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Editor-in-Chief: Kevin N. Vander Meulen, Department of Mathematics, Redeemer University College, Ancaster, ON, Canada, L9K 1J4 (kvanderm@redeemer.ca).

Managing Editor: Louis Deaett, Department of Mathematics, Quinnipiac University, Hamden, Connecticut 06518, USA. (Louis.Deaett@quinnipiac.edu).

Contributing Editors: Minerva Catral (catralm@xavier.edu), Michael Cavers (mcavers@ucalgary.ca), Douglas Farenick (Doug.Farenick@uregina.ca), Carlos Fonseca (carlos@sci.kuniv.edu.kw), Rajesh Pereira (pereirar@uoguelph.ca), Naomi Shaked-Monderer (nomi@tx.technion.ac.il), David M. Strong (David.Strong@pepperdine.edu), and Amy Wehe (awehe@fitchburgstate.edu).

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FEATURE INTERVIEW

“Don’t shy away from problems in industry - they lead to fantastic new Linear Algebra”

Volker Mehrmann Interviewed by Christian Mehl¹

C.M. - Applied as well as core Linear Algebra has been an important part of your research. How did you get started in this area?

V.M. - I actually got started writing a master’s thesis in Numerical Linear Algebra with Ludwig Elsner in Bielefeld, working on eigenvalue problems. After I finished it I told him that I wanted to go to the U.S. to experience a little bit outside of East Westphalia, the local region around Bielefeld. He suggested a few possible names to me and finally I got an offer from Richard Varga from Kent State University to be accepted into the Ph.D. programme. Richard suggested a topic on M -matrices and their generalizations and I wrote my Ph.D. thesis in this field under the supervision of him and Ludwig Elsner. After I finished my thesis, a funny incident brought me back to eigenvalue problems. I was attending the conference at Argonne National Laboratory in honor of James H. Wilkinson’s 65th birthday, where I met Ralph Byers. He was talking about the Hamiltonian eigenvalue problem, the topic of his Ph.D. thesis, and all of a sudden I found a nice application for an otherwise useless algorithm from my master’s thesis. Because of this incident, I started to work on control problems and eigenvalue problems.



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C.M. - Since then, you have done research in so many different areas, like Numerical Analysis, Control Theory, and Analysis of differential-algebraic equations (DAEs), etc. How important is the role that Linear Algebra plays in your research today?

V.M. - In the background it’s all Linear Algebra. One thing that I learned in my many years of studying and doing research is the following. If you have a good understanding of the finite-dimensional setting in an abstract framework then you can apply it to almost all kinds of problems in the infinite-dimensional setting. You may have to extend and modify the theory, but nevertheless the basic ideas typically come from Linear Algebra.

C.M. - Do you think that this central role of Linear Algebra is well enough appreciated in other communities?

V.M. - Fortunately, there has been a change, in particular in Germany. People in all areas of mathematics appreciate and acknowledge the importance of Linear Algebra and Matrix Theory much more than previously. As an example, people from the PDE community or people working in Control Theory have realized that if they want their algorithms to work, then they better do the Linear Algebra right. Typically they want their algorithms not only to be accurate, but also to be fast, and both of those together one cannot get without a detailed and constructive way to deal with the Linear Algebra problem in the background. Everywhere one has to solve a large-scale linear system, or an eigenvalue problem, or one has to do a singular value decomposition or something like that, and typically the bottleneck is this Linear Algebra problem. And usually a good theoretical analysis of the structure and properties of the matrices and the corresponding perturbation theory is the key to success.

In a certain sense, my whole career is built on the following basic principle. If you are dealing with a project from applications then you have to learn where the problems come from, that is, you have to learn about the underlying physics, you have to learn about the models, and you have to learn about the corresponding model equations. But if you finally come to implementing your algorithms, then you have to analyze the properties of the Linear Algebra problem. You can prove many nice theorems about convergence in abstract spaces, but if you do not do the Linear Algebra right, then it does not work in practice. In a certain sense it is a big success that has been achieved in the last few years that people have learned that this is the case. Many people are now using ideas from Linear Algebra directly or indirectly to really make progress in something that at first sight may be completely unrelated to Linear Algebra.

C.M. - Are there specific examples that you have in mind here?

V.M. - For example, take compressed sensing. It is applied Fourier analysis, but in the background it is Linear Algebra. Or take the numerical solution of complicated PDEs in industry. They lead to large-scale linear systems or eigenvalue

¹Technische Universität Berlin, Institut für Mathematik, Berlin, Germany. mehl@math.tu-berlin.edu

problems that you have to solve and if you don't deal in the right way with these Linear Algebra problems in the background, then you cannot get a good solution.

C.M. - Besides working on these problems, in your opinion what are the most important topics in Linear Algebra to be considered in the future?

V.M. - At the moment I see four important topics, where progress is urgently needed and where Linear Algebra plays a major role. The first is the solution of PDE eigenvalue problems. There is a lot of recent work on adaptive finite element methods, and in the symmetric/self-adjoint case very good results are available, but if you go to the non-self-adjoint case then there is essentially nothing. The analysis on the Linear Algebra side in this case is very good already, though not complete yet, but on the PDE side there is not much. This is an important topic, in particular if the modeling is done partially with finite elements and partially with phenomenological models which in many cases you have to use, because it is very hard to model friction or damping in a systematic way except going to the atomistic level.

The second topic that I see is tensor analysis, thus Multilinear Algebra. Here, you have to extract important features of operators in high-dimensional spaces, and this dimension is not the size of a matrix, but the dimension of the underlying physical problem. These problems are not only 3D, but easily 1000D or even higher-dimensional. Our classical matrix formulation is definitely not the right approach here. In recent years, important work on this topic has been made, but significant progress is hindered by the fact that the algebraic geometry behind these problems is not yet clear. I think that this challenging problem brings many different areas of mathematics together. If people working in these areas want to contribute to real-world problems, here is their chance.

The third big problem I have in mind is perturbation theory, which is still mainly restricted to problems that we understand well. Either it is general perturbation theory for matrices and eigenvalue problems or related topics, or it is perturbation theory hidden in the context of PDEs, typically in asymptotic analysis. But if we consider, for example, complicated eigenvalue problems, then nobody yet understands how the perturbation theory, in terms of parameters, in terms of noise, or in terms of uncertainties in the data, really works. Also, it is not clear how to bring this together with additional structure in the problems.

Finally, I think a fourth major problem is that in all the areas of signal and image processing, the Linear Algebra is still the bottleneck. If you look at it from this side, you have to solve an optimization problem by minimizing $Ax - b$ in the L_1 -norm subject to some extra constraints. This is a Linear Algebra problem, but we do not know how to solve it well when the size is very large.

C.M. - It seems there is a lot for us to do in the future. But let us come back to your early career. Which colleagues had the greatest influence on your development as a researcher?

V.M. - I must say that Richard Varga, Ludwig Elsner, and Hans Schneider really were role models in many different directions: Richard Varga with his really broad research from iterative methods, PDEs, complex variables, and approximation theory; Hans Schneider bringing me into graph theory and nonnegative matrices, and Ludwig Elsner being extremely well acquainted with almost all these problems, knowing their background and all important results. He corrected me quite often when I got ahead too enthusiastically and thought that I could solve a problem. In that case he came up very quickly with counterexamples to my ideas. These three clearly had an important influence on my career, as well as had Angelika Bunse-Gerstner who was assistant at the University of Bielefeld when I was a student there.

But I must also say that a very important factor in my life was the very good relationship and very successful research cooperation with Ralph Byers with whom I had been working together in Numerical Linear Algebra until he passed away in 2007. Another strong influence is the long-term cooperation with Peter Kunkel. Our joint work started out with a Linear Algebra problem, a Riccati equation which we could not solve. Or, let me say that we both solved it using different methods and we obtained different results. Then we realized that we had to look into this topic in more detail, and this brought us into the theory of differential-algebraic equations, which in the beginning was also interesting from the pure Linear Algebra point of view, because it involved canonical forms and things like that, but then it went pretty much into Analysis.

C.M. - Among your many contributions to Linear Algebra, which are the ones that you like the most?

V.M. - Actually, one has to scale this within time, because the view changes. One of the problems that have been bothering me for many years, where the progress has been small, and which is still not fully solved, is the Hamiltonian eigenvalue problem and to find an $O(n^3)$ structure-preserving, structurally backward stable method for it. I must say that I am very proud of the result that finally came out and that was published, even though it may not be the perfect

answer – if the perfect answer exists at all, which I do not believe anymore. But at least it is very close to the perfect answer. I consider this to be one of my major contributions.

Other important contributions in the Numerical Linear Algebra setting may be strongly related to the fact that we contributed in bringing computable canonical form under unitary transformations into the Control Theory community. For example, there are staircase forms and transformations that you can do to make a control problem well treatable by numerical methods. This has come up over and over again in my theoretical research and I like these contributions. Concerning the contributions to differential-algebraic equations (DAEs) I must say that I did not think that the impact would be so large in the long run when we started with DAEs in the beginning. It seemed that you just take the Kronecker canonical form, beef it up a little bit, and use unitary transformations instead of general ones. But all this has really emerged into a whole theory and a whole new way of dealing with it.

Then another result that I like was joint work with you and the Mackeys when we brought nonlinear eigenvalue problems back into the world of Linear Algebra where they had essentially vanished since the 1900s and when we brought additional structure in there. This was actually a very interesting experience. Everybody knows that the quadratic eigenvalue problem is important in Mechanics, and all of a sudden we had this quadratic eigenvalue problem coming from applications in joint work with a company here in Berlin. This eigenvalue problem had a funny structure that we had never thought about before and which is now called palindromic structure. At the beginning we didn't know how to linearize it, how to efficiently solve the eigenvalue problem, and how to do the perturbation theory for it, but when we figured that out, it turned out to be a real success. Giving this interview, I would really like to tell the community: Don't shy away from mathematical problems in industry – they can lead to fantastic new Linear Algebra! From time to time it is really worthwhile to work on practical problems in order to get new ideas.

C.M. - Doing your research in Linear Algebra, what was the most surprising moment you experienced?

V.M. - Maybe a very nice story is when I was working with Hongguo Xu (another long-term collaborator) in Chemnitz on the Hamiltonian eigenvalue problem. We tried for months and months to solve this problem, but could not get anywhere. One day in the afternoon, we had worked on an approach which ended up in something that looked like a singular value decomposition with symplectic transformations, but it did not seem to help in our problem. So I said: "Let's give it up. Let us put this Hamiltonian eigenvalue problem to rest and let us write a paper on the SVD with symplectic transformations." Then we both went home. The next morning Hongguo came to my office and said: "Well I think by using the SVD with symplectic transformations, we now can deal with the Hamiltonian eigenvalue problem much better." In a certain sense this was an enlightening moment for me. We had decided to write a paper about the SVD. It was a nice theoretical result, but it seemed to be useless. Surprisingly, the next day it turned out to be a solution to the problem that we actually wanted to solve.

C.M. - Do you have any other anecdotes that you want to share?

V.M. - Together with Greg Ammar, I was driving in a car to a conference in Bremen from Bielefeld. While we were driving through some town, we were enthusiastically discussing mathematics and how to prove one of our theorems. At the town exit, I got stopped by the police and the officer was asking rigorously: "Did you know how fast you were driving?" I answered: "No, I have no clue, we were just proving a theorem." Then the police officer answered: "That's the most extraordinary excuse I have ever heard." Finally, in LAA an acknowledgement to the German police has appeared that we got a speeding ticket during the proof of Theorem 2.1. The interesting thing is that the year after that incident I was visiting Greg in DeKalb, Illinois, and we were stopped by the police. I suggested to Greg to tell him the story about the theorem, but he said that in the United States such an excuse would not work well.

C.M. - Do you have any advice for young researchers in Linear Algebra?

V.M. - Yes, I have: Learn the basics from Linear Algebra both in a matrix-oriented and in an abstract way and try to understand the concept of structures in matrices and linear mappings and things like that. Also try to talk to people in engineering, chemistry, physics, etc., about what their matrix or Linear Algebra problems are and learn their language. You will find that the whole world will be happy to talk to you if you speak their language.

C.M. - That rule certainly does not apply only to mathematics. Thank you very much for the interview.

MATRICES

ALGEBRA, ANALYSIS AND APPLICATIONS

by **Shmuel Friedland** (*University of Illinois at Chicago, USA*)

This volume deals with advanced topics in matrix theory using the notions and tools from algebra, analysis, geometry and numerical analysis. It consists of seven chapters that are loosely connected and interdependent. The choice of the topics is very personal and reflects the subjects that the author was actively working on in the last 40 years. Many results appear for the first time in the volume.

596pp	Dec 2015	
978-981-4667-96-8	US\$148	£98
978-981-3141-03-2(pbk)	US\$69	£46

Linear Algebra as an Introduction to Abstract Mathematics

by **Isaiah Lankham** (*California State University, USA*),
Bruno Nachtergaele & **Anne Schilling** (*UC Davis*)

This is an introductory textbook designed for undergraduate mathematics majors with an emphasis on abstraction and in particular, the concept of proofs in the setting of linear algebra. The purpose of this book is to bridge the gap between the more conceptual and computational oriented undergraduate classes to the more abstract oriented classes.

208pp	Jan 2016	
978-981-4730-35-8	US\$70	£46
978-981-4723-77-0(pbk)	US\$36	£24

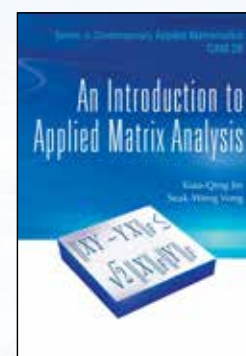
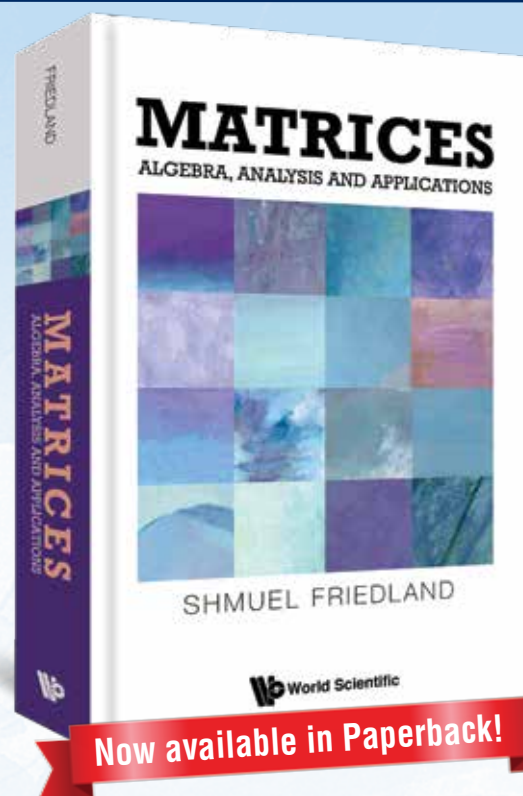
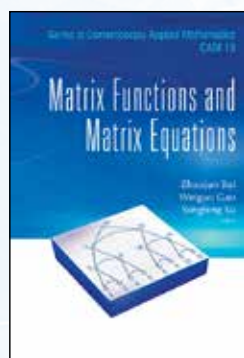
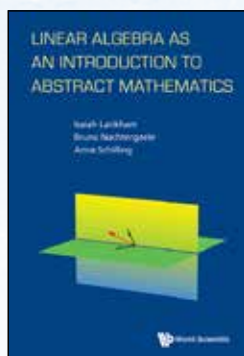
Series in Contemporary Applied Mathematics - Vol 19

Matrix Functions and Matrix Equations

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Contents: Matrix Functions: A Short Course (**Nicholas J Higham** & **Lijing Lin**); A Short Course on Exponential Integrators (**Marlis Hochbruck**); Matrix Equations and Model Reduction (**Peter Benner**, **Tobias Breiten** and **Lihong Feng**); Rayleigh Quotient Based Optimization Methods for Eigenvalue Problems (**Ren-Cang Li**); Factorization-Based Sparse Solvers and Preconditioners (**Xiaoye Sherry Li**);

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(*University of Macau, China*)

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978-981-4749-46-6	US\$48	£32



ARTICLES

The Development of Linear Algebra Research in Japan

Hiroshi Nakazato, Hirosaki University, Japan, nakahr@hirosaki-u.ac.jp

The main purpose of this article is to describe the relationship between modern research in the area of linear algebra and the history of mathematics in Japan before 1945.

Motivation to recall the history of mathematics before 1945. The author often refers to Horn and Johnson's book [7] for the study of the numerical range. To treat such a subject, the author depends on (a) an algebraic elimination, (b) the resolution of an algebraic equation. The ancient Japanese mathematicians also dealt with these methods. Their approaches are of interest to modern research.

Wasan and Linear Algebra. The ancient Japanese learned mathematics from Chinese literature. The Chinese book *The Nine Chapters on the Mathematical Art* was introduced to Japan in the 7th century. In this book, Chapter 8 provided the method to solve simultaneous linear equations by using elimination. Before the 17th century, the study of mathematics in Japan was very limited. The contents of traditional Japanese mathematics were contained in those of Chinese mathematics. The period 1603–1868 was the golden age for the development of the native Japanese mathematics called Wasan. After the Meiji modernization, the Japanese government adopted the European style of mathematics in place of Wasan. Although Wasan is not used nowadays, the mathematical results obtained in that golden age, written in Chinese characters, were rich. Part II of Mikami's book [12] provides classical literature on this subject written in English. In Wasan, plane geometry and solid geometry were the most popular subjects. You may find many beautiful geometrical problems in Fukagawa's book [5]. Japanese mathematicians favored circles over squares. The problem of approximating π was very hot. Takakazu Seki (about 1642–1708) and Takahiro Takebe (1664–1739) are recognized as champions of Wasan. In 1683, Seki wrote *Kai Fukudai no Hô* (the method to solve simultaneous equations) and introduced the determinant of an $n \times n$ matrix for $n = 2, 3, 4$ without any explanation (cf. Takenouchi [19]). In 1712, his student published *Katsuyo Sampo*, a collection of writings of Seki. In this book, Seki's series acceleration method was presented. Seki used this method for accelerating the rate of convergence of a series to obtain a better numerical approximation of π by first taking L_n to be half of the perimeter of a regular n -gon inscribed in a unit circle. He then computed the values of L_n , L_{2n} , and L_{4n} for $n = 2^{15}$ and gave the modified value

$$\tilde{L}_n = L_{2n} + \frac{(L_{2n} - L_n)(L_{4n} - L_{2n})}{(L_{2n} - L_n) - (L_{4n} - L_{2n})}$$

as a candidate for a better approximation of π . He did not provide any reasoning for this method (cf. Morimoto [13]). Takebe obtained the expansion of the function $(\arcsin x)^2$ as

$$(\arcsin x)^2 = \frac{2}{2!}x^2 + \frac{2 \cdot 2^2}{4!}x^4 + \frac{2 \cdot 2^2 \cdot 4^2}{6!}x^6 + \frac{2 \cdot 2^2 \cdot 4^2 \cdot 6^2}{8!}x^8 + \dots$$

in his book *Tetsujutsu Sankei* published in 1722 (cf. [13]). In this book, he performed accurate numerical computations and conjectured the rational coefficients of the expansion by using regular continued fractions. In 1674, Seki published *Hatsubi-Sampo*, presenting 15 questions and their answers. Many questions were related to elimination. For instance, Question 6 asks for a right triangle with sides ℓ , m , and n satisfying $\ell^2 + m^2 = n^2$ and the two additional conditions (1) $n^3 = 700 - \ell^3$ and $m^3 = 900 - n^3$ and hence (2) $m^3 = 200 + \ell^3$. According to Ogawa's article [14], we eliminate n and m to obtain an equation in ℓ . We expand the cube of n^2 as $n^6 = \ell^6 + 3\ell^4m^2 + 3\ell^2m^4 + m^6$. This is modified as (3) $n^6 - (\ell^6 + m^6) = 3\ell^4m^2 + 3\ell^2m^4$. Since the right-hand side of (3) is equal to $3\ell^2m^2(\ell^2 + m^2) = 3\ell^2m^2n^2$, we obtain (4) $n^6 - (\ell^6 + m^6) = 3\ell^2m^2n^2$. We take the cube of both sides of (4) and obtain (5) $(n^6 - (\ell^6 + m^6))^3 = 27\ell^6m^6n^6$. We substitute (1) and (2) into (5) and obtain the equation $((700 - \ell^3)^2 - (\ell^6 + (200 + \ell^3)^2))^3 - 27\ell^6(200 + \ell^3)^2(700 - \ell^3)^2 = 0$.

