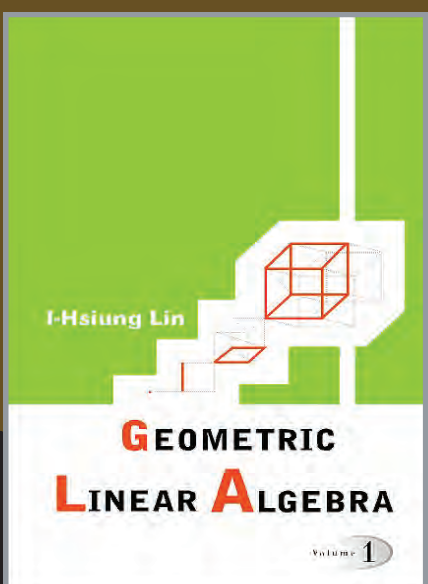
ABSTRACT ALGEBRA
An Introduction to Groups, Rings and Fields
by **Clive Reis**
(University of Western Ontario, Canada)



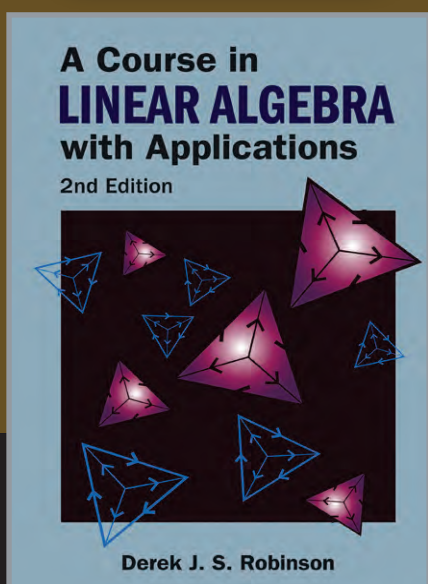
BASIC COMMUTATIVE ALGEBRA
by **Balwant Singh**
(Indian Institute of Technology Bombay, India & UM-DAE Centre for Excellence in Basic Sciences, India)

Series in Algebra - Vol. 11
RINGS RELATED TO STABLE RANGE CONDITIONS
by **Huanyin Chen**
(Hangzhou Normal University, China)

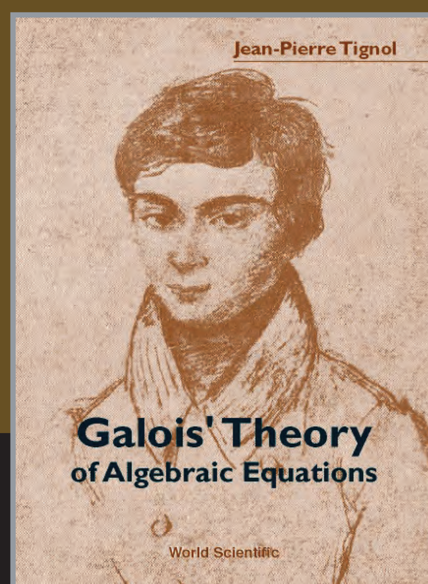
Bestsellers



GEOMETRIC LINEAR ALGEBRA
(Volume 1 & Volume 2)
by **I-Hsiung Lin**
(National Taiwan Normal University, Taiwan)



A COURSE IN LINEAR ALGEBRA WITH APPLICATIONS
(2nd Edition)
by **Derek J S Robinson**
(University of Illinois at Urbana-Champaign, USA)



GALOIS' THEORY OF ALGEBRAIC EQUATIONS
by **Jean-Pierre Tignol** *(Université Catholique de Louvain, Belgium)*

LINEAR ALGEBRA EDUCATION

Report on Linear Algebra Education Minisymposium at ILAS 2010 in Pisa, Italy

By Steve Leon, University of Massachusetts Dartmouth

One of the many highlights of ILAS 2010 was this minisymposium. Guershon Harel (UC San Diego, USA) discussed research about the algebraic ways of thinking necessary for success in beginning college mathematics courses, how well high school texts prepare students for these ways of thinking, how well high school teachers teach to promote them, and how to train teachers to promote these ways of thinking among students.

Boris Koichu (Technion, Israel) discussed strategies for using clickers, and their effect. Clickers allow students to give immediate feedback during a lecture. Edgar Possani (Inst. Tecnologico Autotonomo, Mexico) outlined an approach to teaching linear algebra using a problem from economics and a method called APOS for analyzing students' learning processes. APOS says that the learning process for formulating a model involves identifying *actions*, *processes* and *objects*, and then organizing these into *schema*. Jane Day (San Jose State University, USA) summarized insights about how people learn that she had acquired over time from the theories of Maria Montessori, Jean Piaget, Carl Jung's theory of personality, and Guershon Harel, as well as the recommendations of the Linear Algebra Curriculum Study Group (LACSG) in 1990 and feedback from her own students.

Frank Uhlig (Auburn University, USA) posed a number of provoking questions that could lead us to rethink much of what we do in our courses. Sang-Gu Lee (Sungkyunkwan University, Korea) talked about the linear algebra course that he teaches using Sage Math software. Sage Math provides a very new mobile, multimedia and flexible learning environment for students. It utilizes technology and the internet in innovative in innovative ways and is ideal for teaching linear algebra. Avi Berman spoke on "Principles and Tools in Teaching Linear Algebra". He discussed the objectives of the course in terms of student understanding and skills, ways of achieving those objectives through motivation, visualization, interaction and challenge, and examples of how this is being done at the Technion. Steve Leon emphasized the importance of requiring a second course in linear algebra for all mathematics majors, as recommended by the LACSG, and outlined a number of alternative courses that could be offered. He also described one he has taught in which students work in teams on projects, applying linear algebra to areas such as digital imaging, computer animation, and coordinate metrology. Many of the projects involved original undergraduate research. The final speaker was Kyung-Won Kim (Sungkyunkwan University, Korea), who spoke about "The Development of Excel and Sage Math Tools for Linear Algebra".

For pdf files with more details of these talks, visit the ILAS Education web page http://galois09.skku.sc.kr/newilas/.

Report on Linear Algebra Education Activities at the 2011 Joint AMS/MAA Meetings

By David Strong, Pepperdine University

The MAA contributed paper session on Innovative and Effective Ideas in Teaching Linear Algebra was one of the genuine delights at the Joint Meetings of the American Mathematical Society and the Mathematical Association of America held in January 2011 in New Orleans, Louisiana, USA. This was the fifth year that this session has been organized by David Strong, Gil Strang and David Lay. Fourteen presentations on applications, examples, ideas and philosophies in teaching undergraduate linear algebra were given by speakers from various parts of the world.

The session began with two talks on linear algebra in specific applications, ray-based tomography (Murphy Waggoner) and quaternions in computer animation (Paul Bouthellier). Four talks focused on specific examples to illustrate and better understand particular ideas: discrete dynamical systems, using Mathematica (Thomas Polaski); a geometric view of orthogonal diagonalization of symmetric matrices (Robert Sachs); a first-day-of-class example for introducing a variety of fundamental linear algebra ideas (David Strong); and the "Magic Carpet Ride" and other intuition-developing examples (Megan Wawro).

The bulk of the session was devoted to general thoughts on and practices in the teaching of linear algebra: topics in a second undergraduate linear algebra course (Steven Leon); using Wolfram's *Alpha* for linear algebra (Aldo Maldonado); teaching first with computation, then application, then theory (Jason Grout); the ILAS education homepage (Kim-Kyung Won); Mobile Sage (Sang-Gu Lee); an evaluation of student experiences in a technology-based linear algebra course (Karsten Schmidt); oral interviews to complement written exams (Steven Hetzler); and an "inverted" linear algebra classroom (Robert Talbert).

The talks were well attended, nearly to the capacity of the 100+ person room for some talks. Needless to say, there is great interest amongst mathematicians in linear algebra and how to more effectively teach its many interesting and useful ideas to undergraduates. Links for presentations can be found at http://faculty.pepperdine.edu/dstrong/LinearAlgebra/2011/index.html.

This session will again be held at the January 2012 Joint Meetings in Boston, Massachusetts, USA.

ILAS AND OTHER LINEAR ALGEBRA NEWS

New ILAS Officers

By Steve Kirkland, ILAS President

Steve Kirkland has been elected to the position of President for a three-year term, beginning on March 1, 2011. Francoise Tisseur and David Watkins have been elected to three-year terms as members of the ILAS Board, beginning on March 1, 2011.

On behalf of ILAS, I take this opportunity to thank the members of the Nominating Committee, Ravi Bapat, Avi Berman (Chair), Leiba Rodman, Valeria Simoncini and Hugo Woerdeman, and also to thank all of the nominees for their participation in the election. Finally, I extend thanks to Raphael Loewy, who helped to count the ballots.

