The Bulletin of the International Linear Algebra Society

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Serving the International Linear Algebra Community

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About *IMAGE*

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FEATURE ARTICLE

Linear Algebra and Operator Theory: Hand in Hand, Israel Gohberg's Mathematics

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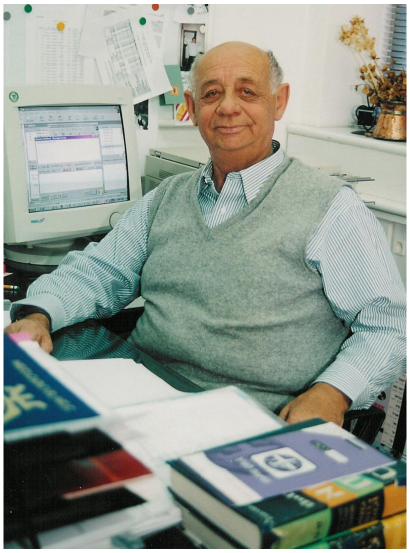

Gohberg at Tel Aviv University

In *IMAGE* 44 of Spring 2010, Avi Berman wrote "One cannot talk about linear algebra in Israel without mentioning the very strong operator theory and linear algebra school of Israel Gohberg." This undoubtedly is true, if only because instead of mentioning a single country one could just as well mention various continents. Indeed, in the past half century, the work of Israel Gohberg on the aforementioned topics had its impact on a global scale.

Two topics: operator theory and linear algebra. For Israel Gohberg they went hand in hand; each serving as a source of inspiration for the other. In an obituary[1] published in 2010, Harry Dym wrote: "One of Israel's favorite questions at the end of a seminar or lecture was: What can you say in the finite dimensional case? In my younger years I used to think that this was just a nice way to eliminate the embarrassment of the speaker in the all too often deathly silence at the end of the lecture when the speaker turns to the audience for questions. It was only much later that I realized that this was really a very serious question (...)". Gohberg himself often illustrated the intriguing relationship between operator theory and linear algebra by emphasizing that inverting an infinite Toeplitz matrix is simpler than inverting a finite one.

Gohberg's seminal contributions to linear algebra and matrix theory are numerous. Of the 26 books he co-authored, nine are in the area of linear algebra, with topics ranging from finite dimensional combinatorial geometry and theory of matrix polynomials and rational matrix functions, to matrices and indefinite scalar products. Israel Gohberg received numerous honors and awards. Here it is appropriate to mention that he was one of the recipients of the first Hans Schneider Prize in Linear Algebra.[2]

In the present article we use a few of Gohberg's key contributions to illustrate one aspect of his mathematical work: the 'hand in hand principle' referred to above.

Characteristic operator functions. In his Soviet Union period, Gohberg contributed to the theory of characteristic operator functions as initiated by M.S. Livšic in the fifties. The leading idea is to investigate a Hilbert space operator via the analysis of a certain analytic function associated to it, which opens the possibility of applying complex analysis. After his immigration to Israel, Gohberg started to cooperate with Peter Lancaster and Leiba Rodman on the theory of matrix polynomials and at about the same time he came into contact with mathematical systems and control theory. In the transfer function from systems theory, often written in the form

$$W(l) = D + C(lI - A)^{-1}B,$$

with A, B, C, D being matrices, Gohberg recognized an affine finite dimensional analogue of the characteristic operator function mentioned above. In systems theory, pairs of complementary invariant subspaces associated with the state matrix A can be used to factor the transfer functions into factors of lower degree, and to construct the corresponding system as a cascade connection of more elementary systems. But surprisingly, in the theory of characteristic operator functions, where A is usually an infinite dimensional Hilbert space operator, the converse situation appears because in some cases the structure of the factors is known and can be employed to describe the invariant subspaces of the state operator A. The first paper[3] in *Integral Equations and Operator Theory*, the new journal founded by Gohberg in 1978, brought the three subjects together. It introduced the inverse of a monic matrix polynomial as a new characteristic function and as the transfer function of a special system.

The state space method in problems of analysis. The different points of view referred to in the previous paragraph were very inspiring and led to various state space methods to solve problems in mathematical analysis. Famous analytical results, such as the limit theorems of Szegő, Kac, and Achiezer, got a new linear algebra interpretation and solving systems of Wiener-Hopf integral equations with rational symbols became a pure matrix problem. At the same time, Gohberg

[1] H. Bart, H. Dym, R. Kaashoek, P. Lancaster, A. Markus, and L. Rodman, "In memoriam Israel Gohberg August 23, 1928 – October 12, 2009," *Linear Algebra and its Applications* 443(5) 2010, 877–892.

[2] Awarded to Shmuel Friedland, Israel Gohberg and Miroslav Fiedler. The prize was presented to Israel Gohberg at the *Fourth ILAS Conference* (Rotterdam, 1994).

[3] H. Bart, I. Gohberg, and M. Kaashoek, "Stable factorizations of monic matrix polynomials and stable invariant subspaces," *Integral Equations and Operator Theory*, 1(4) 1978, 496–517.

noted that this linear algebra approach to Wiener-Hopf equations had an infinite dimensional operator theory version which worked well in the analysis of the transport equation from mathematical physics. Surprisingly, a similar remark holds true for the analysis of canonical differential systems with a rational spectral density; this line of research resulted in beautiful explicit formulas with a linear algebra flavor for non-linear integrable systems. Through contacts with Tom Kailath, Patrick Dewilde and other electrical and control engineers, new issues of computability and stability appeared. It was necessary to analyze stable invariant subspaces of matrices, not only in the standard inner product context, but also for spaces with an indefinite scalar product. The theory of stable invariant subspaces became a new problem area of research, for matrices and operators as well.

Structured matrices and operators. Gohberg's contributions to the area of structured matrices, finite or infinite dimensional, have been pioneering. The inversion formulas of Gohberg-Semencul and Gohberg-Heinig for finite Toeplitz and finite block Toeplitz matrices are celebrated results. In the present context, to underline the 'hand in hand principle', it is interesting to note that these formulas have operator versions where the role of finite Toeplitz matrices is taken over by a convolution integral operators on a finite interval. Other classical structured matrices, such as the Sylvester resultant, were also turned into operator form, and for the Sylvester resultant both the matrix and operator version turned out to be very useful in the analysis of Szegő-Kreĭn orthogonal polynomials and Kreĭn orthogonal entire matrix functions. In the theory of the matrix analogue of semi-separable integral operators a numerical flavor manifested itself. Fast algorithms for inversion were developed. The further elaboration of these ideas led to a theory of structured matrices and operators which had a strong impact on numerical linear algebra.

Completion problems. The reconstruction of matrices and operators on the basis of partial information is another theme that was very prominent in the work of Gohberg after his emigration, and it again combined beautifully finite and infinite dimensional problems. Just as for the state space method, this theme was highly motivated by problems from systems and control theory. A particular feature that emerged from it (in work with Harry Dym and colleagues from Amsterdam) is the so-called band method, an abstract scheme, algebraic in nature, which allows for the treatment of positive and contractive completion problems from one point of view, and which presents a natural strategy to solve such problems by reduction to linear equations. It led to elegant explicit formulas for the solutions of various completion, and interpolation problems, which were worked out in detail for the Carathéodory problem, the Nehari problem and its four block generalization; time-varying (nonstationary) analogues of these problems were also solved. The connection between maximum entropy solutions and central interpolants or central completions was one of the other highlights in this area. Still another one was concerned with the study of the similarity invariants of various partially given transformations, matrices or operators, with remarkable applications to eigenvalue completion problems, to stabilization problems in mathematical system theory and to problems of Wiener-Hopf factorization.

Epilogue 1. Besides being a great mathematician himself, Israel Gohberg was a source of inspiration to others. He was also generous with open problems, and on many occasions would share them within his large circle of collaborators and friends. Many of his students inherited the interplay between matrix and operator theory as an important theme in their work.

Epilogue 2. In the Bible, when one of the kings of Israel has died, it is often written: "The rest of his acts and every thing he did, is that not written in the book of the Chronicles of the kings of Israel?" Such a chronicle for Israel Gohberg is the book *Israel Gohberg and Friends*, published in 2008 by Birkhäuser Verlag on the occasion of Gohberg's 80$^{\text{th}}$ birthday. It provides a fascinating portrait of this great mathematician. To work with him as we both did has been a privilege.

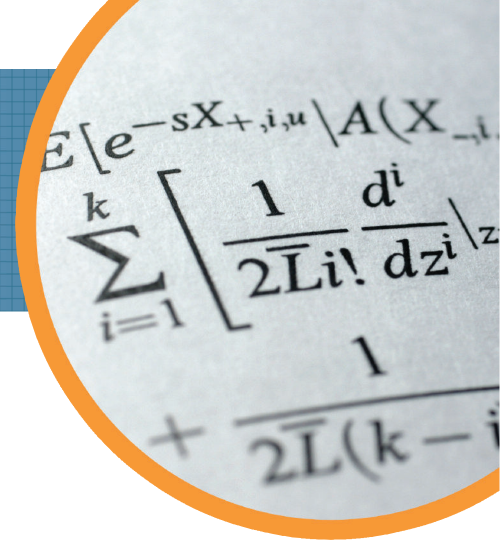
Supplementary Notes on Some Previous Articles

By Richard William Farebrother, bill.farebrother@manchester.ac.uk

- [*IMAGE* 25, p. 22] Correction: The last sentence of Section 2 should read: 'There is a statue of Lewis Carroll in Llandudno (where Alice spent her summer holidays) and a monument to Alice's Adventures in Wonderland in Guildford.'
- [*IMAGE* 40, p. 21] Correction: '1929-1932' should be '1933-1941'.
- [*IMAGE* 41, p. 27] Correction: Garry Tee (personal communication, January 8, 2009) notes that:
  "Albert C. Lewis [1] describes Jean Argand's 1806 graphical representation of complex numbers as 'the first to be printed'. But Caspar Wessel's paper was printed in 1798 and published in the memoirs of the Royal Academy of Denmark in 1799. That Danish paper remained almost unknown until a French translation was published in 1897. A (partial) English translation from the original Danish paper was published in [2]."

[1] Albert C. Lewis (1994). "Complex numbers and vector algebra," in Ivor Gratten-Guinness (Ed.), *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences*, Vol. I. Routledge, London, 722–729.

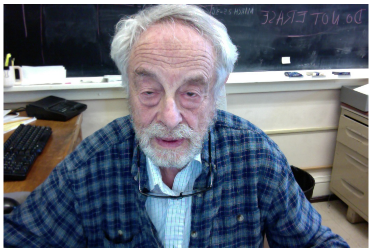
[2] David Eugene Smith (1929). *A Source Book in Mathematics.* McGraw-Hill, New York. Reprinted (in two volumes) by Dover, New York, 1959, Vol. I, pp. 55–66.



FEATURE INTERVIEW

"Know What You Are Good At, Keep At It, and Keep At It"

Hans Schneider Interviewed by Olga Holtz



Hans Schneider

O.H. - *IMAGE* readers want to hear about your career in linear algebra. How did you get started in this area?

H.S. - It was purely accidental. In 1950 I was fired from the Royal Observatory, Edinburgh because I had broken an expensive instrument the first time I used it. This ended my intended career as an astronomer. I was married with one child and the second on its way. My father offered to support me for a Ph.D. I applied to Max Born, then the professor of Applied Mathematics in Edinburgh and one of the founders of quantum theory. But he was about to retire and was taking no more students. So I turned to Prof. A.C. Aitken.

O.H. - What propelled your research and influenced the development of your ideas?

H.S. - It was again serendipity. In the fall term of 1950 Prof. Aitken gave a series of lectures to the small staff (faculty) of the Maths Department of U. Edinburgh. His theme was "Linear Operations in Probability", in effect Markov Chains in terms of stochastic matrices. I still have my notes of these lectures; you can see the first page on my home page. The date is 17 October 1950, which I celebrate as the date of my birth as a research mathematician. As I have often related, one day I asked Prof. Aitken a question to which he replied: "Read Frobenius". I saw this as dismissal; it took me years to realize what excellent advice I had been given. There is a mystery here; on the one hand I had very little contact with Aitken as a graduate student, on the other hand he determined much of my mathematical life of over 60 years.

I need to mention several other mathematicians who influenced me at the beginning of my career: Helmut Wielandt, Alexander Ostrowski, Alston Householder and, most of all, Olga Taussky. I am extremely grateful to all of them and I am proud that they noticed my early work.

O.H. - What were the most important events that affected your career?

H.S. - Two other new mathematics Ph.D.'s were hired by Queen's University, Belfast, when I got my first job in 1952 (thanks to Prof. S. Verblunsky). They had degrees from Cambridge and Princeton. I realized that my education lacked training in modern mathematics. That was the main motivation for pulling up my roots and moving my family to Wisconsin. But, surrounded as I was there by excellent mathematicians in subjects like functional analysis and abstract algebra, what really helped me was ease of contact in the U.S. with mathematicians of an older generation who had a background rather like mine.

O.H. - How has linear algebra changed in the 60 years that you have been involved?

H.S. - It is vastly different. In the 1950s an 1960s it was generally seen as a dead subject which all mathematicians must know, but hardly a topic for research. In fact, a well known mathematician told me this to my face. After I became Editor-in-Chief of LAA in 1972 I realized there was another way to look at the field, as an essential ingredient of many mathematical areas (at least during some stage of their development) and that this would lead to new results in linear algebra. An example of such an applied area active in the 1970s is control theory. Now (in 2012) I am amazed at the vigor of the field and new results arising because of connections to such applied topics as compressed sensing and quantum information. But I have a regret about one development. In the 1960s numerical and theoretical linear algebra formed a continuum and one group; now there is a split, at least in the U.S. Perhaps this change was inevitable.

O.H. - What do you see as your main contributions to linear algebra and mathematics as a whole?

H.S. - I have worked in several aspects of linear algebra. I think my choices were not determined by consideration of the importance of an area, but rather by an emotional need to make contributions where I felt that I could. I won't provide a list here. Sorry.

But I will mention my favorite among my papers. It is a paper that links Lyapunov inertia theory with Perron-Frobenius theory published in *Numerische Mathematik* in 1965. This may seem a curious choice of journal as the paper is not in the least computational. I used to be disappointed that this paper was hardly noticed, until a chance meeting with a young mathematician in the 1990s revived interest in it.

O.H. - What do you see as today's main challenges in linear algebra?

H.S. - Linear algebra is like a vast terrain with lots of hills and few high mountains. You cannot see all of it even if you are prepared to travel. I have to admit I do not have a good answer to this question.

O.H. - Looking back, what would you have done differently in your career? Do you regret any of your decisions?

H.S. - I cannot speak of regret, but looking back I realize that I had several offers from other universities I considered seriously and ultimately rejected. I sometimes wonder what would have happened to me and my family if I had decided differently.

O.H. - Did you enjoy teaching?

H.S. - I was not particularly good at teaching elementary courses to undergraduates and I did not greatly enjoy it. I liked working with my graduate students and postdocs.

O.H. - You are now 85 and still active in mathematical research. Do you see changes in how you do mathematics?

H.S. - I think I still have ideas of what can be explored within the mathematics that I am familiar with. But I now need younger coauthors to push through the proofs and organize the papers. I am extraordinarily fortunate that I have found such help.

O.H. - You are now in your 40$^{\text{th}}$ and last year as Editor-in-Chief of *Linear Algebra and its Applications*. How do you see your role here?

H.S. - The journal was founded in 1968 by a group of people around Ostrowski, all associated with the Gatlinburg (now Householder) meetings. I took over in 1972 when the journal was not doing well. The turn around came when I realized that my function was not that of a guardian angel keeping out poor papers. Rather, it was to establish linear algebra as a field of research and to motivate good mathematicians to publish in the journal. The main problem was that there was (and is) much linear algebra which is seen as a sort of helpmate to other fields. It was necessary to turn this from problem to advantage. But I've already talked about this.

O.H. - What advice can you give to young algebraists?

H.S. - Know what you are good at, keep at it, and keep at it.

Let me end with an anecdote - a true story about a great mathematician, Alexander Ostrowski. Shortly after I arrived in Madison in 1959, I worked with Ostrowski then in his seventies. I would go to his office in the Mathematics Research Center at appointed times. One day I noticed that he was studying a certain paper. So I checked it out and read it. Next time I went to his office I said "Professor Ostrowski, why are you reading this paper. It seems pretty bad to me". He answered "That's right. It's the first paper by a fresh Ph.D. It is likely to contain a good new idea which is badly executed. There may be a paper in it for you!". Now there's advice!

UPCOMING CONFERENCES AND WORKSHOPS

International Workshop on Accurate Solution of Eigenvalue Problems (IWASEP) IX
Embassy Suites, Napa Valley, California, U.S.A., June 4–7, 2012

The purpose of this workshop is to bring together experts on accuracy issues in the numerical solution of eigenvalue problems for four days of research presentations and discussions. This is the ninth such workshop. The most recent was held in Berlin, Germany, June 28 to July 1, 2010.

The themes of this workshop arise in the intersection of two fields, eigenvalue/singular value computation, and fast, accurate matrix computation. The organizers encourage submissions for presentations or posters that relate to these two themes. A prize will be given for the best poster.

The organizers are: Chair - Jesse Barlow (The Pennsylvania State University, *SIAM* representative), Zlatko Drmač (University of Zagreb), Volker Mehrmann (Technical University of Berlin), Ivan Slapničar (University of Split), and Krešimir Veselić (FernUniversität Hagen). The local organizing committee members are: Jim Demmel (UC Berkeley), Esmond G. Ng (Lawrence Berkeley National Laboratory), and Oded Schwartz (UC Berkeley).

For more information, go to `http://www.eecs.berkeley.edu/~odedsc/IWASEP9/`.

SIAM Applied Linear Algebra Conference 2012
Valencia, Spain, June 18–22, 2012

The SIAM Conferences on Applied Linear Algebra, organized by SIAM every three years, are the premier international conferences on applied linear algebra, which bring together diverse researchers and practitioners from academia, research laboratories, and industries all over the world to present and discuss their latest work and results on applied linear algebra. This will be the second time the conference is held outside the United States.

Themes include: iterative methods and preconditioning, randomized linear algebra algorithms, inverse and ill-posed problems, structured matrices, numerical linear algebra for analysis of complex networks, eigenvalue problems, optimization (including PDE-constrained optimization), numerical linear algebra of compressed sensing, and applications.

The plenary speakers for 2012 include: Hans De Sterck (University of Waterloo), Petros Drineas (Rensselaer Polytechnic Institute), Lars Grasedyck (RWTH Aachen University), Anshul Gupta (Thomas J. Watson Research Center), Roland Herzog (Chemnitz University of Technology), Des Higham (University of Strathclyde), Misha Kilmer (Tufts University), Juan Manuel Peña (Universidad de Zaragoza), Jared Tanner (University of Edinburgh), Françoise Tisseur (University of Manchester), Michael Tsatsomeros (Washington State University), and Chao Yang (Lawrence Berkeley National Laboratory). For more information, please visit `http://siamla2012.webs.upv.es/`.

ALAMA 2012 (3rd Biennial Meeting of the Spanish Thematic Network on Linear Algebra, Matrix Analysis and Applications)
Leganés (Madrid), Spain, June 27–29, 2012

The ALAMA Meetings are held every two years since the Thematic Network ALAMA (Linear Algebra, Matrix Analysis and Applications) was created in 2007. These meetings are an opportunity to gather scientists interested in linear algebra in its widest sense, whether they are members of the Network or not. The goal is to bring together for three days, in an environment favoring work and exchange of ideas, researchers whose work has some connection, one way or the other, with linear algebra, matrix analysis, matrix theory and/or their possible applications in different contexts.