Yoshizane Tanaka (1651–1719) published *Sangaku Funkai* around 1690. In this book, he treated the elimination of X from the two algebraic equations $f(X) = a + bX + cX^2 = 0$ and $g(X) = p + qX + rX^2 + sX^3 = 0$. He developed a slightly different method than the one introduced by Sylvester in 1840. We introduce his idea, as described by Komatsu [11]. He expressed the resultant $R(f, g : X)$ as a linear combination $\sum C_i \phi_i$ of the 16 monomials ϕ_i , namely a^3s^2 , a^2brs , a^2cqs , a^2cr^2 , ab^2qs , ab^2r^2 , $abcps$, $abcqr$, b^3ps , b^3qr , ac^2pr , ac^2q^2 , b^2cpr , b^2cq^2 , bc^2pq , and c^3p^2 , with integral coefficients C_i . He determined the coefficients C_i under the assumption that $C_1 = 1$. The result is the following: $C_6 = C_{10} = C_{14} = 0$, $C_1 = C_4 = C_5 = C_{12} = C_{13} = C_{16} = 1$, $C_2 = C_8 = C_9 = C_{15} = -1$, $C_3 = C_{11} = -2$, $C_7 = 3$.

The period 1868–1945. Modern mathematical education and research in Japan were established after the Meiji modernization around 1868. The foundation of modern mathematics in Japan was laid by Dairoku Kikuchi (1855–1917).

The victories of Japan in the three main wars against other countries raised the international position of this country, and Japan quickly became an industrial country and got some colonies in East Asia. In 1931 Japan began a war against China and in 1941 against the USA and its European allies. But in 1945, Japan was completely defeated by the Allied Forces. We briefly mention the contributions of Japanese mathematicians to linear algebra during this period. Teiji Takagi (1875–1960), who is world-famous mostly for his contributions to class field theory, is also well-known for his Japanese textbooks in analysis and algebra. In his paper [17] of 1925, Takagi proved that an arbitrary Toeplitz matrix is unitarily similar to a complex symmetric matrix. Matsusaburō Fujiwara (1881–1946) is known as the founder of the *Tohoku Mathematical Journal*. In 1911, he and T. Hayahi began to publish the first European-style mathematical journal in Japan. T. Hayashi, Fujiwara and Yoshio Mikami (1875–1950) were leaders of the study of Wasan. In [4], Fujiwara characterized the algebraic equations whose roots lie in a circular disc or a half-plane. In [16], Kenjiro Shoda (1902–1977) proved that a square matrix X with $\text{tr}(X) = 0$ is the commutator $AB - BA$ of two matrices A and B . Soichi Kakeya (1886–1947) proved the following theorem [9]. If $p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0$ is a polynomial of degree n with real coefficients satisfying $a_n \geq a_{n-1} \geq \cdots \geq a_1 \geq a_0 \geq 0$, then all the roots of $p(z)$ lie in $|z| \leq 1$.

The period 1945–2016. After the 6-year occupation by the Allied Forces, Japan recovered its independence in 1952. The author lists some mathematical topics after 1945 which are impressive to him. In 1954, Kunihiko Kodaira (1915–1997) was awarded the Fields Medal. He developed the theory of complex manifolds. Riemann’s theta functions are related to his work. The study of linear algebra in Japan was developing in relation to its neighboring areas. In the core area, Iri’s textbook [8] is useful for studying advanced topics in linear algebra. Concerning the subject (b) mentioned above, H. Umemura [21] provided a method to construct solutions of an algebraic equation by using Riemann’s theta functions. Fujii-Kubo [3] obtained bounds on the roots of a polynomial. The classical subject of linear algebra deals with linear operators on finite dimensional vector spaces, especially Euclidean spaces. Linear operators on a Hilbert space, forming an operator algebra, give an interesting generalization of this classical subject. The positivity $\langle Tx, x \rangle \geq 0$ of an operator T is useful for various applications. Kubo and Ando [10] gave a basis for the study of operator means. Unitary dilations are discussed in some areas (cf. [1]). Various new classes of operators were introduced in connection with the Furuta inequality (cf. [6]). Tomita-Takesaki modular theory solved difficulties in non-semifinite von Neumann algebras [18]. The first mathematical subject to interest the author was derivations in operator algebras. S. Sakai and J. Tomiyama introduced this subject to the author (cf. [15, 20]). The author has collaborated with M. T. Chien for 18 years, studying the numerical range from a viewpoint of algebraic curve theory. Their latest work [2] is related to Riemann’s theta functions and elimination methods.

We learn many things from history.

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Linear Algebra in the Netherlands

M.A. Kaashoek, Vrije Universiteit, Amsterdam, Netherlands, m.a.kaashoek@vu.nl

As in many other countries, linear algebra as a separate area of research is not common in the Netherlands. But linear algebra provides and has provided basic tools for various research projects of Dutch mathematicians. Often this has resulted in new applications of Linear Algebra which are of general interest, or in new matrix theory problems. For the Netherlands I am thinking of fields like numerical analysis, mathematical systems theory, graph theory, operations research, and of mathematicians like Willem Haemers, Lex Schrijver, Mark Spijker, Henk van der Vorst, Jan van Schuppen, the late Jan Willems, and from the younger generation, Aart Blokhuis, Jan Brandts, Hein van der Holst, Rob Zuidwijk, and others. Both Willem Haemers and Aart Blokhuis are former Ph.D. students of Jaap Seidel (1919–2001) who was a Dutch pioneer in developing linear algebraic methods for discrete mathematical problems.

So far I did not mention operator theory or any operator theory people. The connection between operator theory and linear algebra is of a special nature. There is a two-way relation between those two fields that is much more complex than in other cases. In the Netherlands, this two-way relation is based on the fact that operator theory, matrix theory, analytic functions, control theory, and electrical engineering share common ground and common problems. Since the late seventies, when Israel Gohberg got a regular visiting professorship at the VU in Amsterdam, these relations were developed further. Beautiful matrix results served as motivation or inspiration for new operator theory problems and new applications. Think of matrix completion problems, the area of inversion of structured matrices, which has its roots in the work of Gohberg-Heinig-Semencul-Trench on Toeplitz matrices, and the classical Szegő inverse problems. All these matrix problems have deep and beautiful infinite-dimensional operator theory analogs involving the work of several Dutch mathematicians: Harm Bart, André Ran, Freek van Schagen, Philip Thijssen, Sanne ter Horst (presently in Potchefstroom SA), Hugo Woerdeman (presently in Philadelphia, PA, USA), and myself. Many chapters in the Bart-Gohberg-Kaashoek-Ran books of 2008 and 2010 can be viewed as chapters in linear algebra. Various infinite-dimensional mathematical analysis problems involving rational matrix data turned out to be reducible to linear algebra problems which led to interesting applications to, among others, direct and inverse spectral problems for Dirac differential systems, in joint work with Alexander Sakhnovich. Another beautiful example of the reverse direction, from an infinite-dimensional setting to a matrix problem, is the recent work of André Ran, jointly with Christian Mehl, Volker Mehrmann and the late Leiba Rodman, which concerns a pure linear algebra problem, namely the behavior of eigenvalues under rank one perturbations. Their work has its roots in a 1995 paper of L. Hörmander and A. Melin on the behavior of the spectrum of compact operators on infinite-dimensional Banach spaces.

Given the various Dutch contributions to linear algebra, it will not be a surprise that Dutch matrix and operator theory people have also served the linear algebra community in other ways, including as editors of the journal *Linear Algebra and its Applications*, as ILAS board members, and by twice organizing an International Linear Algebra Society conference (Rotterdam in 1994 and Amsterdam in 2006).

Acknowledgement. I gratefully acknowledge the useful remarks I got from Harm Bart, Lex Schrijver, André Ran, and Freek van Schagen on an earlier version of this article.

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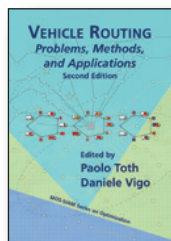
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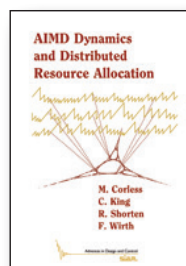
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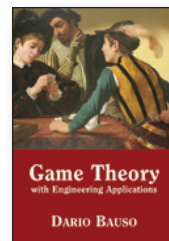
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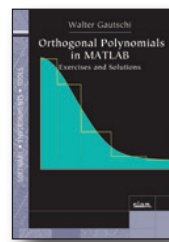
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An Honorary Doctorate for John Francis

Frank Uhlig

Department of Mathematics, Auburn University
Auburn, AL 36849-5310, USA, uhligfd@auburn.edu

This article touches on our history as linear algebraists and should be of interest to *IMAGE* readers since John Francis, the founder of the QR algorithm, had been almost completely unknown for about 50 years until a few years ago (see [10]). We are including here the acceptance speech of John Francis for its historical interest. It includes John's lovely intuitive explanation of eigenvalues. The ceremony took place as part of the University of Sussex summer graduation in July 2015. Peter Coles, the department head, gave the laudatio, followed by John's acceptance speech. Peter Coles' words highlight John's importance for us and for the world of computing in a clear way. Both speeches were transcribed from the video [12] made of the occasion and compared with the original speakers' notes. They were only slightly redacted for factual accuracy and readability.

1. The laudatio presented by Peter Coles, Head of Mathematics and Physical Sciences at Sussex.

"Chancellor, it is both a pleasure and an honour to present John Francis for the award of the degree of Doctor of Science.

John Francis is a pioneer in the field of mathematical computation, where his name is more-or-less synonymous with the so-called "QR algorithm," an ingenious factorization procedure used to calculate the eigenvalues and eigenvectors of linear operators represented as matrices.

Now the mathematics and physics graduates in the audience will be well aware perhaps of the importance of eigenvalue problems which crop up in a huge variety of contexts in these and other scientific disciplines, from geometry to graph theory to quantum mechanics to geology to molecular structure to statistics to engineering; the list is almost endless and there can be few people working in such fields who haven't at one time or another turned to the QR algorithm in the course of their calculations. I know I have, in my own field of astrophysics! Indeed this algorithm has become a standard component of any scientist's mathematical toolkit.

The algorithm was first derived by John Francis in two papers [3, 4] submitted in 1958/59 and published in 1961 and, independently [7, 8] a bit later, by the Russian mathematician Vera Kublanovskaya who sadly passed away in 2012. Interestingly, the problem that John Francis was trying to solve when he devised the QR algorithm concerned "flutter," or a form of vibrations in the wings of aircraft.

But it is perhaps in the world of the World Wide Web that the QR algorithm has had its greatest impact. Many of us who were using the internet way back in 1998 were astonished when Google arrived on the scene because it was so much faster and more effective than all the other search engines available at the time. The secret of this success was the PageRank algorithm (named after one of its inventors, Larry Page) which actually involved applying the SVD algorithm [5], a complicated but straightforward application of the principles underlying QR, to calculate numerical factors expressing the relative importance of elements within a linked set (such as pages on the World Wide Web) as measured by the nature of their links to other elements. Using the SVD or the thus-adapted QR algorithm was originally suggested by Gene Golub [10] to the founders of Google. It is not the only technique exploited in the PageRank algorithm and by Google, but it is safe to say that it is what gave Google its edge.

The achievements of John Francis are indeed impressive, but even more so when you read his biography, because he did all this pioneering work in numerical analysis without even having a degree in mathematics.

John Francis actually left school in 1952 and obtained a place at Christ's College, Cambridge for entry in 1955, after two years of National Service during which he served in Germany and Korea with the Royal Artillery. On leaving the army in 1954 he worked for a time at the National Research Development Corporation, which was set up in 1948 by the Attlee government. He went to university as planned but did not complete his degree, and instead returned to the NRDC in 1956 after only a year or so of study. It was while working there in 1958 and 1959 that he devised the QR algorithm.

John Francis left the NRDC in 1961 to work at Ferranti Ltd, after which, in 1967, he moved to Brighton and took up a position at the University of Sussex in the Laboratory of Experimental Psychology, providing computing services. He then left the University in 1972 to work in various private sector computer service companies in Sussex. He has now retired but still lives locally, in Hove.

Now having left the field of numerical analysis in the early 1960s, John Francis had absolutely no idea of the impact that his work had had, especially the QR algorithm. Nor was he aware that it was widely recognized as one of the Top Ten Algorithms of the Twentieth Century [1, 2] until he was traced and contacted in 2007 by Gene Golub and Frank Uhlig who jointly with Andy Wathen then organized a mini-symposium to celebrate the 50-year anniversary of Francis' work. John Francis became the opening speaker at that meeting in Glasgow which took place in 2009. For more information



on this chain of events see [6, 10].

More recently still, in 2011, after what he describes as sporadic study over a number of years, John Francis was eventually awarded an undergraduate degree from the Open University, 56 years after he started at Cambridge.

I am very glad that there will be no similar delay in proceeding to a doctorate! Chancellor, I present to you for the degree of Doctor of Science, *honoris causa*, John Francis."

2. John Francis' speech on receiving an Honorary Doctorate from the University of Sussex, July 2015.



"Chancellor, Vice-Chancellor, Professor Peter Coles, graduands, families and friends:

I wish to thank the University of Sussex and express how greatly honoured I am to have this degree bestowed upon me.

How has this come about? I am very much the exception to the general rule – most honorary degrees are awarded to illustrious well-known public figures or to people well established in their field with a lifetime of notable work – just look at the others [11] who have received honorary degrees from Sussex! So on receiving the official letter from the Vice Chancellor I was absolutely amazed and still can hardly believe it. Why should an obscure retired elderly resident of this city, with an entirely unremarkable working life, be so honoured?

You will see from the programme booklet (if you had time to look at it) that this is for – as Professor Coles said – my contribution to computational mathematics 56 years ago, specifically my invention of the QR algorithm for finding the eigenvalues and eigenvectors of matrices. And Caroline Lehany, who organised this ceremony, said to

me, perhaps half jokingly, 'I hope you are going to be telling us what these eigenvalues and eigenvectors are.' So you can blame her for the following maths lesson!

Now maths is a fun subject! Isn't it? It's fun! And people generally get unnecessarily frightened of things like matrices. So what are eigenvalues and eigenvectors? Now, no abstruse maths concepts are needed.

Hopefully you graduands out there will have some knowledge of them. Matrices are just tables (or arrays) of numbers that arise in engineering and many other fields. The numbers in the array are typically the coefficients of variables belonging to a set of equations. Parents and friends, you may remember the x 's, y 's, and z 's in your school algebra – the variables. Students, too, you will be familiar with linear equations that come up in all sorts of problems. Now you don't just get an x , y and z , you may get tens or hundreds of variables and just solving the equations to find these needs computers – but this is generally quite straightforward.

The eigenvalues and vectors are, however, more difficult to compute. This is the technical bit: Associated with matrices we have vectors – which is another word that frightens people. You picture a vector as a single column of numbers. Geometrically, the numbers forming a vector are the coordinates of a point and the vector is the line going from the origin to this point – in two dimensions (with just two variables) the point might be a position on a map. Now, matrices can multiply vectors by simple rules of arithmetic and this is called performing a linear transformation – the vector will perhaps be displaced or changed in length or direction. But all square matrices (when they have the same number of rows as columns) have associated with them the special vectors called eigenvectors. And they are only changed in length when multiplied by the matrix, but not in direction, except possibly in the opposite direction if the eigenvalue is negative. The change in length as a multiplying factor is called the eigenvalue. That's the end of the maths lesson!

Eigenvalues are of great practical importance and come into all sorts of fields and Professor Coles has just told you about the Google search engine. So I won't repeat that. But in those early days – the 1950s – processes for computing them for a given matrix could be unstable – or 'badly behaved' in the jargon – and serious errors could occur in the computation or else it could be very slow, and there was a great need for a better method.

And how did I get involved? Well, I am afraid I am going to give you a potted history and repeat some of what Professor Coles has said but I hope it won't take too long.

I left school in 1952 and went into the army for National Service. I got my A-levels for a place in Cambridge in 1955. When I left the army I was 20, and having a gap year, I took a job at the National Research Development Corporation, the NRDC. The NRDC was promoting the development of computers and they had employed Christopher Strachey, later to become Professor of Computer Science at Oxford. I was to assist in that, in particular the design of the multiply and divide unit. And he advertised for a clerk to help in drawing up wiring lists and this was the job that I got – in those days there weren't even printed circuit boards and you needed bits of wire and a soldering iron! If my memory serves me right, Strachey gave a programming course at the London Polytechnic (now City University), which I went to. And that was my introduction to computers.

In 1955, I went to Cambridge but to my shame I gave up the degree course after just two terms. My excuse was that my three years away had made it hard to get back into studying. I had remained in touch with Strachey and was very fortunate that he was willing to take me on then as his assistant – essentially as a programmer. He had formed a small research group at the NRDC on the use of computers in solving matrix problems. In particular, as Professor Coles mentioned, the aircraft flutter problem. The modes of vibration of a structure can be found as the eigenvalues of certain equations, differential equations – and the matrices involved can be quite large. Under Strachey we devised methods for detecting the eigenvalues that would result in unstable vibration – in other words the aircraft falling apart! Working in this group was very intellectually exciting and the subject of finding eigenvalues was very much on the agenda. A new method published, I think it was in '58, which was absolutely seminal for me, by Heinz Rutishauser [9] was the LR eigenvalue algorithm. It was a very elegant method. However, for all its elegance, the method had the usual problems of instability that I've mentioned. In fact there were some matrices it just would not work for at all. Rutishauser's method involves a process of eliminating matrix elements below the diagonal and this is where the instabilities can arise. I was very familiar with various elimination techniques, particularly what are called unitary rotations, which are inherently stable. I thought perhaps that a similar algorithm could be devised using these. Even today I remember my excitement and elation on proving that my algorithm would work theoretically in the same way. That was in '58/'59. By the way, all this work that I did on this QR algorithm at that time was done unofficial and on-the-side, because it was not my proper job! I followed that up by very successful practical work (sanctioned by the NRDC) and my two research papers [3, 4] which involved a lot of laborious computer programming (in machine code; there were no computer languages in those days) and many hours spent testing the algorithm, as computers were perhaps a million or even ten million times slower than today's computers! Incidentally, for the eigenvectors, once you have found the eigenvalues, calculating them is relatively straightforward.

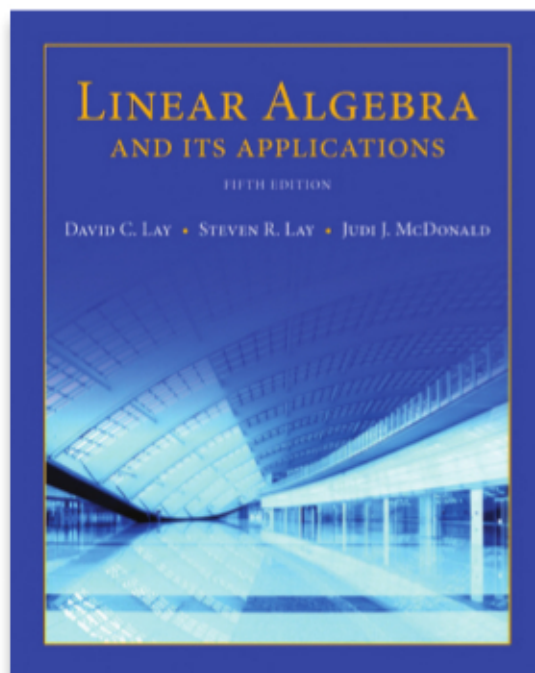
As Professor Coles mentioned, I left the NRDC and never worked in that field again; and I hadn't realised that the QR algorithm was an important technique until I was contacted by Professor Gene Golub and Professor Frank Uhlig (who is with us on the platform) in 2007, see [6].

Although I can't claim to have studied at Sussex I did work there for five years in Experimental Psychology. That is how I came to live in this part of the world with my family and I have lived in Hove for many years now. Thank you!"

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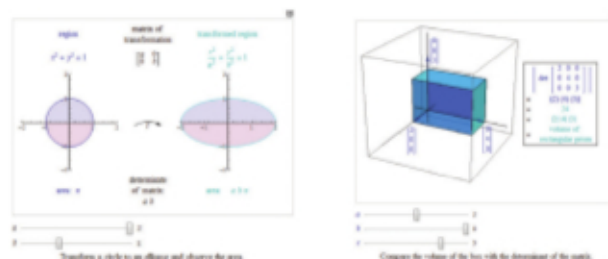
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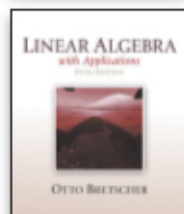
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LINEAR ALGEBRA EDUCATION

Pixar's Linear Algebra

David Borland, Tim Chartier and Tabitha Peck

RENCI, The University of North Carolina at Chapel Hill, NC, 27517, USA, Borland@renci.org

Davidson College, Davidson NC, 28035, USA, tichartier@davidson.edu, tapeck@davidson.edu

At the 2016 Academy Awards, a movie won an Oscar after dazzling millions of moviegoers with eigenvectors. Hard to believe? It is if you're imagining theaters crammed full with audiences watching the image in Figure 1(a). It's understandable when you see how eigenanalysis has helped create beloved Pixar characters like those in Figure 1(b). Indeed, essentially every frame of a Pixar film makes extensive use of linear algebra, and this includes *Inside Out*, winner of the 2016 Oscar for Best Animated Feature Film.