Call for Proposals for ILAS Lecturers at non-ILAS Meetings

By Steve Kirkland, ILAS President

As part of ILAS's commitment to supporting activities in Linear Algebra, the Society maintains a program of ILAS Lectureships at non-ILAS conferences. In a typical year, ILAS supports two such Lectureships, and in an exceptional year, ILAS may support as many as four Lectureships.

This announcement serves as a call for proposals for ILAS Lectureships to take place in 2012. The deadline for submitting proposals for ILAS Lectureships held in 2012 is September 30, 2011. Further details regarding the format of proposals and submission process can be found at http://www.ilasic.math.uregina.ca/iic/misc/non_ilas_guidelines.html.

Send News for IMAGE Issue 47 by October 1, 2011

Send news for Issue 47 by October 1, 2011. This issue will appear online December 1, 2011. All items of interest to the linear algebra community are welcome, including photos and longer articles as well as short notes like historical tidbits. Suggestions are also welcome, such as ideas for survey articles, history topics, books to review, people to interview, etc.

Send your items directly to the appropriate editor:

* Problems and Solutions to Fuzhen Zhang (zhang@nova.edu)
* History of Linear Algebra to Peter Semrl (peter.semrl@fmf.uni-lj.si)
* Book Reviews to Oskar Baksalary (baxx@amu.edu.pl)
* Linear Algebra Education news to Steve Leon (sleon@umassd.edu)
* Advertisements to Steve Kirkland (stephen.kirkland@nuim.ie)

Send all other news to Jane M. Day, Editor in Chief (Jane.Day@sjsu.edu). This includes:

* Announcements and reports of conferences, workshops
* News from journals, brief announcements of new books
* Honors and awards, funding opportunities, job openings
* Brief references to articles appearing elsewhere, of interest to the linear algebra community
* Feature articles other than history and book reviews
* Transitions (new positions, obituaries)
* Letters to the Editor

New SIAM Fellows

We congratulate Ilse C.F. Ipsen and Volker Mehrmann, members of ILAS who have recently been inducted as Fellows of SIAM, Class of 2011. Beresford Parlett, who was a co-winner with Cleve Moler of the ILAS Hans Schneider Prize in 2008, is another new SIAM Fellow in 2011. For details, visit http://fellows.siam.org/.

ILAS 2010 - 2011 Treasurer's Report

April 1, 2010 - March 31, 2011

By Leslie Hogben

Net Account Balance on March 31, 2010			
Vanguard (ST Fed Bond Fund ST 7196.162 Shares)	$77,358.74		
Checking Account - First Federal	$18,782.93		
Certificate of Deposit	$20,000.00		
Checking Account - First National	$1,600.00		
Variances for Currency Transfers	$382.72		
	___________		
			$118,124.39
			===========
General Fund	$54,532.76		
Conference Fund	$12,096.01		
ILAS/LAA Fund	$12,840.57		
Olga Taussky Todd/John Todd Fund	$10,832.14		
Frank Uhlig Education Fund	$4,914.34		
Hans Schneider Prize Fund	$22,908.57		
	___________		
			$118,124.39
			===========
INCOME:			
Dues	$9,980.00		
Corporate Sponsorship	$5,121.90		
General Fund	$4,505.87		
Conference Fund	$10.00		
Taussky-Todd Fund	$310.00		
Uhlig Education Fund	$30.00		
Schneider Prize Fund	$75.00		
Interest - First Federal	$134.07		
Vanguard - Dividend Income	$1,087.86		
- Short Term Capital Gains	$770.94		
- Long Term Capital Gains	$-		
Misc Income	$67.83		
	___________		
Total Income		$22,093.47	
EXPENSES:			
Citi Bank - Credit Card Service Charges	$571.27		
American Express - Credit Card Service Charges	$16.13		
Speaker Fees	$1,950.00		
Treasurer's Report Preparation	$392.50		
Conference Expenses – Pisa	$5,982.32		
Conference Student Travel Support – Pisa	$1,000.00		
ILAS Help	$150.37		
IMAGE Costs	$1,043.50		
Wire Transfer Fees	$16.05		
American Express - Outstanding Unpaid Charges	$320.00		
Less variances in bond market	$580.28		
	___________		
Total Expenses		$12,022.42	
Net Account Balance on March 31, 2011			

Vanguard (ST Fed Bond Fund Admiral ST 7373.167 Shares)	$79,187.81	
Checking Account – First Federal	$29,007.63	
Certificate of Deposit	$20,000.00	
		$128,195.44

General Fund	$64,178.81	
Conference Fund	$12,106.01	
ILAS/LAA Fund	$12,840.57	
Olga Taussky Todd/John Todd Fund	$11,142.14	
Frank Uhlig Education Fund	$4,944.34	
Hans Schneider Prize Fund	$22,983.57	
		$128,195.44

OPPORTUNITIES

Postdoctoral Position in Parallel Scientific Computing at CIMNE/UPC (Spain)

By Dr. Santiago Badia, CIMNE, Barcelona, Spain

The International Center for Numerical Methods in Engineering (CIMNE) at the Universitat Politècnica de Catalunya (UPC) is seeking a postdoctoral candidate in parallel sparse linear algebra for saddle-point type linear systems, arising from the finite element approximation of Navier-Stokes, Maxwell and Magnetohydrodynamics problems. The candidate will belong to the newly created Computational Mathematics and Physics Department, which will be focused on the development of computational methods for the numerical approximation of Partial Differential Equations.

The creation of this group is funded by the Starting Grant COMFUS, recently awarded to Dr. Santiago Badia by the European Research Council (ERC). COMFUS is a five-year project that aims to advance in the field of numerical methods for fusion reactors technology. It involves the numerical approximation of magnetohydrodynamics (MHD), electromagnetics and plasma physics.

We are looking for excellent candidates holding a PhD in Computer Science, Applied Mathematics or Computational Mechanics/Physics. Strong background on sparse linear algebra, parallel computing and domain decomposition techniques is required. The candidate would collaborate in the development of parallel linear solvers for saddle-point problems.

The position will remain opened till a suitable candidate is found. The appointment can be extended to five years, and the salary, among 32,000-40,000 EUR, will depend on experience.

Please contact sbadia@cimne.upc.edu or visit http://badia.rmee.upc.es/ for further information. Send your application, providing CV, to: comfus@cimne.upc.edu. Recommendation letters will likely be requested.

Other Sources for Open Mathematics Positions

Websites that list open college and university positions in mathematics are maintained by the AMS (https://www.mathjobs.org/jobs) and SIAM (http://jobs.siam.org/home/index.cfm?site_id=686). To find a broad spectrum of academic positions, including mathematics, google Academic Keys for Sciences.

IMAGE Problem Corner: Old Problems With Solutions

We present solutions to Problems 45-1 through 45-7. Seven new problems are on the back cover; solutions are invited.

Problem 45-1: Column Space Counterparts Of The Known Conditions For Orthogonal Projectors
Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.tu-dortmund.de

Let $\mathbb{C}_n^{\mathsf{OP}}$ denote the set of orthogonal projectors in $\mathbb{C}_{n,1}$ (Hermitian idempotent matrices of order n), i.e.,

$$\mathbb{C}_n^{\mathsf{OP}} = \{K \in \mathbb{C}_{n,n}\colon\ K^2 = K = K^*\},$$

where K^* means the conjugate transpose of the complex matrix K. It is known that $P, Q \in \mathbb{C}_n^{\mathsf{OP}}$ satisfy:

(i) $PQ \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = QP$,
(ii) $P + Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = 0$,
(iii) $P - Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ PQ = Q$.

Provide other forms of the equalities on the right-hand sides of these equivalences, now as subspace inclusions of the form $\mathcal{R}(X) \subseteq \mathcal{R}(Y)$, where $\mathcal{R}(\cdot)$ is a column space of a matrix argument, and X and Y are functions of P and Q.

Solution 45-1.1 by the proposers Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.tu-dortmund.de

Let $\overline{P} = I_n - P$, where I_n is the identity matrix of order n. The solution to the problem is given in the theorem below.

Theorem. Let $P, Q \in \mathbb{C}_n^{\mathsf{OP}}$. Then:

(a) $PQ \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ \mathcal{R}(PQ) \subseteq \mathcal{R}(Q)$,
(b) $P + Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ \mathcal{R}(Q) \subseteq \mathcal{R}(\overline{P}Q)$,
(c) $P - Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ \mathcal{R}(Q) \subseteq \mathcal{R}(PQ)$.