ALAMA 2012 is the Third ALAMA Meeting since the creation of the ALAMA Network in 2007, and this third edition is celebrated in honor of Rafael Bru. ALAMA 2012 takes place at the Universidad Carlos III de Madrid, Legans (Madrid), Spain.

The plenary speakers are: Carlos Beltrán (Universidad de Cantabria), Rafael Bru (Universidad Politécnica de Valencia), Richard A. Brualdi (University of Wisconsin-Madison), and Volker Mehrmann (Technische Universität Berlin - NICONET Speaker). For more information, please visit `http://www.red-alama.es/encuentro2012/`.

Workshop on Parallel Matrix Algorithms and Applications
London, England, June 28–30, 2012

The 7th International Workshop on Parallel Matrix Algorithms and Applications (PMAA 2012) will take place at the Birkbeck University of London, U.K., June 28–30, 2012. This International Workshop aims to be a forum for an exchange of ideas, insights and experiences in different areas of parallel computing (Multicore, Manycores and GPU) in which matrix algorithms are employed. The Workshop will bring together experts and practitioners from diverse disciplines with a common interest in matrix computation.

Keynote speakers include: Iain Duff (Rutherford Appleton Laboratory, U.K.), Costas Bekas (IBM Research, Zurich, Switzerland), and Bo Kågström (Umeå University, Sweden).

For more information, please visit `http://www.dcs.bbk.ac.uk/pmaa2012/`.

The Eleventh Workshop on Numerical Ranges and Numerical Radii
National Sun Yat-sen University, Kaohsiung, Taiwan, July 9–12, 2012

The study of numerical ranges and numerical radii has a long and distinguished history. There has been a great deal of active research conducted by different research groups around the world. Its relations and applicability are tied to many different branches of pure and applied science such as operator theory, functional analysis, $\mathbb{C}^*$-algebras, Banach algebras, matrix norms, inequalities, numerical analysis, perturbation theory, matrix polynomials, systems theory, quantum information science, etc. Moreover, a wide range of mathematical tools including algebra, analysis, geometry, combinatorial theory, and computer programming are useful in the study.

The first workshop on the subject was in 1992 at Williamsburg. The workshop has since been conducted every other year and in different countries: at Coimbra, Portugal in 1994; at Sapporo, Japan in 1996; at Madison, U.S.A. in 1998, at Nafplio, Greece in 2000, at Auburn, U.S.A. in 2002, at Coimbra, Portugal in 2004; at Bremen, Germany in 2006; at Williamsburg, U.S.A. in 2008; at Kraków, Poland in 2010.

For information about speakers, registration, etc. please visit: `http://www.math.nsysu.edu.tw/~wong/WONRA2012/`.

2012 Workshop on Matrices and Operators
Harbin Engineering University, China, July 13–16, 2012

The purpose of the workshop is to stimulate research and foster interaction of researchers interested in matrix theory, operator theory, and their applications. Hopefully, the informal workshop atmosphere will ensure the exchange of ideas from different research areas. For information about previous workshops, confirmed participants, and registration, please visit: `http://people.wm.edu/~cklixx/mao2012.html`.

International Conference on Trends and Perspectives in Linear Statistical Inference and the 21st International Workshop on Matrices and Statistics
Będlewo, Poland, July 16–20, 2012

The Mathematical Research and Conference Center

The International Conference on Trends and Perspectives in Linear Statistical Inference, LINSTAT 2012, and the 21st International Workshop on Matrices and Statistics, IWMS 2012, will be held on July 16–20, 2012 at Będlewo near Poznań, Poland. LINSTAT 2012 is the follow-up of the 2008 and 2010 editions held in Będlewo, Poland and in Tomar, Portugal.

The purpose of the meeting is to bring together researchers sharing an interest in a variety of aspects of statistics and its applications as well as matrix analysis and its applications to statistics, and offer them a possibility to discuss current developments in these subjects. Special sessions on the following topics will be held: high-dimensional data, robust statistical methods, optimum design for mixed effects regression models, experimental designs, multivariate analysis, and matrices for linear models.

A session to celebrate George P. H. Styan's 75th birthday will be also organized.

Researchers and graduate students in the area of linear algebra, statistical models and computation are particularly encouraged to attend the workshop. The format of this meeting will involve plenary talks and sessions with contributed talks. The invited speakers are Rosemary A. Bailey (U.K.), Narayanaswamy Balakrishnan (Canada), Rajendra Bhatia (India), Somnath Datta (U.S.A.), Valerii Fedorov (U.S.A.), Thomas Mathew (U.S.A.), Paulo E. Oliveira (Portugal), K. Manjunatha Prasad (India) and Peter Šemrl (Slovenia), as well as the winners of the Young Scientists Awards of LINSTAT 2010: Olivia Bluder (Austria), Eero Liski (Finland), Chengcheng Hao (Sweden), and Paulo C. Rodrigues (Portugal).

The Scientific Committee will award the best presentation and best poster of young scientists. The awardees will be invited speakers at the next edition of the LINSTAT or IWMS. The Scientific Committee of LINSTAT 2012 is chaired by Augustyn Markiewicz (Poland), the International Organizing Committee of IWMS 2012 is chaired by Simo Puntanen (Finland), and the chair of the Local Organizing Committee is Katarzyna Filipiak (Poland).

Special issues of *Communications in Statistics - Theory and Methods*, as well as *Simulation and Computation* (Taylor & Francis), devoted to LINSTAT 2012 and IWMS 2012 will include selected papers strongly correlated to the talks of the conference, with emphasis on advances on linear models and inference.

For more information, please visit: `http://linstat2012.au.poznan.pl/`.

Graduate Student Modeling Workshop (IMSM 2012)
Raleigh, North Carolina, U.S.A., July 16–24, 2012

The 18th Industrial Mathematical and Statistical Modeling (IMSM) Workshop for Graduate Students will take place at North Carolina State University, July 16–24, 2012. The workshop is sponsored by the Statistical and Applied Mathematical Science Institute (SAMSI) together with the Center for Research in Scientific Computation (CRSC) and the Department of Mathematics at North Carolina State University.

The IMSM workshop exposes graduate students in mathematics, engineering, and statistics to exciting real-world problems from industry and government. The workshop will provide students with experience in a research team environment and exposure to possible career opportunities. Local expenses and travel expenses will be covered for students at U.S. institutions.

Information is available at `http://www.samsi.info/IMSM12` and questions can be directed to imsm_12@ncsu.edu.

CRM Conference on Applications of Graph Spectra in Computer Science
Barcelona, Spain, July 16–20, 2012

The main aim of the conference is to bring together diverse collection of researchers in graph theory, computer science and complex networks, interested in the applications of graph spectra, to discuss current trends and future directions in this area. The conference is organized and sponsored by Centre de Recerca Mathemàtica (CRM), Barcelona and co-sponsored by Ministry of Science and Innovation (MICINN), Spain.

Abstracts can still be submitted until June 10, 2012. There will be a special issue of *Discrete Applied Mathematics* devoted to the conference theme, with guest editors: Francesc Comellas, Robert Elsasser and Dragan Stevanović.

The keynote speakers for the conference are: Fan Chung (University of California at San Diego), Dragoš Cvetković (Serbian Academy of Sciences and Arts), Anirban Banerjee (Indian Institute of Science Education and Research, Kolkata), Ernesto Estrada (University of Strathclyde), Miguel Àngel Fiol (UPC BarcelonaTech), Edwin Hancock (University of York), Jürgen Jost (Max Planck Institute for Mathematics in Sciences, Leipzig), Nathan Linial (Hebrew University of Jerusalem), Ulrike von Luxburg (University of Hamburg), Piet Van Mieghem (Delft University of Technology), and Daniel Spielman (Yale University).

The scientific committee for the conference includes: Francesc Comellas (UPC BarcelonaTech), Fan Chung (University of California at San Diego), Dragoš Cvetković (Serbian Academy of Sciences and Arts), Robert Elsässer (University of Paderborn), Miguel Àngel Fiol (UPC BarcelonaTech), Steve Kirkland (Hamilton Institute), Nair Maria Maia de Abreu (Federal University of Rio de Janeiro), Piet Van Mieghem (Delft University of Technology), Dieter Mitsche (ETH Zurich), Vladimir Nikiforov (University of Memphis), Maria José Serna (UPC BarcelonaTech), and Dragan Stevanović, chair, (University of Primorska, Slovenia and University of Niš, Serbia).

Please visit: `http://www.crm.es/Activitats/Activitats/2011-2012/GraphSpectra/web-graphspectra/` or contact secreteria@crm.cat for more information.

Summer Research Workshop on Quantum Information Science
Taiyuan University of Technology, Taiyuan, Shanxi, China, July 17–27, 2012

The purpose of the summer workshop is to promote collaborative research on the rapidly developing interdisciplinary area of quantum information science. The focus will be on the mathematical aspects of the subject. Basic background and current research topics will be assumed. There will be four to five themes on subjects including entanglement, quantum operations, quantum error correction, etc. Researchers will present their recent results and open problems on these topics. Participants will share their experiences and insights to solve the open problems. Researchers and graduate students from mathematics, physics, computer sciences, etc. are most welcome to participate.

For further information about the workshop, please visit: `http://people.wm.edu/~cklixx/qc2012.html`.

The 10$^{\text{th}}$ International Conference on Matrix Theory and its Applications
Guiyang, Guizhou, China, July 20–23, 2012

The 10$^{\text{th}}$ International Conference on Matrix Theory and its Applications will be held at Guizhou Normal University, Guiyang, Guizhou, China. It aims to bring together researchers interested in matrix theory to understand and discuss recent advances on matrix analysis, matrix computation and their applications. The Conference will encourage the exchange of ideas and cooperation among researchers in similar research areas.

The conference organizing committee consists of Erxiong Jiang (Co-Chair, Shanghai University), Pengcheng Wu (Co-Chair, Guizhou Normal University), Linzhang Lu (Execute Chair, Guizhou Normal University & Xiamen University), Taijie You (Guizhou Normal University), Zhongzhi Bai (Chinese Academy of Science), and Liping Huang (Changsha Polytechnic University).

The invited speakers include Avi Berman (Technion – Israel Institute of Technology, Israel), Chi-Kwong Li (College of William & Mary, U.S.A.), Liqun Qi (The Hong Kong Polytechnic University, China,) Xiaoqin Jin (Macao University, China), Yangfeng Su (Fudan University, Shanghai, China), and Zhongyun Liu (Changsha Polytechnic University, Hunan, China).

For more information about the conference, please visit the conference website: `http://mc.gznu.edu.cn/ICMTA2012/`.

Shanghai Conference on Algebraic Combinatorics
Shanghai, China, August 17–22, 2012

The Shanghai Conference on Algebraic Combinatorics will take place August 17–22, 2012 at Shanghai Jiao Tong University in Shanghai, China. The topics of this conference are algebraic combinatorics in the widest sense and many related directions in pure and applied mathematics and sciences.

The academic committee consists of: Richard Stanley (MIT), Zhe-Xian Wan (Chinese Acad. Sci.), and Qing Xiang (U. Delaware). The organizing committee consists of: Eiichi Bannai (bannai@sjtu.edu.cn), Hao Shen (haoshen@sjtu.edu.cn), Yaokun Wu (ykwu@sjtu.edu.cn), and Xiao-Dong Zhang (xiaodong@sjtu.edu.cn).

All talks in this conference are by invitation. Those who are interested to attend the conference are invited to drop a message to any member of the academic committee and/or organizing committee to register for the conference and to indicate whether or not they want to give a talk. The list of invited speakers will be fixed in June 2012.

For more information, please visit the conference webpage: `http://math.sjtu.edu.cn/Conference/SCAC/`.

Rome-Moscow School of Matrix Methods and Applied Linear Algebra
Moscow and Rome, Russia and Italy, September 2012

Faculty of Sciences, University of Rome

The Rome-Moscow School of Matrix Methods and Applied Linear Algebra is a form of didactic-scientific cooperation between Rome and Moscow academic and research institutions. The third edition of the school is planned for the next whole month of September 2012, two weeks in Lomonosov Moscow State University and Institute of Numerical Mathematics of the Russian Academy of Sciences (LMSU-INM-RAS), and two weeks in the Department of Mathematics of the University of Rome "Tor Vergata" (TV).

The main purpose of the school is to encourage the exchange of ideas and scientific collaborations between Italian and Russian universities and institutions in the fields of matrix methods and applied linear algebra. The school offers to advanced undergraduate, masters and Ph.D. students a venue for learning and thinking over scientific research topics, not limited to short courses and seminars, and the opportunity of entering in direct contact with people and institutions of excellence in the field.

The various scientific backgrounds of the lecturers and their consequent different approaches to research problems turn out to be an opportunity for students to increase their knowledge and experience. Moreover, the participants could deepen the study experience offered to them by the school in the partner university by applying for suitable Student Exchange Programs in force between LMSU-INM-RAS and TV.

For complete information and instructions on how to apply to the school, go to `www.mat.uniroma2.it/~tvmsscho` (edition 2012).

Note that applications from students of any academic or research institution (not only of TV or LMSU-INM-RAS, and not only of Italian or Russian institutions) are welcome.

Lomonosov Moscow State University

3rd IMA Conference on Numerical Linear Algebra and Optimisation
Birmingham, U.K., September 10–12, 2012

The IMA (Institute of Mathematics and its Applications) and the University of Birmingham are pleased to announce the 3rd IMA Conference on Numerical Linear Algebra and Optimisation.

The success of modern codes for large-scale optimisation is heavily dependent on the use of effective tools of numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimisation problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest. Conference topics include any subject that could be of interest to both communities, such as: direct and iterative methods for large sparse linear systems, eigenvalue computation and optimisation, large-scale nonlinear and semidefinite programming, effect of round-off errors, stopping criteria, embedded iterative procedures, optimisation issues for matrix polynomials, fast matrix computations, compressed/sparse sensing, PDE-constrained optimisation, and applications and real time optimisation.

The invited speakers are: Roland Freund (University of California, Davis), Philip Gill (University of California, San Diego), Serge Gratton (CERFACS, Toulouse), Anne Greenbaum (University of Washington, Seattle), Michael Hintermueller (Humboldt University, Berlin), Sabine van Huffel (K.U. Leuven), Mike Powell (University of Cambridge), and Andrea Walther (University of Paderborn).

Organizing Committee: Chair - Michal Kočvara (University of Birmingham, SIAM Representative) , Co-chair - Daniel Loghin (University of Birmingham), Jacek Gondzio (University of Edinburgh), Nick Gould (Rutherford Appleton Laboratory), Jennifer Scott (Rutherford Appleton Laboratory), and Andy Wathen (University of Oxford). The conference is held in cooperation with the Society for Industrial and Applied Mathematics (SIAM) and SIAM Activity Group on Linear Algebra.

If you would like to register your interest in this conference and join the mailing list, please email conferences@ima.org.uk. General inquiries concerning conference arrangements should be sent to: Lizzi Lake (email: conferences@ima.org.uk). The Institute of Mathematics and its Applications, Catherine Richards House, 16 Nelson Street, Southend-on-Sea, Essex, England SS1 1EF Tel: +44 (0)1702 354020. For more details, please visit the conference webpage: `www.ima.org.uk/conferences/conferences_calendar/numerical_linear_algebra_and_optimisation.cfm`.

The 4th International Symposium on Positive Systems (POSTA 2012) Maynooth, Ireland, September 2–4, 2012

Registration for the 4th International Symposium on Positive Systems (POSTA 2012) is now open at `http://www.hamilton.ie/posta2012`. The symposium will take place at the Hamilton Institute, National University of Ireland Maynooth, from September 2 to September 4, 2012. As with previous symposia, POSTA 2012 will cover a broad range of topics from the theory and applications of Positive Systems and consist of a mix of plenary and contributed presentations as well as social events at which participants will have the opportunity to mingle and discuss their work in a relaxed setting. The organizing committee includes: Buket Benek Gursoy, Steve Kirkland, Ollie Mason and Robert Shorten.

Interested parties are invited to contribute shorter talks (20 minutes in duration) on any topic that falls within the broad scope of the symposium. If you wish to present a talk at the symposium, please submit an extended abstract (up to one page in length) using the latex template available at `http://www.hamilton.ie/ollie/Downloads/POSTA_Abstract.tex` to oliver.mason@nuim.ie, writing POSTA 2012 in the subject line.

There will be a special issue of the journal *Linear and Multilinear Algebra* devoted to papers presented at POSTA 2012. All submissions to the special issue are expected to fall within the journal's usual aims and scope; see the following link for details `http://www.tandf.co.uk/journals/journal.asp?issn=0308-1087`.

LIA-SGT (Latin Ibero-American Spectral Graph Theory) Workshop 2012 Rio de Janeiro, Brazil, September 27–28, 2012

The theory of graph spectra is now a well established field of research in mathematics and in several applied sciences (e.g. chemistry, computer science and operational research), and many results have been published over the last few decades. In recognition of the strong developments in the subject, this workshop has been organized as a forum for researchers around the Latino-Ibero-American region. However, every researcher in graph spectra is welcome to attend the workshop.

The conference will take place in Rio de Janeiro, from September 27 to 28, 2012. For more information visit one of two links: `www.sobrapo.org.br/claiosbpo2012/` or `http://sobrapo.org.br/lia-sgt/`.

There will be a special issue of *Linear Algebra and its Applications* (LAA) dedicated to the workshop. All papers will be refereed, and acceptance will follow the standards of LAA. The deadline for journal submissions is expected to be eight weeks after the conference.

Haifa Matrix Theory Conference Haifa, Israel, November 12–15, 2012

The 2012 Haifa Matrix Theory Conference will take place at the Technion, November 12–15, 2012, under the auspices of the Technion's Center for Mathematical Sciences.

This will be the sixteenth in a sequence of matrix theory conferences held at the Technion since 1984. The ILAS lecturer in the conference will be Thomas Laffey. As in the past, all talks will be of 30 minutes duration, and will cover a wide spectrum of theoretical and applied linear algebra. In particular, the conference will host special sessions in memory of Michael (Miki) Neumann and Uriel G. Rothblum.

The organizing committee consists of Abraham Berman (berman@technion.ac.il), Raphael Loewy (loewy@technion.ac.il) and Naomi Shaked-Monderer (nomi@technion.ac.il). Further information can be obtained at: `www.math.technion.ac.il/cms/decade_2011-2020/year_2012-2013/matrix/`

Matrices and Operators - Conference in honor of the 60th birthday of Rajendra Bhatia Bangalore, India, Dec. 27–30, 2012

The Mathematics Department of Indian Institute of Science, Bangalore

Rajendra Bhatia just turned sixty in May this year. As he has made many beautiful contributions to mathematics, we take this opportunity to hold a conference in his honor in the southern Indian city of Bangalore. Bangalore is widely known as the science capital of India. Apart from the century old Indian Institute of Science, Bangalore has a centre of the Indian Statistical Institute, the Raman Research Institute, Indian Academy of Sciences, Indian Institute of Astrophysics and many other seats of education. As we know, Rajendra's work is very diverse and encompasses many areas of mathematics. Hence this milieu for the conference. The dates and venue are December 27–30, 2012 at the Indian Institute of Science.