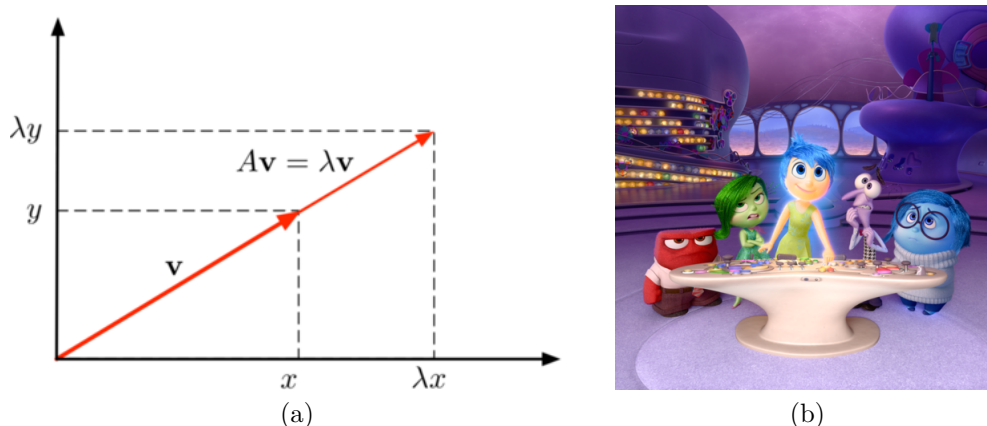


Figure 1: Eigenvectors relate to the images in (a) and (b). Image (b) ©Disney/Pixar.

Before learning how Pixar uses eigenvectors, let's discuss the overarching steps in making computer-animated movies. Animation begins with storyboarding, and with the story come characters, as seen in Figure 2(a). The characters must be designed and then built in a computer so they can be animated. The models of the characters are created with wireframe meshes as seen in Figure 2(b), and a previsualization of the characters, using flat shading, is rendered as seen in Figure 2(c). The characters are then given textures and more detailed shading, which adds a very important element of realism to Pixar's animation, as seen in Figure 2(d).

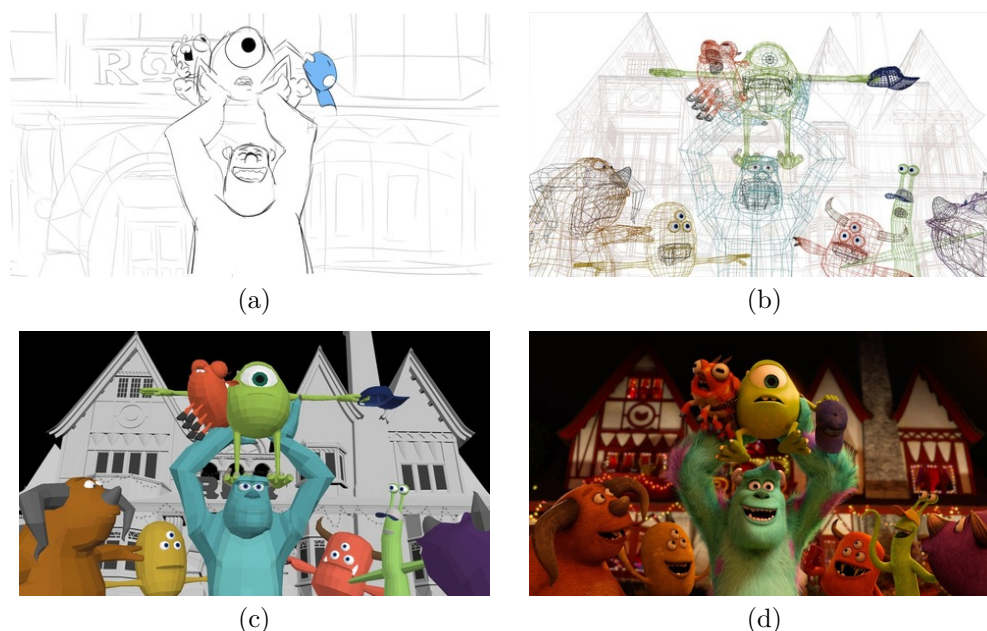


Figure 2: Steps in the process of making an animated film. Images ©Disney/Pixar.

Pixar used eigenvectors in the 1997 short film *Geri's Game*, which demonstrates their subdivision technique that continues to be used in Pixar films today. Subdivision smooths a wireframe mesh by subdividing each polygonal face of the mesh into smaller and smaller faces, giving the appearance of a smooth surface to provide greater realism. To understand subdivision, let's see how it works in 2D.

We'll begin by creating a square, as seen in Figure 3(a). Then, for each line segment, we find the midpoint. This step is called *split* and is seen in Figure 3(b). Next, we replace the original vertices of our square with points found by a weighted *average*. To begin, we'll use a 1-1 weighted average, also known as the 1-1 rule. That is, our original vertices and midpoints are each given the same weight. In the clockwise direction, we replace each vertex of our square with the average of its value and that of the next midpoint. This creates a polygon with eight points as seen in Figure 3(c). We can repeat this process. We *split* the new line segments to find the midpoints, then in the clockwise direction we replace the vertices of our current polygon with an *average* of the vertex and the next midpoint. Each *split* and *average* produces a new *subdivision*.

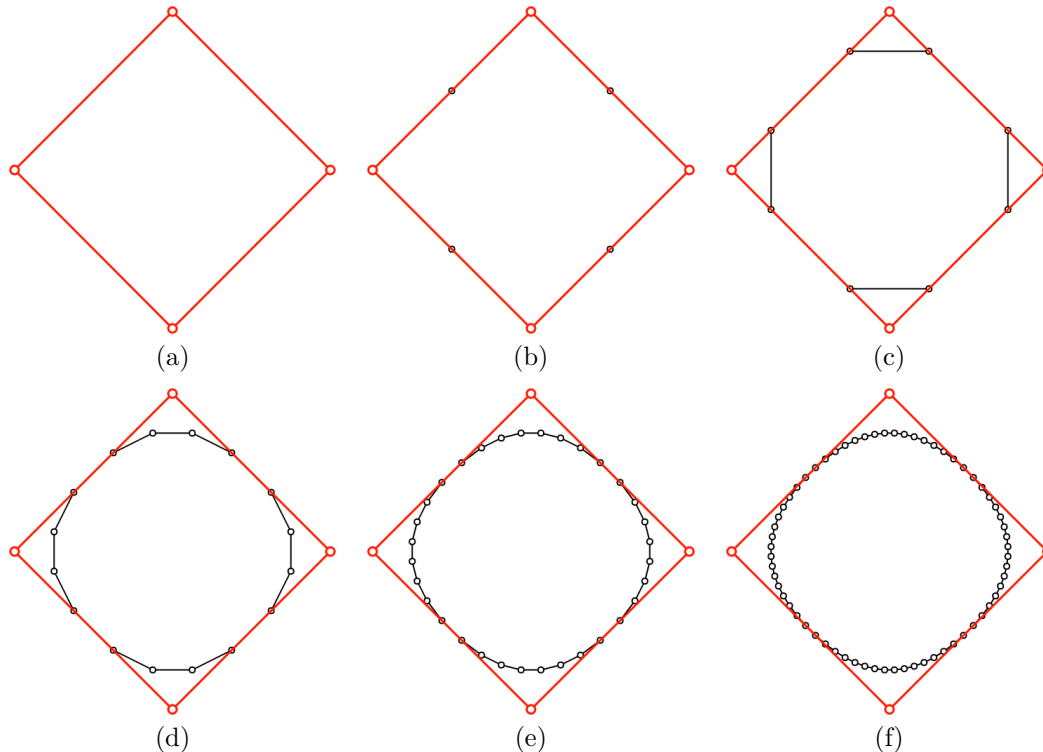


Figure 3: Starting square for subdivision (a), splitting step (b), and averaging step (c). Additional subdivision iterations produce (d), (e) and (f).

If we continue to loop through this process, we get a smoother and smoother polygon, as seen in Figure 3(d), (e) and (f), which depict successive iterates of this algorithm. In the limit, we have the smooth curve we are looking for. What can we say about the curve that would be produced after infinitely many steps? For one thing, two consecutive points get closer and closer in each iteration. So, in their limit, they meet. If you look carefully, we know at every step where two points will meet. Specifically, they'll meet at the midpoint of a line segment on the current polygon.

So we know, at every step, points that lie on that final, smooth curve. The first step gives us the midpoints of the four line segments of our polygon. At the second step, we have eight midpoints, four of which are the original four midpoints, giving us an additional four points on the final curve.

In this previous example, we used the 1-1 rule, and we have yet to use eigenanalysis. We'll get to eigenvectors in a moment, but first let's look at a 1-2-1 weighted average, a.k.a., the 1-2-1 rule. That is, for each vertex, let's take the average of the two adjacent midpoints, one in each of the clockwise and counterclockwise locations, along with two parts of the location of the vertex itself.

With this new rule, can we find a point on the final curve? With our 1-1 rule, we saw this would be the midpoint. This isn't as easily seen with the 1-2-1 rule, at least until you see how to use eigenvectors.

Let's look at a segment of the curve seen in Figure 4(a). Applying the 1-2-1 rule of subdivision creates the points A_1 ,

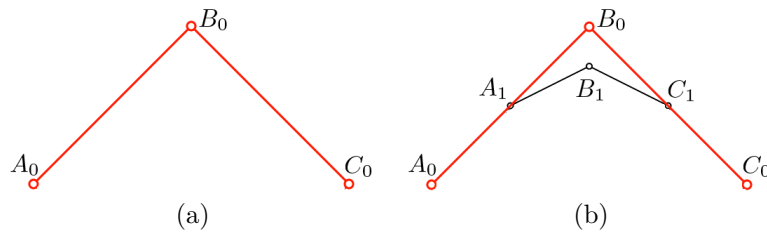


Figure 4: Steps of subdivision using the 1-2-1 rule.

B_1 , and C_1 as seen in Figure 4(b). Points A_1 and C_1 are obtained by *splitting* and are defined in Equations 1 and 2.

$$A_1 = (A_0 + B_0)/2 \quad (1)$$

$$C_1 = (B_0 + C_0)/2 \quad (2)$$

What about B_1 ? This is created with our weighted average, in this case the 1-2-1 rule. That is, B_1 is created as an average of 1 part A_1 , 2 parts B_0 , and 1 part C_1 , which equals $(A_1 + 2B_0 + C_1)/4$, or

$$B_1 = \frac{(A_0 + B_0)/2 + 2B_0 + (B_0 + C_0)/2}{4} = \frac{A_0 + 6B_0 + C_0}{8}. \quad (3)$$

Notice that Equations 1, 2, and 3 define A_1 , B_1 , and C_1 in terms of A_0 , B_0 , and C_0 .

Using our 1-2-1 rule and the Equations 1, 2, and 3, let's find A_n , B_n , and C_n with matrices. We'll compute

$$\begin{bmatrix} A_n \\ B_n \\ C_n \end{bmatrix} = P \begin{bmatrix} A_{n-1} \\ B_{n-1} \\ C_{n-1} \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/8 & 6/8 & 1/8 \\ 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} A_{n-1} \\ B_{n-1} \\ C_{n-1} \end{bmatrix}.$$

Notice, for example, that this computes A_n as an average of A_{n-1} and B_{n-1} .

We see that $P[A_0, B_0, C_0]^T = [A_1, B_1, C_1]^T$ and $P[A_1, B_1, C_1]^T = [A_2, B_2, C_2]^T$. Through substitution, we have $[A_2, B_2, C_2]^T = P^2[A_0, B_0, C_0]^T$. Continuing the pattern, you see that $[A_n, B_n, C_n]^T = P^n[A_0, B_0, C_0]^T$.

We want to know where A_n , B_n , and C_n converge as n gets bigger and bigger. First, note that as n increases, A_n , B_n , and C_n get closer together. That is, A_n , B_n , and C_n are going to converge to the same point in their limit. But where will that point be?

Here is where eigenanalysis comes in. Our matrix P can be diagonalized, such that $P = R\Lambda L$, where the columns of R are the right eigenvectors of P , the rows of L are the left eigenvectors of P , and Λ is a diagonal matrix of the eigenvalues of P . In particular,

$$R = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}, \quad \text{and} \quad L = \begin{bmatrix} 1/6 & 4/6 & 1/6 \\ -1/2 & 0 & 1/2 \\ 1/6 & -2/6 & 1/6 \end{bmatrix}.$$

Remember that we derived $[A_n, B_n, C_n]^T = P^n[A_0, B_0, C_0]^T$, but what we are interested in is what happens as $n \rightarrow \infty$. First, note that $R = L^{-1}$. Then, if we solve for P^2 , we have

$$P^2 = (RAL)^2 = RALRAL = R\Lambda^2L.$$

Following the same pattern, we see that $P^n = R\Lambda^nL$, which can be rewritten as

$$P^n = R\Lambda^nL = R \begin{bmatrix} 1^n & 0 & 0 \\ 0 & 1/2^n & 0 \\ 0 & 0 & 1/4^n \end{bmatrix} L.$$

So, as $n \rightarrow \infty$,

$$P^n = R\Lambda^nL \rightarrow R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} L.$$

Using our 1-2-1 rule, as $n \rightarrow \infty$,

$$P^n \rightarrow \begin{bmatrix} 1/6 & 4/6 & 1/6 \\ 1/6 & 4/6 & 1/6 \\ 1/6 & 4/6 & 1/6 \end{bmatrix}.$$

This is just a matrix of the left eigenvector of P with the largest eigenvalue! This eigenvector becomes the coefficients of a weighted average to find values on the final, smooth curve. To see how, consider $(A_1 + 4B_1 + C_1)/6$. Using Equations 1, 2, and 3 and making substitutions, we get

$$\frac{A_1 + 4B_1 + C_1}{6} = \frac{(A_0 + B_0)/2 + (A_0 + 6B_0 + C_0)/2 + (C_0 + B_0)/2}{6} = \frac{A_0 + 4B_0 + C_0}{6}.$$

So, $(A_1 + 4B_1 + C_1)/6 = (A_0 + 4B_0 + C_0)/6$. This implies that $(A_2 + 4B_2 + C_2)/6 = (A_0 + 4B_0 + C_0)/6$, which further implies $(A_n + 4B_n + C_n)/6 = (A_0 + 4B_0 + C_0)/6$.

Let's call A_∞ the point that A_n is converging to as n grows. Our work shows that $(A_\infty + 4B_\infty + C_\infty)/6 = (A_0 + 4B_0 + C_0)/6$. Now, remember, A_∞ and C_∞ are converging to B_∞ since the points get closer and closer to each other with every step. So, $(B_\infty + 4B_\infty + B_\infty)/6 = (A_0 + 4B_0 + C_0)/6$, or, $B_\infty = (A_0 + 4B_0 + C_0)/6$. In other words, we know in one step what point will lie on the final, smooth curve. For the 1-1 rule we saw earlier, this was the midpoint. Now we have a formula for the 1-2-1 rule, which came from computing an eigenvector.

This is how Pixar creates surfaces for its movies. In what parts of a Pixar film do we see the result of eigenanalysis? Every character you've seen since *Geri's Game* has been a product of eigenanalysis. This significant step in animation changed how characters were designed. Initial wireframes, like that of Geri's hand in Figure 5(a), which is analogous to the images in Figure 2(b) and (c), could be smoothed through subdivision. Applying subdivision to Geri's hand in Figure 5(a) produces the image in (b).

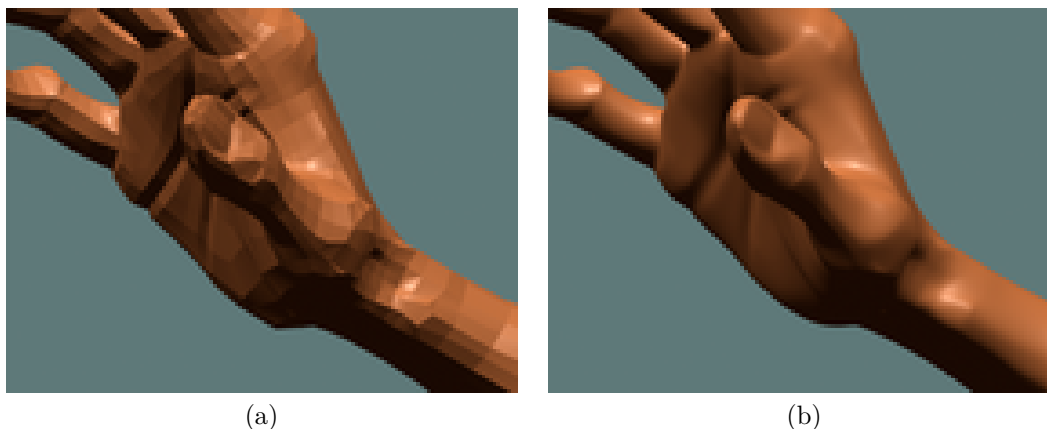


Figure 5: The hand of Geri in Pixar's animated short *Geri's Game*, before (a) and after (b) subdivision. Images ©Disney/Pixar.

Grab a Pixar movie like *The Incredibles*, *Finding Nemo*, *Up*, *Brave*, *Toy Story 3*, or *Monsters University*. Or go to the theater and see one of the recent releases. Enjoy the film and, of course, enjoy seeing eigenvectors, or at least the result of eigenanalysis on the screen! Want to try it yourself? Visit <http://math365.org/lifeislinear/Subdivision/Subdivision.html> and you can experiment with subdivision, and even animate your shapes and become, in a sense, your own animation studio.

BOOKS

Books on Linear Algebra and Related Topics Published in 2015 & 2016

Compiled by *IMAGE* Book Review Editor Douglas Farenick¹

1. Aron, Richard M.; Bernal González, Luis; Pellegrino, Daniel M.; Seoane Sepúlveda, Juan B. *Lineability: The Search for Linearity in Mathematics*. Monographs and Research Notes in Mathematics. CRC Press, Boca Raton, FL, 2016. xix+308 pp. ISBN: 978-1-4822-9909-0
2. Axler, Sheldon. *Linear Algebra Done Right. Third edition*. Undergraduate Texts in Mathematics. Springer, Cham, 2015. xviii+340 pp. ISBN: 978-3-319-11079-0; 978-3-319-11080-6
3. Bean, William Clifton. *Certain Extensor Structures in the Calculus of Variations*. YBK Publishers, New York, 2015. xxv+206 pp. ISBN: 978-1-936411-40-5
4. Björck, Ake. *Numerical Methods in Matrix Computations*. Texts in Applied Mathematics, 59. Springer, Cham, 2015. xvi+800 pp. ISBN: 978-3-319-05088-1; 978-3-319-05098-8
5. Cooperstein, Bruce N. *Advanced Linear Algebra. Second edition*. Textbooks in Mathematics. CRC Press, Boca Raton, FL, 2015. xxvii+594 pp. ISBN: 978-1-4822-4884-5
6. Duan, Guang-Ren. *Generalized Sylvester Equations. Unified Parametric Solutions*. CRC Press, Boca Raton, FL, 2015. xxiii+441 pp. ISBN: 978-1-4822-4396-3
7. Ford, William. *Numerical Linear Algebra with Applications. Using MATLAB*. Elsevier/Academic Press, Amsterdam, 2015. xxvi+602 pp. ISBN: 978-0-12-394435-1
8. Friedland, Shmuel. *Matrices—Algebra, Analysis and Applications*. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2016. xii+582 pp. ISBN: 978-981-4667-96-8
9. Gatto, Letterio. *Generic Linear Recurrent Sequences and Related Topics*. Publicações Matemáticas do IMPA. [IMPA Mathematical Publications] 30o Colóquio Brasileiro de Matemática. [30th Brazilian Mathematics Colloquium] Instituto Nacional de Matemática Pura e Aplicada (IMPA), Rio de Janeiro, 2015. x+131 pp. ISBN: 978-85-244-0405-4
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11. Hackbusch, Wolfgang. *Hierarchical Matrices: Algorithms and Analysis*. Springer Series in Computational Mathematics, 49. Springer, Heidelberg, 2015. xxv+511 pp. ISBN: 978-3-662-47323-8; 978-3-662-47324-5
12. Janežič, Dušanka; Miličević, Ante; Nikolić, Sonja; Trinajstić, Nenad. *Graph-Theoretical Matrices in Chemistry. Second edition*. CRC Press, Boca Raton, FL, 2015. xiv+160 pp. ISBN: 978-1-4987-0115-0
13. Kanel-Belov, Alexei; Karasik, Yakov; Rowen, Louis Halle. *Computational Aspects of Polynomial Identities. Vol. 1. Kemer's Theorems. Second edition*. Monographs and Research Notes in Mathematics. CRC Press, Boca Raton, FL, 2016. xxvi+418 pp. ISBN: 978-1-4987-2008-3
14. Kapitula, Todd. *Ordinary Differential Equations and Linear algebra. A Systems Approach*. Society for Industrial and Applied Mathematics, Philadelphia, PA, 2015. xi+300 pp. ISBN: 978-1-611974-08-9
15. Klawitter, Daniel. *Clifford Algebras. Geometric Modelling and Chain Geometries with Application in Kinematics. With a foreword by Gunter Weiss*. Dissertation, TU Dresden, Dresden, 2014. Springer Spektrum, Wiesbaden, 2015. xviii+216 pp. ISBN: 978-3-658-07617-7; 978-3-658-07618-4
16. Liesen, Jörg; Mehrmann, Volker. *Linear Algebra*. Springer Verlag, 2015. xi + 324 pp. ISBN: 978-3-319-24344-3
17. Maclagan, Diane; Sturmfels, Bernd. *Introduction to Tropical Geometry*. Graduate Studies in Mathematics, 161. American Mathematical Society, Providence, RI, 2015. xii+363 pp. ISBN: 978-0-8218-5198-2