Proof. Let $P \in \mathbb{C}_n^{\mathsf{OP}}$ be of rank r. By spectral theorem, there exists a unitary $U \in \mathbb{C}_{n,n}$ such that

$$P = U \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} U^*. \tag{1}$$

Representation (1) can be used to determine partitioning of $Q \in \mathbb{C}_n^{\mathsf{OP}}$. Namely, with the use of the same matrix U, we can write

$$Q = U \begin{pmatrix} A & B \\ B^* & D \end{pmatrix} U^*, \tag{2}$$

with $A \in \mathbb{C}_{r,r}$ and $D \in \mathbb{C}_{n-r,n-r}$ being Hermitian. Two particular versions of representation (2) are obtained when $r = 0$, in which case matrices A and B are absent, and when $r = n$, in which case matrices D and B are absent. From $Q^2 = Q$ it follows that

$$A = A^2 + BB^* \quad \text{and} \quad D = D^2 + B^*B. \tag{3}$$

In view of the equivalences recalled in the statement of the problem, direct calculations with the use of representations (1) and (2), as well as relationships (3), show that

(a) $PQ \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ B = 0$,
(b) $P + Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ A = 0$,
(c) $P - Q \in \mathbb{C}_n^{\mathsf{OP}} \ \Leftrightarrow\ D = 0$.

Below we show that the inclusions on the right-hand sides of the equivalences (a)–(c) listed in the theorem are equivalent to the requirements that B, A, and D are equal to zero matrices, respectively. A crucial role in the derivations is played by the known fact that any matrices X and Y having the same number of rows, satisfy $\mathcal{R}(X) \subseteq \mathcal{R}(Y)$ if and only if $P_Y X = X$, where P_Y is the orthogonal projector onto $\mathcal{R}(Y)$.

To establish equivalence (a) of the theorem, note that the left-hand side condition in (3) entails

$$\mathcal{R}(PQ) \subseteq \mathcal{R}(Q) \;\Leftrightarrow\; QPQ = PQ \;\Leftrightarrow\; B = 0.$$

Since

$$\mathcal{R}(Q) \subseteq \mathcal{R}(\overline{P}Q) \;\Leftrightarrow\; P_{\overline{P}Q}Q = Q \quad \text{and} \quad \mathcal{R}(Q) \subseteq \mathcal{R}(PQ) \;\Leftrightarrow\; P_{PQ}Q = Q, \tag{4}$$

to prove equivalences (b) and (c) we need projectors onto $\mathcal{R}(\overline{P}Q)$ and $\mathcal{R}(PQ)$. The expressions for these projectors were provided in formulae (3.2) and (3.14), respectively, in Baksalary and Trenkler (2009), and show to be of the forms

$$P_{\overline{P}Q} = U \begin{pmatrix} 0 & 0 \\ 0 & P_D \end{pmatrix} U^* \quad \text{and} \quad P_{PQ} = U \begin{pmatrix} P_A & 0 \\ 0 & 0 \end{pmatrix} U^*. \tag{5}$$

In view of (3), substituting projectors given in (2) and (5) into (4) yields

$$\mathcal{R}(Q) \subseteq \mathcal{R}(\overline{P}Q) \;\Leftrightarrow\; A = 0 \quad \text{and} \quad \mathcal{R}(Q) \subseteq \mathcal{R}(PQ) \;\Leftrightarrow\; D = 0. \qquad \square$$

Reference
O.M. Baksalary & G.Trenkler, Column space equalities for orthogonal projectors, *Applied Mathematics and Computation* 212(**2009**) 519-529.

Solution 45-1.2 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

(i) We show that $PQ = QP$ if and only if $\mathcal{R}(PQ) \subseteq \mathcal{R}(Q)$. If $PQ = QP$, then $\mathcal{R}(PQ) = \mathcal{R}(QP) \subseteq \mathcal{R}(Q)$.

For the converse, take any $u \in \mathbb{C}^n$, and let $u = u_1 + u_2$, where $u_1 \in \mathrm{Ker}(Q)$ and $u_2 \in \mathcal{R}(Q)$. Since $\mathcal{R}(PQ) \subseteq \mathcal{R}(Q)$, $PQu_2 = Qv$ for some $v \in \mathbb{C}^n$. Hence $QPu_2 = QPQu_2 = Q^2v = Qv = PQu_2$.

Furthermore, from $\mathcal{R}(PQ) \subseteq \mathcal{R}(Q)$ we also have $\mathrm{Ker}(Q) \subseteq \mathrm{Ker}((PQ)^*) = \mathrm{Ker}(QP)$ by taking orthogonal complements. Hence $QPu_1 = 0 = Qu_1 = PQu_1$, and therefore $QPu = PQu$.

(ii) Using the fact that $\mathrm{Ker}(P) = \mathcal{R}(I - P)$ for any orthogonal projection P, we have

$$PQ = O \Leftrightarrow \mathcal{R}(Q) \subseteq \mathrm{Ker}(P) \Leftrightarrow \mathcal{R}(Q) \subseteq \mathcal{R}(I - P)$$

(iii) Using the fact that $I - P$ is also an orthogonal projection, we have by (ii)

$$PQ = Q \Leftrightarrow (I - P)Q = O \Leftrightarrow \mathcal{R}(Q) \subseteq \mathcal{R}(I - (I - P)) = \mathcal{R}(P).$$

Also solved by Hans Joachim Werner.

Problem 45-2: Nilpotent Matrix Commuting With A Nilpotent Matrix And With Its Conjugate Transpose
Proposed Johanns de Andrade Bezerra, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let N_0 and N in $\mathbf{C}^{n\times n}$ be nilpotent matrices, where N is nonderogatory, i.e., $dimKerN = 1$. Let N^* be the conjugate transpose of N. If $N_0N = NN_0$ and $N_0N^* = N^*N_0$, show that $N_0 = 0$.

Solution 45-2.1 by Roger A. Horn, *University of Utah*, rhorn@math.utah.edu

The conclusion follows from a weaker hypothesis. We reformulate the problem as follows: Let A and B be square complex matrices of the same size. Suppose that A is nonderogatory and that B commutes with both A and A^*. Show that B is normal and conclude that $B = 0$ if it is nilpotent.

Solution: Examination of the Jordan canonical form shows that A^* is nonderogatory if and only if A is nonderogatory. Any matrix that commutes with a nonderogatory matrix X is a polynomial in X, [1, (3.2.4.2)] so there are polynomials $p(t)$ and $q(t)$ such that $B = p(A) = q(A^*)$. Let U be a unitary matrix such that $A = UTU^*$ and T is upper triangular. [1, (2.3.1)] Then $U^*BU = U^*p(A)U = p(U^*AU) = p(T)$ is upper triangular and $U^*BU = U^*q(A^*)U = q(U^*A^*U) = q(T^*)$ is lower triangular. Thus, $U^*BU = D$ is diagonal and $B = UDU^*$ is normal. If every eigenvalue of B is zero, then $D = 0$ and $B = 0$.

Reference
[1] R.A. Horn and C.R. Johnson, *Matrix Analysis*, Cambridge University Press, New York, 1985.

Solution 45-2.2 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca
and Henry Wolkowicz, *University of Waterloo, Ontario, Canada,* hwolkowicz@uwaterloo.ca

Note that $N_0N^* = N^*N_0$ implies $NN_0^* = N_0^*N$. With $N_0N = NN_0$, we have

$$N(N_0 + N_0^*) = (N_0 + N_0^*)N. \tag{6}$$

Since N, $(N_0 + N_0^*)$ commute, and $rankN = n - 1$, there exists a unitary matrix U such that

$$U^*NU = \begin{pmatrix} 0 & a_1 & \cdots & \cdots & \cdots \\ & 0 & a_2 & \cdots & \cdots \\ & & \ddots & \ddots & \cdots \\ & & & 0 & a_{n-1} \\ & & & & 0 \end{pmatrix}, \quad U^*(N_0 + N_0^*)U = \text{diag}(\lambda_1, \cdots, \lambda_n),$$

where $a_i \neq 0$ for $i = 1, \ldots, n$ and $\lambda_1, \ldots, \lambda_n$ are eigenvalues of $N_0 + N_0^*$.

Pre-/post-multiplying both sides of (6) by U^* and U and comparing the super-diagonal entries of both sizes, we have $a_i\lambda_{i+1} = \lambda_i a_i$, i.e., $\lambda_1 = \cdots = \lambda_n$, which implies that

$$N_0 + N_0^* = \lambda_1 I. \tag{7}$$

From (7), we know that N_0 is normal. A normal nilpotent matrix is zero. This completes the proof.

