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FEATURE INTERVIEWS

Russ & Karen Merris

"Luck, location, and leaves of absence"

Russell Merris Interviewed by Wasin So[1]

Russell Merris is best known for his early work in multilinear algebra, most notably inequalities for immanants related to Elliott Lieb's permanental dominance conjecture, and later for contributions to algebraic graph theory. He is the author of four mathematical textbooks and a historical novel about the life and turbulent times of Hypatia of Alexandria.

W.S. - Where are you from, originally?

R.M. - I was born a few miles from the northern terminus of the Los Angeles aqueduct, grew up watching San Fernando Valley orange groves give way to urban sprawl, and entered high school a month before the launch of Sputnik – an event that revolutionized U.S. science education, much to my benefit. It was at San Fernando High, by the way, that I met life-long friend Bill Watkins. Following an outstanding public school education, I had the great good fortune to be accepted into the fourth full class of Harvey Mudd, the then newest member of the Claremont Colleges.

W.S. - I knew that you had a bachelor's degree in engineering. Can you share the story of how you became a mathematician after all?

R.M. - Graduating from Mudd in 1964, I followed my future wife Karen to UC Berkeley to do graduate work in civil engineering. At the same time, Watkins, who had graduated from UC Santa Barbara, was admitted into Berkeley's Ph.D. program in mathematics. That academic year witnessed the Berkeley Free Speech Movement, an episode that launched the political career of Hollywood actor Ronald Reagan. It was also an *el niño* year, meaning that every step of the mile walk from my apartment to the engineering circle was made in pouring rain. But all that is incidental at best and evasive at worst. The fact of the matter is, I didn't like Berkeley and I didn't like civil engineering.

W.S. - So, what did you do?

R.M. - Floundered; looked into switching to physics; wondered about mathematics; did well enough on the LSAT to get into law school; even took a vocational interest test that suggested I should be an orchestra conductor – less than useful given my scanty musical talent. Fortunately for me, Bill didn't much like Berkeley either. When he made an appointment with the chair of the Santa Barbara mathematics department to work out the mechanics of returning to his *alma mater*, he invited me along for the ride and introduced me to Marvin Marcus. When I asked Marvin whether I had any chance of being admitted to the UCSB graduate program on the strength of an undergraduate degree in engineering, his one word response was: "Transcript!" Happily, I had one in my shirt pocket. I don't know if it was Bill's implicit endorsement, my having come prepared, or the "A" in complex variables that did it but, with a vague allusion to remedial work, Marvin admitted me and offered me a teaching assistantship. In the fall of 1965, Karen and I moved to Santa Barbara, she to teach Mathematics and Spanish at La Colina Junior High School and I to prepare for comprehensive exams in mathematics. Upon discovering decades later that the University of California had only recently authorized its Santa Barbara campus to grant Ph.D.'s in mathematics, I surmised that Marvin had been hired to build up the program! So, once again, I had the great good fortune to be in the right place at the right time.

W.S. - So you were a student of Marvin Marcus at UCSB, a center of Linear Algebra with many well-known people. Do you have any unforgettable memories from that time?

R.M. - One of my favorites involves a colloquium by your thesis advisor, R.C. Thompson. Then a young assistant professor, Bob arrived a few minutes late for his talk, carrying a cardboard box. Without a word, he upended the box spilling maybe 20 reprints out onto the front table. Picking one at random, he glanced at it, turned to the board, gave a riveting lecture (presumably) about the contents, returned to the table, and repeated the process two or three more times. Hard as it may be to believe, Bob appeared to think his pending promotion to Associate Professor was in some jeopardy. If so, his performance may have been decisive.

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W.S. - What was your first job?

R.M. - Watkins and I both earned degrees under Marcus in 1969. Luckily, I finished in the spring and he in the summer, which meant I didn't have to compete with him for a National Research Council postdoc at the Bureau of Standards. Like most freshly minted Ph.D.'s, my research program was in its infancy to say the least. Two years at the Bureau with no duties and marvelous mentor Morris Newman available whenever I wanted was a godsend! Throw in sharing an office with fellow Marcus student Steve Pierce my first year and by the time my postdoc ended in the summer of 1971 I had half a dozen articles accepted for publication, others in the pipeline, and more than enough momentum to get me through the notorious first year of full-time teaching.

W.S. - Where was that?

R.M. - Again, I had the good fortune to be in the right place at the right time. Between 1969 and 1971, the academic job market had pretty much tanked – not to the extent it has these days of course, but it was significantly worse than a few years earlier. Independently of that, for personal reasons, I had to return to California. Fortunately, enrollments were surging at the still relatively new Hayward campus of the California State University system and I was hired on as an assistant professor.

W.S. - Teaching 12 units a semester?

R.M. - The Hayward campus was on the quarter system, but yes.

W.S. - For how long were you, yourself, expected to teach 12 units a quarter?

R.M. - Until I retired in 2005.

W.S. - Wow, 34 years! How, then, did you manage to be so productive?

R.M. - Three L's: Luck, Location, and Leaves of absence.

W.S. - Tell us about your leaves of absence.

R.M. - Among the more interesting was a Fulbright Lectureship at the University of Islamabad in Pakistan where, during the winter and spring of 1973, I experienced an Islamic world that included Karachi, the Swat Valley, Peshawar, Kabul, and the Khyber Pass. On the heels of the Pakistani adventure, I cheerfully accepted an invitation from Graciano de Oliveira to visit the Instituto de Física e Matemática, stopping in Beirut, Cairo, and Rome on the way to Lisbon where my duties amounted to conducting a research seminar for 12-15 advanced graduate students – a group that included Eduardo Marques de Sá and José A. Dias da Silva. (Given the CSU's lack of a Ph.D. program, José is the closest I ever came to having a student of my own.) A third especially significant leave came in 1979 when, under the auspices of a National Academy of Sciences exchange program with the Czech Academy, I visited Miroslav Fiedler in Prague for five months. You would be very hard pressed to name a kinder, more generous, more genuine human than Mirek Fiedler. It was he, by the way, who introduced me to Laplacian matrices of graphs.

W.S. - Your trip to Czechoslovakia must have taken place between the Soviet suppression of the Prague Spring and the fall of the Berlin Wall.

R.M. - Yes. I took my heroic wife and two young children to Czechoslovakia in the midst of an especially repressive era, to Pakistan a year after the December 1971 Indo-Pakistani war, and to Portugal a year before the April 1974 Carnation Revolution. Indeed, we lived in Lisbon during the Arab Oil Embargo and experienced first-hand the heavy price Portugal paid for allowing USAF planes to refuel in the Azores on their way to resupply Israel during the Yom Kippur war.