There will be a series of three talks by Rien Kaashoek. These will be survey style talks on a common theme. Then there will be individual talks. The speakers include Kalyan Bidhan Sinha, Rajeeva Karandikar, Peter Šemrl, Ren-Cang Li, Shmuel Friedland, Tsuyoshi Ando, Fumio Hiai, José Dias da Silva, Chi-Kwong Li, Takashi Sano, Dario Bini, Stefano Serra, Mitsuru Uchiyama, Mehdi Radjabalipour, Abbas Salemi Parizi and Rien Kaashoek.

The contact details are: Tirthankar Bhattacharyya (main organizer). Phone: +91 80 2293 2710, Fax: +1 509 561 1264, email: rajendra.conference@gmail.com. For more details and registration, see `http://math.iisc.ernet.in/~bhatia60`.

18th ILAS Meeting
Providence, Rhode Island, U.S.A., June 3–7, 2013

The 18th Conference of the International Linear Algebra Society (ILAS) will be held in Providence, Rhode Island, U.S.A. on June 3–7, 2013. Confirmed plenary speakers include: Alan Edelman (MIT), Maryam Fazel (University of Washington), Ravi Kannan (Microsoft Research), Jean Bernard Lassere (Centre National de la Recherche Scientifique, France), Thomas Laffey (University College Dublin) (Hans Schneider prize winner), Dianne O'Leary (University of Maryland), Ivan Oseledets (Institute of Numerical Mathematics, RAS), Leiba Rodman (College of William and Mary), Dan Spielman (Yale), Gilbert Strang (MIT), Raymond Sze (Hong Kong Polytechnic University), Fuzhen Zhang (Nova Southeastern University). More information may be found at `http://www.ilas2013.com/`.

4th International Conference on Matrix Analysis and Applications
Konya, Turkey, July 2–5, 2013

The 4th International Conference on Matrix Analysis and Applications will be held in Konya, Turkey, July 2–5, 2013. The purpose of this conference is to stimulate research, to provide an opportunity for researchers to present their newest results, and to meet for informal discussions. The previous meetings of the workshop series have taken place in China and the U.S.A.

The keynote speaker of the 4th conference is Steve Kirkland. The scientific organizing committee (SOC) consists of Peter Šemrl, Tin-Yau Tam, Qingwen Wang, and Fuzhen Zhang, and local organizing committee (LOC) includes Durmuş Bozkurt, Vehbi Paksoy, Ramazan Türkmen, and Fatih Yilmaz.

More detailed information about the conference along with a list of invited speakers will be provided at a later date.

CONFERENCE REPORTS

International Conference on Matrix Analysis and Its Applications - MatTriad 2011
Tomar, Portugal, July 12–16, 2011

Report By Francisco Carvalho and Katarzyna Filipiak

Group photo at the MatTriad dinner

The 2011 edition of MatTriad, fourth in a series of international conferences on Matrix Analysis and Its Applications, was held at the Polytechnic Institute of Tomar (Portugal), from the 12th to the 16th of July 2011. Tomar is a small city in the center of Portugal, 130km north of Lisbon, and welcomed the first edition of MatTriad held outside of Poland, holding beautiful surroundings and privileged conditions to promote an informal get-together of highly considered specialists and researchers interested in the topics selected for this conference.

The conference was co-organized by the Polytechnic Institute of Tomar and by the Faculty of Sciences and Technology (New University of Lisbon) through its Mathematics Department and the Center of Applied Mathematics.

The scientific committee was chaired by Tomasz Szulc (Adam Mickiewicz University, Poznań, Poland) and comprised João T. Mexia (Faculty of Sciences and Technology, New University of Lisbon, Portugal), Ljiljana Cvetković (University of Novi Sad, Serbia), Heike Faßbender (Technical University Braunschweig, Germany) and Simo Puntanen (University of Tampere, Finland). The organizing committee was chaired by Francisco Carvalho (Polytechnic Institute of Tomar, Portugal) and comprised Katarzyna Filipiak (Poznań University of Life Sciences, Poland), Miguel Fonseca (Nova University of Lisbon, Portugal) and Paulo C. Rodrigues (Nova University of Lisbon, Portugal, Wageningen University, The Netherlands).

The aim of this conference was to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications in other parts of mathematics and offer them a possibility to discuss current developments in these subjects. The conference consisted of three invited lectures, invited plenary talks, and contributed talks. The lectures were delivered by N. Balakrishnan (Canada): *Permanents and Applications*; Ali S. Hadi (Egypt): *Applications of Matrix Algebra in Various Fields*; and Volker Mehrmann (Germany): *Nonlinear Eigenvalue Problems: Analysis and Numerical Solution.* The invited speakers were Natália Bebiano (Portugal), Jeff Hunter (New Zealand), Charles R. Johnson (U.S.A.), Juan M. Peña (Spain), and the winners of the MatTriad 2009 Young Scientists Awards: Ricardo Covas (Portugal) and Aneta Sawikowska (Poland).

Young Scientists Awards announcement: S. Puntanen, P. Rodrigues, T. Szulc, L. Cvetković and O. Walch

The 67 contributing speakers delivered their talks in special sessions devoted to the topics chosen for this conference: matrix theory, applied and numerical linear algebra, matrix computations, matrices and graphs, matrices and Markov chains, linear algebra in industry and sciences, linear algebra in statistics, and projectors and their applications. A poster session was also organized providing a more informal atmosphere to discuss the topics presented.

During the conference dinner, the scientific committee proudly announced the winners of the Young Scientists Awards for the best talks presented by Ph.D. students and young scientists: Paulo C. Rodrigues (Portugal) and Olivia Walch (U.S.A.). The winners will be invited speakers at the 2013 edition of MatTriad.

A special issue of ELA - *The Electronic Journal of Linear Algebra*, will be devoted to MatTriad 2011 with selected papers strongly related with the presentations given at the conference.

The participants shared the common opinion that the conference was extremely fruitful and well organized with friendly and warm atmosphere. The list of the participants of MatTriad 2011, abstracts of the talks and posters, the gallery of photographs, and other information can be found at `http://www.mattriad2011.ipt.pt`.

Rome-Moscow School of Matrix Methods and Applied Linear Algebra Lomonosov, Rome, Russia & Italy, September 2011

Report By Carmine di Fiore

Students concentrating on a lecture

The second edition of the Rome-Moscow School of Matrix Methods and Applied Linear Algebra was held in Lomonosov Moscow State University (LMSU), Institute of Numerical Mathematics of the Russian Academy of Sciences (INM-RAS) and University of Rome Tor Vergata (TV) in September 2011. Lecturers from TV, LMSU, INM-RAS, Udine and Leipzig taught 106 hours of lectures/seminars which were attended by 13 Italian and 12 Russian M.Sc. and Ph.D. students. A wide range of topics was taught. In particular, we mention: Prof. D. Fasino (Italy, University of Udine), "Numerical linear algebra with quasiseparable matrices"; Prof. S. A. Goreinov (Russia, Moscow, INM-RAS), "Chebyshev's ideas in matrix approximation problems"; Prof. M. A. Picardello (Italy, University of Rome Tor Vergata), "Signals reconstruction from samples"; Prof. N. Zamarashkin (Russia, Moscow, INM-RAS), "Design and analysis of error-correcting codes". For more details see `www.mat.uniroma2.it/~tvmsscho` (edition 2011).

Rome-Moscow group photo

Professor Ikramov and his students

Special Session on Matrices and Graphs at the Joint Mathematics Meetings Boston, MA, U.S.A, January 4–7, 2012

A special session on matrices and graphs was held at the Joint Mathematics Meetings of the American Mathematical Society (AMS) and the Mathematical Association of America (MAA) in Boston, Massachusetts, January 4–7, 2012.

The organizers of the session were Leslie Hogben (Iowa State University and American Institute of Mathematics) and Bryan Shader (University of Wyoming). There were 16 scheduled talks on topics that included minimum rank and zero forcing, sign patterns, graph energy, graph Laplacians, average mixing on graphs, loopy graphs, and the inverse inertia problem for graphs.

The speakers were: Wayne Barrett, Adam Berliner, Richard A. Brualdi, Steve Butler, Minnie Catral, Louis Deaett, Chris Godsil, Jason Grout, In-Jae Kim, Jillian McLeod, Simone Severini, Pauline van den Driessche, Kevin N. Vander Meulen, Ulrica Wilson, and Michael Young.

The full program can be found at:
`http://jointmathematicsmeetings.org/meetings/national/jmm2012/2138_program_ss50.html#title`.

Participants of the Special Session on Graphs and Matrices at the Joint Meetings

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MAA Session on Innovative and Effective Ideas in Teaching Linear Algebra Boston, MA, U.S.A, January 4–6, 2012

Report By David Strong

One of the real pleasures of the Joint Meetings of the American Mathematical Society (AMS) and Mathematics Association of America (MAA), held in January 2012 in Boston, was the MAA contributed paper sessions on "Innovative and Effective Ideas in Teaching Linear Algebra". This was the sixth year that this session has been organized by David Strong, Gil Strang and David Lay.

Nearly 20 presentations were given on applications, examples, ideas and philosophies in teaching undergraduate linear algebra. Speakers (two of which included ILAS Education Committee Members, Sung-Gu Lee and David Strong) represented a variety of institutions ranging from small teaching-focused schools to large research universities. Topics discussed included a number of pedagogical techniques and tools such as technology and projects, as well as a variety of applications and examples which are both interesting and useful tools with which to discover and discuss specific ideas.

The talks were generally well attended, averaging close to 100 people for many of the talks. It is rewarding to be reminded that there is great interest in the mathematical community regarding how to more effectively teach the many interesting and useful ideas found in linear algebra.

This session will again be held at the January 2013 Joint Meetings in San Diego. A call for papers will appear in the late spring.

Links for presentations can be found at `http://faculty.pepperdine.edu/dstrong/LinearAlgebra/`.

International Workshop and Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices
Manipal, India, January 2–7 and 10–11, 2012

Report By Ravindra B. Bapat and K. Manjunatha Prasad

The workshop and conference was held at the Department of Statistics, Manipal University in Manipal, Karnataka, India. The scientific committee for the events consisted of Professors Ravindra B. Bapat, Steve J. Kirkland, K. Manjunatha Prasad and Simo Puntanen.

Generalized inverses of matrices and combinatorial matrix theory are among areas of matrix theory with a strong theoretical component as well as applications in diverse areas that have seen rapid advances in recent years. The interaction between graph theory and matrix theory is known to be very fruitful, and examples abound where generalized inverses of matrices arising from graphs play an important role. The two-day conference provided an ideal opportunity for participants from India and abroad to interact and exchange ideas in pleasant and academic surroundings. The objective of the workshop was to provide a platform for the young generation to have an exposure to lectures from leading mathematicians working in the area of combinatorial matrix theory and generalized inverses and their applications. Apart from tutorial lectures on these topics, the organizers planned to provide exposure to applied linear algebra topics through some special lectures. To meet this objectives, several tutorial lectures, special lectures, and discussion hours were arranged.

CGTGIM Workshop Group Photo

Having an initial response of 150 individuals (32 speakers, 56 students and 63 other participants), the actual attendance for the workshop and conference was 114 (97 for workshop and 102 for the conference). Apart from India, delegates and participants from Bangladesh, Canada, Estonia, Finland, Germany, Ireland, Nepal, New Zealand, Poland, Portugal, the United States of America were in attendance.

The events were endorsed by ILAS and SIAM. The International Centre for Theoretical Physics (ICTP), Central Statistical Office, Ministry of Statistics and Programme Implementation (`http://www.mospi.nic.in`), Council of Scientific and Industrial Research (`http://www.csir.res.in`) and National Board for Higher Mathematics (`http://www.nbhm.dae.gov.in`) provided support with grants. Manipal University, as host institution, sponsored local hospitality for the international delegates.

Lecture notes devoted to the workshop and proceedings of the papers presented in the conference will be published in two separate volumes after a review process.

CMTGIM Conference Group Photo

JOURNAL REPORTS AND ANNOUNCEMENTS

LAA Memorial Issue for Michael Neumann and Uriel Rothblum

Within the last year, we have tragically lost, at an early age, two major linear algebraists and long term editors of *Linear Algebra and its Applications* (LAA), Miki Neumann (1946–2011) and Uri Rothblum (1947–2012). LAA will publish an issue in their memory to emphasize their contributions.

Papers within the scope of LAA should be submitted, beginning April 15, 2012, via the Elsevier Editorial System (EES): `http://ees.elsevier.com/laa/` choosing the special issue called "Neumann/Rothblum Memorial Issue." There are no special editors for this memorial issue, and authors may choose any of the four Editors-in-Chief to be responsible for their paper. The deadline for submission is December 1, 2012.

Electronic Journal of Linear Algebra (ELA)

By Bryan Shader, Editor-in-Chief, ELA

2011 was a great year for ELA - *The Electronic Journal of Linear Algebra.* In 2011, ELA published 77 papers and received over 250 submissions. Two special conference issues are in the works: one for the 2011 Directions in Matrix Theory Conference, and the other for the MatTriad 2011 Conference.

Much thanks goes to the editorial board and the referees for evaluating the papers in a thoughtful and timely fashion, and to the authors for sharing their mathematical insights through carefully prepared submissions and revisions. It is great to see the area of linear algebra continuing to prosper and evolve.

Kudos to Panagiotis (Panos) Psarrakos, Michael Cavers, In-Jae Kim and Sarah Carnochan Naqvi for their efforts in copy-editing and typesetting that make a published ELA paper something that the entire community can be proud of.

As of December 31, 2011, the ELA website (`http://www.math.technion.ac.il/iic/ela`) has been given a new look. The next task is to develop a fresher ELA style for formatting papers.

HONOURS AND AWARDS

Hans Schneider Prize Awarded to Thomas J. Laffey

By Stephen Kirkland

Thomas J. Laffey

The Hans Schneider Prize is for research, contributions, and achievements at the highest level of Linear Algebra. The ILAS Executive Board is pleased to announce that the 2012 Hans Schneider Prize in Linear Algebra has been awarded to Thomas J. Laffey, of University College Dublin, Ireland. The award is made in recognition of Laffey's lifetime profound contributions to many areas of Linear Algebra, including integral matrices, simultaneous similarity, linear preservers, and, in particular, his breakthrough results on the inverse eigenvalue problem for nonnegative matrices.

The following are just a couple of examples of Laffey's achievements. He proves (in a joint paper with Reams, 1994) that every square matrix over the integers with trace zero is a commutator of two integral matrices, thus providing an analogue to the classical result over fields. The nonnegative inverse eigenvalue problem (NIEP) asks for a characterization of those lists of n complex numbers (possibly with repetitions) that comprise the spectra (with multiplicities) of $n \times n$ entrywise nonnegative matrices. Laffey (in a joint paper with Šmigoc, 2006) completely solves the NIEP if the lists are selfconjugate and consist of one positive real number and $n - 1$ complex numbers with nonpositive real parts. Further, in a recent paper, Laffey brings new insight to the NIEP by giving an ingenious, elementary and constructive proof of the Boyle - Handelman theorem in an important special case.

Laffey is well known in the mathematical community outside of Linear Algebra. He is heavily involved with the Irish team for the International Mathematical Olympiad. Laffey has been a member of the National Committee for Mathematics and represented Ireland at the International Congress of Mathematicians. A member of the Royal Irish Academy since 1981, he served as its Vice President for the period 1990–1992. Laffey's encyclopedic knowledge and enthusiasm for mathematics of all kinds have made him an invaluable source of information and advice for his colleagues and students alike.

The Hans Schneider Prize will be presented to Professor Laffey at the next ILAS meeting in Providence, Rhode Island, U.S.A., June 3–7, 2013, and Laffey will also be invited to give a lecture at that meeting. The ILAS Executive would like to express its sincere thanks to the members of the 2012 Hans Schneider Prize Committee: Shmuel Friedland, Leiba Rodman (chair), Peter Šemrl, Bryan Shader, and Valeria Simoncini. An outstanding group of people were nominated for the prize, and the Prize Committee performed its duties with appropriate thoroughness and careful consideration. The Prize Committee's efforts are most appreciated.

ILAS Members Named SIAM Fellows

By Leslie Hogben, Chi-Kwong Li, and Steven Kirkland

The Society for Industrial and Applied Mathematics (SIAM) has recently announced its SIAM Fellows for 2012, and among those named are ILAS members Michele Benzi and Richard A. Brualdi. Richard Brualdi is also a former ILAS President and Hans Schneider Prize recipient. In addition, Richard Varga, another past winner of ILAS's Hans Schneider Prize, was also named as a SIAM Fellow. On behalf of ILAS, we congratulate Michele, Richard and Richard on this well-deserved acknowledgement of their mathematical contributions.

Benzi, Brualdi and Varga join a list of distinguished ILAS members who are also SIAM Fellows, including Carl de Boor, Iain Duff, Graham Gladwell, Nicholas Higham, Ilse Ipsen, Thomas Kailath, Peter Lancaster, Volker Mehrmann, Michael Overton, Seymour Parter, G. W. Stewart, Gilbert Strang, Paul Van Dooren, and Olof Widlund. Other SIAM Fellows with ILAS connections are Cleve Moler and Beresford Parlett, both of whom are past winners of the Hans Schneider Prize.

OBITUARIES

Uriel George Rothblum, 1947–2012

Uriel George Rothblum

Professor Uriel George Rothblum died on March 26 due to cardiac arrest. He is survived by his wife Naomi and his three sons.

Uri obtained his B.Sc. Magna Cum Laude in Applied Mathematics in 1969 from Tel Aviv University, obtaining an M.Sc. Summa Cum Laude in Mathematics from the same institution in 1971. He received his Ph.D. in Operations Research from Stanford University in 1974.

Uri served as a consultant to the Rand Corporation in Santa Monica, California in 1974, and also consulted for AT&T Bell Laboratories, Murray Hill, NJ for a number of years. Before settling at the Technion - Israel Institute of Technology, Haifa, Israel, he served as a professor at several universities in the United States, including the Courant Institute of Mathematical Sciences (New York University), the State University of New York at Stony Brook, Yale University in New Haven CT, Rutgers University in New Brunswick NJ, Stanford University in CA, and Columbia University in NY. While at the Technion he also served some terms in an administrative capacity, including as a Dean, a Deputy Provost and an Executive Vice-President for Academic Affairs.

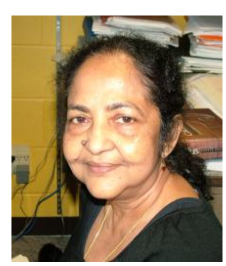
Uri published more than 160 research papers and was recognized for his excellent work: he received various awards and honours, including the *ORSIS Prize for Excellence in Research* in operations research.

Uri was a Senior Editor of *Linear Algebra and its Applications* and Editor-in-Chief of *Mathematics of Operations Research*. He previously served on the editorial boards of other journals, including *SIAM Journal on Matrix Analysis and Applications* and *SIAM Journal on Algebraic and Discrete Methods*.

Personal notes can be read and posted on a blog at `http://ie.technion.ac.il/Home/Users/rothblum0.html`.

Karabi Datta, 1943–2012

Contributed Announcement from Richard A. Brualdi



Karabi Datta

I am very sad to report that Karabi Datta passed away on May 8, 2012, after a very courageous battle with cancer. Dr. Datta was the author of many papers on matrix theory (eigenvalue computations and algorithms, matrix equations, control). Our heartfelt sympathies go out to her family, in particular, to Karabi's devoted husband Biswa Datta.

Dr. Datta served in the Dept. of Math. Sciences at Northern Illinois University, DeKalb, IL, U.S.A.