Solution 45-2.3 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

Let N and N_0 be nilpotent matrices from $\mathbb{C}^{n\times n}$, and let N be non-derogatory. Then it is known that the matrices N and N_0 have only eigenvalue 0, both with algebraic multiplicity n. Moreover, because N is non-derogatory, N has only one Jordan block J with that eigenvalue. In other words, there exists a nonsingular matrix $P \in \mathbb{C}^{n\times n}$ such that

$$P^{-1}NP = J := \begin{pmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{pmatrix},$$

where J is a nilpotent matrix having ones on the superdiagonal and zeros elsewhere. Needless to say, $N_1 := P^{-1}N_0P$ and $N_2 := P^*N_0(P^*)^{-1}$ are both nilpotent matrices because the matrix N_0 is nilpotent. In virtue of $N_0N = NN_0 \Leftrightarrow N_1J = JN_1$, it follows that N_1 is necessarily a strict upper triangular Toeplitz matrix. Recall that Toeplitz matrices have constant entries on each diagonal parallel to the main diagonal. Since N_1 is nilpotent, there are hence only zeros on and below the main diagonal in matrix N_1. Because $N_0N^* = N^*N_0 \Leftrightarrow N_2J^* = J^*N_2$, it likewise follows that N_2 is necessarily a strict lower triangular Toeplitz matrix with zeros on and above the main diagonal. So $N_2^* = P^{-1}N_0^*P$ is a strict upper triangular Toeplitz matrix. Consequently, $W := N_1 + N_2^* = P^{-1}(N_0 + N_0^*)P$ is also a strict upper triangular Toeplitz matrix, where all entries on and below the main diagonal are zeros. This implies that W has only eigenvalue 0 with algebraic multiplicity n. The matrices W and $N_0 + N_0^*$ have the same characteristic polynomial and thus the same eigenvalues. Because the matrix $N_0 + N_0^*$ is Hermitian, this matrix is unitary similar to the diagonal matrix of its eigenvalues. Consequently, $N_0 + N_0^* = 0$. Hence, $N_0N_0^* = -N_0N_0$. This in turn implies that $\mathcal{R}(N_0) = \mathcal{R}(N_0N_0^*) = \mathcal{R}(N_0^2) \subseteq \mathcal{R}(N_0)$, with $\mathcal{R}(\cdot)$ indicating the range (column space) of matrix $(\cdot)$. Therefore, $\mathcal{R}(N_0) = \mathcal{R}(N_0^2)$. Because N_0 is nilpotent, $\mathcal{R}(N_0) = \mathcal{R}(N_0^2)$ can happen if and only if $N_0 = 0$, and our solution is complete.

Also solved by the proposer Johanns de Andrade Bezerra, Eugene Herman, and Éric Pité.

Problem 45-3: PSD Solution To A Matrix Equation

Proposed by Christopher Hillar, *Mathematical Sciences Research Institute, Berkeley, California, USA* chillar@msri.org

Fix an $n \times n$ positive definite matrix A. Determine all positive semidefinite solutions X to the equation $XAX^3AX = 0$.

Solution 45-3.1 by Jan Hauke, *Adam Mickiewicz University, Poznań, Poland,* jhauke@amu.edu.pl

Let $\mathbf{A}$ be an $n \times n$ positive semidefinite matrix. Observe that using left and right cancellation rules [by presenting $\mathbf{A}$ in the form $\mathbf{A} = (\mathbf{A}^{1/2})^2$] we obtain

FACT 1. Let k and $l \geq 2k$ be natural numbers. The set of Hermitian solutions of the matrix equation

$$\mathbf{X}^k\mathbf{A}\mathbf{X}^l\mathbf{A}\mathbf{X}^k = \mathbf{0} \tag{8}$$

coincides with the set of Hermitian solutions of the equation $\mathbf{A}\mathbf{X}^l\mathbf{A} = \mathbf{0}$.

FACT 2. The set of positive semidefinite solutions of the equation (8) coincides with the set of positive semidefinite solutions of the equation $\mathbf{A}\mathbf{X} = \mathbf{0}$, the solution of which (see Khatri and Mitra, 1976) is $\mathbf{X} = (\mathbf{I}_n - \mathbf{A}^-\mathbf{A})\mathbf{U}(\mathbf{I}_n - \mathbf{A}^-\mathbf{A})^*$, where $\mathbf{U}$ is an arbitrary $n \times n$ positive semidefinite matrix and $\mathbf{A}^-$ an arbitrary generalized inverse of $\mathbf{A}$.

PROOF. The result follows from Fact 1 by writing the positive semidefinite $\mathbf{X}$ as $\mathbf{X} = \mathbf{X}^{1/2}\mathbf{X}^{1/2}$ and then using the fact that

$$\mathbf{A}\mathbf{X}^l\mathbf{A} = \mathbf{0} \Longleftrightarrow trace(\mathbf{A}\mathbf{X}^{l/2}\mathbf{X}^{l/2}\mathbf{A}) = 0 \Longleftrightarrow \mathbf{A}\mathbf{X} = \mathbf{0}.$$

REMARK. For $k = 1$ and $l = 3$, equation (8) is a version of Problem 45-2.

COROLLARY. For a positive definite matrix $\mathbf{A}$ the only solution of (8) is $\mathbf{X} = \mathbf{0}$.

Reference

C.G. Khatri and S.K. Mitra, Hermitian and nonnegative definite solutions of linear matrix equations, *SIAM J. Appl. Math.* 31(**1976**)579–585.

Solution 45-3.2 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

The only solution is $X = O$. Suppose, to the contrary, that X is a nonzero positive semidefinite solution. We may replace X by its canonical form $\left[\begin{smallmatrix} D & O \\ O & O \end{smallmatrix}\right]$, where D is diagonal and its diagonal entries are all positive, since X is unitarily similar to this canonical form, and we may replace A by its transformation under the same similarity. Partition A to match the above partitioning of X: $A = \left[\begin{smallmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{smallmatrix}\right]$. The equation $XAX^3AX = O$ becomes $\left[\begin{smallmatrix} DA_{11}D^3A_{11}D & O \\ O & O \end{smallmatrix}\right] = \left[\begin{smallmatrix} O & O \\ O & O \end{smallmatrix}\right]$. This is a contradiction, since D and A_{11} are nonsingular and so $DA_{11}D^3A_{11}D$ is nonsingular.

Solution 45-3.3 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca
and Henry Wolkowicz, *University of Waterloo, Ontario, Canada,* hwolkowicz@uwaterloo.ca

Let $X = Y^2$, where Y is the unique square root of X. Then $XAX^3AX = 0$ is the same as $(Y^3AY^2)^*(Y^3AY^2) = 0$, which implies $Y^3AY^2 = 0$. Let $U^*YU = diag(d_1, \ldots, d_n)$ be the spectral decomposition of Y, then $Y^3AY^2 = 0$ implies $diag(d_1^3, \ldots, d_n^3)U^*AUdiag(d_1^2, \ldots, d_n^2) = 0$. Since U^*AU is positive definite, compare the diagonal entries of $diag(d_1^3, \ldots, d_n^3)U^*AUdiag(d_1^2, \ldots, d_n^2)$, we know $d_i = 0$ for all i. That is, $X = 0$.

Solution 45-3.4 by Edward L. Pekarev, *Lenmorniiproekt-Ukraine, Ukraine,* edpekarev@yandex.ru

Let $\mathcal{H}$ be a Hilbert space with scalar product (. , .), $\dim H \leq \infty$, and A, X be positive semidefinite operators in $\mathcal{H}$, $kerA = 0$. Then an operator X satisfies the equation (if and) only if $X = O$. Really, in this case , for any vector $f \in \mathcal{H}$ we have: $\|X^{1/2}XAXf\|^2 = (X^{1/2}XAXf, X^{1/2}XAXf) = (XAX^3AXf, f) = 0$, and therefore $X^{1/2}XAX = O, X^{1/2}XAXX^{1/2} = O$. Then again for any vector $f \in \mathcal{H}$, we have: $\|A^{1/2}XX^{1/2}f\|^2 = (X^{1/2}XAXX^{1/2}f, f) = 0$, hence $A^{1/2}XX^{1/2} = O$ and $X = O$.

Remark 1. Let $X_1AX_2AX_1 = O$, where A, X_1, X_2 are positive semidefinite operators in $\mathcal{H}$, and $kerA = 0$, $X_1X_2 = X_2X_1$. Then $X_1X_2 = O$. In fact, denoting $Y = X_2^{1/2}AX_1$ one can find $Y^*Y = O$, then $Y = O$. So, $X_1X_2AX_1X_2 = X_1X_2^{1/2}(X_2^{1/2}AX_1)X_2 = X_1X_2^{1/2}YX_2 = 0$, and denoting $Z = A^{1/2}X_1X_2 (= A^{1/2}X_2X_1)$ one can find $Z^*Z = O, Z = O$. In view of the condition $kerA = 0$, we have $X_1X_2 = O$. Note that if also $kerX_1 = kerX_2$, then $X_1 = X_2 = O$. Therefore, in the case $X_1 = X, X_2 = X^3$, we can find again $X = O$.