¹A list of linear algebra books dating back to 2002 is posted on the ILAS website: <http://www.ilasic.org/IMAGE/IMAGES/booklist.pdf>

18. Nassif, Nabil; Erhel, Jocelyne; Philippe, Bernard. *Introduction to Computational Linear Algebra*. CRC Press, Boca Raton, FL, 2015. xxiv+235 pp. ISBN: 978-1-4822-5869-1
19. Ricci, Paolo Emilio. *Forme Canoniche e Funzioni di Matrice*. (Italian) [Canonical forms and functions of matrices] Teoria de Jordan. [Jordan theory] Aracne Editrice, Rome, 2015. x+92 pp. ISBN: 978-88-548-8039-9
20. Schäffer, Juan Jorge. *Linear Algebra*. World Scientific Publishing Co. Pte. Ltd., Hackensack, NJ, 2015. viii+132 pp. ISBN: 978-981-4623-49-0
21. Seagar, Andrew. *Application of Geometric Algebra to Electromagnetic Scattering*. The Clifford-Cauchy-Dirac technique. Springer, Singapore, 2016. xxii+179 pp. ISBN: 978-981-10-0088-1; 978-981-10-0089-8
22. Shapiro, Helene. *Linear Algebra and Matrices. Topics for a Second Course*. Pure and Applied Undergraduate Texts, 24. American Mathematical Society, Providence, RI, 2015. xv+317 pp. ISBN: 978-1-4704-1852-6
23. Strang, Gilbert. *Introduction to Linear Algebra. Fifth Edition*. Wellesley-Cambridge Press, Wellesley, MA, 2016. ISBN: 978-09802327-7-6 (Book website: <http://math.mit.edu/linearalgebra>)
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27. *The Oxford Handbook of Random Matrix Theory*. Edited by Gernot Akemann, Jinho Baik and Philippe Di Francesco. Paperback edition of the 2011 original [MR2920518]. Oxford University Press, Oxford, 2015. xxxi+919 pp. ISBN: 978-0-19-874419-1
28. *Matrix Functions and Matrix Equations*. Summaries from the lectures of the 4th Gene Golub SIAM Summer School held at Fudan University, Shanghai, July 22–August 2, 2013. Edited by Zhaojun Bai, Weiguo Gao and Yangfeng Su. Series in Contemporary Applied Mathematics CAM, 19. Higher Education Press, Beijing; World Scientific Publishing Co., Singapore, 2015. vii+137 pp. ISBN: 978-981-4675-76-5
29. *Numerical Algebra, Matrix Theory, Differential-Algebraic Equations and Control Theory. Festschrift in honor of Volker Mehrmann*. Edited by Peter Benner, Matthias Bollhöfer, Daniel Kressner, Christian Mehl and Tatjana Stykel. Springer, Cham, 2015. xlv+608 pp. ISBN: 978-3-319-15259-2; 978-3-319-15260-8

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We thank Bojan Kuzma for his service to *IMAGE* as Problem Corner Editor since issue 48.



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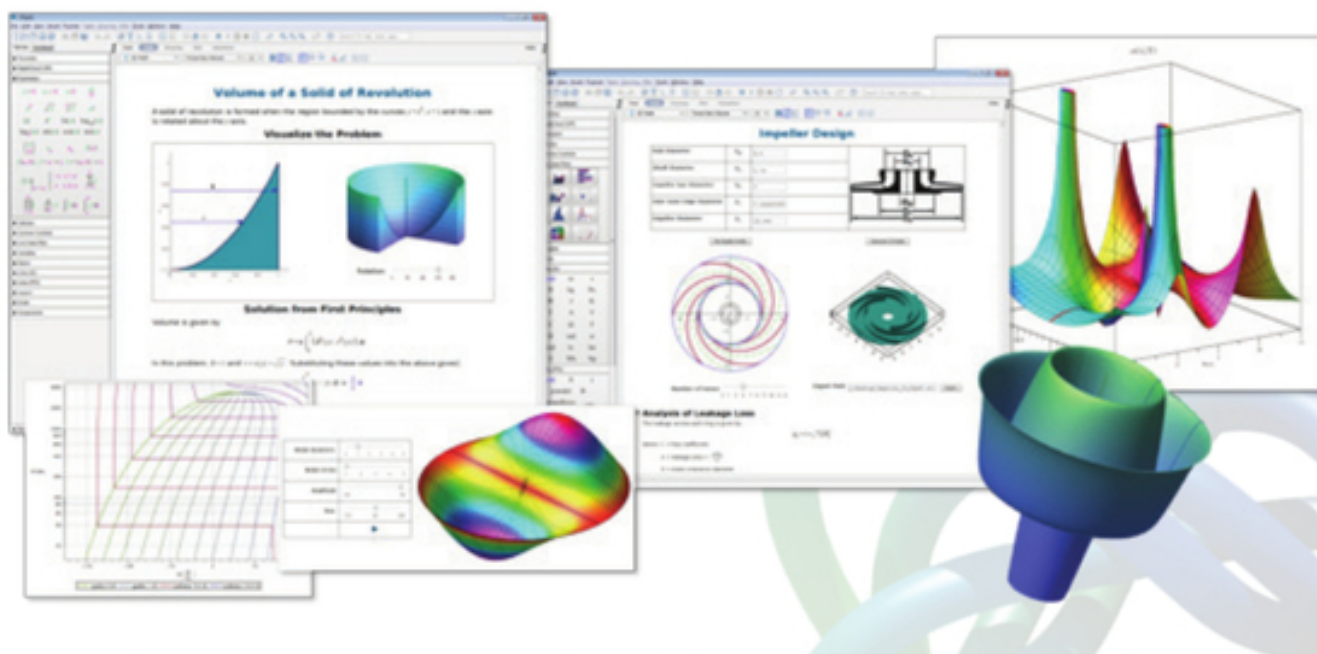
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OBITUARY

Ingram Olkin (1924–2016): Some Personal Memories¹

Simo Puntanen and George P. H. Styan²



Left to right: At IWMS-2011 Tartu, Estonia (photograph by Jeffrey J. Hunter); at IWMS-1995 Montréal (with Gene H. Golub and T. W. Anderson, photograph by Simo Puntanen); at IWMS-1990 Tampere (with Jerzy K. Baksalary & Yadolah Dodge, photograph courtesy University of Tampere); at IWMS-1995 Montréal (photograph by Simo Puntanen).

Ingram Olkin, Professor Emeritus of Statistics and Education at Stanford University, master of multivariate statistical analysis, linear algebra, inequalities, majorization, and meta-analysis, passed away on 28 April 2016 at home in Palo Alto, California, after complications from colon cancer. In the words of his daughter, Julia Olkin [4]:

“My father, Ingram Olkin, died peacefully on Thursday evening, April 28, 2016, with his daughter Rhoda and wife Anita by his side. He had absolutely no regrets ... both personally and professionally, and led a full, wonderful life. He valued all his friendships with everyone. Thank you for being a part of his life ...”

Richard W. Cottle, Professor Emeritus of Management Science & Engineering and a close friend of Olkin, said [2]:

“He was a man of remarkable intelligence and affability. His nearly boundless energy was generously used for the welfare of others. It is hard to capture in words the goodness that Ingram showed in his everyday life.”

In the conversation part of the *Olkin Festschrift* [5], Ingram described himself:

“You also know that I’m generally a people person, which is one of the reasons why I’ve enjoyed students and collaborators. Over the years, the professional contacts have merged with the personal contacts.”

We deeply miss you, a truly outstanding and unforgettable *People Person*, Ingram Olkin.

1. IWMS.

Now let’s go back to some personal memories of Ingram and joint experiences that we shared with him. One important activity for us was his role in the International Workshop on Matrices and Statistics (IWMS) series [23]. Ingram was a frequent participant at IWMS meetings, and at the IWMS-2004 in Poland we celebrated Ingram’s 80th Birthday. On 4 June 2003 his reply to our invitation was this e-mail: (Ingram usually used only lower-case letters in his e-mails.)

“dear all ... wow !!!! how about celebrating my 80th but call it my 60th ... thanks so much to all of you ... would be pleased to attend.”

When Ingram learnt that the IWMS-2014 was to be held in Ljubljana, he immediately, on 22 October 2013, sent this e-mail:

“ ... in any case next year is my 90th and what better than to visit ljubljana ... so i do hope to attend. as i see my strength at this point i should be in good shape by then. so please include me in the program.”

¹A slightly longer version has been prepared for the *Abstracts Booklet* for IWMS-2016 in Funchal, Madeira, Portugal, June 6–9, 2016.

²Simo Puntanen, School of Information Sciences, FI-33014 University of Tampere, Finland simo.puntanen@uta.fi. George P. H. Styan, Department of Mathematics and Statistics, McGill University, 805 ouest, rue Sherbrooke Street West, Montréal (Québec), Canada H3A 0B9 geostyan@gmail.com.

It was always great news for the event organizers that Ingram would be around: a guarantee of lively colorful sessions, Ingram sitting in the front row and asking questions after each talk. Ingram's role in meetings is nicely described in the Olkin-biography article [6]:

“At most statistics meetings, you will find Ingram in constant conversation—perhaps promoting a new journal, encouraging progress of a key committee, or giving advice about seeking grants or allocating funds. His public accomplishments are many and impressive, but equally important are his behind-the-scenes contributions.”

The first IWMS was held in Tampere, Finland, 6–8 August 1990. Ingram gave an invited talk entitled *Interface between statistics and linear algebra*, which was one of his favorite topics and he practically knew everything about it [17, 19]. For the IWMS-2013 in Toronto he prepared an excellent “linear algebra biography”, which was presented there as a poster; see also [20] (2015):

“I gave a brief biography of my introduction to linear algebra and my interaction with some of the linear algebraists at that time.”

At the IWMS-1990 in Tampere, Ingram also gave a talk about Gustav Elfving (1908–1984), a famous Finnish statistician, probabilist and mathematician who was a frequent visitor to Stanford. Ingram's performances in Tampere in 1990 can be seen in videos online at YouTube [29]. When we asked for Ingram's permission to show these videos, he replied:

“these are wonderful ... an absolutely great addition to the conference archives. however, you ask for me to give permission to make these public. the answer is in the negative unless you can add some hair and make me look more like james bond. of course, if you do that then i would be glad to grant permission !!!!!”

The IWMS-2008 was held in Tomar, Portugal (July 22–26, 2008) in celebration of the 90th birthday of T. W. Anderson, mentor of George and grand-mentor of Simo, and a long-time Stanford colleague of Ingram's. We invited Ingram as an after-dinner speaker. Unfortunately, Ingram was unable to attend the IWMS-2008 in Tomar.

Section 20.5 in our 2011 book *Matrix Tricks* [24] deals with *How Deviant Can You Be?* — the deviation of any particular observation from the mean, building on Ingram's paper [18, (1992)] and Jensen & Styan [11].

In December 2011 we (Simo & George) had an interesting and pleasant task: we were to prepare a supporting letter to nominate Ingram Olkin for the Hans Schneider Prize in Linear Algebra. For additional support, we contacted Grace Wahba, Professor of Statistics at the University of Wisconsin–Madison, and on December 31, 2011 she wrote us:

“I wholeheartedly support the proposal that Ingram Olkin be considered for the Hans Schneider Prize in Linear Algebra. Absolutely he has to get it!”

Though Ingram did not ultimately receive this particular Prize, on 2 August 2012, he kindly sent us a thank-you e-mail:

“simo: thanks for your message and in particular i forgot about the award ... however, i am signing George up to write my obituary (assuming he outlives me !!!!!!!) ... I can always count on him. my best, ingram.”

2. Inequalities: Theory of Majorization.

In our supporting letter for the Hans Schneider Prize we pointed out that in our view Ingram's most significant contribution in linear algebra was the book *Inequalities: Theory of Majorization and Its Applications*, with Albert W. Marshall, first published in 1979 [14]. We now have the second edition, with Barry Arnold [15], of the highly-praised classic, without which we know that some people never leave home; now these faithful ones must take into account that the second edition has 909 pages (vs. 569) and its shipping weight is 3.2 pounds (vs. 2.2).

At the end of the first edition of *Inequalities: Theory of Majorization and Its Applications* [14] there is a section on “Biographies” with a photograph of Issai Schur (1875–1941) on page 525. This was the first photograph of Schur that we found and George used it, with the permission of the “publisher and the authors” of [14], in his article on “Schur complements and linear statistical models” [28, (1983/1985)]; see also [21, 22].

Fuzhen Zhang wrote us on 11 May 2016:

“Dating back to 1984, I went to Beijing Normal University as a graduate student. The first math book in English we used as a text was Ingram's (with Marshall), the 1st edition. I learned and benefited so much from the book. The book has become classical, famous and standard as a reference in this area of research. In 2012, I had the privilege of writing a review for the 2nd edition of the book (published in [30]).”

Ingram had a number of Chinese connections, among them was Kai-Tai Fang who in [13, p. 16] tells the following, which is a nice example of Ingram's organizational generosity!

"During my visit to Stanford University (1981–1982), Professor Ingram Olkin organized a small seminar group on 'multivariate multiple comparisons' which met every week. The participants included T. W. Anderson, Mary Ellen Bock, Zhongguo Cheng and me. ... Then in 1985–1986, upon Professor Ingram Olkin's recommendation, I taught two subjects in the Swiss Federal Institute (ETH, Zürich) as a Guest Professor."

George thinks that he first met Ingram at a colloquium in the Department of Mathematical Statistics at Columbia University in the mid-1960s, and at that time may well have served Ingram a cup of tea! Ingram then introduced George to "correlation structure", such as when all the correlation coefficients are equal (intraclass correlation) but the variances are not necessarily all equal. This led to George's Ph.D. thesis [26, (1969)]. See also Ingram's paper on "correlations revisited" (with discussion) [16].

George spent the summer of 1970 at Stanford and he believes it was probably there that Ingram introduced him to the seminal paper by Fan & Hoffman [3, (1955)] in which it is proved that for any $n \times n$ matrix A

$$\lambda_j \left(\frac{A + A^*}{2} \right) \leq \lambda_j^{1/2}(AA^*), \quad j = 1, 2, \dots, n. \quad (4)$$

Here $\lambda_j(A)$ denotes the j th largest eigenvalue of A . See also Marshall & Olkin [14, p. 240, eq. 4]. The inequalities (4) were then used by Grossman & Styan in their article on Theil's BLUS residuals [7, (1972)]. And last, but not least, George is most grateful to Ingram for supporting George's appointment as Editor of *The IMS Bulletin*, 1987–1992 [27].

In the "Biographies" section (pp. 528–529) of *Inequalities: Theory of Majorization and Its Applications* [14] there are three photographs of Godfrey Harold Hardy (1877–1947), who with John Edensor Littlewood (1885–1977) and George Pólya (1887–1985) wrote the seminal book *Inequalities* [8], first published in 1934. G. H. Hardy is featured in *The Man Who Knew Infinity* [1], a recent British biographical drama film based on the book [12, (1991)] by Robert Kanigel. The film stars Dev Patel as Srinivasa Ramanujan (1887–1920) and Jeremy Irons as G. H. Hardy. In his movie review, Allan Hunter [9] wrote:

"It tells such a good story that it is hard to resist."

We agree! Would a movie about Ingram, *The Man Who Knew Inequalities: Theory of Majorization*, similarly make a good story, hard to resist?

Acknowledgements.

Warm thanks go to Kai-Tai Fang, Michael Greenacre, Jeffrey J. Hunter, Peter Šemrl, Evelyn Matheson Styan, Kimmo Vehkalahti, Grace Wahba, and Fuzhen Zhang for their help.

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Obituary Note: Marvin David Marcus (July 31, 1927–February 20, 2016)

Marvin David Marcus (July 31, 1927–February 20, 2016) passed away in his sleep after a long struggle with Alzheimer's. He received his Ph.D. in mathematics from the University of California, Berkeley in 1950. While Marvin was a young professor at the University of British Columbia, he supervised the master's degree of R.C. Thompson. Marvin joined the University of California, Santa Barbara in 1962, first in the Department of Mathematics, then with joint appointments in Mathematics and Computer Science. He founded the Microcomputer Laboratory in 1978 and served as Director until he retired in 1991. He was a prolific author of over forty books, including the well-known *A Survey of Matrix Theory and Matrix Inequalities* with H. Minc, and over 200 articles. He is known for foundational work in several areas of linear algebra, including linear preserver problems, matrix inequalities, and permanents. Along with R.C. Thompson, he founded the journal *Linear and Multilinear Algebra* in 1973. He supervised 18 Ph.D. students, including current ILAS members Russell Merris and William Watkins.

JOURNAL ANNOUNCEMENTS

LAMA Special Issue in Honor of Marvin Marcus

The linear and multilinear algebra community is sad to lose Marvin Marcus, who had made substantial contributions on various topics on the subject, developed human resources by training and attracting many researchers to the research area, and co-founded the journal *Linear and Multilinear Algebra*. A special issue of *Linear and Multilinear Algebra* will be devoted to the memory of Marvin Marcus. Colleagues are encouraged to submit a paper to the special issue by September 30, 2016.

The special editors for this issue are: Shmuel Friedland, University of Illinois - Chicago, friedlan@uic.edu, Thomas Pate, Auburn University, pate_tom@bellsouth.net, and Yiu-Tung Poon, Iowa State University, ytpoon@iastate.edu.

Submissions should be done through the website: <https://mc.manuscriptcentral.com/glma>.

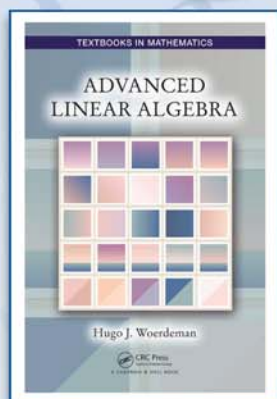
ELA Editorial Board Changes

ELA is pleased to announce that upon the recommendation of the ILAS Journal Committee, the ILAS Board of Directors appointed Michael Tsatsomeros (Washington State University) to join Bryan Shader (University of Wyoming) as co Editor-in-Chief starting March 1, 2016.

In addition, recent additions to the *ELA* Editorial Board are: Dario Bini (Universita di Pisa), Sebastian M. Cioabă (University of Delaware), Geir Dahl (University of Oslo), Froilán Dopico (Universidad Carlos III de Madrid), Torsten Ehrhardt (University of California–Santa Cruz), Zejun Huang (Hunan University), Sergei Sergeev (University of Birmingham), Ilya Spitkovsky (New York University Abu Dhabi and the College of William and Mary), and Françoise Tisseur (University of Manchester).

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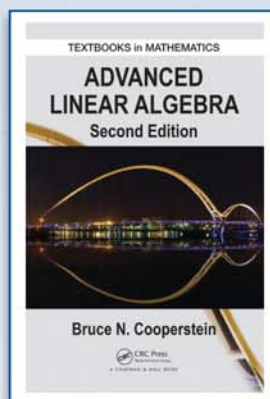
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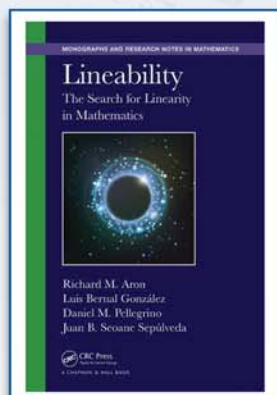
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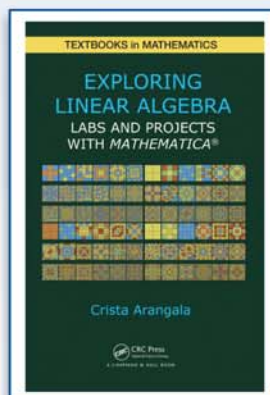
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Richard M. Aron, Luis Bernal-Gonzalez, Daniel M. Pellegrino, Juan B. Seoane Sepulveda

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ISBN: 9781482290900, **\$109.95 / £77.90**



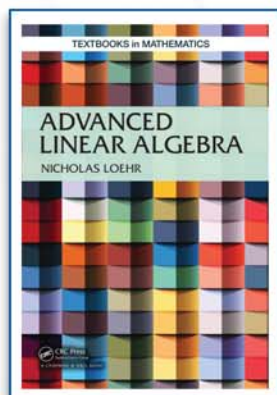
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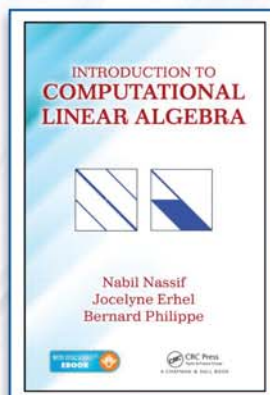
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UPCOMING CONFERENCES AND WORKSHOPS

IWMS 2016 – International Workshop on Matrices and Statistics Funchal, Madeira, Portugal, June 6–9, 2016



The Conference will be held in Funchal, Madeira (Portugal) from June 6 to 9, 2016. Up-to-date information is available at <http://www.iwms.ipt.pt>.