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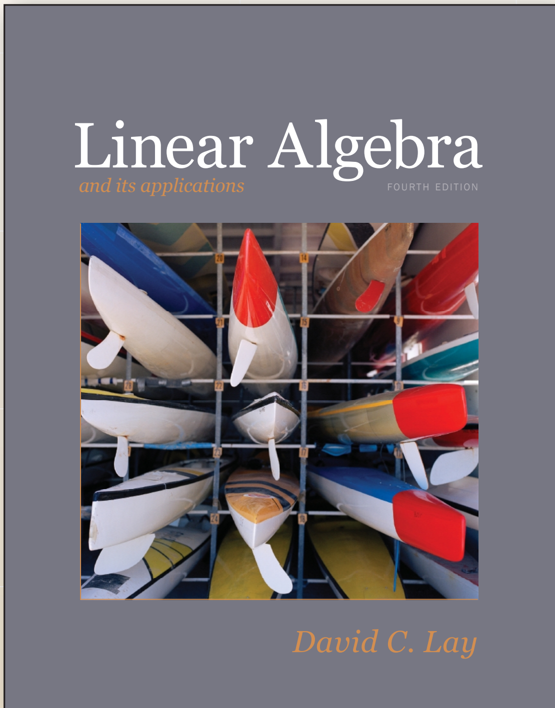

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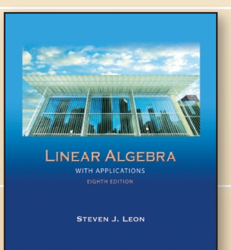

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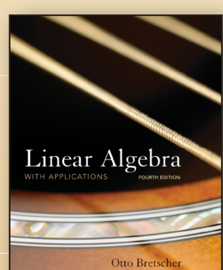

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LINEAR ALGEBRA EDUCATION

Recent Linear Algebra Education Events

By Steven Leon

Among the highlights of the 2012 Joint Meetings of the AMS and MAA were the two sessions on Innovative and Effective Ways to Teach Linear Algebra that were organized by David Strong of Pepperdine University. The sessions featured nineteen talks offering a wealth of constructive ideas for improvements in the teaching of linear algebra. These sessions have become an popular feature at the annual meetings. A listing of all the talks given in these sessions during the last five years is available at the site: `http://faculty.pepperdine.edu/dstrong/LinearAlgebra/index.html`. This site features links to files containing the transparencies from many of the talks that were presented at the meetings.

Linear Algebra Education Events at Future Meetings

By Steven Leon

The 12$^{\text{th}}$ International Congress of Mathematics Education (ICME-12), July 8–15, 2012, Seoul, Korea, will feature a Discussion Group on Issues Surrounding the Teaching of Linear Algebra. The co-chairs of the discussion group are Avi Berman and Kazuyoshi Okubo. The 16$^{\text{th}}$ Haifa Matrix Theory Conference, November 12–15, 2012, will feature a session devoted to the teaching of linear algebra. The Annual ILAS Conference, June 3–7, 2013, Providence, Rhode Island will feature an invited minisymposium on Linear Algebra Education Issues. The organizers for the minisymposium are Avi Berman, Sang-Gu Lee, and Steven Leon

BOOK REVIEWS

Matrix Tricks for Linear Statistical Models: Our Personal Top Twenty By Simo Puntanen, George P.H. Styan and Jarkko Isotalo

Springer, Heidelberg, 2011, xvii+486 pages, ISBN 978-3-642-10472-5

Reviewed by Radosław Kala, Poznań University of Life Sciences, kalar@up.poznan.pl

During the last decade, more than 50 monographs and textbooks have been published in the area of matrix algebra and statistical inference based on linear models. Among this collection, the book of Puntanen, Styan and Isotalo is exceptional. Of course, as it follows from the title, this new book is closely connected with matrices and linear models, but its organization is unique.

First of all, the authors extract from the matrix algebra and theory of linear models twenty key results or ideas which they call "tricks". The tricks are formulated as theorems and are placed at the beginning of each chapter, together with proofs, necessary references and specific remarks. At the end of each chapter, except the shortest ones, there is a list of exercises of various difficulty. The other sections of each chapter contain results connected with the leading trick of the chapter. Usually, they are formulated as propositions with formal proofs and frequently are supplemented by additional comments, citations, historical remarks and precise references from a collection of 661 items listed at the end of the book. The proofs of these propositions, however, contain cross-references to tricks of the other chapters. Consequently, the organization of the material is nonlinear. In addition, certain problems from the theory of linear models are recalled and revisited in many places of the book.

To help the reader, the authors begin the book with a rather long Introduction which contains preliminaries to linear algebra, statistics, random vectors, data matrices and linear models. Here the reader finds the definitions of the concepts, some examples and remarks, as well as basic methods which are discussed in detail in subsequent main chapters. The first main chapter, Chapter 1, contains the first trick, which is concerned with the column space of a matrix. The chapter begins with a collection of 13 simple and well-known facts; the rest of the chapter deals with applications derived from focusing on the column space. One section is devoted to the estimability of linear functions in a simple ANOVA model and the other to showing properties of the covariance matrix of a random vector.

Chapter 2 presents properties of orthogonal projectors. These properties are used to establish solutions to some minimization problems. The properties are extended to orthogonal projectors with respect to an arbitrary inner (or semi-inner) product. Chapter 3 is devoted to the sample correlation coefficient from a geometrical point of view. Moreover, the chapter develops the connections with the coefficient of determination and partial correlation coefficient in a simple regression model. Chapter 4 contains basic results on generalized inverses of a matrix. The chapter is supplemented with an exploration of their relations with the singular value decomposition, oblique projectors, the general solution and minimum norm solution to consistent linear equations, as well as with the least squares solution to inconsistent linear equations.

A collection of results concerning the rank of a partitioned matrix and the matrix product constitute the trick of Chapter 5. Among other results, the reader finds here a nonstandard column space decomposition of a partitioned matrix into disjoint subspaces, a condition for equality of column subspaces of two matrices, and an explicit expression for the intersection of two column spaces.

Chapters 6 and 7 are the shortest. The first contains the rank cancellation rule which leads to the simplification of some matrix equations. In turn, Chapter 7 contains the conditions under which the sum (or difference) of two orthogonal projectors is an orthogonal projector.

Chapter 8 recalls the orthogonal decomposition of the column space of a partitioned matrix and then presents a decomposition of the orthogonal projector. This result is then applied to particular problems connected with estimation of parameters in linear models of various forms, related with testing of linear hypotheses or with expressing the coefficient of determination in different but equivalent forms. Chapter 9 considers the problem of finding the minimum – in Löwner sense – for the covariance matrix of a special linear transform of the partitioned vector random variable. This trick is presented in many statistical applications. Other sections in this chapter show its usefulness in establishing the best linear predictor (BLP) and its relations with linear regression and principal component analysis.

Chapter 10 is the longest one. It deals with the best linear unbiased estimation (BLUE) of linear functions of unknown parameters. This fundamental concept of linear models is characterized by a solution to an appropriate matrix equation. The main problems considered here concern the equality between the BLUE and the ordinary least squares estimator (OLSE), the efficiency of OLSE and its relation with canonical correlations, as well as the equality between the best linear unbiased predictor (BLUP) and the OLSE.

The trick of Chapter 11 is concerned with the basic matrix equation. Its general solution, when consistent, is utilized in discussion of connections between the BLUEs following from two fixed models with the same expectation but with different covariance structures. Similar problems concerning links between BLUEs and BLUPs under mixed models are also considered.

Chapter 12 presents the necessary and sufficient condition for invariance of matrix products involving generalized inverses with respect to the choice of the inverses. The results establishing invariance of the column space and rank of such products are also discussed.

Chapter 13 deals with the block partitioned symmetric nonnegative definite matrices. The main trick here concerns the block-diagonalization of such matrices in which the Schur complements of diagonal blocks appear. Formulas for generalized inverses, determinants and ranks of such matrices are also exhibited. Contrary to the trick of the previous chapter, Chapter 14 contains the necessary and sufficient condition under which a block partitioned symmetric matrix appears to be nonnegative definite. In this context, some consequences related with correlation and covariance matrices and also with the Löwner order in a set of nonnegative definite matrices are also presented. Properties of three specific nonnegative definite matrices form the trick of Chapter 15. Successive subsections show their usefulness in presenting solutions to many problems related with the BLUE and OLSE in linear models of various structures. The trick of Chapter 16 is composed of fourteen equivalent conditions for the disjointness of the column spaces of two matrices. These results are used to express the estimability condition in a partitioned linear model and in two problems concerning the BLUE and OLSE.

Chapters 17, 18 and 19 are devoted to three decompositions: full rank decomposition, the eigenvalue decomposition of a symmetric matrix, and the singular value decomposition of any rectangular matrix. The usefulness of these techniques in matrix algebra is well-known, but the authors focus their attention mainly on various statistical implications. Chapter 20 presents the last trick, the Cauchy–Schwarz inequality. This trick is supplemented by its specific versions, some statistical consequences, and their relations with inequalities of Kantorovich and of Wielandt.

The whole material is enriched with 30 figures illustrating the geometry of some statistical concepts and methods, with 31 photographs of statisticians, mainly from a collection of the first author, and with several images of very interesting philatelic items from a collection of the second author.

This exceptional book can be recommended to all interested students who wish to improve their skills in linear models and matrix manipulations. It can be recommended also to professors teaching statistics who may utilize this book as a rich source of various exercises, references and interesting historical notes.

Quantum Computing: From Linear Algebra to Physical Realizations
By Mikio Nakahara and Tetsuo Ohmi

CRC Press, Taylor & Francis, 2008, xvi+421 pages, ISBN-13: 978-0750309837

Reviewed by Hugo J. Woerdeman, Drexel University, Hugo.Woerdeman@drexel.edu

The book covers a broad range of topics, starting from fundamentals of quantum mechanics and basics of quantum computing, to real-world applications in different implementations.

The first part "From linear algebra to quantum computing" starts out with a review of some basic linear algebra where a linear algebraist can become familiar with the notation typically used in quantum computing (such as the bra and ket notation). Next, in Chapter 2 Framework of Quantum Mechanics, basic notions in quantum mechanics are reviewed starting with the fundamental postulates. The following chapters contain key applications of quantum computing including quantum key distribution (Chapter 3), quantum teleportation (Chapter 4), Grovers search algorithm (Chapter 7), and the groundbreaking factorization algorithm by Peter Shor (Chapter 8). The latter two algorithms are preceded with an introduction to quantum algorithms in Chapter 5 and a treatment of quantum integral transforms in Chapter 6. The first part of the book ends with the topic of decoherence (Chapter 9), which presents the main challenge in the realization of quantum computers, and with a treatment of quantum error correcting codes (Chapter 10) which are designed to circumvent the decoherence problem.

The second part Physical Realizations of Quantum Computing covers several implementations including the Nuclear Magnetic Resonance (NMR) quantum computer (Chapter 12), a low-temperature implementation using trapped ions (Chapter 13), superconducting quantum computing using Josephson Junction Qubits (Chapter 15), and others. As this part of the book is very far from my expertise I asked my colleague Roberto Ramos in Physics to comment. While he is overall very complimentary about the book, and has actually used it in a hybrid undergraduate/graduate course, he comments "The Josephson phase qubit is one of the front-runners in solid-state implementation of quantum computing but the authors focused more on the charge and flux qubit. They should expand their treatment of the phase qubit. At the very least, they should mention the paper that proposed the idea: *IEEE Trans. On Appl. Supercond.*, vol. 11, pp. 998–1001 (2001). Secondly, they discuss the idea of entanglement but focused more on the flux qubit. However, quantum entanglement between two solid state qubits was proposed and measured in the Josephson phase qubit before the flux qubit. It can also be conceptually easy to explain."

I would be very happy to use this book for an upper level undergraduate special topics course on quantum computing: the book is clearly written and very accessible, contains a good number of exercises, and best of all, solutions to many of the exercises! I suspect that the die-hards in the area will prefer the already classical book by Nielsen and Chuang, especially for a graduate course, but for a novice like me, this book provides excellent access to the material. Finally, it needs to be mentioned that the second part presents material not covered in earlier books.

ILAS AND OTHER LINEAR ALGEBRA NEWS

ILAS President/Vice President Annual Report: May 15, 2012

Respectfully submitted by Steve Kirkland, ILAS President, stephen.kirkland@nuim.ie
and Chi-Kwong Li, ILAS Vice-President, ckli@math.wm.edu

1. The following were elected in the ILAS 2011 elections to offices with terms that began on March 1, 2012, and will end on February 28, 2015:

- Board of Directors: Dale Olesky and Peter Šemrl
- Secretary/Treasurer: Leslie Hogben

The ILAS election in the fall/winter of 2011 marked the first time that ILAS has used electronic voting. By all accounts the electronic balloting went smoothly, and an increased number of people voted in the election.

The following continue in the ILAS offices to which they were previously elected:

- President: Steve Kirkland (term ends February 28, 2014)
- Vice President: Chi-Kwong Li (term ends February 28, 2013)

Board of Directors:

- Christian Mehl (term ends February 28, 2013)
- T-Y. Tam (term ends February 28, 2013)
- Françoise Tisseur (term ends February 28, 2014)
- David Watkins (term ends February 28, 2014)

Dario Bini and Shaun Fallat completed their three-year terms on the ILAS Board of Directors on February 29, 2012. We thank Dario and Shaun for their valuable contributions as Board members; their service to ILAS is most appreciated.

We also thank the members of the Nominating Committee - Wayne Barrett, Froilan Dopico, Helena Šmigoc, Jeff Stuart (chair) and Bit-Shun Tam - for their work on behalf of ILAS, and to all of the candidates that agreed to have their names stand for election.

2. The following ILAS-endorsed meetings have taken place since our last report:
   - International Workshop and Conference on Combinatorial Matrix Theory and Generalized Inverses of Matrices, Manipal University, Manipal, India, January 2–7 and 10–11, 2012.
   - Western Canada Linear Algebra Meeting, University of Lethbridge, Lethbridge, Canada, May 12–13, 2012.

3. ILAS has endorsed the following conferences of interest to ILAS members:
   - SIAM/ALA 2012 Conference, Valencia, Spain, June 18–22, 2012.
   - ALAMA 2012 (3rd Biennial Meeting of the Spanish Thematic Network on Linear Algebra, Matrix Analysis and Applications), Universidad Carlos III de Madrid, Leganés (Madrid), Spain, June 27–29, 2012.
   - Workshop on Parallel Matrix Algorithms and Applications, Birkbeck University, London, England, June 28–30, 2012.
   - 2012 Workshop on Matrices and Operators, Harbin Engineering University, China, July 13–16, 2012.
   - International Workshop on Matrices and Statistics, Będlewo, Poland, July 16–20, 2012.
   - The Eleventh Workshop on Numerical Ranges and Numerical Radii, National Sun Yat-sen University, Kaohsiung, Taiwan, August 9–12, 2012.
   - Haifa Matrix Theory Conference, Technion University, Haifa, Israel, November 12–15, 2012.

4. The following ILAS Lectures at non-ILAS conferences have been delivered since the last report.
   - Chi-Kwong Li, Western Canada Linear Algebra Meeting, University of Lethbridge, Lethbridge, Canada, May 12–13, 2012.

5. The following ILAS conferences are scheduled:
   - 18th ILAS Conference, June 3–7, 2013, Providence, Rhode Island, U.S.A. The organising committee consists of: Tom Bella (co-chair), Paul Van Dooren, Heike Faßbender, Bill Helton, Olga Holtz, Steve Kirkland, Vadim Olshevsky (co-chair), Victor Pan, Panos Psarrakos, Hugo Woerdeman, Ljiljana Cvetković, Chen Greif, Tin-Yau Tam.
   - 19th ILAS Conference, August 6–9, 2014, Suwon, Korea. The organising committee consists of: Nair Abreu, Tom Bella, Rajendra Bhatia, Richard A. Brualdi, Man-Duen Choi, Nick Higham, Leslie Hogben, Suk-Geun Hwang (chair), Steve Kirkland, Sang-Gu Lee, Helena Šmigoc, Fuzhen Zhang.

6. The *Electronic Journal of Linear Algebra* (ELA) is now in its 23rd volume. The Editor-in-Chief is Bryan Shader. A redesigned version of the ELA web site was launched in January 2012. ELA has also replaced its copyright transfer form for accepted papers with an author declaration and consent to publish form. The consent to publish form grants ELA the right to publish a paper, but leaves the copyright for that paper with the authors.

- ELA Volume 22 (2011) is complete, and contains 79 papers (1184 pages).
- ELA Volume 23 (2012) is in progress, and contains 29 papers (444 pages) so far.

7. *IMAGE* is the semi-annual newsletter for ILAS, and the Editor-in-Chief is Kevin N. Vander Meulen.

Issue 47 of *IMAGE*, was published on December 1, 2011, the last issue to be developed under the leadership of Jane Day. ILAS extends special thanks to Jane for her excellent work as Editor-in-Chief of *IMAGE* for 5 and a half years. Many members of ILAS are now receiving *IMAGE* in electronic form; the printed copies Vol. 46 and 47 were mailed to fewer than 50 ILAS members soon after the issues were posted on the ILAS website.

There have been some changes to the editorial staff of *IMAGE*. Fuzhen Zhang finished his term as editor for the Problems Section with Issue 47, and at the completion of Issue 48, Oskar Baksalary is ending his term as a contributing editor after serving *IMAGE* for twelve years. ILAS extends its appreciation to Fuzhen and Oskar for their dedicated efforts on behalf of *IMAGE*. Douglas Farenick of the University of Regina, Canada, will be the new contributing editor in charge of soliciting book news and reviews. Bojan Kuzma of the University of Ljubljana, Slovenia, is the new editor of the Problems Section. Michael Cavers of the University of Calgary, Canada, has joined *IMAGE* as a general contributing editor. He is helping to switch *IMAGE* over to a LaTeX format, including the preparation of style files for contributions.

8. ILAS-NET is a moderated newsletter for mathematicians worldwide, with a focus on linear algebra; it is managed by Sarah Carnochan Naqvi. As of May, 2012, there are 795 subscribers to ILAS-NET. In order to keep email addresses current, please send any updates to owner-ilas-net@math.technion.ac.il.

9. The ILAS Information Centre (IIC) is hosted at the University of Regina. Discussions are underway on the topics of securing a domain name for ILAS, and of having professional technical support and hosting for the IIC.

10. The 2012 Hans Schneider Prize has been awarded to Thomas Laffey of University College Dublin, Ireland. In recommending Tom for the award, the Prize Committee made special mention of Tom's contributions in the areas of integral matrices, simultaneous similarity, linear preservers, and the inverse eigenvalue problem for nonnegative matrices. The Prize Committee consisted of Shmuel Friedland, Steve Kirkland (ex-officio), Leiba Rodman (chair), Peter Šemrl, Bryan Shader and Valeria Simoncini. Thanks are due to the Prize Committee for its careful evaluation of the candidates, and to the nominators of all of the outstanding mathematicians who were under consideration for the Prize.