Remark 2. If $(X_1AX_1)X_2(X_1AX_1) = O$, where A, X_1, X_2 are positive semidefinite operators in $\mathcal{H}$ and $kerA = 0$, then, similarly to the *Remark 1*, one can find that $X_1X_2 = O$.

Solution 45-3.5 by by Alicja Smoktunowicz, *Warsaw University of Technology, Poland*, smok@mini.pw.edu.pl

We prove that there is only one positive semidefinite solution X to the equation $XAX^3AX = 0$, that is, $X = 0$. Clearly, $X = 0$ satisfies the above conditions.

Now assume that $X \in \mathbb{C}^{n\times n}$ is positive semidefinite solution to the equation $XAX^3AX = 0$. Then, using the spectral decomposition, we have $X = UDU^*$, where U is a unitary matrix and $D = \operatorname{diag}(d_i)$ is a diagonal matrix with $d_i \geq 0$ for $i = 1, 2, \ldots, n$.

Let $B = U^*AU$. Then the equation $XAX^3AX = 0$ can be rewritten as follows $UDU^*AUD^3U^*AUDU^* = 0$, which is equivalent to $DBD^3BD = 0$, since U is unitary.

Let us define the matrix $Y = D^{3/2}BD$. We see that $Y^*Y = 0$, so $Y = 0$. In particular, $y_{ii} = d_i^{3/2}b_{ii}d_i = 0$ for all i. Since B is positive definite, its diagonal elements are positive, i.e. $b_{ii} > 0$ for $i = 1, \ldots, n$. From this it follows that all diagonal elements d_i are equal to 0, hence $D = 0$ and $X = 0$.

Solution 45-3.6 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

For any given positive semidefinite matrix B we have $Y^*BY = 0$ if and only if $BY = 0$. Furthermore, if B is positive definite, then $Y^*BY = 0$ if and only $Y = 0$. With this in mind, the following chain of implications is straightforward: $XAX^3AX = 0 \Rightarrow X^2AX = 0 \Rightarrow X^2AX^2 = 0 \Rightarrow AX^2 = 0 \Rightarrow AX^2A = 0 \Rightarrow XA = 0 \Rightarrow X = 0$. In other words, for given positive definite matrix A the only positive semidefinite solution X to the equation $XAX^3AX = 0$ is given by the zero matrix of appropriate order.

Also solved by the proposer Christopher Hillar.

Problem 45-4: Nilpotent 0-1 Matrix

Proposed by Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu
and Fuzhen Zhang, *Nova Southeastern University, Ft. Lauderdale, USA,* zhang@nova.edu

Show that every nilpotent 0-1 matrix is permutation-similar to a strict upper-triangular 0-1 matrix.

Solution 45-4.1 by Carlos Fonseca, *University of Coimbra, Coimbra, Portugal,* cmf@mat.uc.pt

Let A be a nilpotent 0-1 matrix. Then A is the adjacency matrix of a (simple) digraph, say D. So, the original question is equivalent to asking whether we can relabel the vertices (nodes) of D in such a way that (ij) is a directed edge in D if and only if $i < j$. Moreover, since A is nilpotent, there are no directed cycles in D.

Let us use induction on the number of vertices. If every vertex has a positive outdegree then there is a directed cycle. Therefore there is at least one vertex with outdegree zero. *Remark:* These vertices are nonadjacent.

Suppose that V is the set of vertices of D and denote the non-empty set of vertices with outdegree zero by Z.

The subgraph generated by $V - Z$ satisfies the condition, since it has a smaller number of vertices, and therefore, it has a proper ordering. Using this order it is straightforward to relabel the elements of Z.

Solution 45-4.2 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

This is a proof by induction on n, where $n \times n$ is the size of the matrix. The result is trivial for $n = 1$. So we assume that $n > 1$ and that the result is true for $n-1 \times n-1$ matrices. Let A_n be any $n \times n$ nilpotent 0-1 matrix, and let $\mathbf{e}_1, \ldots, \mathbf{e}_n$ denote the standard unit basis vectors in $\mathbb{R}^n$. We claim that $A_n\mathbf{e}_j = \mathbf{0}$ for some j, $j = 1, \ldots, n$. If not, an induction argument shows that for each k, $k = 0, \ldots, n$, we have $A_n^k\mathbf{e}_1 = \sum_{j=1}^n c_{jk}\mathbf{e}_j$ for some non-negative integers $c_{1k}, \ldots, c_{nk}$ where at least one of these coefficients is positive. This is a contradiction, since $A_n^n\mathbf{e}_1 = O\mathbf{e}_1 = \mathbf{0}$. Hence, $A_n\mathbf{e}_j = \mathbf{0}$ for some j. Let P by any $n \times n$ permutation matrix such that $P\mathbf{e}_1 = \mathbf{e}_j$. Then $P^{-1}A_nP\mathbf{e}_1 = P^{-1}A_n\mathbf{e}_j = \mathbf{0}$. Thus, $P^{-1}A_nP$ is a nilpotent 0-1 matrix who first column is $\mathbf{0}$. Hence, A_{n-1}, the bottom-right $n-1 \times n-1$ submatrix of $P^{-1}A_nP$, is a nilpotent 0-1 matrix. By our induction assumption, there is an $n-1 \times n-1$ permutation matrix Q such that $Q^{-1}A_{n-1}Q$ is a strict upper-triangular 0-1 matrix. Let $R = \begin{bmatrix} 1 & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & Q \end{bmatrix}$. Therefore PR is a permutation matrix and

$$(PR)^{-1}A_nPR = R^{-1}(P^{-1}A_nP)R = \begin{bmatrix} 1 & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & Q^{-1} \end{bmatrix} \begin{bmatrix} 0 & \mathbf{v}^{\mathrm{T}} \\ \mathbf{0} & A_{n-1} \end{bmatrix} \begin{bmatrix} 1 & \mathbf{0}^{\mathrm{T}} \\ \mathbf{0} & Q \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{v}^{\mathrm{T}}Q \\ \mathbf{0} & Q^{-1}A_{n-1}Q \end{bmatrix}$$

which is a strict upper-triangular 0-1 matrix.

Solution 45-4.3 by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca
and Henry Wolkowicz, *University of Waterloo, Ontario, Canada,* hwolkowicz@uwaterloo.ca

For a nonnegative matrix A, if it is irreducible, then $\rho(A) > 0$, where ρ denotes the spectral radius of A. Thus every nonnegative nilpotent matrix must be reducible. It is obvious that a 0-1 matrix is permutation preserving. We proceed by induction on the size of the 0-1 matrix. When $n = 2$, there is nothing to prove. Suppose the assertion is true for all $k \leq n$, then when $k = n+1$, there exists a permutation matrix, say P, such that $PAP^T = \left(\begin{smallmatrix} X & Y \\ 0 & Z \end{smallmatrix}\right)$, where X, Z are square 0-1 matrices of size less than n. By assumption, there are permutation matrices P_1, P_2 such that both $P_1XP_1^T$, $P_2ZP_2^T$ are strict upper triangular. Therefore,

$$\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} \begin{pmatrix} X & Y \\ 0 & Z \end{pmatrix} \begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix}^T = \begin{pmatrix} P_1XP_1^T & P_1YP_2^T \\ 0 & P_2ZP_2^T \end{pmatrix}$$

is a strict upper triangular 0-1 matrix.

Solution 45-4.4 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

Let $N_1 \in \mathbb{C}^{n,n}$ be an arbitrary but fixed nilpotent 0-1 matrix. Then at least one column of N_1 consists entirely of zeros. If the j_1-th column of N_1 is such a zero column, put $P_1 := I_{(j_1 \to 1)}$, where $I_{(j_1 \to 1)}$ is the matrix obtained from the identity matrix I_n just by moving the j_1-th column e_{j_1} of I_n to the first column position. Observe that $N_2 := P_1^t N_1 P_1$ is nilpotent and has following block structure

$$N_2 := P_1^t N_1 P_1 = \begin{pmatrix} 0 & * \\ 0 & \tilde{N}_2 \end{pmatrix},$$

where $\tilde{N}_2 \in \mathbb{C}^{n-1,n-1}$ is also a nilpotent 0-1 matrix. So, as before, $\tilde{N}_2$ must contain at least one zero column. If the j_2-th column of $\tilde{N}_2$ is zero, put $P_2 := I_{(j_2+1 \to 2)}$, where $I_{(j_2+1 \to 2)}$ is the matrix obtained from the identity matrix I_n just by moving the (j_2+1)-th column e_{j_2+1} of I_n to column position 2. Observe that the matrix $N_3 := P_2^t N_2 P_2$ is a nilpotent 0-1 matrix with the following block structure

$$N_3 := P_2^t N_2 P_2 = \begin{pmatrix} 0 & * & * \\ 0 & 0 & * \\ 0 & 0 & \tilde{N}_3 \end{pmatrix},$$

where $\tilde{N}_3 \in \mathbb{C}^{n-2,n-2}$ is a nilpotent 0-1 matrix. After at most $n-1$ iterations of the preceding step, we obtain a strict upper triangular 0-1 nilpotent matrix N_n. In other words, after at most $n-1$ such iterations it turns out that $P := \prod_{i=1}^{n-1} P_i$ is a permutation matrix and $P^t N_1 P$ is strict upper triangular and nilpotent. So, N_1 is indeed permutation-similar to a strict upper-triangular 0-1-matrix. Of course this result also holds true for any nonnegative nilpotent matrix.