The purpose of the workshop is to bring together researchers sharing an interest in a variety of aspects of statistics and its applications as well as matrix analysis and its applications to statistics, and to offer them an opportunity to discuss current developments in these subjects. The workshop will bridge the gap among statisticians, computer scientists and mathematicians in understanding each other's tools.

We anticipate that the workshop will stimulate research, in an informal setting, and foster the interaction of researchers in the interface between matrix theory and statistics.

Some emphasis will be put on related numerical linear algebra issues and numerical solution methods relevant to problems arising in statistics.

The workshop will include invited talks and special sessions devoted to cutting-edge research topics.



Contact: Francisco Carvalho | E-mail: iwms@ipt.pt | Website: <http://www.iwms.ipt.pt>

IMA Math-to-Industry Boot Camp University of Minnesota, USA, June 20 – July 29, 2016

The Institute for Mathematics and its Applications is hosting a six-week “boot camp” designed to provide graduate students with training and experience that is valuable for employment outside of academia. The program is targeted at Ph.D. students in pure and applied mathematics and consists of courses in the basics of programming, data analysis, and mathematical modeling. Students will work in teams on projects, be provided with soft skills training, and interact with industry participants.

More information and the online application are available at <http://www.ima.umn.edu/2015-2016/SW6.20-7.29.16/>.

5th ALAMA Conference on Linear Algebra, Matrix Analysis and Applications University of Leon, Spain, June 20–22, 2016

The 5th ALAMA Meeting will be held June 20–22, 2016 in Leon, Spain. ALAMA is the Spanish network that gathers together scientists interested in Linear Algebra, Matrix Analysis and Applications. Like in the previous editions, the aim of the conference is to bring together for three days, in an environment favouring work and exchange of ideas, researchers, whether they are members of the Network or not, whose work has some connection, one way or the other, with linear algebra, matrix analysis, matrix theory and/or their possible applications in different contexts. We thus hope to cover a wide spectrum of interests, both from a theoretical point of view and from a more numerical and/or computational viewpoint, not forgetting the eventual practical applications.

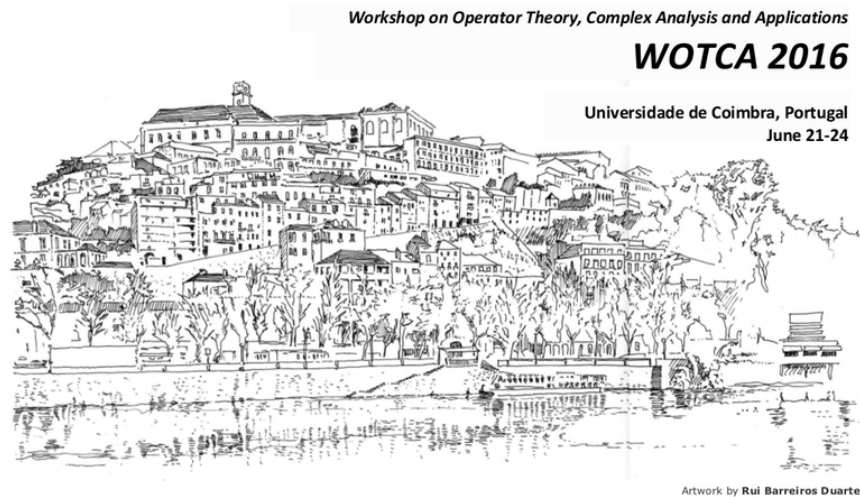
The invited speakers are: André Ran (Vrije Universiteit of Amsterdam), Valeria Simoncini (Università di Bologna), Steve Mackey (Western Michigan University), Raquel Pinto (Universidade de Aveiro), and Alberto Hernández (INCIBE - Spanish Cybersecurity National Institute).

The Scientific Committee consists of: Miguel Carriegos (Universidad de Leon), Andreas Frommer (University of Wuppertal), José Angel Hermida (Universidad de Leon), Montserrat Lopez (Universidad de Leon), Françoise Tisseur (University

of Manchester), Paul Van Dooren (University of Leuven), and Ion Zaballa (University of Basque Country).

The conference will be hosted by the University of Leon. Talks will take place at La Real Colegiata de San Isidoro. Further information can be found at the conference web page: <http://alama.unileon.es>.

2016 International Workshop on Operator Theory, Complex Analysis and Applications (WOTCA 2016) Coimbra, Portugal, June 21–24, 2016



The Workshop on Operator Theory, Complex Analysis and Applications 2016 –WOTCA 2016– will take place at the University of Coimbra, Portugal, on 21–24 June 2016.

It aims to bring together researchers working in Operator Theory, Complex Analysis, and their applications, and to create an opportunity to highlight the current state of the art in these fields, present open problems and engage in fruitful discussions. Connections with Linear Algebra will also be emphasized.

This meeting follows the line of four previous workshops, WOTCA 2010, WOTCA 2012, 2-Day OTCA 2013 and WOTCA 2014.

The programme will consist of several invited 25+5 minute talks and a poster session.

WOTCA 2016 has no registration fee and is open to everybody interested, but the number of participants will be restricted to a maximum of 50, in order to foster useful dialogue in a friendly atmosphere. Participation of young researchers and students is most welcome, and we hope this will also encourage contact between leading mathematicians and younger ones.

This meeting is supported by CAMGSD – Center for Mathematical Analysis, Geometry and Dynamical Systems of IST and CMUC – Center for Mathematics of the University of Coimbra.

The Organizing Committee consists of Cristina Câmara (Instituto Superior Técnico, University of Lisbon), Cristina Diogo (ISCTE – Lisbon University Institute), Teresa Malheiro (University of Minho) and Ana Nata (Polytechnic Institute of Tomar).

The invited speakers are Natália Bebiano (Portugal), Chafiq Benhida (France), Gabriel Lopes Cardoso (Portugal), Gustavo Corach (Argentina), Raul Curto (USA), Michael Dritschel (UK), Piotr Dymek (Poland), Eva Gallardo Gutiérrez (Spain), Celeste Gouveia (Portugal), João Gouveia (Portugal), Andreas Hartmann (France), Håkan Hedenmalm (Sweden), Zenon Jablonski (Poland), Charles Johnson (USA), Karim Kellay (France), László Kérchy (Hungary), Kamila Kliš-Garlicka (Poland), David Krejcirik (Czech Republic), Alejandra Maestripieri (Argentina), Sérgio Mendes (Portugal), Boris Mityagin (USA), Susana D. Moura (Portugal), Vladimir Müller (Czech Republic), Lina Oliveira (Portugal), Jonathan Partington (UK), Pawel Pietrzycki (Poland), Artur Planeta (Poland), Marek Ptak (Poland), João Filipe Queiró (Portugal), William T. Ross (USA), Petr Siegl (Switzerland), Frantisek Stampach (Sweden), Jan Stochel (Poland), Franciszek Hugon Szafraniec (Poland), Brett D. Wick (USA), and Rongwei Yang (USA).

For further details, please visit the conference webpage at <https://wotca16.math.tecnico.ulisboa.pt>.

2016 Workshop on Matrices and Operators (MAO 2016) Jeju-do, Korea, July 3–6, 2016

The 2016 Workshop on Matrices and Operators (MAO 2016) will be held July 3–6, 2016. The purpose of the workshop is to stimulate research and foster interaction of researchers interested in matrix theory, operator theory, and their applications. Hopefully, the informal workshop atmosphere will ensure the exchange of ideas from different research areas. The conference venue will be the Suites Hotel Jeju, Saekdal-dong, Seogwipo-si, Jeju-do, Korea. The MAO workshops have been held annually since 2007.

The organizing committee consists of Chi-Kwong Li, College of William and Mary, USA; Yongdo Lim, Sungkyunkwan University, Korea; Hyunmin Kim, Pusan National University, Korea; Hosoo Lee, Sungkyunkwan University, Korea; and Sejong Kim, Chungbuk National University, Korea.

Further information can be found at <http://shb.skku.edu/mao2016>.

International Workshop on Operator Theory and Operator Algebras (WOAT 2016) University of Lisbon, Portugal, July 5–8, 2016

The International Workshop on Operator Theory and Operator Algebras (WOAT 2016) will be held July 5–8, 2016 at the Instituto Superior Técnico, University of Lisbon, Portugal. This workshop continues a series of conferences organized in Lisbon since 2006 and aims to stimulate communication between researchers in Operator Theory and Operator Algebras. WOAT 2016 will also include two special sessions on Matrix Theory and Applications.

More details can be found at the conference webpage: <https://woat2016.math.tecnico.ulisboa.pt>.

The 20th ILAS Conference Leuven, Belgium, July 11–15, 2016

The 20th Conference of the International Linear Algebra Society (ILAS) will take place July 11–15, 2016 at KU Leuven, Leuven, Belgium.

The Invited Plenary Speakers are

- Marco Donatelli, Insubria University (Italy)
- Paul van Dooren, Catholic University of Louvain (Belgium)
- Lieven De Lathauwer, KU Leuven (Belgium)
- Monique Laurent, Centrum Wiskunde & Informatica (Netherlands)
- Elizabeth Meckes, Case Western Reserve University (USA)
- Lajos Molnar [*LAMA* speaker, supported by Taylor & Francis], Bolyai Institute (Hungary)
- Pablo A. Parrilo [*LAA* speaker, supported by Elsevier], Massachusetts Institute of Technology (USA)
- André Ran, Vrije Universiteit of Amsterdam (Netherlands) and
- Fernando de Terán [*SIAG/LA* speaker] Universidad Carlos III de Madrid (Spain).



The current list of Invited Minisymposia with organizers is:

- Data-Driven Model Reduction, by Athanasios C. Antoulas;
- Matrix Equations, by Peter Benner and Beatrice Meini;
- Tropical Algebra in Numerical Linear Algebra, by James Hook, Jennifer Pestana, and Françoise Tisseur;
- Matrix Inequalities and Operator Means, by Jean-Christophe Bourin and Takeaki Yamazaki;
- Linear Algebra and Quantum Computation, by Chi-Kwong Li, Raymond Sze, and Yiu Tung Poon;
- Matrix Methods for Solving Systems of Multivariate Polynomial Equations, by Bernard Mourrain, Vanni Noferini, and Marc Van Barel;
- Image Restoration and Reconstruction, by Marco Donatelli and Jim Nagy;
- Matrix Methods in Network Analysis, by Francesco Tudisco and Dario Fasino; and
- Low-Rank Tensor Approximations, by André Uschmajew and Bart Vandereycken.

The current list of Contributed Minisymposia with organizers is:

- Combinatorial Matrix Theory, by Adam Berliner, Louis Deaett, and Craig Erickson;

- Crouzeix's Conjecture, by Michael Overton;
- Eigenvalue Computations and Applications, by Ren-Cang Li, Wen-Wei Lin, and Lei-Hong Zhang;
- Generalizations of the Strong Arnold Property and the Inverse Eigenvalue Problem of a Graph, by Leslie Hogben and Bryan Shader;
- Generalized Inverse and its Applications, by Dragana Cvetkovic Ilic and Yimin Wei;
- Geometry and Order Structure in Matrices and Operators, by Fumio Hiai;
- Ideas and Issues in Linear Algebra Education, by Sepideh Stewart and David Strong;
- Matrix structures and univariate polynomial rootfinding, by Jared L. Aurentz and Paola Boito;
- Nonnegative Matrices and Majorization, by Richard A Brualdi and Geir Dahl;
- Numerical Solution and Preconditioning for Augmented Linear Systems, by Yu-Mei Huang, Rui-Ping Wen, and Xi Yang;
- Perturbation theory and distance problems associated with eigenvalues, by Shreemayee Bora and Christian Mehl;
- Polynomial and Rational Eigenvalue Problems, by Javier Perez Alvaro and Nikta Shayanfar;
- Preconditioning for PDEs, Optimisation and Data Assimilation, by Melina Freitag and John Pearson;
- Recent Advances in the Solution of Least Squares Problems, by Ken Hayami and Yimin Wei;
- Recent Developments in Non-linear Preservers, by Gyorgy Pal Geher and Lajos Molnar;
- Tensor Methods: Numerical and Computational Aspects, by Lieven De Lathauwer;
- Tensors for Signals and Systems, by Kim Batselier, Philippe Dreesen, Mariya Ishteva, and Konstantin Usevich;
- Total Positivity, by Shaun Fallat and Jürgen Garloff; and
- Tropical Linear Algebra and Beyond, by Marianne Johnson and Sergei Sergeev.

The Scientific Committee consists of: Raf Vandebril (KU Leuven, Belgium), Wim Michiels (KU Leuven, Belgium), Peter Šemrl (University of Ljubljana, Slovenia), Froilán Dopico (Universidad Carlos III de Madrid, Spain), Heike Faßbender (Technische Universität Braunschweig, Germany), Françoise Tisseur (The University of Manchester, UK), Dario Bini (University of Pisa, Italy), Hugo Woerdeman (Drexel University, United States), Fumio Hiai (Tohoku University, Japan), Douglas Farenick (University of Regina, Canada), and Steve Kirkland (University of Manitoba, Canada). The scientific organizing committee can be reached by e-mail at ilas2016.soc@cs.kuleuven.be.

The Local Organizing Committee consists of: Raf Vandebril (Chairman, Department of Computer Science, KU Leuven), Thomas Mach (Department of Computer Science, KU Leuven), Karl Meerbergen (Department of Computer Science, KU Leuven), Wim Michiels (Department of Computer Science, KU Leuven), Wim Vanroose (Department of Mathematics and Computer Science, University of Antwerp), and Marc Van Barel (Department of Computer Science, KU Leuven). The organizing committee can be reached by e-mail at ilas2016@cs.kuleuven.be.

There will be a special issue of *Linear Algebra and its Applications (LAA)* for papers corresponding to talks given at the conference. The special editors of this issue are: Bas Lemmens, Douglas Farenick, Marc Van Barel, and Raf Vandebril. Papers will be refereed according to the usual high standards of *LAA*. The submission deadline will be December 15, 2016. More details will follow on when submission of papers will be open and how to submit papers.

For additional information, go to <http://ilas2016.cs.kuleuven.be>.

Graduate Student Modeling Workshop (IMSM 2016) North Carolina State University, USA, July 17–27, 2016

The 22nd Industrial Mathematical & Statistical Modeling (IMSM) Workshop for Graduate Students will take place at North Carolina State University, 17–27 July 2016. The workshop is sponsored by the Statistical and Applied Mathematical Sciences Institute (SAMSI) together with the Center for Research in Scientific Computation (CRSC) and the Department of Mathematics at North Carolina State University.

The IMSM workshop exposes graduate students in mathematics, engineering, and statistics to exciting real-world problems from industry and government. The workshop provides students with experience in a research team environment and exposure to possible career opportunities. On the first day, a Software Carpentry bootcamp will bring students up to date on their programming skills in Python, Matlab and R, and introduce them to version control systems and software repositories.

Information is available at <http://www.samsi.info/IMSM16> and questions can be directed to grad@samsi.info.

Rocky Mountain–Great Plains Graduate Research Workshop in Combinatorics 2016 (GRWC 2016) University of Wyoming, USA, July 17–29, 2016

GRWC is a 2-week collaborative research workshop for experienced graduate students from all areas of combinatorics, broadly interpreted (combinatorial matrix theory has been an integral part of GRWC since its beginning). Students will work in collaborative groups with faculty and postdocs on research problems from across the discipline. The workshop will also host a variety of professional development workshops to prepare students and postdocs for the job hunt and their transition to careers in academia, government, or industry. All graduate students applying to participate in the GRWC will submit an open problem in combinatorics (broadly interpreted). Organizing faculty will engage in one-on-one mentoring with participants to help them refine their submissions. These problems will form the basis of the collaborative research undertaken at the workshop.

More information is available at the workshop website: <http://sites.google.com/site/rmgpgrwc>. Please direct all inquiries to grwc2016@gmail.com.

The organizing institutions for GRWC are Iowa State University, University of Colorado Denver, University of Denver, University of Nebraska–Lincoln, and University of Wyoming. The workshop is funded in part by these institutions, the NSA, the Institute for Mathematics and its Applications (IMA), the Combinatorics Foundation, and NSF funding is anticipated.

International Workshop on Operator Theory and Applications (IWOTA) St. Louis, Missouri, USA, July 18–22, 2016

The International Workshop on Operator Theory and Applications (IWOTA) will take place July 18–22, 2016 on the campus of Washington University in St. Louis.

For further details, please visit <http://openscholarship.wustl.edu/iwota2016>. The conference e-mail address is iwota2016@gmail.com.

The Twelfth International Conference on Matrix Theory and Applications Lanzhou City, China, July 22–27, 2016

The Twelfth International Conference on Matrix Theory and Applications will be held at Lanzhou University, Lanzhou City, Gansu Province, P.R. China, during July 22–26, 2016 (Registration: July 22, Departure: July 27). The conference aims at enhancing academic communication and promoting research among scholars in China and abroad interested in matrix theory methods and computations as well as their applications, and provides a friendly platform for senior researchers and young scientists to exchange ideas on recent developments in the relevant areas. The conference is co-organized by Shanghai University and the Academy of Mathematics and Systems Science of Chinese Academy of Sciences (CAS), P.R. China. The local organizing institution is Lanzhou University.

For further details, please visit <http://icmta.lzu.edu.cn>.

5th IMA Conference on Numerical Linear Algebra and Optimization University of Birmingham, UK, September 7–9, 2016

The IMA and the University of Birmingham are pleased to announce the Fifth IMA Conference on Numerical Linear Algebra and Optimization. The meeting is co-sponsored by SIAM, whose members will receive the IMA members' registration rate.

The success of modern codes for large-scale optimization is heavily dependent on the use of effective tools from numerical linear algebra. On the other hand, many problems in numerical linear algebra lead to linear, nonlinear or semidefinite optimization problems. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest. Conference topics include any subject that could be of interest to both communities, such as: direct and iterative methods for large sparse linear systems, eigenvalue computation and optimisation, large-scale nonlinear and semidefinite programming, effect of round-off errors, stopping criteria, embedded iterative procedures, optimisation issues for matrix polynomials, fast matrix computations, compressed/sparse sensing, PDE-constrained optimisation, and applications and real time optimisation.

The invited speakers are: Tim Davis (Texas A&M University), Anders Forsgren (KTH Stockholm), Andreas Frommer (University of Wuppertal), Jacek Gondzio (University of Edinburgh), Laura Grigori (INRIA Paris-Rocquencourt),

Jennifer Scott (Rutherford Appleton Laboratory), and Lieven Vandenberghe (UCLA).

The Organising Committee consists of: Michal Kočvara (University of Birmingham, co-chair), Daniel Loghin (University of Birmingham, co-chair), Iain Duff (Rutherford Appleton Laboratory), Nick Gould (Rutherford Appleton Laboratory), Julian Hall (University of Edinburgh), and Françoise Tisseur (University of Manchester).

The conference will be hosted by the University of Birmingham. Talks will take place in the School of Mathematics.

For further information or to register your interest, please contact Lizzi Lake (e-mail: conferences@ima.org.uk) The Institute of Mathematics and its Applications, Catherine Richards House, 16 Nelson Street, Southend-on-Sea, Essex, England SS1 1EF. Tel: +44 (0)1702 354020.

For more details, please visit the conference webpage at <http://tinyurl.com/IMANLA02016>.

The Householder Symposium XX on Numerical Linear Algebra Virginia Tech, Blacksburg, Virginia, USA, June 18–23, 2017

The Householder Symposium XX on Numerical Linear Algebra will be held at Virginia Tech in Blacksburg, Virginia, USA, June 18–23, 2017. This symposium is the twentieth in a series, previously called the Gatlinburg Symposia, and will be hosted by the Virginia Polytechnic Institute and State University (VA Tech), in cooperation with the Society for Industrial and Applied Mathematics (SIAM) and the SIAM Activity Group on Linear Algebra. Details are available at <http://www.math.vt.edu/HHXX>.