W.S. - What about the other two L's, Luck and Location?

R.M. - In the early 1970s, research was viewed with some suspicion on the Hayward campus. When, for example, I applied for a National Science Foundation grant in the fall of 1974, I had to obtain signatures from no fewer than eight administrators – during the first week of classes no less – before the President would forward my application to Washington. I thought my chances of being successful could be as high as 1 in 10. Had I been turned down, I might have been willing to go through the hassle of applying a couple of more times before giving up for good. I will never know, because Fortune smiled on me again and I was awarded a modest summer research grant. It is, of course, easier to get a second grant than a first, a third than a second, and so on. In my case, NSF continued to fund my research until 1985, the year I landed a huge 3-year contract from the Office of Naval Research that included funds for student assistants and postdoctoral research associates! Between 1985 and 1989, I was able to bring Bob Grone and V.S. Sunder to campus for a summer, help sponsor Ravi Bapat during an academic year, and invite Mirek Fiedler, Steve Pierce, and Bill Watkins to campus for short visits. Independently funded long-term visitors during that period were José da Silva and M. Ishaq ... which brings me to Location. The Hayward campus is midway between Berkeley and Stanford. When

Rajendra Bhatia visited Berkeley in hopes of collaborating with Chandler Davis and found that Chandler had gone back to Toronto, I was available to help fill the void. When Sunder was granted a leave of absence from the Indian Statistical Institute to visit Vaughan Jones only to discover that Jones, himself, would be away from Berkeley on leave, he came to Hayward instead. When K. Balasubramanian, a brilliant young postdoc at the Lawrence Berkeley National Lab, learned that the author of a result he'd found useful in analyzing Nuclear Magnetic Resonance spectra lived just down the road in Hayward, he came to see me. Balu, as his friends call him, became an eager aficionado of frisbee golf on the Cal State campus years before gaining international recognition for his work in theoretical chemistry ... which reminds me of the Internet. I think it was at the ILAS meeting in Atlanta that I met Onn Chan. After his talk, I mentioned to Onn that one of his results could be expressed in terms of the "third immanant." A few months later, Chan emailed from Singapore inviting me to collaborate with him and T.-K. Lam on an article about the famous Wiener Index from chemistry. Subsequently, world-renowned Serbian theoretical chemist Ivan Gutman emailed us to propose collaborating on an article about algebraic connections between chemical indices. Two years after that, I published a joint article with Gutman on the number of high-energy bands in the photoelectron spectrum of alkanes. The point is that I never met Lam or Gutman!

W.S. - Your participation in the Atlanta meeting seems to have paid off handsomely! How would you describe the value of mathematics meetings in general?

R.M. - That's an interesting question. Have some meetings been a waste of time? Absolutely. Others? The opposite. Take the 1986 Auburn matrix theory conference for example. Bill Watkins and I were on our way to rent a car in the Atlanta Airport when I spotted Gene Golub and offered him a ride. I spent much of the drive to Auburn complaining to Bill about how the linear algebra community was being short-changed by the AMS, and discussing a rumor that Hans Schneider was trying to organize a group that would be more responsive to our needs. Meanwhile, Gene sat in the back all but forgotten. When we dropped him at his hotel, he thanked us for the ride and suggested we investigate SIAM. Unbeknownst to Bill and me, Gene had just been elected president of SIAM! How much or even whether my diatribe contributed to SIAM's investment in its Linear Algebra SIAG, I'll never know, but I feel certain it helped me land an appointment to the editorial board of the new SIMAX journal. The conference itself afforded numerous highlights, not the least of which was meeting Tom Pate who, inspired as much by George Soules as by my survey talk on the subject, went on to become the world's leading expert on Elliott Lieb's permanental dominance conjecture. Then there was ILAS 2, the 1992 Lisbon conference organized by José da Silva. As measured by requests for reprints, downloads from the Elsevier web site, and MathSciNet citations, the Laplacian matrix survey I published in connection with my Lisbon talk is the most influential article I've ever written. Other career highlights occurred at meetings in Edinburgh and Hong Kong in 1993, Prague in 1996, Haifa in 1998, and Cancun in 2008.

W.S. - Among your many contributions to Linear Algebra, which are the ones that you like the most?

R.M. - The first thing that comes to mind is a result that blends several of my mathematical interests, namely, multilinear algebra, group representation theory, algebraic graph theory, and giving Cal State students meaningful opportunities to participate in mathematical research. The student in this case was Phil Botti who helped me with the computational aspects of some work related to the graph isomorphism problem: An early hope that the adjacency spectrum might characterize isomorphism, i.e., that similar graph adjacency matrices must necessarily be permutation similar, had long since been refuted. In 1973 Allen Schwenk went further when he proved that, for almost all trees T, there is a nonisomorphic tree T^* such that T and T^* are adjacency cospectral. All right, what about something like the permanental polynomial which is generically preserved only under monomial similarity? Already, in 1968, J. Turner exhibited two nonisomorphic 12-vertex trees T and T^* for which the immanantal polynomial $d_\chi(xI - A(T)) = d_\chi(xI - A(T^*))$ for all 77 irreducible characters χ of the symmetric group S_{12}. (Determinant and permanent correspond to $\chi = \epsilon$ and $\chi = 1$, resp.) In 1993, finding what I needed in a 1977 Brendan McKay computer search, I managed to unite the results of Schwenk and Turner: two graphs on n vertices are co-immanantal mates if for every character of S_n their adjacency immanantal polynomials and their Laplacian immanantal polynomials are both equal; then almost all trees have a nonisomorphic co-immanantal mate. (Left unanswered is whether a similar result holds for 2-connected graphs.)

W.S. - Do you have any advice for young researchers in Linear Algebra?

R.M. - Don't let yourselves be intimidated by fear of failing. If you're not failing from time to time you're not living up to your full potential. That's not the same thing as never giving up. Indeed, nothing is more useful to a working mathematician than knowing when to give up and switch to another problem – or even to another subdiscipline.