Editorial note: This problem is equivalent to American Mathematical Monthly Problem 11487, February 2010.

Problem 45-5: Comparison Of The Trace Of Two Inverse Matrices

Proposed by Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Let A and B be $n \times n$ positive definite matrices. It is easy to see $Tr(A^2 + AB^2A) = Tr(A^2 + BA^2B)$. Show a less trivial inequality

$$Tr[(A^2 + AB^2A)^{-1}] \geq Tr[(A^2 + BA^2B)^{-1}].$$

Solution 45-5 by the proposer Minghua Lin, *University of Regina, Saskatchewan, Canada,* lin243@uregina.ca

Observe that if X, $X+Y$, $X-Y$ are positive matrices of the same size, then $(X+Y)^{-1} + (X-Y)^{-1} \geq 2X^{-1}$. Note that

$$(A + iAB)(A - iBA) = A^2 + AB^2A,$$

$$(A - iBA)(A + iAB) = A^2 + BA^2B - i(BA^2 - A^2B),$$

$$(A - iAB)(A + iBA) = A^2 + AB^2A,$$

$$(A + iBA)(A - iAB) = A^2 + BA^2B + i(BA^2 - A^2B).$$

We find

$$\begin{aligned}&Tr[(A^2+AB^2A)^{-1}]\\&=\frac{1}{2}\left\{Tr[((A+iAB)(A-iBA))^{-1}]+Tr[((A-iAB)(A+iBA))^{-1}]\right\}\\&=\frac{1}{2}\left\{Tr[((A-iBA)(A+iAB))^{-1}]+Tr[(A+iBA)(A-iAB)^{-1}]\right\}\\&\geq Tr[(A^2+BA^2B)^{-1}].\end{aligned}$$

Problem 45-6: Matrix Unitary Decomposition

Proposed by Dennis Merino, *Southeastern Louisiana University, Hammond, USA,* dmerino@selu.edu

Let U and Q be given $n\times n$ complex matrices.

(i) Suppose that U is unitary. Show that for every integer $k\geq 2$, U can be written as a sum of k unitary matrices.

(ii) Suppose that Q is orthogonal. When do there exist orthogonal Q_1 and Q_2 such that $Q=Q_1+Q_2$?

Solution 45-6.1 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA,* eaherman@gmail.com

(i) An induction argument shows that it suffices to prove the result for $k=2$. The matrices $D_1=e^{i\pi/3}I$ and $D_2=e^{-i\pi/3}I$ are unitary. Since $e^{i\pi/3}+e^{-i\pi/3}=1$, we have $I=D_1+D_2$ and therefore $U=UD_1+UD_2$, where UD_1 and UD_2 are unitary.

(ii) We claim that, when n is even, every orthogonal matrix can be written as a sum of two orthogonal matrices; and, when n is odd, no orthogonal matrix can be so expressed. First, suppose we have $Q=Q_1+Q_2$, where Q,Q_1,Q_2 are orthogonal. As with real orthogonal matrices, the transpose and product of complex orthogonal matrices are complex orthogonal. Hence

$$I=Q^{\mathrm{T}}Q=Q_1^{\mathrm{T}}Q_1+Q_1^{\mathrm{T}}Q_2+Q_2^{\mathrm{T}}Q_1+Q_2^{\mathrm{T}}Q_2=2I+P+P^{\mathrm{T}}$$

and so $P+P^{\mathrm{T}}+I=O$, where $P=Q_1^{\mathrm{T}}Q_2$ is orthogonal. Furthermore, multiplying this equation by P yields $P^2+I+P=O$. In summary, the matrix P satisfies

$$P^{\mathrm{T}}=-(P+I)\quad\text{and}\quad P^2+P+I=O \tag{9}$$

The second result in (9) implies that the only eigenvalues of P are $e^{\pm i2\pi/3}=-\frac{1}{2}\pm i\frac{\sqrt{3}}{2}$. Hence, there is a nonsingular matrix X such that $X^{-1}PX=J$, where J is a Jordan form of P and so has these two eigenvalues along its diagonal. By the first result in (9),

$$X^{-1}P^{\mathrm{T}}X=-X^{-1}(P+I)X=-(J+I)$$

Since $-(-\frac{1}{2}\pm\frac{\sqrt{3}}{2}+1)=-\frac{1}{2}\mp\frac{\sqrt{3}}{2}$, we see that P^{T} has the same eigenvalues as P but with their algebraic multiplicities switched. In n is odd, this is a contradiction, since a matrix and its transpose have the same eigenvalues with the same algebraic multiplicities.

On the other hand, one may readily check that $P=\begin{pmatrix}-\frac{1}{2}&\frac{\sqrt{3}}{2}\\-\frac{\sqrt{3}}{2}&-\frac{1}{2}\end{pmatrix}$ is orthogonal and satisfies $P^2+P+I=O$ as in (9). Hence, if P_n is the $2n\times 2n$ block-diagonal matrix in which each block is P, then P_n as well is orthogonal and satisfies $P_n^2+P_n+I=O$. Therefore, if Q is any $2n\times 2n$ orthogonal matrix, then

$$Q=QI=Q(-P_n^2-P_n)=-QP_n^2-QP_n$$

and each term in this sum is orthogonal.

Solution 45-6.2 by Éric Pité, *Paris, France,* eric.pite@telecom-paristech.org

(i) There exist a unitary matrix P and $\theta_1,\ldots,\theta_n\in\mathbb{R}$ such that $U=P^*\mathrm{diag}(e^{i\theta_1},\ldots,e^{i\theta_n})P$.

We have $e^{i\theta_j}=e^{i(\theta_j+\pi/3)}+e^{i(\theta_j-\pi/3)}$. Let $U_1=P^*\mathrm{diag}(e^{i(\theta_1+\pi/3)},\ldots,e^{i(\theta_n+\pi/3)})P$ and $U_2=P^*\mathrm{diag}(e^{i(\theta_1-\pi/3)},\ldots,e^{i(\theta_n-\pi/3)})P$. Hence $U=U_1+U_2$, where U_1 and U_2 are unitary.

By induction, for every integer $k\geqslant 2$, U can be written as a sum of k unitary matrices.

(ii) For $\theta\in\mathbb{R}$, denote by $R_\theta=\left(\begin{smallmatrix}\cos\theta&-\sin\theta\\\sin\theta&\cos\theta\end{smallmatrix}\right)$. We have $I_2=R_{\pi/3}+R_{-\pi/3}$ and $-I_2=R_{2\pi/3}+R_{-2\pi/3}$.

Hence, if there exist $p,q\in\mathbb{N}$ with $2p+2q=n$ and $P\in O_n(\mathbb{R})$ such that $Q=P\mathrm{diag}(I_{2p},-I_{2q})P^T$, there exist two orthogonal matrices Q_1 and Q_2 such that $Q=Q_1+Q_2$. We show that these are the only orthogonal matrices that can be the sum of two orthogonal matrices (in particular there is no solution when n is odd).

Assume that there exist two orthogonal matrices Q_1 and Q_2 such that $Q=Q_1+Q_2$. We have $I_n=2I_n+R^T+R$, with $R=Q_1^TQ_2$.

So R is orthogonal, hence there exist $k, \ell, m \in \mathbb{N}$, and $P \in O_n(\mathbb{R})$ such that $R = P\text{diag}(I_k, -I_\ell, R_{\theta_1}, \ldots, R_{\theta_m})P^T$ and $\theta_i \not\equiv 0 \pmod{\pi}$. Feeding into the equation $R^T + R = I_n$, we obtain that $k = \ell = 0$ (hence n is even), and $\theta_i = \pm\pi/3$, which implies that Q is of the form $Q = S\text{diag}(I_{2p}, -I_{2q})S^T$ for some $S \in O_n(\mathbb{R})$.