Send News for *IMAGE* Issue 49

IMAGE Issue 49 is due to appear online on December 1, 2012. Send your news for this issue to the appropriate editor by October 1, 2012: *IMAGE* seeks to publish all news of interest to the linear algebra community. Photos are always welcome, as well as suggestions for improving the newsletter. Send your items directly to the appropriate person:

- Problems and Solutions to Bojan Kuzma (bojan.kuzma@upr.si)
- History of Linear Algebra to Peter Šemrl (peter.semrl@fmf.uni-lj.si)
- Book Reviews to Douglas Farenick (Doug.Farenick@uregina.ca)
- Linear Algebra Education news to Steve Leon (sleon@umassd.edu)
- Announcements and Reports of Conferences, Workshops and Journals to Minerva Catral (catralm@xavier.edu)
- Interviews of Senior Linear Algebraists to Carlos Fonseca (cmf@mat.uc.pt)
- Advertisements to Amy Wehe (awehe@fitchburgstate.edu)

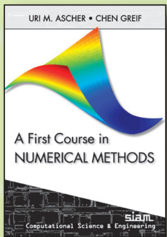
All other news to Kevin N. Vander Meulen (kvanderm@redeemer.ca). This includes:

- Feature articles such as surveys, interesting applications, new developments.
- Honors and awards, funding opportunities, job openings.
- Brief references to articles appearing elsewhere, of interest to the linear algebra community.
- Announcements of new books, software, websites.
- Transitions (new positions, obituaries).
- Letters to the Editor.
- Small items like clever proofs, historical tidbits, cartoons, etc.
- Suggestions for *IMAGE*

For past issues of *IMAGE*, please visit `http://www.ilasic.math.uregina.ca/iic/IMAGE/`.

We thank Oskar Maria Baksalary for 12 years of service to *IMAGE* as a contributing editor!

New Linear Algebra Titles *from* SIAM



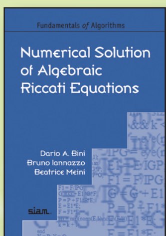
A First Course in Numerical Methods

Uri M. Ascher and Chen Greif

Computational Science and Engineering 7

Designed for students and researchers who seek practical knowledge of modern techniques in scientific computing, this book provides an in-depth treatment of fundamental issues and methods, the reasons behind the success and failure of numerical software, and fresh and easy-to-follow approaches and techniques.

2011 • xxii + 552 pages • Softcover • ISBN 978-0-898719-97-0
List Price $95.00 • SIAM Member Price $66.50 • CS07



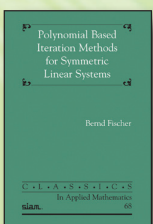
Numerical Solution of Algebraic Riccati Equations

Dario A. Bini, Bruno Iannazzo, and Beatrice Meini

Fundamentals of Algorithms 9

This concise and comprehensive treatment of the basic theory of algebraic Riccati equations describes the classical as well as the more advanced algorithms for their solution in a manner that is accessible to both practitioners and scholars. It is the first book in which nonsymmetric algebraic Riccati equations are treated in a clear and systematic way.

2012 • xii + 256 pages • Softcover • ISBN 978-1-611972-08-5
List Price $69.00 • SIAM Member Price $48.30 • FA09



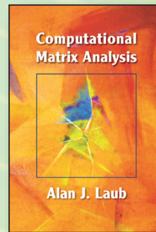
Polynomial Based Iteration Methods for Symmetric Linear Systems

Bernd Fischer

Classics in Applied Mathematics 68

This is the only textbook that treats iteration methods for symmetric linear systems from a polynomial point of view. This particular feature enables readers to understand the convergence behavior and subtle differences of the various schemes, which are useful tools for the design of powerful preconditioners.

2011 • 283 pages • Softcover • ISBN 978-1-611971-91-0
List Price $85.00 • SIAM Member Price $59.50 • CL68

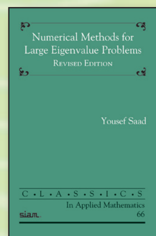


Computational Matrix Analysis

Alan J. Laub

Using an approach that its author calls "matrix analysis for grown-ups," this new textbook introduces fundamental concepts of numerical linear algebra and their application to solving certain numerical problems arising in state-space control and systems theory. It provides readers with a one-semester introduction to numerical linear algebra; an introduction to statistical condition estimation in book form for the first time; and an overview of certain computational problems in control and systems theory.

2012 • xiv + 154 pages • Softcover • ISBN 978-1-611972-20-7
List Price $47.00 • SIAM Member Price $32.90 • OT125



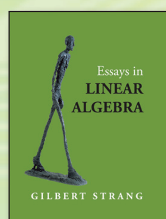
Numerical Methods for Large Eigenvalue Problems, *Revised Edition*

Yousef Saad

Classics in Applied Mathematics 66

This revised edition discusses numerical methods for computing eigenvalues and eigenvectors or large sparse matrices. It provides an in-depth view of the numerical methods that are applicable for solving matrix eigenvalue problems that arise in various engineering and scientific applications. Each chapter was updated by shortening or deleting outdated topics, adding topics of more recent interest, and adapting the Notes and References section.

2011 • xvi + 276 pages • Softcover • ISBN 978-1-61197-072-2
List Price $70.00 • Member Price $49.00 • CL66



Essays in Linear Algebra

Gilbert Strang

Wellesley-Cambridge Press 11

This book brings together 27 classic articles by Gilbert Strang on linear algebra, computational science, applied mathematics, and calculus, each introduced by a new essay. The essays present historical background, the current state, and unsolved problems.

2012 • x + 342 pages • Hardcover • ISBN 978-0-9802327-6-9
List Price $50.00 • SIAM Member Price $35.00 • WC11

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3/12

ILAS 2010-2011 Treasurer's Report

April 1, 2011 - March 31, 2012
By Leslie Hogben

Net Account Balance on March 31, 2011			
Vanguard (ST Fed Bond Fund Admiral 7373.167 Shares)		$79,187.81	
Checking Account - First Federal		$29,007.63	
Certificate of Deposit		$20,000.00	
			$128,195.44
General Fund		$64,178.81	
Conference Fund		$12,106.01	
ILAS/LAA Fund		$12,840.57	
Olga Taussky Todd/John Todd Fund		$11,142.14	
Frank Uhlig Education Fund		$4,944.34	
Hans Schneider Prize Fund		$22,983.57	
			$128,195.44
INCOME:			
Dues	$8,700.00		
CorporateSponsorship	$200.00		
General Fund	$3,235.00		
Conference Fund	$75.00		
LAMA for Speaker	$-		
Taussky-Todd Fund	$70.00		
Uhlig Education Fund	$ 5.00		
Schneider Prize Fund	$85.00		
Interest - First Federal	$61.14		
Loan at Conference (approx)	$700.00		
Interest on First Federal Certificate of Deposit	$331.40		
Vanguard - Dividend Income	$1,057.35		
- Short Term Capital Gains	$834.65		
- Long Term Capital Gains	$260.81		
Variances in the Market	$259.04		
Misc Income	$-		
Total Income		$15,874.39	
EXPENSES:			
Citi Bank - Credit Card Service Charges	$682.37		
American Express - Credit Card Service Charges	$27.71		
First Data - Deposit on credit card terminal	$459.29		
Speaker Expenses	$1,000.00		
Treasurer's Report Preparation	$532.50		
Treasurer's Assistant	$100.00		
Conference Expenses	$564.00		
Student/PostDoc Travel Support	$2,000.00		
Business License	$61.25		
IMAGE Costs	$857.57		
Wire Transfer Fees	$53.50		
Misc Expenses	$89.39		
Ballot Costs	$400.00		
Total Expenses		$6,827.58	
Net Account Balance on March 31, 2012			
Vanguard (ST Fed Bond Fund Admiral 7548.536 Shares)		$81,599.66	
Checking Account - First Federal		$35,311.19	
Certificate of Deposit		$20,331.40	
Accounts Payable		$(700.00)	
			$136,542.25
General Fund		$74,854.62	
Conference Fund		$9,617.01	
ILAS/LAA Fund		$12,840.57	
Olga Taussky Todd/John Todd Fund		$11,212.14	
Frank Uhlig Education Fund		$4,949.34	
Hans Schneider Prize Fund		$23,068.57	
			$136,542.25

IMAGE Problem Corner: Old Problems With Solutions

We present solutions to Problems 47-1 through 47-7. Solution to Problem 47-3 is partial, we invite readers to present a short solution for 3×3 matrices. Seven new problems are on the back cover; solutions are invited.

Editorial note: It was pointed out by Minghua LIN that *IMAGE* Problem 46.3 can have different interpretations as regards the underlying algebraic structure. The solution by Hans Joachim WERNER assumed the $\{0,1\}$-vectors are embedded in a field $\mathbb{Z}_2$. We later learned from the proposer Christopher HILLAR that, although not explicitly mentioned, linear independence of $\{0,1\}$-vectors over fields of characteristic zero was asked for. In this case, Solution 46.3 is still valid provided that the last displayed equation reads

$$p(m,n) \geq \frac{1}{2^{mn}} \prod_{j=1}^{m} (2^n - 2^{j-1})$$

if $1 \leq m \leq n$ and $p(m,n) = 0$ if $m > n$.

Problem 47-1: Idempotent Matrices

Proposed by Johanns DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let A, B be $n \times n$ idempotent matrices, I_n be the $n \times n$ identity matrix, and $X = \mathrm{Im}(A) + \mathrm{Im}(B)$. Show that

$$\dim\bigl(\mathrm{Ker}(I - AB)\bigr) = \dim\bigl(\mathrm{Im}(A) \cap \mathrm{Im}(B)\bigr) + \dim\bigl(X \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)\bigr).$$

Solution 47-1.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, U.S.A.*, eaherman@gmail.com

Since an idempotent matrix is the identity map on its range, we have

$$\mathrm{Im}(A) \cap \mathrm{Im}(B) \subseteq \mathrm{Ker}(I_n - AB).$$

Hence, we can choose a basis $\{u_1, \ldots, u_m\}$ for $\mathrm{Ker}(I_n - AB)$ such that $\{u_1, \ldots, u_k\}$ is a basis for $\mathrm{Im}(A) \cap \mathrm{Im}(B)$ for some $k \leq m$. We show that

$$\mathcal{B} = \{(I_n - B)u_{k+1}, \ldots, (I_n - B)u_m\}$$

is a basis for $X \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)$, which implies the desired dimension formula. Note that $\mathrm{Ker}(I_n - AB) \subseteq \mathrm{Im}(A)$. Hence, for all j with $1 \leq j \leq m$, $(I_n - B)u_j = u_j - Bu_j \in X$. Also, $u_j \in \mathrm{Ker}(I_n - AB)$ implies that $A(I_n - B)u_j = Au_j - ABu_j = u_j - u_j = 0$ and $B(I_n - B)u_j = Bu_j - B^2u_j = 0$, which proves that $(I_n - B)u_j \in X \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)$. Next, suppose $\sum_{j=k+1}^{m} c_j(I_n - B)u_j = 0$ for some scalars $c_{k+1}, \ldots, c_m$. Then $\sum_{j=k+1}^{m} c_j u_j = B\sum_{j=k+1}^{m} c_j u_j \in \mathrm{Im}(B) \cap \mathrm{Im}(A)$, and so $c_j = 0$ for $j = k+1, \ldots, m$. Thus, $\mathcal{B}$ is linearly independent. Finally, let $v \in X \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)$. Then $v = u + w$, where $Au = u$ and $Bw = w$, and

$$0 = Av = u + Aw, \qquad 0 = Bv = Bu + w.$$

Hence $(I_n - AB)u = u + Aw = 0$, and so there exist scalars $c_1, \ldots, c_m$ such that $u = \sum_{j=1}^{m} c_j u_j$. Therefore

$$v = u - Bu = (I_n - B)\sum_{j=1}^{m} c_j u_j = \sum_{j=k+1}^{m} c_j (I_n - B)u_j$$

which proves that $\mathcal{B}$ spans $X \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)$. ∎

Solution 47-1.2 by Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com
and Henry WOLKOWICZ, *University of Waterloo, Ontario, Canada*, hwolkowicz@uwaterloo.ca

Let us recall some well-known facts for matrices X, Y, Z from [1, p. 52] (we denote by $[X|\,Y]$ the block matrix with one block row, consisting of matrices X, Y):

1. $\mathrm{Im}[X|\,Y] = \mathrm{Im}(X) + \mathrm{Im}(Y), \qquad \mathrm{Ker}\begin{bmatrix} X \\ Y \end{bmatrix} = \mathrm{Ker}(X) \cap \mathrm{Ker}(Y).$
2. $\dim(\mathrm{Im}(XY)) + \dim(\mathrm{Im}(Y) \cap \mathrm{Ker}(X)) = \dim \mathrm{Im}(Y).$
3. Frobenius inequality: $\mathrm{rank}(XYZ) \geq \mathrm{rank}(XY) + \mathrm{rank}(YZ) - \mathrm{rank}(Y).$

Returning to our problem, we express each component in terms of rank.

$$\dim(\mathrm{Ker}(I-AB)) = n - \mathrm{rank}(I-AB).$$

$$\dim(\mathrm{Im}(A)\cap\mathrm{Im}(B)) = \dim(\mathrm{Im}(A)) + \dim(\mathrm{Im}(B)) - \dim(\mathrm{Im}(A)+\mathrm{Im}(B)) = \mathrm{rank}(A)+\mathrm{rank}(B)-\mathrm{rank}\big[A|\,B\big].$$

$$\begin{aligned}\dim(X\cap\mathrm{Ker}(A)\cap\mathrm{Ker}(B)) &= \dim\left(\mathrm{Im}\big[A|\,B\big]\cap\mathrm{Ker}\begin{bmatrix}A\\B\end{bmatrix}\right) = \dim\mathrm{Im}\big[A|\,B\big] - \dim\mathrm{Im}\left(\begin{bmatrix}A\\B\end{bmatrix}\big[A|\,B\big]\right)\\ &= \mathrm{rank}\big[A|\,B\big] - \mathrm{rank}\begin{bmatrix}A & AB\\ BA & B\end{bmatrix} = \mathrm{rank}\big[A|\,B\big] - \mathrm{rank}\begin{bmatrix}A-ABA & 0\\ BA & B\end{bmatrix}\\ &= \mathrm{rank}\big[A|\,B\big] - \mathrm{rank}(A-ABA) - \mathrm{rank}(B).\end{aligned}$$

Therefore, the desired inequality follows if we can show

$$n - \mathrm{rank}(I-AB) = \mathrm{rank}(A) - \mathrm{rank}(A-ABA).$$

Let $X = I-AB, Y = I$, and $Z = A$. By the Frobenius inequality, $n-\mathrm{rank}(I-AB) \geq \mathrm{rank}(A)-\mathrm{rank}(A-ABA)$. To prove the other direction, we let $X = I-A, Y = I-AB$ and $Z = A$. Again by the Frobenius inequality, $0 = \mathrm{rank}(XYZ) \geq \mathrm{rank}(XY) + \mathrm{rank}(YZ) - \mathrm{rank}(Y) = \mathrm{rank}(I-A) + \mathrm{rank}(A-ABA) - \mathrm{rank}(I-AB) = n - \mathrm{rank}(A) + \mathrm{rank}(A-ABA) - \mathrm{rank}(I-AB)$, i.e., $n - \mathrm{rank}(I-AB) \leq \mathrm{rank}(A) - \mathrm{rank}(A-ABA)$. ∎

Reference

[1] F. Zhang, *Matrix Theory: Basic Results and Techniques*, Second Edition, Springer, 2011.

Solution 47-1.3 by Leo LIVSHITS, *Colby College, Waterville, Maine, U.S.A.*, llivshi@colby.edu

Lemma 1. If A and B are idempotents then $\mathrm{Ker}(I-AB) = \mathrm{Ker}(I-ABA)$.

Proof. Since $A^2 = A$ is an identity on $\mathrm{Im}(A)$, $x \in \mathrm{Ker}(I-AB) \Leftrightarrow x = ABx \Leftrightarrow x = ABAx \Leftrightarrow x \in \mathrm{Ker}(I-ABA)$. ∎

Lemma 2. If A and B are idempotents then $\mathrm{Nullity}(I-AB) := \dim(\mathrm{Ker}(I-AB)) = \mathrm{Nullity}(A(I-B)A) - \mathrm{Nullity}(A)$.

Proof. From Lemma 1 we know that $\mathrm{Ker}(I-AB) = \mathrm{Ker}(I-ABA)$. Writing A and B as block matrices with respect to the direct sum decomposition $\mathrm{Im}(A) \dotplus \mathrm{Ker}(A)$ we get $A = \left(\begin{smallmatrix} I & 0\\ 0 & 0\end{smallmatrix}\right)$ and $B = \left(\begin{smallmatrix} T & P\\ Q & R\end{smallmatrix}\right)$, so that

$$I - ABA = \begin{pmatrix} I-T & 0\\ 0 & I\end{pmatrix}.$$

Hence, $\mathrm{Nullity}(I-AB) = \mathrm{Nullity}(I-ABA) = \mathrm{Nullity}(I-T) = \mathrm{Nullity}\left(\begin{smallmatrix} I-T & 0\\ 0 & 0\end{smallmatrix}\right) - \mathrm{Nullity}(A) = \mathrm{Nullity}(A(I-B)A) - \mathrm{Nullity}(A)$. ∎

A proof of the following standard fact is straightforward, and can be found on page 42 of [1].

Lemma 3. For any matrices C, D of appropriate size, $\mathrm{Nullity}(CD) = \dim\big(\mathrm{Im}(D)\cap\mathrm{Ker}(C)\big) + \mathrm{Nullity}(D)$. ∎

Lemma 4. If A and B are idempotents then

$$\mathrm{Im}\big((I-B)A\big)\cap\mathrm{Ker}(A) = \big(\mathrm{Im}(A)+\mathrm{Im}(B)\big)\cap\big(\mathrm{Ker}(A)\cap\mathrm{Ker}(B)\big). \tag{1}$$

Proof. Since $(I-B)A = A + B(-A)$ and $B(I-B)A = 0$, it follows that $\mathrm{Im}\big((I-B)A\big) \subseteq \big(\mathrm{Im}(A)+\mathrm{Im}(B)\big)\cap\mathrm{Ker}(B)$, and so the left hand side (LHS) of (1) is a subset of the right hand side (RHS). For the reverse inclusion, suppose z is an element of the RHS. To show that $z \in$ LHS we only need to show that

$$z \in \mathrm{Im}\left((I-B)A\right).$$

To that end, note that $z = Ax + By$ for some x, y, and since $Bz = 0$ we have

$$BAx + By = 0.$$

Thus $z = Ax - BAx = (I - B)Ax \in \mathrm{Im}((I - B)A)$ as required. ∎

We can now prove the required identity.

$$\begin{aligned}
\mathrm{Nullity}(I - AB) &= \mathrm{Nullity}(A(I - B)A) - \mathrm{Nullity}(A) \\
&= \Big(\dim\big(\mathrm{Im}\big((I - B)A\big) \cap \mathrm{Ker}(A)\big) + \mathrm{Nullity}\big((I - B)A\big)\Big) - \mathrm{Nullity}(A) \\
&= \dim\big(\mathrm{Im}\big((I - B)A\big) \cap \mathrm{Ker}(A)\big) + \Big(\dim\big(\mathrm{Im}(A) \cap \mathrm{Ker}(I - B)\big) + \mathrm{Nullity}(A)\Big) - \mathrm{Nullity}(A) \\
&= \dim\big(\mathrm{Im}(I - B)A \cap \mathrm{Ker}(A)\big) + \dim\big(\mathrm{Im}(A) \cap \mathrm{Im}(B)\big) \\
&= \dim\Big(\big(\mathrm{Im}(A) + \mathrm{Im}(B)\big) \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)\Big) + \dim\big(\mathrm{Im}(A) \cap \mathrm{Im}(B)\big),
\end{aligned}$$

where the first equality is by Lemma 2, the second and third follow from Lemma 3, and the last is a consequence of Lemma 4. ∎

Reference

[1] V. V. Prasolov, *Problems and theorems in linear algebra* (Translated from the Russian manuscript by D. A. Leĭtes), American Mathematical Society, 1994.