"An atmosphere full of unwavering support and encouragement"

Heike Faßbender Interviewed by Beatrice Meini[1]

Heike Faßbender and Beatrice Meini

Heike Faßbender is known for her book *Symplectic Methods for the Symplectic Eigenproblem* and her many papers in numerical linear algebra, structured eigenvalue problems and non-linear eigenvalue problems. She has supervised four Ph.D. students. She currently serves as the president of GAMM (the International Association of Applied Mathematics and Mechanics) and serves as an associate editor for both *Linear Algebra and its Applications* and the *SIAM Journal on Matrix Analysis and Applications.*

B.M. - Did you always know that you wanted to become a mathematician?

H.F. - Well, I do not recall a specific event which got me interested in mathematics or even made me say I wanted to be a mathematician. It just happened this way. Even so, in 6th grade I told my parents that I wanted to become a mathematics professor. Our mathematics teacher at that time held a Ph.D. in mathematics, but seemed more interested in her heavy make-up, her clothes and her chihuahua (which accompanied her to every class she was teaching) than in explaining mathematics to us. I explained to my parents that I wanted to prove to everyone that one can do better than that.

B.M. - I hope you had more inspiring mathematics teachers afterwards.

H.F. - Yes, definitely. During the final three years of high school we had a great mathematics teacher. She not only was able to explain mathematics to us, but she also raved about how fantastic life as a student had been. Other teachers organized extracurricular activities in mathematics. I really enjoyed these. But I also liked Biology and was fairly good in English and Latin. So when it was time to decide about enrollment, I was not sure about my major.

B.M. - So how did you decide to enroll for mathematics?

H.F. - I was the first person in our family who wanted to go to a university. My parents were a bit skeptical about this. They wanted me to choose a subject that promised a secure job. I ruled out Biology (which would have meant cutting up a frog – a no-go for me at that time) and Languages (very boring fill-in exercises) quite quickly and was left with my interest in mathematics. My father (a very typical conservative, suit-wearing public officer) had arranged a meeting with a young mathematician at the University of Bielefeld for me. This young mathematician turned out to be Volker Mehrmann, with wild hair, a beard and hand-knitted jumper and socks. Volker described mathematics in such a fascinating way that I decided to study mathematics. (Or was it just his very different attire?) I am still grateful to Volker for taking time to talk me into studying mathematics! But as mathematics itself did not seem to promise a secure job, I looked into mathematical finance. That would have meant leaving Bielefeld and moving to a different city. My parents did not like that idea. As the coursework for the major in mathematical finance and the one in mathematics was almost identical for the first two years, we agreed that I could start my studies at the University of Bielefeld. That way I would be able to live at home (no additional costs) and my family would be able to see whether the university will have some kind of negative effect on me. (I am still not quite sure what they thought might happen to me at the university. My two younger brothers were kind of forced to leave home when they started to go to university.) It was further agreed that I would be allowed to attend a different university after the first two years of study in order to continue the education in mathematical finance.

B.M. - But, as far as I know, you never left Bielefeld before you got your first degree.

H.F. - Right. In retrospect, I am really glad that my parents were so anxious: mathematical finance would never have met my interests. In Bielefeld my minor was economics and I did not really like that. For passing an exam you just had to memorize 100 pages of text and repeat the correct part of it. I did not see the point of that. But in my third term in Bielefeld I met the first mathematics professor who really impressed me, Wolfgang Dahmen. He taught our Numerical Analysis class. He had just joined the faculty at the University of Bielefeld and it was his first term of teaching. So he was much younger than our professors in Linear Algebra and Calculus. He was the first professor who was really enthusiastic for what he was teaching. His four-hour class kept me busy for the entire term and introduced me to the

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fascinating world of numerical analysis. Motivated by his class, I attended seminars offered by the numerical mathematics group, in particular by Volker Mehrmann and Angelika Bunse-Gestner. Moreover, I attended a number of classes on numerical linear algebra by Ludwig Elsner. The atmosphere within the group was great. I think besides the fact that I like numerical linear algebra, this atmosphere was a major point in getting me really into research and mathematics.

B.M. - Can you describe what you mean by this atmosphere?

H.F. - An atmosphere full of unwavering support and encouragement. It is this atmosphere which is very special to our field. I think Roger Horn put it as "this culture of supporting, nurturing and sharing with our colleagues." From day one, no one (in particular none of the prominent researchers visiting Bielefeld) made me feel to be just a stupid student; I was allowed to be part of the team and taken as seriously as anyone else.

B.M. - What made you stay in academia?

H.F. - During my work on my diploma thesis, Angelika arranged for me to get to know Pat Eberlein, who invited me to come to Buffalo to work with her. I gladly took up that chance and spent 17 months at SUNY Buffalo, where, among many other positive events, I got to know Nil and Steve Mackey, two of my later coworkers. When it was time to return to Germany, Angelika offered me a Ph.D. position in her newly-established group at Bremen University. From that point on, things just went very smoothly. I got my Ph.D. and my habilitation at Bremen, moved to TU Munich for my first professor position, and just a few years later to TU Braunschweig.

B.M. - What do you enjoy most in your research work?

H.F. - Well, a couple of things. The freedom to decide which research questions to work on, the opportunity to decide which new subjects to learn about, and the possibility to work with eager bright students. But I also enjoy getting into a flow, being fully immersed in my research, this complete absorption that is so special (and hard to understand by non-mathematicians).

B.M. - You are a leading researcher in numerical linear algebra, the head of the AG Numerik of the Institute Computational Mathematics and the president of the International Association of Applied Mathematics and Mechanics (GAMM). You have been vice president of your university and dean of your department. There aren't many females having such important roles in the academy and in research. Do you think that gender might influence the scientific and academic career of a female researcher?

H.F. - Yes, I think gender influenced my career. But this is hard to fix. Women are just as gifted when it comes to science as men are! Within the Linear Algebra community I never had the feeling that females are discriminated against. This has been different in the German scientific world. I have been clearly told that 'a female cannot successfully fill a demanding position as a full professor leading a strong institute at a German technical university.' In my opinion, parts of the German scientific system are discriminating by design. For example, in our university 40% of all members of all committees have to be female. This is well-intentioned, to make sure that the old boys networks do not work as they used to. But, on the professor level at my university, only 19% are female. Thus, I have to sit on a lot more committees than my male colleagues, spending (sometimes wasting) time while my colleagues can do research. The very same system then penalizes females for not writing as many papers as their male colleagues.

B.M. - Is there any advice that you would give to young researchers?

H.F. - Find a problem that you think is interesting mathematics and work on it. Get deeper and deeper into the theory. But, being a professor at a technical university working on real applications, I have to point out that this is not enough. At the same time you should be open to people from engineering or life science, listen to their problems and try to see how your research might help them. Learn their language (as well as the language of related mathematical fields). New and interesting problems will show up. Go to workshops and conferences and talk to other researchers. Do not be afraid to ask. Have the courage to go into new areas and fields, even if at the beginning you feel dumb.