Solution 45-6.3 by Hans Joachim Werner, *University of Bonn, Bonn, Germany,* hjw.de@uni-bonn.de

We begin with remembering the following two definitions. A UNITARY matrix is defined to be a COMPLEX $n \times n$ matrix $U \in \mathbb{C}^{n\times n}$ for which $U^*U = I_n$, where U^* is the CONJUGATE TRANSPOSE of U and I_n stands as usual for the identity-matrix of order n. An ORTHOGONAL matrix is defined to be a REAL $n \times n$ matrix $P \in \mathbb{R}^{n\times n}$ for which $P^tP = I_n$, where P' indicates the TRANSPOSE of P. Next, observe that the sets $\mathcal{U}^{n\times n} := \{U \in \mathbb{C}^{n\times n} \mid U \text{ unitary}\}$ and $P^{n\times n} := \{P \in \mathbb{R}^{n\times n} \mid P \text{ orthogonal}\}$ form multiplicative groups (use the ordinary matrix multiplication as the binary group operation). Furthermore, recall that unitary matrices are NORMAL matrices and hence unitarily diagonalizable. If (λ, x) is an eigenpair for the unitary matrix U, then $x^*x = x^*U^*Ux = \lambda\bar{\lambda}\cdot x^*x \Rightarrow \lambda\bar{\lambda} = 1 \Rightarrow \lambda = e^{i\tau}$ for some $\tau \in [0, 2\pi)$, with $\bar{\lambda}$ indicating the COMPLEX CONJUGATE of λ. Consequently, every unitary matrix is unitarily similar to a diagonal matrix D of the form

$$D = \text{diag}(e^{i\tau_1}, e^{i\tau_2}, \cdots, e^{i\tau_n}). \tag{10}$$

Now, let U, U_1, U_2 be unitary $n \times n$ matrices. Then, $U_1 = W_1U$ and $U_2 = W_2U$ for some (unitary) matrices $W_1, W_2 \in \mathcal{U}^{n\times n}$. Consequently, $U = U_1 + U_2$ or, equivalently, $U = W_1U + W_2U = (W_1 + W_2)U$ if and only if $W_1 + W_2 = I_n$, i.e., if and only if $W_2 = I_n - W_1$. Check that $W_1 \in \mathcal{U}^{n\times n}$ satisfies $I_n - W_1 \in \mathcal{U}^{n\times n}$ if and only if $W_1 + W_1^* = I_n$. From the first paragraph we know that there exist a (unitary) matrix $V \in \mathcal{U}^{n\times n}$ and a diagonal matrix D of the form (10) such that $W_1 = VDV^*$. Then, evidently, $W_1 + W_1^* = I_n$ if and only if $D + D^* = I_n$ or, equivalently, if and only if $\tau_i \in \{\frac{\pi}{3}, \frac{-\pi}{3}\}$. The following result is now clear.

Theorem 1. Let $U \in \mathcal{U}^{n\times n}$. Then (U_1, U_2) is a pair of unitary matrices satisfying $U = U_1 + U_2$ if and only if $U_1 = VDV^*U$ and $U_2 = VD^*V^*U$ for some matrix $V \in \mathcal{U}^{n\times n}$ and some diagonal matrix D of the form

$$D = \text{diag}(e^{i\tau_1}, e^{i\tau_2}, \cdots, e^{i\tau_n})$$

with $\tau_i \in \{\frac{\pi}{3}, \frac{-\pi}{3}\}$ for $i = 1, 2, \ldots, n$.

Theorem 1 shows that every unitary matrix U can be written as the sum of two unitary matrices, say U_1 and U_2. Since U_1 is unitary, according to our Theorem 1, it can also be written as the sum of two unitary matrices, say T_1 and T_2. Then, $U = T_1 + T_2 + U_2$, i.e., U can be written as the sum of three unitary matrices. Continuing in this manner (complete induction on k) shows that every unitary matrix can indeed be written, as claimed, as the sum of k unitary matrices, and so our proof of part (i) is complete.

It is well-known (see, e.g., C.D. Meyer, Matrix Analysis and Applied Linear Algebra, SIAM, Philadelphia, 2000, Exercise 7.5.13) that every ORTHOGONAL matrix $P \in \mathcal{P}^{n\times n}$ is orthogonally similar to a block diagonal matrix D of the form

$$D = \text{diag}\left(\pm 1, \cdots, \pm 1, \begin{pmatrix} \cos(\tau_1) & \sin(\tau_1) \\ -\sin(\tau_1) & \cos(\tau_1) \end{pmatrix}, \ldots, \begin{pmatrix} \cos(\tau_\ell) & \sin(\tau_\ell) \\ -\sin(\tau_\ell) & \cos(\tau_\ell) \end{pmatrix}\right) \tag{11}$$

with $\tau_i \in [0, 2\pi)$ for $i = 1, 2, \ldots, \ell$.

Now, let P, P_1, and P_2 be orthogonal $n \times n$ matrices. Then, $P_1 = R_1P$ and $P_2 = R_2P$ for some (orthogonal) matrices $R_1, R_2 \in \mathcal{P}^{n\times n}$. Consequently, $P = P_1 + P_2$ or, equivalently, $P = R_1P + R_2P = (R_1 + R_2)P$ if and only if $R_1 + R_2 = I_n$, i.e., if and only if $R_2 = I_n - R_1$. Check that $R_1 \in \mathcal{P}^{n\times n}$ satisfies $I_n - R_1 \in \mathcal{P}^{n\times n}$ if and only if $R_1 + R_1' = I_n$. From our last paragraph we know that there exist an orthogonal matrix $V \in \mathcal{P}^{n\times n}$ and a block diagonal matrix D of the form (11) such that $R_1 = VDV'$. Consequently, $R_1 + R_1' = I_n$ if and only if $D + D' = I_n$, which evidently happens if and only if the block diagonal matrix D is of the form

$$D = \text{diag}\left(\begin{pmatrix} \cos(\tau_1) & \sin(\tau_1) \\ -\sin(\tau_1) & \cos(\tau_1) \end{pmatrix}, \cdots, \begin{pmatrix} \cos(\tau_\ell) & \sin(\tau_\ell) \\ -\sin(\tau_\ell) & \cos(\tau_\ell) \end{pmatrix}\right) \tag{12}$$

with $\tau_i = \pm\frac{\pi}{3}$ for $i = 1, \ldots, \ell$. So, we have the following result solving part (ii) of this problem.

Theorem 2. Let $P \in \mathcal{P}^{n\times n}$. This orthogonal matrix P can be written as the sum of two orthogonal matrices, say P_1 and P_2, if and only if n is even. In which case, (P_1, P_2) is a pair of orthogonal matrices satisfying $P = P_1 + P_2$ if and only if $P_1 = VDV'P$ and $P_2 = VD'V'P$ for some matrix $V \in \mathcal{P}^{n\times n}$ and some block diagonal matrix D of the form

$$D = \text{diag}\left(\begin{pmatrix} \cos(\tau_1) & \sin(\tau_1) \\ -\sin(\tau_1) & \cos(\tau_1) \end{pmatrix}, \cdots, \begin{pmatrix} \cos(\tau_\ell) & \sin(\tau_\ell) \\ -\sin(\tau_\ell) & \cos(\tau_\ell) \end{pmatrix}\right),$$

where $\tau_i \in \{\frac{\pi}{3}, \frac{-\pi}{3}\}$ *for* $i = 1, 2, \ldots, \ell$.

Needless to say, every orthogonal $n \times n$ matrix, with n even, can also be written as the sum of $k \geq 2$ orthogonal matrices.

Also solved by the proposer Dennis Merino.

Problem 45-7: A Generalization of Roth's Theorem

Proposed by Harald K. Wimmer, *Universität Würzburg, 97074 Würzburg, Germany,* wimmer@mathematik.uni-wuerzburg.de

Let K be a field and let $A \in K^{r\times r}$, $B \in K^{s\times s}$, $C_1, C_2 \in K^{r\times s}$. Prove or disprove that the following statements are equivalent:

(i) The matrices $M_1 = \begin{pmatrix} A & C_1 \\ 0 & B \end{pmatrix}$ and $M_2 = \begin{pmatrix} A & C_2 \\ 0 & B \end{pmatrix}$ are similar.

(ii) There exist $X \in K^{r\times s}$ and nonsingular P and Q with $AP = PA$ and $QB = BQ$ such that $AX - XB = C_2 - PC_1Q$.