Solution 47-1.4 by Hans Joachim WERNER, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

If A is an idempotent $n \times n$ matrix over the field $\mathbb{F}$, i.e., if $A^2 = A$, then $\mathbb{F}^n = \mathrm{Im}(A) \oplus \mathrm{Ker}(A)$, i.e., the set of n-tuples over the field $\mathbb{F}$ is the direct sum of $\mathrm{Im}(A)$ and $\mathrm{Ker}(A)$. Thus, the linear spaces $\mathrm{Im}(A)$ and $\mathrm{Ker}(A)$ have only the origin in common. Moreover, if A is idempotent, then $\mathrm{Im}(I - A) = \mathrm{Ker}(A)$. Our solution to this problem is based on the following result.

Theorem. Let A and B be $n \times n$ idempotent matrices over the field $\mathbb{F}$. Then,

$$\mathrm{Ker}(I - AB) = \mathrm{Im}(A) \cap [\mathrm{Im}(B) \oplus [\mathrm{Ker}(A) \cap \mathrm{Ker}(B)]] . \tag{2}$$

Proof. Clearly, $x \in \mathrm{Ker}(I - AB) \Leftrightarrow (I - AB)x = 0 \Leftrightarrow x = ABx$. Hence, $x \in \mathrm{Im}(A)$ whenever $x \in \mathrm{Ker}(I - AB)$. In other words, $\mathrm{Ker}(I - AB) \subseteq \mathrm{Im}(A)$. So, in what follows, let $x \in \mathrm{Im}(A)$ be such that $x = ABx$. Then $x = Bx + y$, where $y = (I - B)x \in \mathrm{Ker}(B)$. Moreover, $Bx = ABx + (I - A)Bx = x + z$ with $z = (I - A)Bx \in \mathrm{Ker}(A)$. Consequently, $Bx = x - y = x + z$, and so $-y = z \in \mathrm{Ker}(B) \cap \mathrm{Ker}(A)$. Therefore, $x \in \mathrm{Im}(B) \oplus [\mathrm{Ker}(A) \cap \mathrm{Ker}(B)]$, so that $\mathrm{Ker}(I - AB) \subseteq \mathrm{Im}(A) \cap [\mathrm{Im}(B) \oplus [\mathrm{Ker}(A) \cap \mathrm{Ker}(B)]]$. Conversely, let $x \in \mathrm{Im}(A) \cap [\mathrm{Im}(B) \oplus [\mathrm{Ker}(A) \cap \mathrm{Ker}(B)]]$. Then $x = y + z$, where $x \in \mathrm{Im}(A)$, $y \in \mathrm{Im}(B)$, $z \in \mathrm{Ker}(A) \cap \mathrm{Ker}(B)$. Consequently, $ABx = AB(y + z) = ABy = Ay = A(x - z) = Ax = x$, and so $x \in \mathrm{Ker}(I - AB)$. ∎

From (2) it follows that if $x \in \mathrm{Ker}(I - AB)$ is nonzero, then either $x \in \mathrm{Im}(A) \cap \mathrm{Im}(B)$ or there exists a nonzero vector $y \in \mathrm{Im}(B)$ such that $x - y \in [\mathrm{Im}(A) + \mathrm{Im}(B)] \cap [\mathrm{Ker}(A) \cap \mathrm{Ker}(B)]$ is nonzero. Since these two cases are exclusive and exhaustive, we obtain, as claimed,

$$\dim(\mathrm{Ker}(I - AB)) = \dim(\mathrm{Im}(A) \cap \mathrm{Im}(B)) + \dim\big([\mathrm{Im}(A) + \mathrm{Im}(B)] \cap \mathrm{Ker}(A) \cap \mathrm{Ker}(B)\big). \quad ∎$$

Also solved by the proposer Johanns DE ANDRADE BEZERRA.

Problem 47-2: Another Characterization Of Normality

Proposed by Oskar Maria BAKSALARY, *Adam Mickiewicz University, Poznań, Poland,* obaksalary@gmail.com
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Let A be a square matrix with complex entries and let $A^\dagger$ and A^* denote the Moore–Penrose inverse and the conjugate transpose of A, respectively. Show that A is normal if and only if the column space of A^* is contained in that of A and $A^\dagger A^*$ is a contraction.

Solution 47-2.1 by Johanns DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let $A \in M_n(\mathbb{C})$ be a normal matrix. Then $\mathrm{Im}(A) = \mathrm{Im}(A)^*$, and so $\mathrm{Im}(A)^* \subseteq \mathrm{Im}(A)$. Moreover, if $\|\cdot\|$ is a spectral norm then $\|A^\dagger A^*\|^2 = \|(A^\dagger A^*)(A^\dagger A^*)^*\| = \|A^\dagger A^* A(A^\dagger)^*\| = \|(A^\dagger AA^*(A^\dagger)^*)\| = \|(A^\dagger A(A^\dagger A)^*)\| = 1$, because $A^\dagger A$ is a self-adjoint projection. Therefore, $A^\dagger A^*$ is a contraction.

Conversely, any square matrix $A \in M_n(\mathbb{C})$ can be written in the form

$$A = U \begin{pmatrix} \Sigma K & \Sigma L \\ 0 & 0 \end{pmatrix} U^*,$$

where U is a unitary matrix, $KK^* + LL^* = I_r$, $\Sigma = \operatorname{diag}(\sigma_1 I_{r_1}, \ldots, \sigma_t I_{r_t})$, $r_1 + \cdots + r_t = r = \operatorname{rank}(A)$, and $\sigma_1 > \sigma_2 > \cdots > \sigma_t > 0$ being the singular values of A (to see this, use polar decomposition: $A = UP$ and write $P = V(\Sigma \oplus 0)V^*$, Σ diagonal invertible, to get $A = (UV) \cdot (\Sigma \oplus 0)W \cdot (UV)^*$ with $W = V^*UV = \left(\begin{smallmatrix} K & L \\ W_{21} & W_{22} \end{smallmatrix}\right)$ unitary. See also Corollary 6 in [Hartwig and Spindelböck, Matrices for which A^* and $A^\dagger$ commute, *Linear and Multilinear Algebra* 14 (1984) 241–256]). Hence,

$$A^\dagger = U \begin{pmatrix} K^*\Sigma^{-1} & 0 \\ L^*\Sigma^{-1} & 0 \end{pmatrix} U^*, \qquad A^\dagger A = U \begin{pmatrix} K^*K & K^*L \\ L^*K & L^*L \end{pmatrix} U^*, \qquad AA^\dagger = U \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} U^*.$$

As $\operatorname{rank}(A) = \operatorname{rank}(A^*)$, the claim $\operatorname{Im}(A)^* \subseteq \operatorname{Im}(A)$ implies $\operatorname{Im}(A) = \operatorname{Im}(A^*)$, and so $A^\dagger A = AA^\dagger$, because they are self-adjoint projections onto $\operatorname{Im}(A)$ and onto $\operatorname{Im}(A^*)$, respectively. We conclude that $L = 0$ and $KK^* = I_r$. Thus, $A^* = U \left(\begin{smallmatrix} K^*\Sigma & 0 \\ 0 & 0 \end{smallmatrix}\right) U^*$, which implies $A^\dagger A^* = U \left(\begin{smallmatrix} K^*\Sigma^{-1}K^*\Sigma & 0 \\ 0 & 0 \end{smallmatrix}\right) U^*$, and then

$$\|A^\dagger A^*\| = \left\| U \begin{pmatrix} K^*\Sigma^{-1}K^*\Sigma & 0 \\ 0 & 0 \end{pmatrix} U^* \right\| = \left\| \begin{pmatrix} K^*\Sigma^{-1}K^*\Sigma & 0 \\ 0 & 0 \end{pmatrix} \right\|.$$

Since $A^\dagger A^*$ is a contraction, the spectral radius, $\rho(K^*\Sigma^{-1}K^*\Sigma(K^*\Sigma^{-1}K^*\Sigma)^*) = \rho((A^\dagger A^*)(A^\dagger A^*)^*) = \|(A^\dagger A^*)(A^\dagger A^*)^*\| = \|(A^\dagger A^*)\|^2 \leq 1$. Also, $K \in M_r(\mathbb{C})$ is unitary, so the matrices $K^*\Sigma^{-1}K^*\Sigma(K^*\Sigma^{-1}K^*\Sigma)^* = K^*\Sigma^{-1}K^*\Sigma^2 K\Sigma^{-1}K$ and $\Sigma^{-1}K^*\Sigma^2K\Sigma^{-1} = X^*X$ have the same eigenvalues, where $X := \Sigma K \Sigma^{-1}$. Hence, $\rho(X^*X) \leq 1$. Let $\lambda_i(X)$, $i = 1, \ldots, r$ be the eigenvalues of X. As K is unitary, $|\lambda_i(X)| = 1$, so by Schur's inequality [1, Theorem 34.1.1], and as $\rho(X^*X) \leq 1$ implies $\lambda_i(X^*X) \leq 1$,

$$r = \sum_{i=1}^{r} |\lambda_i(X)|^2 \leq \operatorname{trace}(X^*X) = \sum_{i=1}^{r} \lambda_i(X^*X) \leq 1 + \cdots + 1 = r.$$

So, all the eigenvalues of X^*X are equal to 1, hence $X^*X = I_r$. Thus, $\Sigma^{-1}K^*\Sigma = X^* = X^{-1} = \Sigma K^{-1}\Sigma^{-1} = \Sigma K^* \Sigma^{-1}$, and then $\Sigma^2 K^* = K^*\Sigma^2$. Now, notice that $AA^* = U \left(\begin{smallmatrix} \Sigma^2 & 0 \\ 0 & 0 \end{smallmatrix}\right) U^* = U \left(\begin{smallmatrix} K^*\Sigma^2 K & 0 \\ 0 & 0 \end{smallmatrix}\right) U^* = A^*A$ and therefore A is normal. ∎

Reference

[1] V.V. Prasolov, *Problems and Theorems in Linear Algebra*, Translation of Mathematical Monographs. Volume 134, U.S.A., 1994.

Solution 47-2.2 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, U.S.A.*, eaherman@gmail.com

Let $A = U \left[\begin{smallmatrix} D & 0 \\ 0 & 0 \end{smallmatrix}\right] V^*$ with $D = \operatorname{diag}(\sigma_1, \ldots, \sigma_r)$ be the SVD of A, where U and V are unitary and $\sigma_1 \geq \cdots \geq \sigma_r > 0$. Then

$$A^* = V \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} U^*, \qquad A^\dagger = V \begin{bmatrix} D^{-1} & 0 \\ 0 & 0 \end{bmatrix} U^*.$$

Define the unitary matrix $W = U^*V$, and let the partition of $W = \left[\begin{smallmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{smallmatrix}\right]$ be compatible with $\left[\begin{smallmatrix} D & 0 \\ 0 & 0 \end{smallmatrix}\right]$. Then

$$AA^* = A^*A \iff U \begin{bmatrix} D^2 & 0 \\ 0 & 0 \end{bmatrix} U^* = V \begin{bmatrix} D^2 & 0 \\ 0 & 0 \end{bmatrix} V^* \iff \begin{bmatrix} D^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} D^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\iff D^2 W_{11} = W_{11} D^2,\ W_{21} = 0,\ W_{12} = 0.$$

Hence, since D is diagonal,

$$A \text{ is normal} \iff DW_{11} = W_{11}D,\ \ W_{21} = O,\ \ W_{12} = O. \tag{3}$$

Also, $A^\dagger A^* = V \left[\begin{smallmatrix} D^{-1} & 0 \\ 0 & 0 \end{smallmatrix}\right] \left[\begin{smallmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{smallmatrix}\right] \left[\begin{smallmatrix} D & 0 \\ 0 & 0 \end{smallmatrix}\right] U^* = V \left[\begin{smallmatrix} D^{-1}W_{11}D & 0 \\ 0 & 0 \end{smallmatrix}\right] U^*$. Hence

$$A^\dagger A^* \text{ is a contraction} \iff D^{-1}W_{11}D \text{ is a contraction.} \tag{4}$$

Furthermore, if we let $\mathcal{Z} = \operatorname{Im}\left[\begin{smallmatrix} D & 0 \\ 0 & 0 \end{smallmatrix}\right]$, which is $\mathbb{C}^r \oplus 0_{n-r}$, then $\operatorname{Im}(A^*) = V(\mathcal{Z})$ and $\operatorname{Im}(A) = U(\mathcal{Z})$. Hence $\operatorname{Im}(A^*) \subseteq \operatorname{Im}(A) \iff W(\mathcal{Z}) \subseteq \mathcal{Z} \iff W_{21} = 0$. Thus, since W is unitary,

$$\operatorname{Im}(A^*) \subseteq \operatorname{Im}(A) \iff W_{21} = 0,\ \ W_{12} = 0. \tag{5}$$

Therefore, if A is normal, $D^{-1}W_{11}D = W_{11}$ by (3). Since W_{11} is unitary, $A^\dagger A^*$ is a contraction by (4). Also, by (3) and (5), $\operatorname{Im}(A^*) \subseteq \operatorname{Im}(A)$.

Finally, suppose $A^\dagger A^*$ is a contraction and $\operatorname{Im}(A^*) \subseteq \operatorname{Im}(A)$. By (3), (4), and (5), it remains to show that if $D^{-1}W_{11}D$ is a contraction, where W_{11} is unitary and $D = \operatorname{diag}(\sigma_1, \ldots, \sigma_r)$ with $\sigma_1 \geq \cdots \geq \sigma_r > 0$, then $D^{-1}W_{11}D = W_{11}$. If D is a multiple of the identity matrix, the conclusion is clear. Otherwise, there exists k with $1 \leq k < r$ such that $\sigma_1 = \cdots = \sigma_k > \sigma_{k+1}$. Let $W_{11} = [w_{ij}]_{1 \leq i,j \leq r}$ and let $\mathbf{e}_j$ denote the jth standard unit vector in $\mathbb{C}^r$. Then for all j with $k+1 \leq j \leq r$,

$$\|D^{-1}W_{11}D\mathbf{e}_j\|^2 = |w_{1j}|^2 + \cdots + |w_{kj}|^2 + \frac{\sigma_1^2}{\sigma_{k+1}^2}|w_{k+1,j}|^2 + \cdots \frac{\sigma_1^2}{\sigma_r^2}|w_{rj}|^2.$$

Since $\|D^{-1}W_{11}D\mathbf{e}_j\| \leq 1$, $|w_{1j}|^2 + \cdots |w_{rj}|^2 = 1$, and $\sigma_1/\sigma_j > 1$ for $j = k+1, \ldots, r$, we see that $w_{k+1,j} = \cdots = w_{rj} = 0$. We may therefore write

$$W_{11} = \begin{bmatrix} X & Y \\ 0 & Z \end{bmatrix}$$

for some $k \times k$ matrix X. Since W_{11} is unitary, $Y = 0$ and Z is unitary. Also, X commutes with the corresponding block of D, since that block is a multiple of the identity matrix. By a straightforward induction argument, it follows that W_{11} is block diagonal, where each block commutes with the corresponding block of D; hence, W_{11} and D commute. ∎

Solution 47-2.3 by Roger A. HORN, *University of Utah, Salt Lake City, Utah, U.S.A.*, rhorn@math.utah.edu

Let $A \in M_n$ and suppose that $1 \leq \operatorname{rank}(A) = r \leq n$. If A is normal, then $A = U\Lambda U^*$, in which $U \in M_{n,r}$ has orthonormal columns and $\Lambda \in M_r$ is nonsingular and diagonal. Then $\operatorname{Im}(A^*) = \operatorname{Im}(A)$ and $A^\dagger A^* = UDU^*$, in which $D = \Lambda^{-1}\bar{\Lambda}$ is diagonal and unitary. Consequently, $A^\dagger A^*$ has r singular values equal to one and $n - r$ singular values equal to zero, so it is a contraction.

Conversely, let $A = V\Sigma W^*$ be a thin singular value decomposition, in which $V, W \in M_{n,r}$ have orthonormal columns and $\Sigma \in M_r$ is positive diagonal. If $\operatorname{Im}(A^*) \subseteq \operatorname{Im}(A)$ then $\operatorname{Im}(A^*) = \operatorname{Im}(A)$ (both subspaces have dimension r) and there is a unitary $U \in M_r$ such that $W = VU$. We have $A = V\Sigma U^* V^*$, which is normal if and only if ΣU^* is normal, if and only if $\Sigma U^* = U^*\Sigma$ (this is a polar factorization of a matrix, which is normal if and only if the factors commute). If $A^\dagger A^* = V(U\Sigma^{-1}U\Sigma)V^*$ is a contraction, then so is $C = U\Sigma^{-1}U\Sigma$. But $1 = |\det U\Sigma^{-1}U\Sigma| = |\det C| = \sigma_1(C)\cdots\sigma_r(C) \leq \sigma_1(C) \leq 1$ (recall that $\sigma_1(C) = \|C\| \leq 1$ since C is contraction), so *all* of the singular values of C are equal to one, which means that it is unitary. Thus, $I = CC^* = U\Sigma^{-1}U\Sigma^2U^*\Sigma^{-1}U^*$, so $U^*\Sigma^2 = \Sigma^2U^*$, which implies that $U^*\Sigma = \Sigma U^*$ (Σ is a polynomial in Σ^2). ∎

Solution 47-2.4 by Leo LIVSHITS, *Colby College, Waterville, Maine, U.S.A.*, llivshi@colby.edu

If A is normal then $\|Aw\| = \|A^*w\|$ for all w and so $\operatorname{Ker}(A) = \operatorname{Ker}(A^*)$ and $\operatorname{Im}(A) = \operatorname{Im}(A^*)$. Thus

$$\|A^\dagger A^* Aw\| = \|A^\dagger AA^*w\| = \|A^*w\| = \|Aw\|,$$

because $A^\dagger A$ is the orthogonal projection onto $\operatorname{Im}(A^*)$. Hence $A^\dagger A^*$ is isometric on the range (i.e., column space) of A, and since it vanishes on $\operatorname{Ker}(A^*)$ which is the orthogonal complement of the range of A, $A^\dagger A^*$ is a contraction (of norm 1).

To demonstrate the converse implication, suppose that the range of A^* is contained in the range of A and that $A^\dagger A^*$ is a contraction. Since A and A^* must have equal rank, it follows that A and A^* have equal ranges in our case, and consequently equal kernels as well. Since $(A^\dagger)^* A^* = (AA^\dagger)^*$ is the orthogonal projection onto the range of A, we have

$$\|A(Aw)\| = \|A(A^\dagger)^* A^* Aw\| \leq \|A\left(A^\dagger\right)^*\| \cdot \|A^* Aw\| = \|A^\dagger A^*\| \cdot \|A^* Aw\| \leq \|A^* Aw\|,$$

which shows that $\|Az\| \leq \|A^* z\|$ for every z in the range of A. Since $\|Az\| = \|A^* z\| = 0$ for any $z \in \operatorname{Ker}(A) = \operatorname{Ker}(A^*) = (\operatorname{Im}(A))^\perp$, we see that

$$\|Az\| \leq \|A^* z\|$$

for every z, which is equivalent to the statement that $0 \leq A^* A \leq AA^*$. Yet

$$0 = \operatorname{Trace}\left(AA^* - A^* A\right)$$

so that $AA^* - A^* A$ is a positive semi-definite matrix with trace zero, which implies it is the zero matrix, and hence A is normal. ∎

Solution 47-2.5 by Hans Joachim WERNER, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

A matrix $A \in \mathbb{C}^{n \times n}$ is called an EP-matrix if $\mathcal{R}(A) = \mathcal{R}(A^*)$, where $\mathcal{R}(\cdot)$ indicates the range (column space). Then, $\mathcal{R}(A^*) \subseteq \mathcal{R}(A) \Leftrightarrow \mathcal{R}(A^*) = \mathcal{R}(A)$. Since $AA^\dagger$ is the orthogonal projection onto $\mathcal{R}(A)$, A is an EP-matrix $\Leftrightarrow AA^\dagger A^* = A^*$. For the following it is also pertinent to mention that A is an EP-matrix if and only if $AA^\dagger = A^\dagger A$. Next, recall that a complex matrix $C \in \mathbb{C}^{n \times n}$ is a contraction if and only if $I_n - CC^*$ is a positive semidefinite Hermitian matrix (see [F. Zhang, *Matrix Theory,* Springer, New York, 1999, p. 144]); it follows that $B(I - CC^*)B^*$ is also positive semidefinite and Hermitian, irrespective of the choice of $B \in \mathbb{C}^{n \times n}$. With these observations in mind we now prove the following characterization.