The Second Malta Conference in Graph Theory and Combinatorics (2MCGTC 2017) University of Malta, Malta, June 26–30, 2017

The Department of Mathematics, within the Faculty of Science, of the University of Malta is pleased to announce The Second Malta Conference in Graph Theory and Combinatorics. This conference will commemorate the 75th birthday of Professor Stanley Fiorini, who was the first to introduce graph theory and combinatorics at the University of Malta.

The Conference will be held in the seaside resort of Qawra in the north of the island, and will run from Monday 26 to Friday 30 June 2017. The aim of this conference is to bring together experts and researchers in all the areas of graph theory and combinatorics from across the world to share their research findings, and to enhance collaboration between researchers who are in different stages of their careers.

The Plenary Speakers are: Yair Caro (University of Haifa-Oranim, Israel), Peter Dankelmann (University of Johannesburg, South Africa), Patrick W. Fowler, F.R.S. (University of Sheffield, United Kingdom), Peter Frankl (Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary), Chris Godsil (University of Waterloo, Canada), Wilfried Imrich (Montanuniversität Leoben, Austria), Gyula O. H. Katona (Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary), Sandi Klavžar (University of Ljubljana, Slovenia), Mikhail Klin (Ben-Gurion University of the Negev, Israel), Imre Leader (University of Cambridge, United Kingdom), Brendan McKay (Australian National University, Australia), Karen Meagher (University of Regina, Canada), Raffaele Scapellato (Politecnico di Milano, Italy), and Slobodan Simic (Serbian Academy of Sciences and Arts, Serbia).

A number of parallel sessions for talks delivered by participants will be held. A special issue of *Discrete Applied Mathematics*, containing selected full-length papers, refereed according to the high standards of the journal, will be dedicated to the Conference.

The Organising Committee consists of: Peter Borg, John Baptist Gauci, Josef Lauri, and Irene Sciriha.

Further information can be found at <http://www.um.edu.mt/events/2mcgtc2017>; E-mail: 2mcgtc2017@um.edu.mt.

MAT-TRIAD'2017 - International Conference on Matrix Analysis and its Applications Będlewo, Poland, September 25–29, 2017

The 7th conference from the MAT-TRIAD series will be held in Będlewo (neighborhood of Poznań, Poland) at the Mathematical Research and Conference Center of the Polish Academy of Sciences.

MAT-TRIAD provides an opportunity to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications. Researchers and graduate students interested in recent developments in matrix theory and computation, operator theory, applications of linear algebra in statistics, matrices in graph theory, as well as combinatorial matrix theory are particularly encouraged to attend.

The format of the meeting will involve plenary sessions, sessions with contributed talks, and a poster session. The list

of Invited Speakers includes winners of Young Scientists Awards of MAT-TRIAD'2015. We are also planning courses delivered by leading experts. Thematic workshops are welcome.

The work of young scientists will receive special consideration at MAT-TRIAD'2017. The best poster as well as the best talk by a graduate student or scientist with a recently-completed Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference.

The conferences in the MAT-TRIAD series attract a number of national and international participants, provide a high quality scientific program as well as a friendly atmosphere for the discussion and exchange of ideas.

The Scientific Committee consists of Tomasz Szulc (Poland, Chair), Natália Bebiano (Portugal), Ljiljana Cvetković (Serbia), Heike Faßbender (Germany) and Simo Puntanen (Finland).

The Organising Committee is chaired by Augustyn Markiewicz (Poland) and includes Francisco Carvalho (Portugal), Katarzyna Filipiak (Poland), Jan Hauke (Poland) and Dominika Wojtera-Tyrakowska (Poland).

For more information, please visit <https://mattriad.wmi.amu.edu.pl>.

CONFERENCE REPORTS

The 2015 SIAM Conference on Applied Linear Algebra Atlanta, Georgia, USA, October 26–30, 2015

Report by Chen Greif and James Nagy

The 2015 SIAM Conference on Applied Linear Algebra, organized by the SIAM Activity Group on Linear Algebra every three years, took place on October 26–30, 2015 in Atlanta, GA, USA. The main conference themes were: Iterative Methods and Preconditioning, Randomized Linear Algebra Algorithms, Inverse and Ill-posed Problems, Structured Matrices, Numerical Linear Algebra for Analysis of Complex Networks, Eigenvalue Problems, Optimization, Numerical Linear Algebra of Compressed Sensing, and Applications. These themes were discussed in twelve invited lectures, given by Haim Avron (IBM), Raymond Chan (Chinese University of Hong Kong), Geir Dahl (University of Oslo), Zlatko Drmac (University of Zagreb), Howard Elman (University of Maryland), Maryam Fazel (University of Washington), Melina Freitag (University of Bath), Xiaoye Sherry Li (Lawrence Berkeley National Laboratory), Volker Mehrmann (Technical University of Berlin), Michael Overton (New York University), Haesun Park (Georgia Institute of Technology), and Eugene Tyrtshnikov (Russian Academy of Sciences). Dahl and Overton were sponsored by ILAS. In addition to these invited lectures, there were 60 minisymposia (with typically 4 talks each), 91 contributed talks, and 28 poster presentations. The minisymposia included featured topics on Complex Networks (organized by Michele Benzi and Christine Klymko), Finding and Exploiting Structure in Data (organized by Mark Davenport), Linear Algebra for PDE-constrained Optimization (organized by John Pearson and Tyrone Rees), Matrix Equations and Matrix Geometric Means (organized by Bruno Iannazzo and Raf Vandebril), Numerical Methods for Markov Chains and Stochastic Models (organized by Federico Poloni), and Preconditioning and Iterative Methods for Complex Linear Systems (organized by Zeng-Qi Wang).

A novelty for this conference was a poster blitz preceding the poster session. The blitz gave an opportunity for each poster presenter to have 60 seconds to present a single slide and briefly explain why conference attendees should attend their poster. To maximize visibility for all participants, the poster blitz took place in the main lecture hall, with no competing sessions. Thus, the poster blitz provided each poster presenter an opportunity to give a “mini plenary lecture.”

The conference was the second-largest SIAM ALA meeting, with a total of 415 registered participants from about 50 countries covering five continents. One of the highlights of the meeting was the awarding of the SIAG-LA Prize for the best paper in applied linear algebra over the preceding three years. The prize committee consisted of James Nagy (Chair), Mark Embree, Misha Kilmer, Daniel Kressner, and Esmond Ng. The 2015 SIAG-LA Prize was awarded to David Bindel and Amanda Hood for their paper titled “Localization Theorems for Nonlinear Eigenvalue Problems” (*SIAM Journal on Matrix Analysis and Applications*, 34 (2013) 1728–1749).

Another highlight of the conference was a dinner, held on Wednesday night (October 28), which included a wonderful presentation by Iain Duff on a reminiscence about the start of the SIAG-LA and the the SIAM ALA Conference series.

The next SIAM Applied Linear Algebra Conference will take place in Hong Kong, May 4–8, 2018.

5th International Conference on Matrix Analysis and Applications (ICMAA) Nova Southeastern University, Fort Lauderdale, USA, December 18–20, 2015

Report by Shahla Nasserassr and Fuzhen Zhang

The 5th International Conference on Matrix Analysis and Applications (ICMAA) was held at Nova Southeastern University, Fort Lauderdale, Florida, USA, December 18–20, 2015. The conferences in the series prior to this one were held in China (Beijing, Hangzhou), United States (Nova Southeastern University), and Turkey (Konya). Former keynote speakers are Roger Horn, Richard Brualdi, Chi-Kwong Li, Steve Kirkland and Alexander A. Klyachko (ILAS guest speaker). The Keynote Speaker of the 5th ICMAA was Professor Shmuel Friedland, University of Illinois, Chicago, USA.

Sixty-two people, from more than nine different countries participated in the conference; 48 talks were presented. With the generous sponsorship of Shanghai University, the conference waived registration fees for all students and retirees, and provided a meal for a reception on the day of arrival and a social gathering in the evening on the 19th. The conference concluded with an exotic tour to Shark Valley in Everglades National Park on the 20th.

The Scientific Organizing Committee (SOC) of the 5th conference of the series consisted of Shaun Fallat (University of Regina, Canada), Peter Šemrl (University of Ljubljana, Slovenia), Tin-Yau Tam (Auburn University, USA), Qingwen Wang (Shanghai University, Shanghai, China) and Fuzhen Zhang (Nova Southeastern University, USA, Chair). Local Organizing Committee members were Shahla Nasserassr (Chair), Vehbi Paksoy, and the math faculty and students of Nova Southeastern University.

A special session in memory of R.C. Thompson was organized by Wasin So and a special issue of the international mathematical research journal *Special Matrices* (published by De Gruyter) will be devoted to the meeting.



Conference participants at ICMAA 2015

ILAS NEWS

ILAS Lectures at Non-ILAS Conferences

As part of the commitment of ILAS to supporting activities in Linear Algebra, the Society maintains a program of providing some support to non-ILAS conferences. Effective for proposals submitted in 2016 for conferences taking place in 2017, this support may take one of two forms. (A conference may apply for only one of these two forms.)

1. *ILAS Lectureships at non-ILAS conferences.* It is now expected that ILAS Lecturers will be of the stature of plenary speakers at ILAS conferences, and may be supported by up to US\$1000 for expenses, or, in the case of a Hans Schneider Lecturer, US\$1500. Reimbursement guidelines may be obtained from ILAS Secretary/Treasurer Leslie Hogben.
2. *General support of conferences of up to \$750.* Such support could be used for support of student expenses, expenses for participants from developing countries, or plenary speakers. Refreshment costs are not eligible. This is a new program and you are welcome to consult ILAS President Peter Šemrl about what you may propose.

This is a reminder that there is a deadline of September 30, 2016 for receipt of proposals for sponsorship of ILAS Lectures at non-ILAS meetings and for general support of conferences taking place in 2017. Please note that applications submitted after the deadline will not be considered, since it is important to rank all applications at the same time. Each proposal is automatically assumed to be also a request for ILAS endorsement of the conference.

Further details on guidelines for proposals can be found at http://www.ilasic.org/misc/non_ilas_guidelines.html.

2016 Hans Schneider Prize Awarded to Rajendra Bhatia and Paul Van Dooren

Rajendra Bhatia and Paul Van Dooren are the 2016 recipients of the Hans Schneider Prize for lifetime contributions to linear algebra.

Rajendra Bhatia has made very substantial and broad contributions to the theory of perturbation of eigenvalues and eigenvectors, positive definite matrices, and matrix equations and inequalities. He has brought ideas and techniques from other areas, like Riemannian geometry, to matrix analysis. He is a prolific writer, having written several books, three of which (*Perturbation Bounds for Matrix Eigenvalues*, *Matrix Analysis*, and *Positive Definite Matrices*) are now classics and greatly cited.



R. Bhatia

Paul Van Dooren has made very substantial and significant contributions to linear algebra and its applications to other disciplines. His main line of research is numerical linear algebra and matrix theory, with applications to system and control theory. His work has been innovative and has been cited an unusually large number of times. Recently, he has applied numerical linear algebra and matrix theory to large-scale graphs and networks, drawing the attention of many researchers in applied science.



P. Van Dooren

For many years, both Bhatia and Van Dooren have served the linear algebra community, and more generally the mathematics and science community, in many important ways: editorial work, expository writing, committees, and mentorship of young students and researchers. Both are highly respected members of the linear algebra community.

Paul Van Dooren will be awarded the prize at the ILAS Conference in Leuven in 2016, and Rajendra Bhatia will be awarded the prize at the ILAS Conference in Ames in 2017.

The 2016 Hans Schneider Prize Selection Committee consisted of Richard Brualdi (chair), Shmuel Friedland, Ilse Ipsen, Thomas Laffey, Xingzhi Zhan, and Peter Šemrl (ex officio).

ILAS Members Elected to SIAM Executive



I. Ipsen

Two ILAS members were elected to the SIAM Executive. In particular, the President-elect is ILAS member Nicholas Higham of the University of Manchester. And the Vice President at Large is ILAS member Ilse Ipsen of North Carolina State University. For further information, see <http://connect.siam.org/meet-siams-newest-leadership>.



N. Higham

ILAS Members Named SIAM Fellows



F. Tisseur

The Society for Industrial and Applied Mathematics (SIAM) has recently announced its 2016 SIAM Fellows, and among those named are long-time ILAS members Professor Françoise Tisseur from the University of Manchester, and Professor James G. Nagy from Emory University. Françoise was recognized “for contributions to numerical linear algebra, especially numerical methods for eigenvalue problems,” and Jim was recognized “for contributions to the computational science of image reconstruction.” For further information, see <http://connect.siam.org/2016-class-of-siam-fellows-announced/>.



J. Nagy

ILAS Election Results

Hugo Woerdeman has been elected to the position of ILAS Vice-President for a term beginning on May 1, 2016, and ending on February 28, 2019.

ILAS President/Vice President Annual Report: March 31, 2016

**Respectfully submitted by Peter Šemrl, ILAS President, peter.semrl@fmf.uni-lj.si
and Steve Kirkland, ILAS Vice-President, stephen.kirkland@umanitoba.ca**

1. Board approved actions since the last report include:

- The Board decided that ILAS will help the organizers of the ILAS 2016 Conference in Leuven with the support of \$2000 to help to keep registration fees down. Belgium is expensive and it is difficult for the organizers of ILAS 2016 to meet the ILAS low conference fee requests.
- As part of ILAS's commitment to supporting activities in Linear Algebra, the Society maintains a program of providing some support to non-ILAS conferences. Effective with proposals submitted in 2016 for conferences taking place in 2017, this support may take one of two forms (a conference may apply for only one of these two forms):
 - (a) ILAS Lectureships at non-ILAS conferences. It is now expected that ILAS lecturers will be of the stature of plenary speakers at ILAS conferences, and may be supported by up to \$1000 for expenses, or in the case of a Hans Schneider Lecturer, \$1500.
 - (b) General support of conferences of up to \$750. Such support could be used for support of student expenses, expenses for participants from developing countries, or plenary speakers.

2. ILAS elections ran from November 13, 2015 to December 31, 2015 and proceeded via electronic voting. The following were elected to offices with three-year terms that began on March 1, 2016:

- Vice-President: Bryan Shader
- Board of Directors: Ravindra Bapat and Helena Šmigoc.

The following continue in the ILAS offices which they currently hold:

- President: Peter Šemrl (term ends February 28, 2017)
- Second Vice-President (for ILAS conferences): Steve Kirkland (term ends February 28, 2017)
- Secretary/Treasurer: Leslie Hogben (term ends February 28, 2018)
- Board of Directors: Froilán Dopico (term ends February 28, 2017), Beatrice Meini (term ends February 28, 2018), Michael Overton (term ends February 28, 2017), and Henry Wolkowicz (term ends February 28, 2018).

Avi Berman and Volker Mehrmann completed their terms on the ILAS Board of Directors on February 28, 2016. We thank them for their valuable contributions as Board members; their service to ILAS is most appreciated. We also thank the members of the Nominating Committee—Rachel Quinlan (chair), Pauline van den Driessche, Raf Vandebril, David Watkins, Hugo Woerdeman—for their work on behalf of ILAS, and also extend gratitude to all candidates that agreed to have their names stand for the elections.

3. Due to unforeseen family medical issues, Bryan Shader resigned from his post as ILAS Vice-President on January 1, 2016. We thank him for his service to ILAS. In accordance with ILAS bylaws, the ILAS President has appointed Steve Kirkland to serve as ILAS Vice-President in the period from January 1 until April 30, 2016. Nominated for a term, beginning May 1, 2016, and ending February 28, 2019, as ILAS Vice-President are:

- Heike Faßbender, Germany
- Hugo Woerdeman, USA.

We thank the members of the Nominating Committee—Harm Bart, Pauline van den Driessche (chair), Chi-Kwong Li, Françoise Tisseur, Eugene Tyrtyshnikov—for their important service to ILAS. Voting in the ILAS Special Election for Vice-President will conclude on April 15, 2016.

4. The following ILAS-endorsed meetings have taken place since our last report:

- 24th International Workshop on Matrices and Statistics, Haikou, China, May 25–28, 2015, ILAS lecturer: Karl Gustafson
- Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics, Ames, USA, June 1–12, 2015, ILAS lecturer: Franklin Kenter
- MatTriad-International Conference on Matrix Analysis and its Applications, Coimbra, Portugal, September 7–11, 2015
- SIAM Conference on Applied Linear Algebra, Atlanta, USA, October 26–30, 2015, ILAS lecturers: Michael Overton and Geir Dahl
- Workshop on Tensor Decompositions and Applications, Leuven, Belgium, January 18–22, 2016.

5. ILAS has endorsed the following conferences of interest to ILAS members:

- Western Canada Linear Algebra Meeting, University of Manitoba, Winnipeg, Canada, May 14–15, 2016, Kevin Vander Meulen will be the ILAS lecturer
- The International Workshop on Operator Theory and Applications, Washington University, St. Louis, USA, July 18–22, 2016, Gitta Kutyniok will be Hans Schneider ILAS lecturer
- The Twelfth International Conference on Matrix Theory and Applications, Lanzhou University, Lanzhou City, China, July 22–26, 2016, Richard Brualdi will be the ILAS lecturer.

6. The following ILAS conferences are scheduled:

- 20th ILAS Conference, July 11–15, 2016, Leuven, Belgium. The chair of the organizing committee is Raf Vandebril. For more information see <http://ilas2016.cs.kuleuven.be>.
- 21st ILAS Conference: Connections, July 24–28, 2017, Ames, Iowa, USA. The chair of the organizing committee is Leslie Hogben. For more information see <https://ilas2017.math.iastate.edu>.

7. Rajendra Bhatia and Paul Van Dooren are 2016 Hans Schneider prize recipients for lifetime contributions to linear algebra. Paul Van Dooren will be awarded the prize at the ILAS Conference in Leuven in 2016, and Rajendra Bhatia will be awarded the prize at the ILAS Conference in Ames in 2017. The 2016 Hans Schneider Prize Committee consisted of Richard Brualdi (chair), Shmuel Friedland, Ilse Ipsen, Thomas Laffey, Peter Šemrl (ex-officio member) and Xingzhi Zhan.

8. The *Electronic Journal of Linear Algebra (ELA)* is now in its 31st volume. In 2015 *ELA* published 60 papers totaling 973 pages in volume 30; 12 papers totaling 193 pages in volume 29, devoted to the *Proceedings of the International Conference on Linear Algebra and its Applications dedicated to Professor Ravindra B. Bapat*; and 10 papers totaling 123 pages in volume 28, devoted to the *Proceedings of Graph Theory, Matrix Theory and Interactions Conference in honor of David Gregory*.

Over the past year, 34,116 downloads of articles from the *ELA* portal <http://repository.uwo.edu/ela> occurred.

Recent additions to the *ELA* Editorial Board are: Dario Bini (Università di Pisa), Sebastian M. Cioabă (University of Delaware), Geir Dahl (University of Oslo), Froilán Dopico (Universidad Carlos III de Madrid), Torsten Ehrhardt (University of California–Santa Cruz), Zejun Huang (Hunan University), Sergei Sergeev (University of Birmingham), Ilya Spitkovsky (New York University Abu Dhabi and the College of William and Mary), and Françoise Tisseur (University of Manchester).

Upon the recommendation of the ILAS Journal Committee, the ILAS Board of Directors appointed Michael Tsatsomeros (Washington State University) to join Bryan Shader (University of Wyoming) as co-Editor-in-Chief, starting March 1, 2016.

9. *IMAGE* is the semi-annual bulletin for ILAS. The Editor-in-Chief is Kevin N. Vander Meulen. Since the spring issue 54, Louis Deaett has been serving as Managing Editor. Bojan Kuzma is finishing his term as Problem Corner Editor after this spring issue 56. We are very grateful for his four and a half years of service as the Problem Corner Editor, starting with issue 48. Rajesh Pereira will be the new Problem Corner Editor.

10. ILAS-NET is a moderated newsletter for mathematicians worldwide, with a focus on linear algebra; it is managed by Sarah Carnochan Naqvi. In September 2015, ILAS-NET moved to a new service provider, MailChimp. A new layout for ILAS-NET messages was created at that time. An RSS news feed is now available for ILAS-NET.

As of March 21, 2016 there are 894 subscribers to ILAS-NET. An archive of ILAS-NET messages is available at <http://www.ilasic.org/ilas-net/>. To send a message to ILAS-NET, please send the message (preferably in text format) in an e-mail to ilasic@uregina.ca indicating that you would like it to be posted on ILAS-NET. If the message is approved, it will be posted soon afterwards. To subscribe to ILAS-NET, please complete the form at <http://ilasic.us10.list-manage.com/subscribe?u=6f8674f5d780d2dc591d397c9&id=dbda1af1a5>.

The ILAS website, known as the ILAS Information Centre (IIC), is located at <http://www.ilasic.org> and provides general information about ILAS (e.g., ILAS officers, By-laws, Special Lecturers) as well as links to pages of interest to the ILAS community.