B.M. - In order to help our readers to get to know you a little better, please complete the following sentences:

H.F. - My best ideas come to me when I am not expecting them, at night or while taking a walk, definitely not at my desk.

When my computer dies I am stymied. I usually have a second one, but it is not always up-to-date.

When I am frustrated about some administrative stuff I try to take a walk and remind myself that being a professor is one of the greatest jobs on Earth since you can decide freely on what kind of research you want to do and you can work with bright and eager young students from which you can learn so much. When I am frustrated about my current research as I do not make progress as desired, I try to take a break and do something else, sometimes for days, in order to clear my mind and to be able to focus again and see other aspects which I have not seen before. Sometimes it helps to talk to Peter (Benner, my husband).

On my office wall there is a pin board with (almost all) my name badges from the conferences and workshops I have been to.

In Germany, professors usually have to retire at the age of 65. When I retire Peter and I will certainly not sit in our garden and watch the grass grow, but we have not thought about this yet. That is more than ten years ahead of us.

Usually, a day starts with a cup of tea, two daily newspapers and checking my e-mails.

BOOK ANNOUNCEMENT

Exploiting Hidden Structure in Matrix Computations: Algorithms and Applications (Cetraro, Italy 2015) by M. Benzi and V. Simoncini (Editors)

Lecture Notes in Mathematics vol. 2173, CIME Foundation Subseries, Springer, 2016 ix + 406pp., ISBN 978-3-319-49886-7.

This volume collects the text of the lectures given at the CIME Summer Course of the same title in Cetraro, Italy in June 2015. The table of contents is as follows:

M. Benzi and V. Simoncini, *Preface*

C. F. Van Loan, *Structured Matrix Problems from Tensors*

D. A. Bini, *Matrix Structures in Queuing Models*

J. Ballani and D. Kressner, *Matrices with Hierarchical Low-Rank Structures*

M. Benzi, *Localization in Matrix Computations: Theory and Applications*

H. Z. Munthe-Kaas, *Groups and Symmetries in Numerical Linear Algebra*

The volume is also available as an eBook.

For further information, see `http://www.springer.com/us/book/9783319498867`.

ARTICLES

Development of Linear Algebra in Korea

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and
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Linear Algebra and related topics have formed one of the main research fields for several decades in Korea. It has been only 30 years since some Korean mathematicians were exposed to research in the field of linear algebra. The first researcher was Suk-Geun Hwang at Kyungpook National University (KNU) who had studied combinatorial matrix theory under the supervision of Prof. Richard A. Brualdi at the University of Wisconsin–Madison and then came back to Korea with a doctorate degree. In 1987, he published an article titled "On a Conjecture of E. Dittert" in *Linear Algebra and its Applications*. It could be considered the first article on linear algebra ever published by a Korean. Since then he continued his study on permanent theory, and worked as a dissertation advisor. He taught and guided many Ph.D. candidate students at KNU and Sungkyunkwan University (SKKU), starting with Seok-Zun Song (Jeju University) through Suk-Soo Do (Inje University), Si-Ju Kim (Andong University), Hang Kyun Shin (Wooseok University), Gi-Sang Cheon (SKKU), Ik-Pyo Kim (Daegu University), and Seong-Soo Pyo (KNU), to Jin-Woo Park (Daegu University) and Eun-Young Lee (KNU) in the field of combinatorial matrix theory.

Just in time, two new Korean Ph.D.'s returned to Korea after their study in the USA, and led studies in the field of Linear Algebra. Han Hyuk Cho (under R.A. Brualdi), who was appointed as a professor of Seoul National University (SNU) in 1989, studied combinatorial matrix theory and graph theory. Sang-Gu Lee (under LeRoy B. Beasley, Utah State University) became a professor at SKKU in 1991, where he has guided Gwang-Yeon Lee (Hanseo Univ), You-Ho Lee (Daegu Haneui Univ), Se-Won Park (Seonam Univ), Han-Guk Seol (SKKU), Jin-Soo Kim, Min Ae Jin (Tianjin University of Finance and Economics), Jeong-Mo Yang (NRF), Duk-Sun Kim (NHN-Naver), Holodagava (Mongolian Univ of Education), and Kyung-Won Kim (Tekville.com) to their doctorates, so they could diversify research in combinatorial matrix theory and numerical linear algebra in Korea.

In 1990, the Topology and Geometry Research Center, the first prestigious mathematics research center at KNU, was established, where Suk-Geun Hwang served as a senior researcher/director for nine years and contributed greatly in Korea to vitalizing studies in the field of Linear Algebra. Importantly, he founded a Korean branch of the International Linear Algebra Society (KoBILAS) by joining hands with not only researchers who were either his former students or those who came back to Korea after their study abroad, but also their disciples. A joint research group of linear algebra was gradually formed among KNU, SKKU, and SNU, and studies in linear algebra began in earnest in Korea. At that time, some Korean mathematicians who had studied combinatorial matrix theory and graph theory under the guidance of Professor Brualdi came back to Korea after earning their doctorates: Hyeong-Chan Jung (Korea Tech), Geum-Sook Hwang (Busan University of Foreign Studies), and Yoomi Rho (Incheon University). Besides, Suh-Ryung Kim (SNU) also came back home after obtaining a doctorate in graph theory from Rutgers University. Since 1992, operator theorists Seung-Hyeok Kye (SNU) and Woo Young Lee (SNU) began to make contributions in the field of linear operators, positive linear maps and matrix algebras. They also studied the facial structure of a convex set consisting of positive linear maps.

As more and more new researchers got involved in diverse research areas of linear algebra, various academic conferences took place in Korea.

In 2001, the Com2MaC Symposium on Combinatorics, Graph Theory, Algorithms and Matrix Theory was held at POSTECH (Pohang University of Science and Technology). In 2002, the International Conference on Combinatorial Matrix Theory was held at POSTECH as well. In 2009, a Combinatorial Matrix Theory session successfully ended in the first joint meeting between the Korean Mathematical Society and the American Mathematical Society. In 2010, the Workshop on Linear Preserver Problems took place at SKKU.