Theorem. Let $A \in \mathbb{C}^{n \times n}$, and let $C = A^\dagger A^*$. Then A is normal if and only if A is EP and C is a contraction.

Proof of Sufficiency: If A is an EP-matrix and C is a contraction, then $A^* = AA^\dagger A^* = AC$, and $I_n - CC^*$ is positive semidefinite. Consequently, $A(I - CC^*)A^* = AA^* - A^* A$ is also positive semidefinite. Since $\operatorname{trace}(AA^* - A^* A) = 0$, necessarily $AA^* - A^* A = 0$, i.e., A is normal (see Problem 2 on p. 163 in [F. Zhang, *Matrix Theory,* Springer, New York, 1999]). ∎

Proof of Necessity: Now, let $A \in \mathbb{C}^{n \times n}$ be normal. Again, let $C = A^\dagger A^*$. Then, $\mathcal{R}(A) = \mathcal{R}(A^*)$, i.e., A is an EP-matrix. Moreover, $CC^* = A^\dagger A^* A(A^\dagger)^* = A^\dagger AA^*(A^\dagger)^* = (A^\dagger A)(A^\dagger A)^* = A^\dagger A$, i.e., CC^* coincides with the orthogonal projection onto $\mathcal{R}(A^*)$. Therefore, $I - CC^* = I - A^\dagger A$. Since $I - A^\dagger A$ is positive semidefinite, C is a contraction. ∎

Also solved by the proposers Oskar Maria BAKSALARY *and Götz* TRENKLER.

Problem 47-3: Matrices That Commute With Their Conjugate And Transpose
Proposed by Geoffrey GOODSON, *Towson University, MD, U.S.A.*, ggoodson@towson.edu

It is known that if $A \in M_n$ is normal ($AA^* = A^* A$), then $A\bar{A} = \bar{A}A$ if and only if $AA^T = A^T A$. This leads to the question: do both $A\bar{A} = \bar{A}A$ and $AA^T = A^T A$ imply that A is normal? One can give an example to show that this is false when $n = 4$. In fact, any matrix of the form $\left[\begin{smallmatrix} I_{ab} & I_{cd} \\ 0 & I_{ab} \end{smallmatrix}\right]$, where $I_{ab} = \left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right]$, $a, b, c, d \in \mathbb{C}$, $c^2 + d^2 = 0$, c and d are not both zero, has this property. Show that if $A \in M_n$, $n = 2$ or $n = 3$, then $A\bar{A} = \bar{A}A$ and $AA^T = A^T A$ do imply that A is normal.

Solution 47-3.1 by the proposer Geoffrey GOODSON, *Towson University, MD, U.S.A.*, ggoodson@towson.edu

Solution for the case $n = 2$.
Lemma. If $A \in M_2(\mathbb{C})$ with $AA^T = A^T A$, then A is either symmetric or of the form $\left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right]$, $a, b \in \mathbb{C}$.

Proof. Suppose that $A = \left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right]$, $a, b, c, d \in \mathbb{C}$, with $AA^T = A^T A$, then $\left[\begin{smallmatrix} a^2+b^2 & ac+bd \\ ac+bd & c^2+d^2 \end{smallmatrix}\right] = \left[\begin{smallmatrix} a^2+c^2 & ab+cd \\ ab+dc & b^2+d^2 \end{smallmatrix}\right]$. Hence $b^2 = c^2$ and $ab + cd = ac + bd$. If $b = c$, then A is symmetric. If $b = -c$, then $ab - bd = -ab + bd$ or equivalently $ab = bd$. If $b = 0$, then A is symmetric. If $b \neq 0$ then $a = d$ and $A = \left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right]$. ∎

Assume now $A \in M_2(\mathbb{C})$ satisfies $AA^T = A^T A$ and $A\overline{A} = \overline{A}A$. From the Lemma, we have two cases. If $A = \left[\begin{smallmatrix} a & b \\ -b & a \end{smallmatrix}\right]$, $a, b \in \mathbb{C}$, then A is normal. On the other hand, if A is symmetric with $A\overline{A} = \overline{A}A$, then since $A^* = \overline{A}$ in this case, we must have $AA^* = A^* A$, so A is normal. ∎
Editorial note: The proposer has also given a lengthy solution for $n = 3$.

Solution 47-3.2 by Hans Joachim WERNER, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

Let A^t, A^*, and $\bar{A}$ denote the transpose, the conjugate transpose, and the (elementwise complex) conjugate, respectively, of the square complex matrix A. According to Schur's triangularization theorem, every square complex matrix is unitarily similar to an upper-triangular matrix. That is, for each $A \in \mathbb{C}^{n\times n}$, there exist a unitary matrix U (i.e., $U^*U = I_n$) and an upper-triangular matrix T such that $U^*AU = T$.

Let $A \in \mathbb{C}^{2\times 2}$ be such that $A\bar{A} = \bar{A}A$ and $AA^t = A^tA$. Then $A = UTU^*$ for some unitary matrix $U \in \mathbb{C}^{2\times 2}$ and some upper-triangular matrix $T = \left(\begin{smallmatrix} \lambda_1 & \xi \\ 0 & \lambda_2 \end{smallmatrix}\right)$, where λ_1 and λ_2 are the not necessarily distinct eigenvalues of A. Then

$$\bar{A} = \bar{U}\bar{T}(\overline{U^*}) = \bar{U}\begin{pmatrix} \bar{\lambda}_1 & \bar{\xi} \\ 0 & \bar{\lambda}_2 \end{pmatrix} U^t \qquad \text{and} \qquad A^t = (U^*)^t T^t U^t = \bar{U}\begin{pmatrix} \lambda_1 & 0 \\ \xi & \lambda_2 \end{pmatrix} U^t.$$

Therefore,

$$A\bar{A} = \bar{A}A \quad \Leftrightarrow \quad UTU^*\bar{U}\bar{T}U^t = \bar{U}\bar{T}U^tUTU^* \quad \Leftrightarrow \quad T\cdot(U^tU)^{-1}\bar{T}U^tU = (U^tU)^{-1}\bar{T}U^tU\cdot T, \tag{6}$$

and

$$AA^t = A^tA \quad \Leftrightarrow \quad UTU^*(U^*)^tT^tU^t = (U^*)^tT^tU^tUTU^* \quad \Leftrightarrow \quad T\cdot(U^tU)^{-1}T^tU^tU = (U^tU)^{-1}T^tU^tU\cdot T, \tag{7}$$

where $(U^tU)^{-1} = (U^tU)^* = \bar{U}^t\bar{U}$. From (6) and (7) we obtain

$$(A\bar{A} = \bar{A}A, \quad AA^t = A^tA) \quad \Rightarrow \quad TK = KT \quad \text{with} \quad K := (U^tU)^{-1}(\bar{T} + T^t)U^tU.$$

Next, observe that $(\bar{T} + T^t)^* = \bar{T} + T^t$, i.e., $\bar{T} + T^t$ is Hermitian. As $(U^tU)^{-1} = (U^tU)^* = \bar{U}^t\bar{U}$, U^tU is a (symmetric) unitary matrix, and so it follows that $K := (U^tU)^{-1}(\bar{T} + T^t)U^tU$ is Hermitian. Hence K is a 2×2 matrix of the form $K = \left(\begin{smallmatrix} k_1 & \omega \\ \bar{\omega} & k_2 \end{smallmatrix}\right)$. Check that

$$TK = KT \Leftrightarrow \begin{pmatrix} \lambda_1 k_1 + \xi\bar{\omega} & \lambda_1\omega + \xi k_2 \\ \lambda_2\bar{\omega} & \lambda_2 k_2 \end{pmatrix} = \begin{pmatrix} \lambda_1 k_1 & \xi k_1 + \lambda_2\omega \\ \lambda_1\bar{\omega} & \xi\bar{\omega} + \lambda_2 k_2 \end{pmatrix} \Leftrightarrow \xi\bar{\omega} = 0,\ \bar{\omega}(\lambda_2 - \lambda_1) = 0,\ \xi(k_2 - k_1) = \omega(\lambda_2 - \lambda_1).$$

We have $\xi\bar{\omega} = 0$ if and only if $\xi = 0$ or $\bar{\omega} = 0$. If $\xi = 0$, then T is diagonal, in which case A is unitarily similar to the diagonal matrix T. If $\bar{\omega} = 0$, then $\omega = 0$, so $k_2 = k_1$ or $\xi = 0$. If $\omega = 0$ and $k_2 = k_1 =: c$, then $K = cI_2$, and so $\bar{T} + T^t = U^tU(cI_2)(U^tU)^{-1} = cI_2$, in which case $\bar{T} + T^t$ is diagonal. This tells us that again $\xi = 0$, and so T is diagonal. Since a square complex matrix is normal if and only if it is unitarily similar to a diagonal matrix (see [C.D. Meyer, *Matrix Analysis and Applied Linear Algebra*, SIAM, Philadelphia, 2000, p. 547]), our solution is complete. ∎

Problem 47-4: Completely Invertible Matrices

Proposed by Christopher HILLAR, *Mathematical Sciences Research Institute, Berkeley, California, U.S.A.*, chillar@msri.org

Given an $n \times n$ complex matrix A, its *field of values* $\mathcal{F}(A)$ is the set (containing the eigenvalues of A) in the complex plane:

$$\mathcal{F}(A) := \{x^*Ax : x^*x = 1\} \subseteq \mathbb{C}.$$

(Here, x^* for a column vector x is its conjugate transpose $\bar{x}^\top$). Show that $0 \notin \mathcal{F}(A)$ if and only if $0 \notin \mathcal{F}(A^{-1})$.

Solution 47-4.1 by Johanns DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

It suffices to show that $0 \in \mathcal{F}(A)$ if and only if $0 \in \mathcal{F}(A^{-1})$. Let $\mathcal{F}'(A)$ be the angular field of values of A, that is, $\mathcal{F}'(A) = \{x^*Ax : x \in \mathbb{C}^n, x \neq 0\}$. Obviously, $0 \in \mathcal{F}(A)$ if and only if $0 \in \mathcal{F}'(A)$. By [1, Theorem 1.7.5], $\mathcal{F}'(A^{-1}) = \overline{\mathcal{F}'(A)}$, hence $0 \in \mathcal{F}(A) \Leftrightarrow 0 \in \mathcal{F}'(A) \Leftrightarrow 0 \in \mathcal{F}'(A^{-1}) \Leftrightarrow 0 \in \mathcal{F}(A^{-1})$. ∎

Reference

[1] R.A. Horn and C.R. Johnson, *Topics in Matrix Analysis*, 1st edition, Cambridge University Press, New York, 1991.

Solution 47-4.2 by Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com
and Henry WOLKOWICZ, *University of Waterloo, Ontario, Canada*, hwolkowicz@uwaterloo.ca

It suffices to show the "only if" part, since the "if" part can be shown by taking $B = A^{-1}$ (if $0 \notin \mathcal{F}(B) = \mathcal{F}(A^{-1})$, then $0 \notin \mathcal{F}(B^{-1}) = \mathcal{F}(A)$). It is well known that $\mathcal{F}(A)$ is a compact convex set and the spectrum of A is contained in $\mathcal{F}(A)$. Thus if $0 \notin \mathcal{F}(A)$, then A is invertible. Define the numerical radius and the Crawford number of A by

$$w(A) = \sup\{|z| : z \in \mathcal{F}(A)\} \quad \text{and} \quad c(A) = \inf\{|z| : z \in \mathcal{F}(A)\}.$$

Since $0 \notin \mathcal{F}(A)$, we have then $0 < c(A) \le w(A)$. The conclusion follows from the following result (see [1, Theorem 3.6.]):

Theorem. Suppose A is invertible. Then $c(A^{-1}) \ge c(A)/w(A)^2$. ■

Reference

[1] M.-D. Choi and C.-K. Li, Numerical ranges of the powers of an operator, *J. Math. Anal. Appl.* 365 (2010) 458–466.

Solution 47-4.3 by Edward POON, *Embry-Riddle Aeronautical University, Prescott, AZ, U.S.A.,* edward.poon@erau.edu

$$\begin{aligned} 0 \in \mathcal{F}(A) &\iff 0 = x^*Ax = (A^{-1}y)^*A(A^{-1}y) \quad \text{for some unit vector } x = A^{-1}y \\ &\iff 0 = 0^* = (z^*(A^{-1})^*z)^* = z^*A^{-1}z \quad \text{where } z = y/\|y\| \\ &\iff 0 \in \mathcal{F}(A^{-1}). \end{aligned}$$

Hence $0 \notin \mathcal{F}(A)$ if and only if $0 \notin \mathcal{F}(A^{-1})$. ■

Solution 47-4.4 by Denis SERRE, *École Normale Supérieure de Lyon, France*, denis.serre@umpa.ens-lyon.fr

Because $0 \notin \mathcal{F}(A)$, A is invertible. The Toepliz–Hausdorff theorem tells us that $\mathcal{F}(A)$ is compact and convex. By the Hahn–Banach theorem, it is separated from 0 by a line. Up to a rotation in the plane (i.e., multiplying A by suitable unimodular number), we may suppose that this line is vertical; in other words, there exists $\epsilon > 0$ such that the real part, $\Re(x^*Ax) \ge \epsilon$ whenever $\|x\| = 1$. This amounts to saying that $\Re(x^*Ax) \ge \epsilon\|x\|^2$, or that $A + A^* \ge 2\epsilon I_n$. A congruence then gives

$$(A^{-1})^* + A^{-1} \ge 2\epsilon(A^{-1})^*A^{-1}.$$

The matrix $(A^{-1})^*A^{-1} = (AA^*)^{-1}$ is Hermitian positive definite, hence is larger than ηI_n for some $\eta > 0$. We obtain $(A^{-1})^* + A^{-1} \ge 2\epsilon\eta I_n$. Hence $\mathcal{F}(A^{-1})$ is contained in the half-plane $\Re z \ge \epsilon\eta$. In particular, $0 \notin \mathcal{F}(A^{-1})$. ■

Solution 47-4.5 by Jeffrey STUART, *Pacific Lutheran University, Tacoma, WA 98447 U.S.A.*, jeffrey.stuart@plu.edu

All vectors are column vectors in $\mathbb{C}^n$. Since $(\mathbf{x}^*A\mathbf{x})^* = \mathbf{x}^*A^*\mathbf{x}$ for all vectors $\mathbf{x}$, $0 \in \mathcal{F}(A)$ if and only if $0 \in \mathcal{F}(A^*)$. If 0 is an eigenvalue for A with eigenvector $\mathbf{v}$, then $\mathbf{v}$ can be scaled so that $\mathbf{v}^*\mathbf{v} = 1$, and hence, $0 \in \mathcal{F}(A)$.

Suppose that $0 \notin \mathcal{F}(A)$ and assume $0 \in \mathcal{F}(A^{-1})$. Then there is a vector $\mathbf{w}$ with $\mathbf{w}^*\mathbf{w} = 1$ and $\mathbf{w}^*A^{-1}\mathbf{w} = 0$. Let $\mathbf{y} = A^{-1}\mathbf{w} \ne \mathbf{0}$. Then

$$0 = \mathbf{w}^*A^{-1}\mathbf{w} = \mathbf{y}^*A^*A^{-1}A\mathbf{y} = \mathbf{y}^*A^*\mathbf{y},$$

so $0 \in \mathcal{F}(A^*) = \mathcal{F}(A)$, a contradiction. ■

Also solved by Eugene HERMAN, *Roger A.* HORN, *Leo* LIVSHITS, *and Hans Joachim* WERNER.

Editorial note: The same problem also appeared in Amer. Math. Monthly (Problem 11619 in the January 2012 issue).

Problem 47-5: Diagonalizable Matrix With Real Eigenvalues

Proposed by Roger A. HORN, *University of Utah, Salt Lake City, Utah, U.S.A.*, rhorn@math.utah.edu

Let $A_1, \ldots, A_N \in M_n(\mathbb{C})$ be Hermitian and positive definite and let $B = (\sum_{i=1}^N A_i)(\sum_{i=1}^N A_i^{-1})$. Show that B is diagonalizable and has real eigenvalues, and the smallest eigenvalue of B is not less than N^2.

Solution 47-5.1 by Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com
and Henry WOLKOWICZ, *University of Waterloo, Ontario, Canada*, hwolkowicz@uwaterloo.ca

Let $R = \sum_{i=1}^N A_i$, $S = \sum_{i=1}^N A_i^{-1}$. Then $R^{-1/2}BR^{1/2} = R^{-1/2}RSR^{1/2} = R^{1/2}SR^{1/2}$, where $R^{1/2}$ means the unique Hermitian positive semidefinite square root of R. Since $R^{1/2}SR^{1/2}$ is positive definite, we conclude that B is diagonalizable and has real eigenvalues. Let $Q = \frac{1}{\sqrt{N}}[I_n \mid I_n \mid \cdots \mid I_n]$ be a block matrix with N blocks of I_n in a single row, and let $A = A_1 \oplus \cdots \oplus A_N$ be a block-diagonal matrix. Then, $\frac{1}{N^2}B = QAQ^*QA^{-1}Q^*$. Note that $QQ^* = I_n$, hence we have (see [1, Eq. (6)])

$$QA^{-1}Q^* \ge (QAQ^*)^{-1}.$$

Therefore, since $\mathrm{spectrum}\,(XY) = \mathrm{spectrum}\,(YX)$ implies $\mathrm{spectrum}\,(XY) = \mathrm{spectrum}\,(X^{1/2}YX^{1/2})$, by Weyl's Monotonicity Theorem,

$$\frac{1}{N^2}\lambda_{\min}(B) = \lambda_{\min}((QAQ^*)^{1/2}QA^{-1}Q^*(QAQ^*)^{1/2}) \ge \lambda_{\min}\left((QAQ^*)^{1/2}(QAQ^*)^{-1}(QAQ^*)^{1/2}\right) = 1.$$

That is, $\lambda_{\min}(B) \geq N^2$. ∎

Remark. There is no upper bound for largest eigenvalue of B, for example, $N = 2$, $A_1 = \varepsilon I_n$, $A_2 = I$, and let $\varepsilon \to 0^+$.

Reference

[1] A. W. Marshall and I. Olkin, Matrix versions of the Cauchy and Kantorovich inequalities, *Aequationes Math.* 40 (1990) 89–93.