Solution 45-7 by Jakub Kierzkowski, *Warsaw University of Technology, Poland,* J.Kierzkowski@mini.pw.edu.pl

(i)⇒(ii). We prove this implication under the *additional assumption* that A and B have no common eigenvalues. Let $M_1 = \begin{bmatrix} A & C_1 \\ 0 & B \end{bmatrix}$, $M_2 = \begin{bmatrix} A & C_2 \\ 0 & B \end{bmatrix}$ be similar. Then there exist $U \in K^{r\times r}$, $Y \in K^{s\times s}$, $W \in K^{r\times s}$, $Z \in K^{s\times r}$ such that $M = \begin{bmatrix} U & W \\ Z & Y \end{bmatrix}$ is nonsingular and

$$\begin{bmatrix} U & W \\ Z & Y \end{bmatrix}\begin{bmatrix} A & C_1 \\ 0 & B \end{bmatrix} = \begin{bmatrix} UA & UC_1 + WB \\ ZA & ZC_1 + YB \end{bmatrix} = \begin{bmatrix} AU + C_2Z & AW + C_2Y \\ BZ & BY \end{bmatrix} = \begin{bmatrix} A & C_2 \\ 0 & B \end{bmatrix}\begin{bmatrix} U & W \\ Z & Y \end{bmatrix}.$$

From this it follows that $ZA = BZ$, i.e., Z is a solution of the Sylvester equation $BZ - ZA = 0$. It is known (see, e.g. [1], p. 331) that $Z = 0$ is the only solution of this equation if A and B have no common eigenvalues. This means, that

$$UA = AU \text{ and } YB = BY \tag{13}$$

and $AW + C_2Y = UC_1 + WB$, which is

$$AW - WB = UC_1 - C_2Y. \tag{14}$$

Since $Z = 0$ and M is nonsingular, we obtain that U and Y are nonsingular, so we can multiply (13) and (14) by U^{-1} and Y^{-1}:

$$UAU^{-1} = A, \text{ and thus we have } AU^{-1} = U^{-1}A, \tag{15}$$

$$YBY^{-1} = B, \text{ and thus we have } BY^{-1} = Y^{-1}B, \tag{16}$$

$$AWY^{-1} - WBY^{-1} = UC_1Y^{-1} - C_2, \tag{17}$$

From (16) we have

$$WBY^{-1} = WY^{-1}B. \tag{18}$$

Taken together (17) and (18) gives

$$A(-WY^{-1}) - (-WY^{-1})B = C_2 - UC_1Y^{-1}.$$

Finally matrices $X = -WY^{-1}$, $P = U$, $Q = Y^{-1}$ prove statement (ii).

(ii)⇒(i). Now let there exist X and nonsingular $P,\ Q$ of proper dimensions, such that $AP = PA$, $BQ = QB$ and $AX - XB = C_2 - PC_1Q$. It is clear that $\begin{bmatrix} P & -XQ^{-1} \\ 0 & Q^{-1} \end{bmatrix} \cdot \begin{bmatrix} P^{-1} & P^{-1}X \\ 0 & Q \end{bmatrix} = I_{r+s}$. Thus we have

$$\begin{bmatrix} P & -XQ^{-1} \\ 0 & Q^{-1} \end{bmatrix}\begin{bmatrix} A & C_1 \\ 0 & B \end{bmatrix}\begin{bmatrix} P^{-1} & P^{-1}X \\ 0 & Q \end{bmatrix} = \begin{bmatrix} PA & PC_1 - XQ^{-1}B \\ 0 & Q^{-1}B \end{bmatrix}\begin{bmatrix} P^{-1} & P^{-1}X \\ 0 & Q \end{bmatrix}$$

$$= \begin{bmatrix} PAP^{-1} & PAP^{-1}X + PC_1Q - XQ^{-1}BQ \\ 0 & Q^{-1}BQ \end{bmatrix} = \begin{bmatrix} A & AX - XB + PC_1Q \\ 0 & B \end{bmatrix},$$

which, by the assumption, is equal to $\begin{bmatrix} A & C_2 \\ 0 & B \end{bmatrix}$. This means, that $\begin{bmatrix} A & C_1 \\ 0 & B \end{bmatrix}$ and $\begin{bmatrix} A & C_2 \\ 0 & B \end{bmatrix}$ are similar.

Reference

Nicholas J. Higham, *Functions of Matrices: Theory and Computation,* SIAM, Philadelphia, 2008.

A remark by the Proposer: In the case $C_1 = 0$ the statements (i) and (ii) are equivalent. See W.E. Roth, The equations $AX - YB = C$ and $AX - XB = C$ in matrices, Proc. Amer. Math. Soc. 3(1952)392–396.

IMAGE Problem Corner: Problems and Solutions

Problems: We introduce 7 new problems in this issue and invite readers to submit solutions for publication in IMAGE. Solutions: We present solutions to all problems in the previous issue [IMAGE 45 (Fall 2010), p. 48]. Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to IMAGE Problem Corner editor Fuzhen Zhang (zhang@nova.edu). The working team of the Problem Corner consists of Dennis S. Bernstein, Nir Cohen, Shaun Fallat, Dennis Merino, Edward Poon, Peter Šemrl, Wasin So, Nung-Sing Sze, and Xingzhi Zhan.

NEW PROBLEMS:

Problem 46-1: Space Decomposition in Terms of Column and Null Spaces

Proposed by Oskar Maria Baksalary, *Adam Mickiewicz University, Poznań, Poland,* baxx@amu.edu.pl
and Götz Trenkler, *Technische Universität Dortmund, Dortmund, Germany,* trenkler@statistik.tu-dortmund.de

Let $\mathcal{R}(\cdot)$ and $\mathcal{N}(\cdot)$ denote column and null spaces of a matrix argument, respectively. For given $K \in \mathbb{C}_{n,n}$, find a decomposition $U \oplus V = \mathbb{C}_{n,1}$ such that $U, V \subseteq \mathbb{C}_{n,1}$ depend exactly on the four subspaces $\mathcal{R}(K)$, $\mathcal{R}(K^*)$, $\mathcal{N}(K)$, and $\mathcal{N}(K^*)$, where K^* stands for the conjugate transpose of K.

Problem 46-2: Commutator

Proposed by Gerald Bourgeois, *University of French Polynesia, Tahiti*, bourgeois.gerald@gmail.com

If both matrices $A, B \in \mathcal{M}_n(\mathbb{C})$ commute with their commutator $C = AB - BA$, it is known that A and B are simultaneously triangularizable; this is the "Little McCoy" theorem (see N. McCoy, *On quasi-commutative matrices*, Trans. Amer. Math. Soc.36(1934)327–340). If only A commutes with C, show that A and B must be simultaneously triangularizable if $n = 2$, but they need not be simultaneously triangularizable if $n \geq 3$.

Problem 46-3: Lower Bound for Probability

Proposed by Christopher Hillar, *Mathematical Sciences Research Institute, Berkeley, California, USA* chillar@msri.org

Find a good lower bound for the probability $p(m, n)$ that m random binary column vectors in $\{0, 1\}^n$ are linearly independent.

Problem 46-4: Hermitian Matrices with Zero Product

Proposed by Roger Horn, *University of Utah, Salt Lake City, USA,* rhorn@math.utah.edu

Let A and B be $n \times n$ Hermitian matrices with rank$(A) = r$, rank$(B) = s$. Let $p_A(t) = t^n + a_{n-1}t^{n-1} + \cdots$ be the characteristic polynomial of A, and similarly for the characteristic polynomials $p_B(t)$ (coeffs are b_i) and $p_{A+B}(t)$ (coeffs are c_i). (a) Suppose that $Im(A) + Im(B) = Im(A + B)$. If $a_{n-r}b_{n-s} = c_{n-r-s}$, show that $AB = 0$. (b) If $t^n p_{A+B}(t) = p_A(t)p_B(t)$, show that $AB = 0$.

Problem 46-5: A Trace Inequality

Proposed by Minghua Lin, *University of Waterloo, Ontario, Canada,* mlin87@ymail.com

Let A and B be $m \times n$ strictly contractive complex matrices (i.e., their spectral norms are less than 1). Show that

$$\left|Tr(I - B^*A)^{-1}\right|^2 \leq Tr(I - A^*A)^{-1}\, Tr(I - B^*B)^{-1}.$$

Problem 46-6: Singular Matrix as a Product of n Idempotent Matrices

Proposed by Éric Pité, *Paris, France,* eric.pite@telecom-paristech.org

Show that any singular $n \times n$ matrix (over any field) is a product of n idempotent matrices.

Problem 46-7: Trace Inequality of a Positive Semidefinite Matrix with Exponential Function

Proposed by Fuzhen Zhang, *Nova Southeastern University, Fort Lauderdale, USA,* zhang@nova.edu

Let A be an $n \times n$ positive semidefinite (Hermitian) matrix. Show that

$$\mathrm{tr}(e^A) \leq e^{\mathrm{tr}(A)} + (n - 1)$$

with equality if and only if rank$(A) \leq 1$.

Solutions to Problems 45-1 through 45-7 are on page 39.