Yongdo Lim, who got a doctorate under the supervision of Professor Jimmie Lawson at Louisiana State University, made remarkable achievements in research on the Karcher mean, to which he contributed for more than 10 years after returning to Korea. Yongdo Lim was one of the plenary speakers at ILAS 2014. He has been serving as an editor for the journals *Linear and Multilinear Algebra*, *Annals of Functional Analysis* and *J. Korean Math. Society.*

Hyeon-Min Kim obtained his doctorate in numerical linear algebra under the guidance of Prof. Nicholas Higham at Manchester University, England. Now he is a professor of Busan National University.

Over the past 10 years, SKKU was able to build up a research group consisting of Sang-Gu Lee, Gi-Sang Cheon, Yongdo Lim, Jang Soo Kim and Yoon Mo Jung, with more than 10 Ph.D. graduates in the field of Linear Algebra.

With those efforts, the three linear algebraists, Sang-Gu Lee, Yongdo Lim and Seok-Zun Song served as members of the organizing committee of the 2014 International Congress of Mathematicians held in Seoul. Along with Gi-Sang Cheon and Han Hyuk Cho, they also hosted the invited mini-symposia and contributed mini-symposia of the 19$^{\text{th}}$ International Linear Algebra Society Conference held in 2014, at which many Korean linear algebraists delivered presentations.

In 2016, a new science research center, the Applied Algebra and Optimization Research Center (AORC), opened at SKKU and will be sponsored by the Korean Government for seven years. The director is Gi-Sang Cheon, a leading researcher in the Riordan matrix group. One of the main research areas of AORC is Linear Algebra. It is expected, in connection with AORC, that far more research and activity in linear algebra will take place in Korea for the next few decades.

Acknowledgement. The authors gratefully acknowledge help from Suk-Geun Hwang, Gi-Sang Cheon, Han-Hyuk Cho, Yongdo Lim, and Suh-Ryung Kim on writing this article.

Supplementary Notes on Gaussian Elimination

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1. Introduction. In this article we are concerned with various methods for solving the system of n linear equations in n unknowns $Ax = b$ where A and b are $n \times n$ and $n \times 1$ matrices of known elements and x is an $n \times 1$ matrix of unknown coefficients. Farebrother [7, 8, 9] has identified three distinct elimination procedures:

(a) The first, which Farebrother [7] attributes to Carl Friedrich Gauss (1777–1855) [12] in 1811, may be interpreted as premultiplying the system $Ax = b$ by a sequence of $n \times n$ elementary matrices $M_1, M_2, \ldots$, where each of these matrices, their product M, and its inverse $L = M^{-1}$ are $n \times n$ lower triangular matrices with unit elements on their principal primary diagonals. Further, these elementary matrices are modified identity matrices (with a single nonzero off-diagonal element) chosen in such a way that $MA = U$ is an $n \times n$ upper triangular matrix.

(b) The second elimination procedure, which Farebrother [8, 9] attributes to Johann Tobias Mayer (1723–1762) [16] in 1750, is a variant of Gauss's [12] procedure which rescales the rows of the $n \times (n + 1)$ array $[A\ b]$ in such a way that U is an upper triangular matrix with unit elements on its principal diagonal.

(c) The third variant of this procedure Farebrother [7] attributes to the ancient Chinese, as it is featured in the eighth chapter of the *Jiuzhang Suanshu (Nine Chapters on the Mathematical Art)*. This procedure assumes that the elements of A and b are integers and operates in such a way that it preserves integers but at the cost of generating entries that can rapidly exceed the largest integer that can be stored on a computer (unless special provision has been made for the storage of such numbers).

We now outline some supplementary remarks on the contributions of Brezinski and Tournès [1], Farebrother [7, 9] and Grcar [14, 15] in this context. Also see Searle [17, §4.3].

2. Unified Terminology. There is clearly some need to associate a single name with all three variants of this elimination procedure and several authors including Grcar [15] have chosen to name them all for Gauss. However, the standard real number variant of the Gaussian elimination procedure still needs to be carefully distinguished from its scaled and whole number variant as they have distinct numerical properties

As an example of the type of confusion that can follow from the misapplication of such general terminology, we observe that Brezinski and Tournès [1, p. 86] employed the whole number variant of the procedure to solve a 1787 worked example. The lower triangular matrix corresponding to their manipulations is clearly formed as the product of three lower triangular matrices with 2, 2 and 5 on their principal diagonals and a single unit element below the diagonals. But Brezinski and Tournès [1] still claim that Gaussian elimination sets $A = LU$ where L has unit elements on its principal diagonal. That is, they employ the whole number procedure before citing a result that is relevant only for the standard real number procedure.

3. Doolittle, Wallace, Snedecor and Crout. Instead of factoring A as $A = LU$ indirectly as the product of a sequence of elementary matrices, we may seek to factor it directly. This was the approach adopted (apparently independently) by Myrick Hascall Doolittle (1830–1913) [5] in 1881, by Henry Agard Wallace (1888–1965) and George

Waddel Snedecor (1881–1974) [20] in 1925, and by Prescott Durand Crout (1907–1984) [4] in 1941. For further details of these and other related procedures, see Brezinski and Tournès [1, pp. 87–91, 105–106]. And for further details of the history of Wallace and Snedecor's procedure, see Carriquiry and David [2] and Farebrother [11] (quoted by [1, p. 91]).

In this context, Farebrother [7, p. 27] and Grcar [14, p. 206] note that Doolittle's procedure, like that of Gauss [12], is associated with a lower triangular matrix L with unit elements on its principal diagonal. Thus Brezinski and Tournès's [1, p. 88] suggestion, to the contrary, that Doolittle's procedure is associated with an upper triangular matrix U with unit elements on its principal diagonal must be incorrect.

4. Lewis Carroll. Grcar [14, 15] does not examine Johann Tobias Mayer's [16] scaled variant of Gauss's [12] procedure in his otherwise excellent surveys of Gaussian Elimination. Nor does he mention the contribution of Charles Lutwidge Dodgson (1832–1898), better known as Lewis Carroll, towards improving the implementation of the whole number variant of this method. As noted in Farebrother [7, p. 18], this improvement was discussed in Chapters 4 and 5 of Dwyer [6].

5. Cramer's Rule. Of the same vintage as Mayer's elimination procedure and with a precursor for the case $n = 2$ in the eighth chapter of the *Jiuzhang Suanshu*, is the rule of Gabriel Cramer (1704–1752) [3] for solving systems of n linear equations in n unknowns by a technique involving the evaluation of $n + 1$ determinants of size $n \times n$ that Farebrother [10] has interpreted as an orthogonalisation procedure.