Solution 47-5.2 by Suvrit SRA, *Max Planck Institute for Intelligent Systems, Tübingen, Germany*, suvrit@tuebingen.mpg.de

Since B is a product of two positive definite matrices we may write $B = X^2Y^2$, or equivalently, $X^{-1}BX = XY^2X$; but the latter matrix is symmetric and positive, hence diagonalizable with positive real eigenvalues.

Now we prove that $\lambda_{\min}(B) \geq N^2$. To that end, recall that the arithmetic mean for positive operators dominates (in Löwner order) the operator harmonic mean (this fact follows from operator convexity of $x \mapsto x^{-1}$). This result implies in particular that

$$\frac{A_1 + \cdots + A_N}{N} \geq N(A_1^{-1} + \cdots + A_N^{-1})^{-1} \implies \left(\sum\nolimits_i A_i\right) \geq N^2\left(\sum\nolimits_i A_i^{-1}\right)^{-1}. \tag{8}$$

Now appealing to $\lambda_i(X^{1/2}AX^{1/2}) = \lambda_i(AX)$, after multiplying both sides of (8) by $X = (\sum_i A_i^{-1})^{1/2}$ on the left and the right, we obtain the desired result about $\lambda_{\min}(B)$. ∎

Also solved by Jan HAUKE*, the proposer Roger A.* HORN*, and Hans Joachim* WERNER.

Problem 47-6: An Exponential Order

Proposed by Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com

Let A and B be positive invertible operators on a Hilbert space H and let M and m be constants such that $M \geq A \geq m > 0$. If $\log A - \log B$ is a positive operator, show that $e^{\frac{M}{m}A} \geq e^B$.

Solution 47-6.1 by the proposer Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com

Let A and B be positive invertible operators on a Hilbert space H satisfying $M \geq A \geq m > 0$. If $\log A \geq \log B$, then by [2]

$$\left(\frac{M}{m}\right)^p A^p \geq B^p, \tag{9}$$

for all $p \geq 1$. By (9), we have $e^B = \sum\limits_{n=0}^{\infty} \frac{1}{n!}B^n \leq \sum\limits_{n=0}^{\infty} \frac{1}{n!}\left(\frac{M}{m}\right)^n A^n = e^{\frac{M}{m}A}$. ∎

A remark by the proposer: The inequality $e^{\frac{M}{m}A} \geq e^B$ of Problem 47-6 is slightly stronger than [1, Corollary 8].

Reference

[1] S. Izumino, R. Nakamoto, and Y. Seo, Operator inequalities related to Cauchy-Schwarz and Hölder-McCarthy inequalities, *Nihonkai Math. J.* 8 (1997) 117-122.

[2] T. Yamazaki and M. Yanagida, Characterizations of chaotic order associated with Kantorovich inequality, *Sci. Math.* 2 (1999) 37–50.

Solution 47-6.2 by Hans Joachim WERNER, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

We consider the Hilbert space $\mathbb{C}^n$. By $\mathcal{P}^{n\times n}$ we denote the set of all positive semidefinite Hermitian matrices. We write $A \succeq 0$ to indicate that $A \in \mathcal{P}^{n\times n}$. If $A \in \mathcal{P}^{n\times n}$ is nonsingular, then we indicate this by writing $A \succ 0$. As usual, $A \succeq B$ means $A - B \succeq 0$. If $A \in \mathcal{P}^{n\times n}$ is nonsingular then there exists a unitary matrix U (i.e., $U^*U = I_n$) and a nonnegative diagonal matrix D such that

$$A = UDU^*, \tag{10}$$

wherein the diagonal entries $\lambda_i > 0$, $1 \leq i \leq n$, of D are the eigenvalues of A, and the column vectors of U are corresponding eigenvectors. The function $\log(\cdot)$ is the inverse of the exponential function $e:\ \mathbb{R} \to \mathbb{R}$ and, in view of (10), $f(A) = Uf(D)U^* = U\,\mathrm{diag}(f(\lambda_1), \ldots, f(\lambda_n))\,U^*$ for $f(t) = \log(t)$ and for $f(t) = e^t$.

In what follows, let $A, B \in \mathcal{P}^{n\times n}$ be such that $M \cdot I_n \succeq A \succeq m \cdot I_n \succ 0$ and $\log(A) - \log(B) \succeq 0$. Then, $M > m > 0$, $A \succ 0$, and $M \cdot I_n - A \succeq 0$, and so $M \cdot I_n \succeq D \succ 0$. The function log is monotone increasing on the set of positive real numbers. Since $\log(M \cdot I_n) = \log(M) \cdot I_n$ and $\log(A) = U \log(D)U^*$, it follows from $M \cdot I_n \succeq D \succ 0$ that $\log(M) \cdot I_n \succeq \log(D)$ or, equivalently, $\log(M) \cdot I_n \succeq U \log(D)U^* = \log(A)$. So, $\log(M) \cdot I_n - \log(A) \succeq 0$. Since, by assumption, $\log(A) - \log(B) \succeq 0$, we now obtain $\log(M) \cdot I_n - \log(B) \succeq 0$ because the sum of two arbitrary matrices from $\mathcal{P}^{n\times n}$ again belongs to $\mathcal{P}^{n\times n}$. Reasoning similarly, we obtain from $\log(M) \cdot I_n \succeq \log(B)$ that $e^{\log(M\cdot I_n)} = e^{\log(M)} \cdot I_n = M \cdot I_n \succeq e^{\log(B)} = B$. Hence, $M \cdot I_n \succeq B$ which gives

$$e^{M\cdot I_n} = e^M \cdot I_n \succeq e^B, \quad \text{i.e.,} \quad e^M \cdot I_n - e^B \succeq 0. \tag{11}$$

On the other hand, $M \cdot I_n \succeq A \succeq m \cdot I_n \succ 0$ implies $\frac{M}{m}A \succeq M \cdot I_n$, and so we also have

$$e^{\frac{M}{m}\cdot A} \succeq e^{M\cdot I_n} = e^M \cdot I_n, \quad \text{i.e.,} \quad e^{\frac{M}{m}\cdot A} - e^M \cdot I_n \succeq 0. \tag{12}$$

Combining (11) and (12) yields

$$(e^{\frac{M}{m}\cdot A} - e^M \cdot I_n) + (e^M \cdot I_n - e^B) = e^{\frac{M}{m}\cdot A} - e^B \succeq 0$$

or, equivalently, $e^{\frac{M}{m}\cdot A} \succeq e^B$. ∎

Problem 47-7: Singular Value Inequalities

Proposed by Ramazan TÜRKMEN, *Science Faculty, Selcuk University, 42031 Konya, Turkey*, rturkmen@selcuk.edu.tr
and Fuzhen ZHANG, *Nova Southeastern University, Fort Lauderdale, Florida, U.S.A.*, zhang@nova.edu

Let $\lambda_i(X)$, $1 \le i \le n$, denote the ith largest eigenvalue of an $n \times n$ complex matrix X whose eigenvalues are all real. If A and B are $n \times n$ complex matrices, show that

$$\frac{1}{2}\lambda_i(A^*A + B^*B + A^*B + B^*A) \le \lambda_i(AA^* + BB^*), \quad i = 1, 2, \ldots, n.$$

Solution 47-7.1 by Roger A. HORN, *University of Utah, Salt Lake City, Utah, U.S.A.*, rhorn@math.utah.edu

The asserted inequalities are valid even if A and B are not square. Let A, B be $m \times n$ complex matrices, let $q = \min\{m, n\}$, and let $X = [A\ B]$. The q largest eigenvalues of $XX^* = AA^* + BB^*$ are the same as those of

$$X^*X = \begin{pmatrix} A^*A & A^*B \\ B^*A & B^*B \end{pmatrix}.$$

Consider the unitary matrix

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} I_n & I_n \\ -I_n & I_n \end{pmatrix}$$

and let $H = \frac{1}{2}(A^*A + A^*B + B^*A + B^*B)$. Then

$$U(X^*X)U^* = \begin{pmatrix} H & \bigstar \\ \bigstar & \bigstar \end{pmatrix}$$

so Cauchy's eigenvalue interlacing inequalities ensure that $\lambda_i(XX^*) = \lambda_i(X^*X) \ge \lambda_i(H)$, $i = 1, \ldots, q$. ∎

Solution 47-7.2 by Radosław KALA, *Poznań University of Life Sciences, Poland*, kalar@up.poznan.pl

To prove the inequality

$$\frac{1}{2}\lambda_i(A^*A + B^*B + A^*B + B^*A) \le \lambda_i(AA^* + BB^*)$$

first observe that the matrix on the left hand side can be expressed as $(A + B)^*(A + B)$. Observe also that the nonzero eigenvalues of this matrix are the same as those of

$$(A + B)(A + B)^*$$

(see [D.A. Harville, *Matrix Algebra From a Statistician's Perspective*, Springer, 1997, p. 546]). Now, let $R = \frac{1}{2}(A - B)(A - B)^*$ and $S = \frac{1}{2}(A + B)(A + B)^*$. These matrices are nonnegative definite and

$$S + R = AA^* + BB^*.$$

To end the proof it suffices to use the Monotonicity Theorem [C.R. Rao and M.B. Rao, *Matrix Algebra and Its Applications to Statistics and Econometrics*, World Scientific, 2001, p.322, see also Complement 10.3.2.1, p.337] that $\lambda_i(S) \le \lambda_i(S + R)$ for any Hermitian matrix S and nonnegative definite R. ∎

Solution 47-7.3 by Suvrit SRA, *Max Planck Institute for Intelligent Systems, Tübingen, Germany*, suvrit@tuebingen.mpg.de

Define the Hermitian matrices

$$X = \begin{pmatrix} 0 & A \\ A^* & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & B \\ B^* & 0 \end{pmatrix}. \tag{13}$$

Recall now that the map $t \to t^2$ is operator convex. Thus, we have $\left(\frac{X+Y}{2}\right)^2 \leq \frac{X^2+Y^2}{2} \iff (X+Y)^2 \leq 2(X^2+Y^2)$, which in particular implies the block matrix inequality

$$\begin{pmatrix} (A+B)(A+B)^* & 0 \\ 0 & (A+B)^*(A+B) \end{pmatrix} \leq \begin{pmatrix} 2(AA^*+BB^*) & 0 \\ 0 & 2(A^*A+B^*B) \end{pmatrix}.$$

Looking at the first block of the above inequality we immediately obtain $\lambda_i((A+B)(A+B)^*) \leq \lambda_i(2(AA^*+BB^*))$. But since for any matrix X, the matrices X^*X and XX^* have the same eigenvalues, we conclude that

$$\lambda_i((A+B)^*(A+B)) = \lambda_i((A+B)(A+B)^*) \leq 2\lambda_i((AA^*+BB^*)). \quad \blacksquare$$

Solution 47-7.4 by Hans Joachim WERNER, *University of Bonn, Bonn, Germany*, hjw.de@uni-bonn.de

First, observe that the three matrices $(A+B)^*(A+B) = A^*A+B^*B+A^*B+B^*A$, A^*A, and B^*B are all positive semidefinite and Hermitian. So, A^*A+B^*B is also positive semidefinite and Hermitian. Second, recall that the eigenvalues of such matrices are all nonnegative real numbers.

Theorem. Let $A, B \in \mathbb{C}^{n\times n}$. Then, for $1 \leq i \leq n$,

$$\max\{\lambda_i((A+B)^*(A+B)),\, \lambda_i((A-B)^*(A-B))\} \ \leq\ \min\{2\lambda_i(A^*A+B^*B),\, 2\lambda_i(AA^*+BB^*)\}, \tag{14}$$

where $\lambda_i(\cdot)$ denotes the i^{th} largest eigenvalue of a matrix.

Proof. The matrices $(A+B)^*(A+B)$, $(A-B)^*(A-B)$, and A^*A+B^*B are all positive semidefinite and Hermitian. Since $(A+B)^*(A+B)+(A-B)^*(A-B) = 2(A^*A+B^*B)$, according to Weyl's Monotonicity Theorem (see [R. Bhatia, *Matrix Analysis*, Springer-Verlag, New York, 1997, p. 63]), it follows that, for $1 \leq i \leq n$,

$$2\lambda_i(A^*A+B^*B) \geq \lambda_i((A+B)^*(A+B)) \quad \text{and} \quad 2\lambda_i(A^*A+B^*B) \geq \lambda_i((A-B)^*(A-B)).$$

Consequently, for $1 \leq i \leq n$,

$$\max\{\lambda_i((A+B)^*(A+B)),\, \lambda_i((A-B)^*(A-B))\} \leq 2\lambda_i(A^*A+B^*B) \tag{15}$$

and

$$\max\{\lambda_i((A+B)(A+B)^*),\, \lambda_i((A-B)(A-B)^*)\} \leq 2\lambda_i(AA^*+BB^*). \tag{16}$$

Since $\mathrm{spectrum}((A+B)^*(A+B)) = \mathrm{spectrum}((A+B)(A+B)^*)$ and $\mathrm{spectrum}((A-B)^*(A-B)) = \mathrm{spectrum}((A-B)(A-B)^*)$, (14) follows from (15) and (16). $\blacksquare$

Our inequality (14) is sharper than the inequality proposed in this problem. Consider the matrices $A = \left(\begin{smallmatrix} 1 & 0 \\ 1 & 0 \end{smallmatrix}\right)$ and $B = \left(\begin{smallmatrix} 1 & 2 \\ 1 & 0 \end{smallmatrix}\right)$. Then,

$$\lambda_1((A+B)^*(A+B)) = 6+2\sqrt{5} < 2\lambda_1(A^*A+B^*B) = 12 < 2\lambda_1(AA^*+BB^*) = 8+4\sqrt{2},$$

whereas

$$\lambda_2((A+B)^*(A+B)) = 6-2\sqrt{5} < 2\lambda_2(AA^*+BB^*) = 8-4\sqrt{2} < 2\lambda_2(A^*A+B^*B) = 4.$$

Since $\mathrm{trace}(A^*A+B^*B) = \mathrm{trace}(AA^*+BB^*)$, it is no surprise that for $n=2$ we have $\lambda_2(AA^*+BB^*) < \lambda_2(A^*A+B^*B)$ if and only if $\lambda_1(AA^*+BB^*) > \lambda_1(A^*A+B^*B)$. $\blacksquare$

Also solved by Eugene HERMAN, *Minghua* LIN *and Henry* WOLKOWICZ, *Leo* LIVSHITS, *and the proposers Ramazan* TÜRKMEN *and Fuzhen* ZHANG.

Editorial note: Minghua LIN *and Henry* WOLKOWICZ *pointed out that the constant $1/2$ is best possible as one can see by taking $A = B = I$. Johanns* DE ANDRADE BEZERRA *obtained the estimates $\lambda_i(A^*A+B^*B+A^*B+B^*A) \leq 2\lambda_i(A^*A+B^*B)$, which also follows from solution by Hans Joachim* WERNER *or by Suvrit* SRA.

IMAGE Problem Corner: New Problems

Problems: We introduce 7 new problems in this issue and invite readers to submit solutions for publication in *IMAGE*. Solutions: We present solutions to all problems in the previous issue [*IMAGE* 47 (Fall 2011), p. 40]. Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Bojan Kuzma (bojan.kuzma@famnit.upr.si). The working team of the Problem Corner consists of Gregor Dolinar, Nung-Sing Sze, and Rajesh Pereira.

NEW PROBLEMS:

Problem 48-1: Reverse Order Law for the Core Inverse

Proposed by Oskar Maria BAKSALARY, *Adam Mickiewicz University, Poznań, Poland,* obaksalary@gmail.com
and Götz TRENKLER, *Dortmund University of Technology,* trenkler@statistik.tu-dortmund.de

Let A be an $n \times n$ complex matrix and let $A^{\oplus}$ be the Core inverse of A defined in [O.M. Baksalary and G. Trenkler, Core inverse of matrices, *Linear and Multilinear Algebra* 58 (2010) 681–697] to be the unique (when exists) matrix satisfying

$$AA^{\oplus} = P_A \quad \text{and} \quad \mathrm{Im}(A^{\oplus}) \subseteq \mathrm{Im}(A),$$

where P_A is the orthogonal projector onto $\mathrm{Im}(A)$. If $A^{\oplus}$, $B^{\oplus}$, and $(AB)^{\oplus}$ exist, does it follow that $(AB)^{\oplus} = B^{\oplus}A^{\oplus}$?

Problem 48-2: Minimality for the Nonzero Singular Values of an Idempotent Matrix

Proposed by Johanns DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let A be an idempotent $n \times n$ complex matrix. If λ is a nonzero eigenvalue of AA^*, show that $\lambda \geq 1$.

Problem 48-3: A Generalization of Geršgorin's Theorem

Proposed by Frank J. HALL, *Georgia State University, Atlanta, Georgia, USA*, fhall@gsu.edu
and Roger A. HORN, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu

Let λ be an eigenvalue of $A \in M_n(\mathbb{C})$ with geometric multiplicity at least $k \geq 1$ and let $R_i'(A) = \sum_{j \neq i} |a_{ij}|$ denote the i^{th} deleted absolute row sum of A. Show that λ is contained in each union of $n-k+1$ different Geršgorin discs of A, that is,

$$\lambda \in \bigcup_{j=1}^{n-k+1} \{z \in \mathbf{C} : |z - a_{i_j i_j}| \leq R_{i_j}'(A)\} \tag{17}$$

for *any* choice of indices $1 \leq i_1 < \cdots < i_{n-k+1} \leq n$.

Problem 48-4: Characterizations Of Symmetric And Skew-Symmetric Matrices

Proposed by Roger A. HORN, *University of Utah, Salt Lake City, Utah, U.S.A.*, rhorn@math.utah.edu

Let A be a square complex matrix, and let $\bar{A}$ and $A^* = \bar{A}^T$ denote its complex conjugate and conjugate transpose, respectively. Show that: (1) $A = A^T$ if and only if $A\bar{A} = AA^*$, and (2) $A^T = -A$ if and only if $A\bar{A} = -AA^*$.

Problem 48-5: Similarity Invariant Seminorms

Proposed by Nathaniel JOHNSTON, *University of Guelph, Guelph, Canada*, njohns01@uoguelph.ca

Let $n \geq 2$. A semi-norm $\|\cdot\|$ on complex $n \times n$ matrices is similarity invariant if

$$\|X\| = \|SXS^{-1}\| \qquad \text{for all } X, S \in M_n(\mathbb{C}) \text{ with } S \text{ nonsingular.}$$

Find all similarity invariant norms and similarity invariant semi-norms on $M_n(\mathbb{C})$.

Problem 48-6: A determinant inequality

Proposed by Minghua LIN, *University of Waterloo, Ontario, Canada*, mlin87@ymail.com

Let A, B be $n \times n$ positive semidefinite Hermitian matrices. Show that $\det(A^2 + AB^2A) \leq \det(A^2 + BA^2B)$.

Problem 48-7: Block-entry-wise compressions

Proposed by Edward POON, *Embry-Riddle Aeronautical University, Prescott, AZ, U.S.A.,* edward.poon@erau.edu

Let A be an $mn \times mn$ complex matrix with $m \times m$ block matrix entries A_{ij}, $1 \leq i, j \leq n$. Suppose for each unit vector $x \in \mathbb{C}^m$ the matrix U_x, defined by $(U_x)_{ij} = x^* A_{ij} x$, is unitary. Does it follow that each A_{ij} is a scalar multiple of the $n \times n$ identity matrix I_n (that is, $A_{ij} = u_{ij} I_n$ where u_{ij}, $1 \leq i, j \leq n$, are the entries of a unitary matrix)?

Solutions to Problems 47-1 through 47-7 are on pp. 29–39.