Respectfully submitted,

Peter Šemrl, ILAS President (peter.semrl@fmf.uni-lj.si); and

Steve Kirkland, ILAS Vice-President (stephen.kirkland@umanitoba.ca).



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ILAS 2015 - 2016 Treasurer's Report

April 1, 2015 - March 31, 2016

By Leslie Hogben

Net Account Balance on March 31, 2015

Vanguard (7776.346 @ 10.80/share)	\$ 83,984.54	
Checking Account - Great Western	\$ 40,570.77	
Certificate of Deposit #2	\$ 45,395.75	
Accounts Payable	\$ (700.00)	
		<u>\$ 169,251.06</u>
General Fund	\$ 86,872.00	
Conference Fund	\$ 9429.01	
Olga Taussky Todd/John Todd Fund	\$ 11,850.14	
Hans Schneider Lecture Fund	\$ 14,401.00	
Frank Uhlig Education Fund	\$ 5,005.77	
Hans Schneider Prize Fund	\$ 28,801.57	
ELA Fund	\$ 51.00	
ILAS/LAA Fund	\$ 12,840.57	
		<u>\$ 169,251.06</u>

INCOME:

Dues	\$ 9,428.00	
General Fund	\$ 3,115.00	
Conference Fund	\$ 185.00	
Taussky-Todd Fund	\$ 51.00	
Hans Schneider Lecture Fund	\$ 26.00	
Uhlig Education Fund	\$ 101.00	
Schneider Prize Fund	\$ 281.00	
ELA	\$ 21.00	
Elsevier Sponsorship (Flow Through)	\$ 2,500.00	
LAMA for Speaker	\$ -	
Misc Income	\$ 350.00	
Royalty Income	\$ -	
Interest - Great Western	\$ 81.65	
Interest on Great Western Certificate of Deposit (#2)	\$ 450.00	
Vanguard - Income Dividend	\$ 1,035.29	
- Short Term Capital Gains	\$ 206.25	
- Long Term Capital Gains	\$ 70.45	
Variances in the Market	\$ 160.70	
Total Income		\$ 18,062.34

EXPENSES:

First Data - Credit Card Processing Fees	\$ 933.67	
Speaker Fees	\$ 3,396.00	
Hans Schneider Lecture	\$ -	
Treasurer's Assistant	\$ 427.50	
Conference Expenses	\$ -	
2015 ILAS Conference - Elsevier Flowthru	\$ -	
Business License	\$ 61.25	
IMAGE Costs	\$ 1,468.52	
Ballot Costs	\$ 790.00	
Web Hosting & Online Membership Forms	\$ 212.78	
Misc Expenses	\$ 308.00	
Total Expenses		\$ 7,597.72

Net Account Balance on March 31, 2016

Vanguard (ST Fed Bond Fund Admiral 7876.686 Shares)	\$ 85,225.74	
Checking Account - Great Western	\$ 49,112.70	
Certificate of Deposit	\$ 45,450.00	
Accounts Payable	\$ (72.76)	
		<u>\$ 179,715.68</u>
General Fund	\$ 92,417.82	
Conference Fund	\$ 9,864.56	
Olga Taussky Todd/John Todd Fund	\$ 12,151.68	
Hans Schneider Lecture Fund	\$ 14,677.54	
Frank Uhlig Education Fund	\$ 5,357.31	
Hans Schneider Prize Fund	\$ 29,333.11	
ELA Fund	\$ 322.54	
ILAS/LAA Fund	\$ 15,591.11	
		<u>\$ 179,715.68</u>

IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to all Problems in issue 55 of *IMAGE*. Six new problems are on the last page; solutions are invited.

Problem 55-1: The Perfect Aggregation of Linear Equations With M -Matrix

Proposed by Arkady A. BABAJANYAN, *Institute for Informatics and Automation Problems, Armenian Academy of Sciences, Yerevan, Armenia*, arbab@list.ru

Let $F \in M_n(\mathbb{R})$ be an M -matrix (i.e., $F = \alpha I - A$ where $A \geq 0$ entrywise and $\alpha \geq \rho(A)$, the spectral radius) and let C be a $(0, 1)$ -matrix of size $m \times n$ ($m < n$) and full rank, where each column contains a single 1.

(A) If F is non-degenerate, construct an entrywise non-negative $m \times n$ matrix D of full rank so that $\bar{F} = DFC^+$ is again a non-degenerate M -matrix and the condition of consistency $DF = \bar{F}C$ holds (here, C^+ is a Moore-Penrose inverse).

(B) Generalize to the case of a degenerate M -matrix F .

Editorial note: No solution of IMAGE Problem 55-1 was received. The solution of (A) by the proposer can be found in [A. A. Babadzhanyan, Perfect aggregation of systems of linear balance equations, Akad. Nauk Armyan. SSR Dokl. 68 (1979) 213–218 (in Russian)]. As an application, the proposer gives the following supplement: Consider a reduction of a system of real linear M -equations $Fx = y$ into a smaller dimension $\bar{F}\bar{x} = \bar{y}$ with the help of entrywise nonnegative transformations $\bar{x} = Cx$ and $\bar{y} = Dy$, where C is known. Find D so that $\bar{F}$ is again an M -matrix and $\bar{x}' = Cx'$ holds, where x' and $\bar{x}'$ denote a solution to the original and to the modified system of equations, respectively.

We are still looking for a solution.

Problem 55-2: About Equality of Dimensions of Direct Complements

Proposed by Johanns DE ANDRADE BEZERRA, *Natal, RN, Brazil*, pav.animal@hotmail.com

Let $A, B \in M_n(\mathbb{C})$ satisfy $\text{rank}(A) = \text{rank}(B)$. Let V and W be subspaces of $\mathbb{C}^n$ with $\text{Im } A = (\text{Im } A \cap \text{Im } B) \oplus V$ and $\text{Ker } A = (\text{Ker } A \cap \text{Ker } B) \oplus W$. Show that $\dim V = \dim W$ if A and B are range-Hermitian.

Solution 55-2.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Since A and B are range-Hermitian and have the same rank, r , we have

$$\dim(\text{Im } A) = \dim(\text{Im } B) = r \quad \text{and} \quad \dim(\text{Ker } A) = \dim(\text{Ker } B) = n - r.$$

The subspaces $\text{Im } A \cap \text{Im } B$, $\text{Im } A$, and $\text{Im } B$ have bases of the following forms, respectively:

$$\{u_1, \dots, u_k\}, \quad \{u_1, \dots, u_k, v_{k+1}, \dots, v_r\}, \quad \{u_1, \dots, u_k, w_{k+1}, \dots, w_r\}.$$

Then $\{u_1, \dots, u_k, v_{k+1}, \dots, v_r, w_{k+1}, \dots, w_r\}$ is a basis for $\text{Im } A + \text{Im } B$, which therefore has dimension $2r - k$. The orthogonal complement of $\text{Im } A + \text{Im } B$ is $\text{Ker } A \cap \text{Ker } B$, which therefore has dimension $n - (2r - k) = n + k - 2r$. From the given equations involving V and W , we conclude that

$$\dim V = r - k \quad \text{and} \quad \dim W = (n - r) - (n + k - 2r) = r - k. \quad \square$$

Solution 55-2.2 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

Consider first the case that $A^* = A = A^2$ and $B^* = B = B^2$, i.e., both A and B are orthogonal projections. Let $r = \text{rank}(A)$. Then $\text{Ker } A \cap \text{Ker } B = \text{Ker} \begin{bmatrix} A \\ B \end{bmatrix}$, and $\text{Im } A = \text{Ker}(I - A)$, and $\text{Im } A \cap \text{Im } B = \text{Ker} \begin{bmatrix} I - A \\ I - B \end{bmatrix}$, indicating that

$$\dim V = \dim(\text{Im } A) - \dim(\text{Im } A \cap \text{Im } B) = r - (n - \text{rank} \begin{bmatrix} I - A \\ I - B \end{bmatrix}),$$

$$\dim W = \dim(\text{Ker } A) - \dim(\text{Ker } A \cap \text{Ker } B) = (n - r) - (n - \text{rank} \begin{bmatrix} A \\ B \end{bmatrix}).$$

Using block elementary transformations, we have

$$\begin{aligned} 2r + \text{rank} \begin{bmatrix} I - A \\ I - B \end{bmatrix} &= \text{rank} \begin{bmatrix} I - A & 0 & 0 \\ I - B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & B \end{bmatrix} = \text{rank} \begin{bmatrix} I - A & A & 0 \\ I - B & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & B \end{bmatrix} = \text{rank} \begin{bmatrix} I & A & 0 \\ I - B & 0 & 0 \\ A & A & 0 \\ 0 & 0 & B \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} I & A & 0 \\ I - B & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{bmatrix} = \text{rank} \begin{bmatrix} I & A & 0 \\ I - B & 0 & B \\ 0 & 0 & B \end{bmatrix} = \text{rank} \begin{bmatrix} I & A & 0 \\ I & 0 & B \\ B & 0 & B \end{bmatrix} = \text{rank} \begin{bmatrix} I & A & -B \\ I & 0 & 0 \\ B & 0 & 0 \end{bmatrix} \\ &= \text{rank} \begin{bmatrix} 0 & A & -B \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{rank} \begin{bmatrix} 0 & A & B \\ I & 0 & 0 \end{bmatrix} = n + \text{rank} \begin{bmatrix} A & B \end{bmatrix} = n + \text{rank} \begin{bmatrix} A & B \end{bmatrix}^* = n + \text{rank} \begin{bmatrix} A \\ B \end{bmatrix}. \end{aligned}$$

Therefore

$$\dim V = r + \operatorname{rank} \begin{bmatrix} I - A \\ I - B \end{bmatrix} - n = \operatorname{rank} \begin{bmatrix} A \\ B \end{bmatrix} - r = \dim W.$$

For general range-Hermitian A and B , denote by $X^\dagger$ the Moore-Penrose inverse of X . Then we have

$$\operatorname{Im} A = \operatorname{Im}(A^\dagger A), \quad \operatorname{Ker} A = \operatorname{Ker}(A^\dagger A), \quad \operatorname{Im} B = \operatorname{Im}(B^\dagger B), \quad \operatorname{Ker} B = \operatorname{Ker}(B^\dagger B).$$

Also, $\operatorname{rank}(A^\dagger A) = \operatorname{rank}(A) = \operatorname{rank}(B) = \operatorname{rank}(B^\dagger B)$. Since $A^\dagger A$ and $B^\dagger B$ are orthogonal projections, we reduce the general case to the special one which we have already proved. $\square$

Also solved by the proposer.

Problem 55-3: A Matrix Product Characterizing the Samuelson Transformation Matrix

Proposed by Richard William FAREBROTHER, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@hotmail.com

Let $C_n = (c_{ij})$ be a $2^n \times (n+1)$ matrix of ± 1 , with rows indexed by $i = 0, 1, \dots, 2^n - 1$, such that $\sum_{j=0}^n (1 - c_{ij})2^{n-j-1} = i$ for $i = 0, \dots, 2^n - 1$ (i.e., i th row forms a symbolic binary expansion of i).

Does there exist a $2^n \times (n+1)$ matrix E_n , with rows taken from the $(n+1) \times (n+1)$ identity matrix, so that $E_n^T C_n$ equals the $(n+1) \times (n+1)$ Samuelson transformation matrix $S_n = (s_{hj})$ whose j th column contains the coefficients for the expansion of the polynomial

$$\sum_{h=0}^n s_{hj} x^{n-h} y^h = (x+y)^{n-j} (x-y)^j?$$

Solution 55-3.1 by Tamás F. GÖRBE, *University of Szeged, Hungary*, tfgorbe@physx.u-szeged.hu

The answer is affirmative. To see this, start with $n = 0$, when $C_0 = S_0 = (1)$, thus $E_0 = (1)$ solves the problem. For $n = 1$ we have $C_1 = S_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, hence $E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is an appropriate solution. Both matrices defined in the problem, C_n and S_n , are of recursive nature, so we also define E_n recursively. Let $s_{hj}^{(n)}$ stand for the (h, j) component of S_n . The first and last rows of S_n are $s_{0j}^{(n)} = 1$ and $s_{nj}^{(n)} = (-1)^j$, $j = 0, 1, \dots, n$. Due to $(x+y)^{n-j} (x-y)^j = (x+y)(x+y)^{(n-1)-j} (x-y)^j$ we have

$$s_{hj}^{(n)} = s_{hj}^{(n-1)} + s_{h-1,j}^{(n-1)}, \quad 1 \leq h \leq n-1, \quad 0 \leq j \leq n-2. \quad (5)$$

Similarly, from $(x+y)^{n-j} (x-y)^j = (x-y)(x+y)^{(n-1)-(j-1)} (x-y)^{j-1}$ we deduce

$$s_{hj}^{(n)} = s_{h,j-1}^{(n-1)} - s_{h-1,j-1}^{(n-1)}, \quad 1 \leq h \leq n-1, \quad 1 \leq j \leq n. \quad (6)$$

Notice that (5)–(6) only make sense for $n \geq 2$. Let's turn to the matrix C_n with the (i, j) -entry denoted by $c_{ij}^{(n)}$. Note that $c_{i0}^{(n)} = 1$, $0 \leq i \leq 2^n - 1$. The fact that row i contains a symbolic binary expansion of i implies

$$c_{ij}^{(n)} = c_{i,j-1}^{(n-1)}, \quad 0 \leq i \leq 2^{n-1} - 1, \quad 1 \leq j \leq n. \quad (7)$$

Moreover, due to $i + (2^n - 1 - i) = 2^n - 1$ we have $c_{2^n-1-i,j}^{(n)} = -c_{ij}^{(n)}$ for all $j = 1, \dots, n$. This yields the recursion formula

$$c_{ij}^{(n)} = -c_{2^n-1-i,j-1}^{(n-1)}, \quad 2^{n-1} \leq i \leq 2^n - 1, \quad 1 \leq j \leq n. \quad (8)$$

Now we are able to solve $E_n^T C_n = S_n$ for E_n , which has the unknown components $e_{ij}^{(n)}$, $i = 0, \dots, 2^n - 1$, $j = 0, \dots, n$. We already solved the problem for $n = 0$ and $n = 1$, thus we take $n \geq 2$ and assume that E_{n-1} is known. First, we use the relations (6), with $1 \leq h \leq n-1$ and $1 \leq j \leq n$, to get

$$(S_n)_{hj} = (S_{n-1})_{h,j-1} - (S_{n-1})_{h-1,j-1} = (E_{n-1}^T C_{n-1})_{h,j-1} - (E_{n-1}^T C_{n-1})_{h-1,j-1} = \sum_{i=0}^{2^{n-1}-1} (e_{ih}^{(n-1)} - e_{i,h-1}^{(n-1)}) c_{i,j-1}^{(n-1)}.$$

Next, if we write out the (h, j) -entry of $S_n = E_n^T C_n$ and utilize relations (7) and (8) we obtain

$$(S_n)_{hj} = (E_n^T C_n)_{hj} = \sum_{i=0}^{2^{n-1}-1} e_{ih}^{(n)} c_{ij}^{(n)} + \sum_{i=2^{n-1}}^{2^n-1} e_{ih}^{(n)} c_{ij}^{(n)} = \sum_{i=0}^{2^{n-1}-1} (e_{ih}^{(n)} - e_{2^n-1-i,h}^{(n)}) c_{ij}^{(n)} = \sum_{i=0}^{2^{n-1}-1} (e_{ih}^{(n)} - e_{2^n-1-i,h}^{(n)}) c_{i,j-1}^{(n-1)}.$$

Comparing the right-hand sides in the above two equations, we see that they will be equal if we set

$$e_{ih}^{(n)} = e_{ih}^{(n-1)}, \quad e_{2^{n-1}-i,h}^{(n)} = e_{i,h-1}^{(n-1)}, \quad (0 \leq i \leq 2^{n-1} - 1, 1 \leq h \leq n-1). \quad (9)$$

One can check that (9) satisfies (5) as well. Columns 0 and n of E_n are still to be found. For $h = 0, j = 0, \dots, n$ we have

$$\sum_{i=0}^{2^n-1} e_{i0}^{(n)} c_{ij}^{(n)} = (E_n^T C_n)_{0j} = (S_n)_{0j} = s_{0j}^{(n)} = 1, \quad (10)$$

and since row 0 of C_n is all +1's, equations (10) are solved by

$$e_{00}^{(n)} = 1, \quad e_{i0}^{(n)} = 0, \quad i = 1, \dots, 2^n - 1. \quad (11)$$

Similarly, for $h = n, j = 0, \dots, n$ we have

$$\sum_{i=0}^{2^n-1} e_{in}^{(n)} c_{ij}^{(n)} = (E_n^T C_n)_{nj} = (S_n)_{nj} = s_{nj}^{(n)} = (-1)^j, \quad (12)$$

and hence with $k = \sum_{j=0}^n (1 - (-1)^j) 2^{n-j-1}$ the solution of equations (12) is

$$e_{kn}^{(n)} = 1, \quad e_{in}^{(n)} = 0, \quad i = 0, \dots, 2^n - 1, \quad i \neq k. \quad (13)$$

Equations (9), (11), and (13) provide a full solution to the problem. To conclude, we give instructions on how to construct E_n recursively, in a somewhat more expressive manner. Step 1: Write down E_{n-1} . Step 2: Below E_{n-1} write another copy of E_{n-1} but with reversed rows (i.e., flip E_{n-1} under itself). This gives the matrix FE_{n-1} , where F is the $2^n \times 2^{n-1}$ matrix with components $F_{ij} = \delta_{ij}$ for $0 \leq i < 2^{n-1}$ and $F_{ij} = \delta_{i+j, 2^{n-1}}$ for $2^{n-1} \leq i \leq 2^n - 1$. Step 3: Shift the lower half of FE_{n-1} one column to the right. Step 4: Fill in the blanks with 0's to get E_n . $\square$

Remark. A more roundabout and laborious way to prove equations (5)–(6) is through the explicit combinatorial formula $s_{hj}^{(n)} = \sum_{m=\max(0, h+j-n)}^{\min(h,j)} (-1)^m \binom{n-j}{h-m} \binom{j}{m}$, $0 \leq h, j \leq n$.

Solution 55-3.2 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

The answer is affirmative.

By definition, the rows of C_n consist of $(n+1)$ -bit *modified* binary representations of all n -bit integers $i = 0, 1, \dots, 2^n - 1$, in the sense that 0's and 1's in the original binary representation are replaced by 1's and -1 's, respectively. So the first column of C_n only has 1's, while the rest of the columns have half of their entries equal to +1 and the other half equal to -1 .

From the definition of S_n , and as $(x+y)^{n-j}(x-y)^j = (x+y)(x+y)^{(n-1)-j}(x-y)^j = (x-y)(x+y)^{(n-1)-(j-1)}(x-y)^{j-1}$, we can construct S_n recursively in the following way. The first n columns of S_n are given by

$$\begin{bmatrix} S_{n-1} \\ 0_{1 \times n} \end{bmatrix} + \begin{bmatrix} 0_{1 \times n} \\ S_{n-1} \end{bmatrix},$$

while the last n columns of S_n are given by

$$\begin{bmatrix} S_{n-1} \\ 0_{1 \times n} \end{bmatrix} - \begin{bmatrix} 0_{1 \times n} \\ S_{n-1} \end{bmatrix}.$$

We can easily choose E_0 and E_1 as identity matrices because $C_0 = S_0 = 1$ and $C_1 = S_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. For $n > 1$, we define $\hat{C}_n$ recursively as

$$\hat{C}_n = \begin{bmatrix} \hat{C}_{n-1} & x_{n-1} \\ \hat{C}_{n-1} & -x_{n-1} \end{bmatrix}$$

with $\hat{C}_1 = C_1$, where x_{n-1} is the last column of $\hat{C}_{n-1}$. Notice that x_{n-1} consists of ± 1 's half-and-half. The rows of $\hat{C}_n$ are the same as the rows of C_n up to row permutation, that is, there exists a permutation matrix $P_n \in \mathbb{R}^{2^n \times 2^n}$ such that $C_n = P_n \hat{C}_n$.

Let $\hat{E}_1 = E_1$. By induction, assume there exists $\hat{E}_{n-1}$, whose rows are taken from I_n , such that $S_{n-1} = \hat{E}_{n-1}^T \hat{C}_{n-1}$. Let

$$\hat{E}_n = \left[\begin{bmatrix} \hat{E}_{n-1}^T \\ 0_{1 \times 2^n} \end{bmatrix}, \begin{bmatrix} 0_{1 \times 2^n} \\ \hat{E}_{n-1}^T \end{bmatrix} \right]^T.$$

Then the rows of $\hat{E}_n$ are all taken from the rows of I_{n+1} . Notice that the first n columns of S_n are given by

$$\begin{bmatrix} \hat{E}_{n-1}^T \hat{C}_{n-1} \\ 0_{1 \times n} \end{bmatrix} + \begin{bmatrix} 0_{1 \times n} \\ \hat{E}_{n-1}^T \hat{C}_{n-1} \end{bmatrix} = \hat{E}_n^T \begin{bmatrix} \hat{C}_{n-1} \\ \hat{C}_{n-1} \end{bmatrix},$$

which are also the first n columns of $\hat{E}_n^T \hat{C}_n$. Similarly, the last column of S_n is equal to the last column of

$$\begin{bmatrix} \hat{E}_{n-1}^T \hat{C}_{n-1} \\ 0_{1 \times n} \end{bmatrix} - \begin{bmatrix} 0_{1 \times n} \\ \hat{E}_{n-1}^T \hat{C}_{n-1} \end{bmatrix} = \hat{E}_n^T \begin{bmatrix} \hat{C}_{n-1} \\ -\hat{C}_{n-1} \end{bmatrix},$$

which is also the last column of $\hat{E}_n^T \hat{C}_n$. Therefore, we obtain $S_n = \hat{E}_n^T \hat{C}_n$. This is equivalent to the desired result as we can set $E_n = P_n \hat{E}_n$. $\square$

Also solved by the proposer.