In this context, Brezinski and Tournès [1, p. 85] have noted that

> Of course, Cramer's formulae give, in theory, the solution but, from the practical point of view, they cannot be used. Computing determinants by their minors ...
>
> Indeed, on a computer performing 10^7 operations per second, it requires 37 seconds when $n = 10$, 2 hours when $n = 12$, more than one day for $n = 13$, almost one year for $n = 15$, 17 years when $n = 16$, and ... 10^{10} years if $n = 23$ which is more than the age of the Universe! This is why scientists rapidly turn to elimination methods such as Gauss's one.

Simple iterative calculations establish that the above timings are correct for a full Laplace expansion of $n+1$ determinants of size $n \times n$ each of which is evaluated as the sum of $n! = 1 \times 2 \times \cdots \times n$ terms each of which is formed as the product of n elements (and therefore requires $n - 1$ multiplications). [Note that this is not an expansion by minors as stated in the above quotation.]

To help explain a mysterious figure in the final paragraph of Farebrother [10], we augment the above list by inserting the phrase "3 million years when $n = 20$" and deduce that it takes 146,850 years to evaluate a single 20×20 determinant on a machine performing 10^7 operations per second (or 1.4685 million years on an older machine performing 10^6 operations per second).

If instead of using the Laplace expansion, the typical $n \times n$ determinant is evaluated using minors, then the number of multiplications per $n \times n$ determinant is reduced from $(n!)(n-1)$ to about $(n!)(e-1)$ where $e = 2.7182818285$ is the base of natural logarithms. In this context, we find that it takes about 2.78 million years to evaluate 21 determinants of size 20×20 on a machine performing 10^6 operations per second, or about 132,470 years per determinant, if these are evaluated using minors. This last figure is in accord with the crude (if readily available) lower bound of 150,000 years quoted in [10].

6. Scrappy Histories. In the final sentence of his first section, Grcar [15, p. 782] notes: "there may be no other subject that has been molded by so many renowned mathematicians [as Gaussian elimination]." However, I would suggest that Ivor Grattan-Guinness (1941–2014) [13] might have contended that the subject areas of linear programming and matrix theory would have been strong competitors for this title.

Acknowledgements. I am indebted to Claude Brezinski for sending me a copy of Brezinski and Tournès [1] book on the life of André-Louis Cholesky (1875–1918) and to Joseph Grcar for sending me copies of his articles [14, 15]. Despite the minor criticisms recorded in the present article, I strongly recommend all three works to the reader.

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LINEAR ALGEBRA EDUCATION

Each year, a highlight of the happenings in Linear Algebra education is the "Innovative and Effective Ways to Teach Linear Algebra" session at the Joint Meetings of the American Mathematical Society and the Mathematical Association of America. At the meetings held this past January in Atlanta, there were as usual a number of very worthwhile presentations. In this issue of *IMAGE* we highlight two of these, both of which summarize some of the good Linear Algebra teaching taking place at Brigham Young University. Links to talks from past Joint Meetings can be found at `http://seaver-faculty.pepperdine.edu/dstrong/LinearAlgebra/index.html`. On a related note, let me also encourage everyone to consider attending the dozen or so talks on teaching Linear Algebra that will be part of our upcoming ILAS meetings in Iowa.

David Strong
Contributing Editor for Education

Teaching Advanced Spectral Theory to Undergraduates Using Computer Labs

Emily J. Evans[1]

Occasionally in linear algebra, a student will pose the question, "How is this used in the real world?" Other times, students will question the utility of a mathematical method because the calculations involved are tedious and error prone. At Brigham Young University, several faculty members are involved in the development of a curriculum in applied and computational mathematics that seeks to address both of these questions for linear algebra and several other mathematical topics. Students enrolled in this program are very interested in both the practical applications and computational aspects of what they are learning. To this end, we have designed a lab curriculum that is taught in conjunction with the more theory-based class that teaches numerical linear algebra algorithms and provides opportunities for the students to test real-world applications of linear algebra. The goal of these labs is to reinforce student learning and to keep the students engaged in and excited about the curriculum.

We have chosen to use Python as the computing language for our labs because it is relatively simple for the students to pick up, it is an open source language, which removes cost as a barrier to entry, and it is designed for scientific computing. Incoming students have taken an introductory computer programming class (usually in C++) as a prerequisite, but at least half of the incoming students have never used Python (or any other numerical computing languages). For this reason, the labs for the first three weeks of the semester focus on learning Python essentials. For the remainder of the course, students use Python to reinforce the topics they learn in the classroom. As the semester progresses, students "earn" the privilege of using commercial-grade Python software libraries for their assignments.

As an example of how these labs are used to support classroom learning, consider the teaching of eigenvalues, eigenvectors, and the singular value decomposition (SVD). In class, students review eigenvalues, eigenvectors, and invariant subspaces. In addition, the students are taught about simple/semi-simple matrices, a simplified version of the spectral mapping theorem that holds for semi-simple matrices and polynomials, and the Schur decomposition. The classroom portion on eigenvalues takes approximately two weeks to teach and so students do two labs to reinforce the material taught in class. The first lab is a practical lab in which the students

- code the power method to find the dominant eigenvalue/eigenvector,
- code the QR algorithm to find all the eigenvalues, and
- modify the code to use Hessenberg preconditioning.

Completion of this lab "earns" students the privilege of using the scipy.linalg package to perform eigenvalue/eigenvector computations.

The second lab is an application lab where students use eigenvectors to perform image segmentation. The pairing of a practical lab with an application lab is common throughout the first semester. Although students only spend three class periods learning about the SVD, students also do two labs focused on SVD. The lab with a practical focus has the students calculating the SVD for a small matrix, introduces commercial packages to compute the SVD, and then has

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the students perform image compression. The second lab, which is application-focused, is one of the favorite labs of the year. In this lab, students use the SVD and the eigenfaces method to perform facial recognition.

In the classroom and in the lab, more advanced spectral topics are also taught, including the spectral decomposition of matrices, the Perron-Frobenius theorem, the Drazin inverse, and Krylov subspaces. These topics are taught during the second semester of the year-long course, when students are more comfortable with Python and the mathematical topics being presented. The labs typically combine the "how to" with the application. An example of this combination is the lab on the Drazin inverse, a spectral pseudoinverse. The lab involves

- seeing if two matrices are Drazin inverses of each other (reinforces the properties of a group inverse),
- writing an algorithm that calculates the Drazin inverse, and
- using the Drazin inverse to predict friendships in the class.

We have found that providing real-world applications that the students can implement themselves, even if in a simplified manner, increases the motivation and retention levels of linear algebra topics in our class. Moreover, we have found that some of our application labs, such as the facial recognition lab, act as great marketing material for our classes. Our students report that the Python skills and numerical computing confidence they gain in the labs are important factors in obtaining summer internships and post-graduation employment. We have made the Python labs available on github for any instructors who are interested in implementing them in their own classrooms. You can find them at: `https://foundations-of-applied-mathematics.github.io`.

Teaching Non-Math Majors the Theory of Linear Algebra Through Proofs

Martha Lee Kilpack[2]

A common concern among linear algebra professors is how to balance the teaching of computations with the teaching of theory. This is especially true in courses filled with non-math majors. Among our students we want to encourage an understanding of linear algebra beyond the computations. However, many non-math majors shy away from the theory because they do not understand how to turn classroom understanding into personal knowledge. To promote understanding of linear algebra theory, we have students learn to write proofs, some of which will appear on the exams. Below I discuss the process some professors at Brigham Young University have been taking to help students learn to write proofs and gain a better understanding of the theory behind the computations found in linear algebra.

To facilitate and encourage proof-writing, we give the students a list of approximately ten propositions they should prove in preparation for an exam about a week before the exam. The students know that two or three of the proofs from this list will be on the exam. In the week leading up to the exam students could work together and ask the professors for help on the proofs. When a student or group of students feel they have a completed proof, they may show this proof to their professor. We would let them know if it was correct. This gives students an opportunity to receive feedback before a proof is graded. I encourage students to come in for help with proofs and let them know I do not expect them to get how to write proofs the first time around. This helps students understand that we are there to help each of them through this process.

Let us look at one example of a proposition that a student could be given to prove, as well as sample some help that students could get along the way.

Proposition: *Let $T : V \to W$ be a linear transformation from vector space V onto vector space W. Then* $\dim V \geq \dim W$.

If a student comes with no idea of where to start I ask:

1. What is a linear transformation?
2. What does it mean for a transformation to be *onto*?
3. How do you find the dimension of a vector space?

A student who does not know the answers to these questions has to look up the answers and we look at examples of these ideas together.

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After all this, I give a clue to start with a basis for V, $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$, and I ask the student to think about how to use the set $\{T(\mathbf{b}_1), T(\mathbf{b}_2), \ldots, T(\mathbf{b}_n)\}$ in W. I finally point them in the direction of theorems about spanning sets of vector spaces.

From this process the students would get a review of linear transformations, onto functions, and how to tell a set is a spanning set. It is about getting the students to think and come up with ideas. To accommodate the time needed for my students, I hold extra office hours the week the students are working through these proofs. Many students come to the office hours and find others who need help on the same proof and work together on that proof.

If a student brings a written proof, I clean up the wording or explain parts which are incomplete. The following is an example of a proof a student might bring in.

> Let the set $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ be a basis for V. The set $\{T(\mathbf{b}_1), T(\mathbf{b}_2), \ldots, T(\mathbf{b}_n)\}$ will then span W. The spanning set theory says if $W \neq \{\mathbf{0}\}$ then there is a basis which is a subset of $\{T(\mathbf{b}_1), T(\mathbf{b}_2), \ldots, T(\mathbf{b}_n)\}$. Thus dim $V \geq$ dim W.

This proof is exactly the correct idea but some pieces are missing. I start by letting the student know this is the correct idea but they are missing some pieces. I then ask the student, "How do you know $\{T(\mathbf{b}_1), T(\mathbf{b}_2), \ldots, T(\mathbf{b}_n)\}$ will span W?" and "What if W is the zero vector space?" The student then has a chance to correct the missing parts and be fully prepared.

As I have implemented this process it has been rewarding to see the students complete a proof. The students get a sense of accomplishment from the difficult task. They gain confidence in themselves and in their understanding of the material. This process takes time for the professor and for the students, but those students who utilize this to the fullest have progressed in understanding as the semester moved forward.

Of this experience one student said, "I still hate proofs, but they forced me to understand the material," a wonderful byproduct of helping students learn to write proofs.

CONFERENCE REPORT

Matrix Equations and Tensor Techniques Workshop
Pisa, Italy, February 13–14, 2017

Report by Björn Baran

The 7$^{\text{th}}$ workshop on Matrix Equations and Tensor Techniques (METT) took place in Pisa, Italy, February 13–14, 2017. During the last few years, the METT workshops have evolved into an internationally established meeting for researchers in the field of tensor techniques and matrix equations. In 2005, the workshop series started in Leipzig as a regional meeting for scientists with an interest in matrix equations. In a two-year rhythm it was held in 2007 in Chemnitz, in 2009 in Braunschweig, in 2011 in Aachen, in 2013 in Lausanne, Switzerland, and in 2015 in Bologna, Italy. Since 2011, tensor techniques have been included in the list of topics and two years later the workshop took place for the first time outside of Germany. The organizing committee consists of Peter Benner (MPI Magdeburg), Heike Faßbender (TU Braunschweig), Lars Grasedyck (RWTH Aachen), Daniel Kressner (EPF Lausanne), Beatrice Meini (University of Pisa), and Valeria Simoncini (University of Bologna).

The 58 participants came from 15 different countries (4 outside of Europe) to the university city Pisa in Italy, whose university was founded in 1343. Pisa is the capital of the Province of Pisa, has around 90,000 inhabitants, and is well-known for its major touristic attraction—its leaning tower. The scientific program of the workshop was spread over two full days, and separated into 7 sessions, including a poster session. There were four sessions on matrix equations.

The workshop started on Monday with a session entirely devoted to general linear matrix equations, where, in contrast to the usual situation, the unknown appears more than twice, which makes these equations more difficult than standard Lyapunov or Sylvester equations. Later in the afternoon, the focus shifted to differential matrix equations, including presentations regarding applications and numerical solution methods. Moreover, the Tuesday morning session included several studies concentrating on numerical and theoretical aspects of (star)-Sylvester equations. Two sessions were dedicated to tensor techniques. The sessions covered several tensor formats, low-rank tensor completion and tensor decompositions. For example, the Singular Value Decomposition of higher order tensors as well as the quantized-tensor-train decomposition were discussed. The quantized-tensor-train decomposition can be used to approximate polynomial and piecewise-analytic functions. With this decomposition, exponential convergence can be achieved in certain problems with singular and highly oscillatory solutions. The first day's scientific program was completed by a poster session. The topics of tensor techniques and matrix equations were covered with 13 different posters which stimulated lively and open discussions with the authors of the posters. The evening was pleasantly concluded by the social dinner in a traditional Italian restaurant in the old town of Pisa.