Problem 55-4: Orthogonality of Two Positive Semidefinite Matrices

Proposed by Minghua LIN, *Shanghai University, Shanghai, China*, mlin87@ymail.com

Let A, B be positive definite. Assume $M = \begin{pmatrix} A & X \\ X^* & B \end{pmatrix}$, $N = \begin{pmatrix} A & Y \\ Y^* & B \end{pmatrix}$ are positive semidefinite and $MN = 0$. Show that $X = -Y$. What if A and B are only assumed to be positive semidefinite?

Solution 55-4.1 by the proposer Minghua LIN

Only the first part, when A, B are positive definite. As M, N are positive semidefinite, there are contractive matrices U, V such that $X = A^{1/2}UB^{1/2}$, $Y = A^{1/2}VB^{1/2}$. Comparing the entries at positions $(1, 1)$ and $(1, 2)$ of $MN = 0$ yields $A + UBV^* = 0$ and $AV + UB = 0$, which implies $A(I - VV^*) = 0$. Hence, V is unitary. Similarly, U is unitary. Then it follows from $A + UBV^* = 0$ that $A = -UV^*(VBV^*)$. By uniqueness of polar decomposition, this implies that $-UV^* = I$, so $U = -V$ and so $X = -Y$. $\square$

Solution 55-4.2 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

Let $A = U\tilde{A}U^*$ and $B = V\tilde{B}V^*$ be the spectral decompositions of A and B , respectively. Then

$$\tilde{M} := \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}^* M \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} \tilde{A} & \tilde{X} \\ \tilde{X}^* & \tilde{B} \end{pmatrix}, \quad \tilde{N} := \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}^* N \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} = \begin{pmatrix} \tilde{A} & \tilde{Y} \\ \tilde{Y}^* & \tilde{B} \end{pmatrix},$$

where $\tilde{X} = U^*XV$, $\tilde{Y} = U^*YV$. The condition $MN = 0$ becomes $\tilde{M}\tilde{N} = 0$. Consequently, we have $\tilde{A}\tilde{Y} + \tilde{X}\tilde{B} = \tilde{A}\tilde{X} + \tilde{Y}\tilde{B} = 0$. The componentwise form can be rewritten as

$$\tilde{X}_{ij} = -\tilde{A}_{ii}\tilde{Y}_{ij}\tilde{B}_{jj}^{-1} = -\tilde{A}_{ii}^{-1}\tilde{Y}_{ij}\tilde{B}_{jj}.$$

When $\tilde{Y}_{ij} \neq 0$, we obtain $\tilde{A}_{ii} = \tilde{B}_{jj}$ and hence $\tilde{X}_{ij} = -\tilde{Y}_{ij}$. When $\tilde{Y}_{ij} = 0$, $\tilde{X}_{ij} = -\tilde{Y}_{ij}$ still holds. Therefore we obtain $\tilde{X} = -\tilde{Y}$ and therefore $X = -Y$. Note that so far we have not used the positive semidefiniteness of M and N .

When A and B are positive semidefinite, the conclusion $X = -Y$ is still correct. But we require the condition that M and N are positive semidefinite. We can assume that $\tilde{A} = \begin{pmatrix} A_1 & 0 \\ 0 & 0 \end{pmatrix}$ and $\tilde{B} = \begin{pmatrix} B_1 & 0 \\ 0 & 0 \end{pmatrix}$, where A_1 and B_1 are positive definite. Then $\tilde{M}$ and $\tilde{N}$ can be partitioned conformally as

$$\tilde{M} = \begin{pmatrix} A_1 & 0 & X_{11} & X_{12} \\ 0 & 0 & X_{21} & X_{22} \\ X_{11}^* & X_{21}^* & B_1 & 0 \\ X_{12}^* & X_{22}^* & 0 & 0 \end{pmatrix}, \quad \tilde{N} = \begin{pmatrix} A_1 & 0 & Y_{11} & Y_{12} \\ 0 & 0 & Y_{21} & Y_{22} \\ Y_{11}^* & Y_{21}^* & B_1 & 0 \\ Y_{12}^* & Y_{22}^* & 0 & 0 \end{pmatrix}.$$

As $\tilde{M}$ is positive semidefinite, the blocks X_{12} , X_{21} , and X_{22} are all zeros. Thus $\tilde{M}$ is permutationally equivalent to $\begin{pmatrix} A_1 & X_{11} \\ X_{11}^* & B_1 \end{pmatrix} \oplus 0$. Similarly, using the same permutation, $\tilde{N}$ is permutationally equivalent to $\begin{pmatrix} A_1 & Y_{11} \\ Y_{11}^* & B_1 \end{pmatrix} \oplus 0$. Since A_1 and B_1 are positive definite, applying the argument above we obtain $X_{11} = -Y_{11}$. Consequently $X = -Y$. $\square$

Also solved by Eugene A. HERMAN.

Problem 55-5: On the Rank of Integral Matrices

Proposed by Volodymyr PROKIP, *Institute for Applied Problems of Mechanics and Mathematics, Ukraine*, v.prokip@gmail.com

Let R be an integral domain with unit element $e \neq 0$ and let $M_n(R)$ denote the ring of n -by- n matrices over R . Let $\text{adj}(C)$ denote the classical adjoint matrix of $C \in M_n(R)$. Assume $A, B \in M_n(R)$ satisfy $\text{rank}(A) + \text{rank}(B) = \text{rank}(A+B) = n$.

Show that (i) $A \operatorname{adj}(A+B)A = A \det(A+B)$ and (ii) $A \operatorname{adj}(A+B)B = 0$.

Solution 55-5.1 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Let F be the field of fractions generated by R . The R -module R^n has almost the same properties as the vector space F^n . For example, $C(\operatorname{adj} C) = (\operatorname{adj} C)C = (\det C)I$ holds for all $C \in M_n(R)$. Also, if $v_1, \dots, v_k \in R^n$, then $\{v_1, \dots, v_k\}$ is linearly dependent over R if and only if it is linearly dependent over F . However, if $\{v_1, \dots, v_n\}$ is a linearly independent set in R^n and $v \in R^n$, then v need not be a linear combination over R of $v_1, \dots, v_n$; but, for some nonzero $r \in R$, rv is such a linear combination.

(ii). Since $\operatorname{rank}(A+B) = n$, $A+B$ is invertible over F and $\operatorname{adj}(A+B) = \det(A+B)(A+B)^{-1}$. Also, for any $u \in R^n$, there exist $r \in R \setminus \{0\}$ and $v \in R^n$ such that $rBu = (A+B)v$. Hence, $B(ru - v) = Av$. Since $\operatorname{rank}(A) + \operatorname{rank}(B) = n$, we have $\operatorname{Im}(A) \cap \operatorname{Im}(B) = \{0\}$. Thus, $Av = 0$ and $rBu = Bv$. Therefore

$$rA(\operatorname{adj}(A+B))Bu = \det(A+B)A(A+B)^{-1}Bv = \det(A+B)A(A+B)^{-1}(A+B)v = \det(A+B)Av = 0,$$

and so $A \operatorname{adj}(A+B)B = 0$.

(i). By (ii), $A(\operatorname{adj}(A+B))A = A(\operatorname{adj}(A+B))(A+B) = A \det(A+B)$. □

Solution 55-5.2 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

Let F be the quotient field of R . Then $M_n(R)$ can be viewed as a subset of $M_n(F)$ so that all standard techniques in linear algebra built on a field carry over. Let $C = \operatorname{adj}(A+B)$, $\delta = \det(A+B)$. From the identity

$$\begin{bmatrix} \delta A - ACA & 0 \\ A & A+B \end{bmatrix} = \begin{bmatrix} \delta I & -AC \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ I & I \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} I & I \\ 0 & I \end{bmatrix}$$

we conclude that $\operatorname{rank}(A+B) \leq \operatorname{rank} \begin{bmatrix} \delta A - ACA & 0 \\ A & A+B \end{bmatrix} \leq \operatorname{rank} \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} = \operatorname{rank}(A) + \operatorname{rank}(B)$. As the equalities hold, we have $\delta A - ACA = 0$ as desired. The second part follows from $ACB = AC(A+B) - ACA = \delta A - \delta A = 0$. □

Remark. In the proof, we only assume $\operatorname{rank}(A) + \operatorname{rank}(B) = \operatorname{rank}(A+B)$. The conclusion holds even if $\operatorname{rank}(A+B) < n$.

Also solved by the proposer.

Problem 55-6: Singular Value Inequalities for 2×2 Block Matrices

Proposed by Ramazan TÜRKMEN, *Science Faculty, Selçuk University, 42031 Konya, Turkey*, rturkmen@selcuk.edu.tr

Let $\begin{pmatrix} M & K \\ K^* & N \end{pmatrix} \geq 0$ be a positive semidefinite matrix with $M, N, K \in M_n(\mathbb{C})$. Denote by $s^\downarrow(K) = (s_1(K), \dots, s_n(K))$ the vector of singular values of K and denote by $\lambda^\downarrow(P) = (\lambda_1(P), \dots, \lambda_n(P))$ the vector of eigenvalues of $P \geq 0$, both arranged in decreasing order and with counted multiplicities. Prove that, with respect to weak majorization $\prec_w$, one has

$$(s^\downarrow(K), s^\downarrow(K)) \prec_w (\lambda^\downarrow(M), \lambda^\downarrow(N)).$$

Solution 55-6.1 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

We first consider the case that $K = \operatorname{diag}(s_1, \dots, s_n)$ with $s_1 \geq \dots \geq s_n \geq 0$. When $n = 1$, the conclusion follows from $M + N \geq 2\sqrt{MN} \geq 2K$ and $\max\{M, N\} \geq (M + N)/2 \geq K$. When $n > 1$, we assume that the conclusion holds for $1, \dots, n-1$. Let us denote $\operatorname{Sum}_d^\downarrow v = \sum_{i=1}^d v_i^\downarrow$, where $v = (v_1, \dots, v_m) \in \mathbb{R}^m$, $(d = 1, \dots, m)$. The goal is to show that

$$\operatorname{Sum}_d^\downarrow(s^\downarrow(K), s^\downarrow(K)) \leq \operatorname{Sum}_d^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)), \quad (d = 1, \dots, 2n).$$

If $d = 2k$ with $1 \leq k < n$, applying the induction hypothesis to the $2k \times 2k$ principle submatrix

$$\begin{pmatrix} M_{(1:k, 1:k)} & K_{(1:k, 1:k)} \\ K_{(1:k, 1:k)}^* & N_{(1:k, 1:k)} \end{pmatrix}$$

yields that

$$\operatorname{Sum}_d^\downarrow(s^\downarrow(K), s^\downarrow(K)) = 2 \sum_{i=1}^k s_i \leq \operatorname{Sum}_d^\downarrow(\lambda^\downarrow(M_{(1:k, 1:k)}), \lambda^\downarrow(N_{(1:k, 1:k)})) = \sum_{i=1}^k (\lambda_i^\downarrow(M_{(1:k, 1:k)}) + \lambda_i^\downarrow(N_{(1:k, 1:k)})).$$

By the Cauchy interlacing theorem, $\lambda_i^\downarrow(M_{(1:k, 1:k)}) \leq \lambda_i^\downarrow(M)$ and $\lambda_i^\downarrow(N_{(1:k, 1:k)}) \leq \lambda_i^\downarrow(N)$ for $1 \leq i \leq k$. Therefore,

$$\operatorname{Sum}_d^\downarrow(s^\downarrow(K), s^\downarrow(K)) \leq \sum_{i=1}^k (\lambda_i^\downarrow(M) + \lambda_i^\downarrow(N)) \leq \operatorname{Sum}_d^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)).$$

If $d = 2n$, applying the induction hypothesis to the 2×2 principle submatrices $\begin{pmatrix} M_{(i,i)} & s_i \\ s_i & N_{(i,i)} \end{pmatrix}$ yields

$$\text{Sum}_d^\downarrow(s^\downarrow(K), s^\downarrow(K)) = 2 \sum_{i=1}^n s_i \leq \sum_{i=1}^n (M_{(i,i)} + N_{(i,i)}) = \text{Tr}(M) + \text{Tr}(N) = \sum_{i=1}^n (\lambda_i(M) + \lambda_i(N)) = \text{Sum}_d^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)).$$

Thus we have achieved the goal when d is even. When d is odd, the conclusion follows from

$$\begin{aligned} \text{Sum}_d^\downarrow(s^\downarrow(K), s^\downarrow(K)) &= \frac{1}{2} \text{Sum}_{d-1}^\downarrow(s^\downarrow(K), s^\downarrow(K)) + \frac{1}{2} \text{Sum}_{d+1}^\downarrow(s^\downarrow(K), s^\downarrow(K)) \\ &\leq \frac{1}{2} \text{Sum}_{d-1}^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)) + \frac{1}{2} \text{Sum}_{d+1}^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)) \leq \text{Sum}_d^\downarrow(\lambda^\downarrow(M), \lambda^\downarrow(N)). \end{aligned}$$

Consider the case for a general K . Suppose that $K = U\Sigma V^*$ is the singular value decomposition of K . Then

$$\begin{pmatrix} U^*MU & \Sigma \\ \Sigma & V^*MV \end{pmatrix} = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix}^* \begin{pmatrix} M & K \\ K^* & N \end{pmatrix} \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \geq 0.$$

Using the conclusion for a diagonal Σ , we obtain $(s^\downarrow(K), s^\downarrow(K)) = (s^\downarrow(\Sigma), s^\downarrow(\Sigma)) \prec_w (\lambda^\downarrow(U^*MU), \lambda^\downarrow(V^*NV)) = (\lambda^\downarrow(M), \lambda^\downarrow(N))$. $\square$

Solution 55-6.2 by the proposer Ramazan TRKMEN

Let $x, y, z \in \mathbb{R}^n$. If $x \prec_w \frac{1}{2}(y + z)$, then, by [2, Theorem 1],

$$(x, x) \prec_w (y, z). \quad (*)$$

Bapat in [1, Lemma 1] obtained that if $\begin{bmatrix} M & K \\ K^* & N \end{bmatrix} \geq 0$, where M, N are square and of the same order, then

$$s^\downarrow(K) \prec_w \frac{1}{2}(s^\downarrow(M), s^\downarrow(N)).$$

Let X is positive semidefinite, then it is well-known $\lambda(X) = s(X)$. If we put notations of $s^\downarrow(K)$, $s^\downarrow(M)$, $s^\downarrow(N)$ instead of vectors x, y, z in $(*)$, the desired result follows. $\square$

References

- [1] R. Bapat, Majorization and singular values, *Linear and Multilinear Algebra* 21 (1987) 211–214.
- [2] R. Turkmen, V. Paksoy, and F. Zhang, Some inequalities of majorization type, *Linear Algebra Appl.* 437 (2012) 1305–1316.

Linear and Multilinear Algebra

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$$\xi^{\alpha} = \sum_{\pi} \operatorname{sgn} \pi \xi^{\alpha - \operatorname{id} + \pi}$$

$$\xi^{\alpha} \otimes \xi^{\beta} = \sum_{\pi} \operatorname{sgn} \pi \xi^{\alpha - \operatorname{id} + \pi} \otimes \xi^{\beta}$$

$$\xi^{\alpha} \otimes \xi^{\beta} = \sum_{\pi} \operatorname{sgn} \pi (\xi^{\beta} \downarrow S_{\alpha - \operatorname{id} + \pi} \uparrow S_n)$$

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IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce 6 new problems in this issue and invite readers to submit solutions for publication in *IMAGE*. **Solutions:** We present solutions to all problems in the previous issue [*IMAGE* 55 (Fall 2015), p. 44]. **Submissions:** Please submit proposed problems and solutions in macro-free L^AT_EX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Rajesh Pereira (pereirar@uoguelph.ca). Bojan Kuzma has finished his term as *IMAGE* Problem Corner editor. He thanks everyone involved in this column for support and help.

NEW PROBLEMS:

Problem 56-1: Minimizing Problem

Proposed by Janko BRAČIČ, *University of Ljubljana, NTF, Slovenia*, janko.bracic@fmf.uni-lj.si
and Cristina DIOGO, *Departamento de Matemática, Lisbon University Institute, Portugal*, cristina.diogo@iscte.pt

Let $A \in M_n(\mathbb{C})$ be a normal matrix with eigenvalues $\lambda_1, \dots, \lambda_n$, and let $\|\cdot\|$ be the spectral norm. Show that for a complex number $\lambda \neq \lambda_j$ ($j = 1, \dots, n$) there exists a complex number μ such that

$$\|(A - \lambda I)^{-1}(A - \mu I)\| < 1 \quad (*)$$

if and only if λ is not a convex combination of the eigenvalues $\lambda_1, \dots, \lambda_n$. In case of a general matrix $A \in M_n(\mathbb{C})$, find a necessary and sufficient condition for λ such that $(*)$ holds for some complex number μ .

Problem 56-2: Determinant of a Sum of Squares

Proposed by Gérald BOURGEOIS, *Université de la Polynésie française, BP 6570, 98702 FAA'A, Tahiti, Polynésie Française*, bourgeois.gerald@gmail.com

Let $(A_i)_{1 \leq i \leq p}$ be n -by- n real matrices that are simultaneously triangularizable over $\mathbb{C}$. Show that $\det(\sum_{i=1}^p A_i^2) \geq 0$.

Problem 56-3: On Areas of Some Related Triangles

Proposed by László LAJOS, *Eötvös Loránd University, Budapest Dept. Num. Analysis, Hungary*, laszlo@numanal.inf.elte.hu

Let $\text{Vert}(a, b)$ be the triangle with vertices $(a_i, b_i) \in \mathbb{R}^2$, $i = 1, 2, 3$, let $\text{Side}(a, b)$ be the triangle with vertices at crosspoints of lines $y = a_i x + b_i$, $i = 1, 2, 3$, and let $\text{Vand}(a)$ be the Vandermonde determinant of the a_i 's. Prove the formula

$$|\text{Vand}(a)| \cdot \text{Area}(\text{Side}(a, b)) = 2 \cdot \text{Area}(\text{Vert}(a, b))^2$$

for the area of triangles with vertices at points (a_i, b_i) and with side lines $y = a_i x + b_i$, $i = 1, 2, 3$.

Problem 56-4: A Determinant Inequality

Proposed by Minghua LIN, *Shanghai University, Shanghai, China*, mlin87@ymail.com

Let A, B be $n \times n$ positive semidefinite Hermitian matrices. Show that

$$\left| \det(A^{1/2} B^{1/2} + B^{1/2} A^{1/2}) \right| \leq \det(A + B).$$

Under the same assumptions, does it hold that $|\det(A^\nu B^{1-\nu} + B^\nu A^{1-\nu})| \leq \det(A + B)$ for each $0 \leq \nu \leq 1$?

Problem 56-5: Probability That a Large $\mathbb{F}_q$ -Matrix is Singular

Proposed by Denis SERRE, *École Normale Supérieure de Lyon, France*, denis.serre@ens-lyon.fr

Let $\mathbb{F}_q$ be a finite field with $q = p^n$ elements. If matrices inside $\mathbf{M}_n(\mathbb{F}_q)$ are chosen randomly with the entries independent and uniformly distributed over $\mathbb{F}_q$, find a closed formula for the probability $p_n(q)$ that $\det M = 1$ for $M \in \mathbf{M}_n(\mathbb{F}_q)$. Verify that $p_n(q) < \frac{1}{q}$ (hence $\det M = 0$ is more likely). Show that $n \mapsto p_n(q)$ is decreasing and admits a limit $p(q) \in (0, \frac{1}{q})$.

Problem 56-6: An Asymptotic Moment Property for Unitary Matrices

Proposed by Stephen RUSH, *University of Guelph, Guelph, Canada*, srush01@uoguelph.ca

Suppose U, V are unitary matrices (not necessarily of the same dimension) with $\lim_{k \rightarrow \infty} |\text{Tr}(U^k) - \text{Tr}(V^k)| = 0$. Show that U and V are unitarily similar.

Solutions to Problems 55-1 through 55-6 are on pages 41–46.