In his closing remarks, Peter Benner thanked all participants for their attendance. Peter Benner will act as local organizer of the 8^{th} METT workshop that will take place in Magdeburg, Germany, in 2019.

BOOK REVIEW

Introduction to Linear Algebra, 5th Edition, by Gilbert Strang

Wellesley - Cambridge Press, 2016, ISBN 978-0-9802327-7-6, x+574 pages.
Reviewed by Douglas Farenick, University of Regina, douglas.farenick@uregina.ca

Undergraduate mathematics textbooks are not what they used to be, and Gilbert Strang's superb new edition of *Introduction to Linear Algebra* is an example of everything that a modern textbook could possibly be, and more.

First, let us consider the book itself. As with his classic *Linear Algebra and its Applications* (Academic Press) from forty years ago, Strang's new edition of *Introduction to Linear Algebra* keeps one eye on the theory, the other on applications, and has the stated goal of "opening linear algebra to the world" (Preface, page x). Aimed at the serious undergraduate student – though not just those undergraduates who fill the lecture halls of MIT, Strang's home institution – the writing is engaging and personal, and the presentation is exceptionally clear and informative (even seasoned instructors may benefit from Strang's insights). The first six chapters offer a traditional first course that covers vector algebra and geometry, systems of linear equations, vector spaces and subspaces, orthogonality, determinants, and eigenvalues and eigenvectors. The next three chapters are devoted to the singular value decomposition, linear transformations, and complex numbers and complex matrices, followed by chapters that address a wide range of contemporary applications and computational issues. The book concludes with a brief but cogent treatment of linear statistical analysis.

I would like to stress that there is a richness to the material that goes beyond most texts at this level. Included are guides to websites and to OpenCourseWare, which I shall comment upon later in this review. The final page lists "Six Great Theorems of Linear Algebra." Chapter 7 begins with an informative account of image compression, and would be wonderful material for an undergraduate student to present in a seminar to other students.

Strang's experience at writing and teaching linear algebra is apparent in the layout of the typeset. Offset in blue type are topic-specific headings that indicate what is contained in the content of the text to follow. For example, on page 5, after developing material on linear combinations of vectors, we find the heading "The Important Questions." On page 149, after studying the null space, there is a subsection with the heading "Elimination: The Big Picture." Each section contains the headings "Review of the Key Ideas," "Worked Examples," "Problems," and "Challenge Problems." These sections are essential reading for the instructor, not just the student. The Worked Examples include material such as the Gershgorin Circle Theorem, while the Problems and Challenge Problems offer the student a chance to master basic ideas and to think much more mathematically about the concepts under study. For example, Problem 29 of Chapter 6 asks for the computation of the eigenvalues of three matrices (not just generic matrices, but matrices with structure and, thus, a chance to learn something about how the features of the matrix influence the eigenvalues), while Problem 39 of the same chapter asks for the possible values of the determinants, traces, and eigenvalues of the six 3×3 permutation matrices. There is nothing here that can be said to be dry, uninteresting, or irrelevant; rarely does an undergraduate mathematics text feel so alive as this one.

As I indicated earlier, serious students who study all twelve chapters will be well-rewarded for their efforts. However, the level of sophistication grows considerably toward the later chapters. Indeed, while the initial pages of the book provide examples of how to perform addition and scalar multiplication in $\mathbb{R}^2$ (with vectors of *numbers*, not just *symbols*), the book closes with remarks about the dynamic Kalman filter. This book, therefore, will remain in the personal libraries of the students who read it through to the end. The student who requires only very basic facility with linear algebra, emphasizing calculation over understanding, is perhaps better served by a different text; nevertheless, there is no reason why the first few chapters of the book could not be used successfully for any introductory linear algebra course in any university or college.

On its own, Strang's book is outstanding. I can easily imagine a small group of undergraduate students working through a section of the book together, guided by the personal touch of Strang's writing and the clarity of his presentation. I can just as easily imagine an instructor using the book to frame his or her own ideas about how to teach the subject in the lecture hall. But here is a question: Do undergraduate students actually read books anymore? Maybe they still do; I am quite certain, though, that they rely upon media resources just as much as they rely upon the traditional textbook and lecture format.

Thus, as with many modern textbooks, Strang's *Introduction to Linear Algebra* comes with additional content: video lectures on the MIT OpenCourseWare platform, a website devoted to the book (`math.mit.edu/linearalgebra`), and a site of review questions and Java demos. Unlike many competing books sold by commercial publishers, the online content of Strang's edition is not bundled with the book purchase: all of these web-based extras are freely available, as are selected sections from the book. While this approach is good for keeping the price of the book reasonably low, there is the disadvantage that the online material for Strang's book lacks, for example, randomised practice problems that can be completed and graded online. On the other hand, Professor Strang's MIT 18.06 course website has been viewed by nearly 2 million people, and his video lectures have been viewed by over 6 million people. Above all, I believe it is the book that matters most, rather than the bells and whistles, so to speak. In that regard, Gilbert Strang's book *Introduction to Linear Algebra* is a timely update to his early editions, the most recent of which appeared in 2005, and offers tremendous value to all involved in the learning, teaching, and application of linear algebra.

Send News for *IMAGE* Issue 59

IMAGE Issue 59 is due to appear online on December 1, 2017. Send your news for this issue to the appropriate editor by October 2, 2017. *IMAGE* seeks to publish all news of interest to the linear algebra community. Photos are always welcome, as well as suggestions for improving the newsletter. Send your items directly to the appropriate person:

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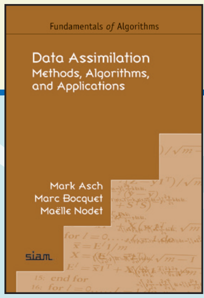

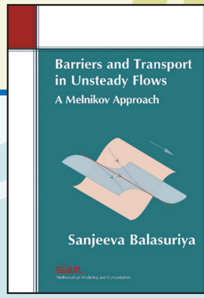

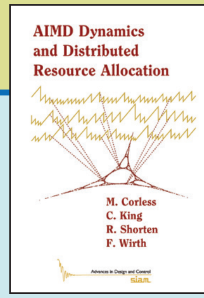

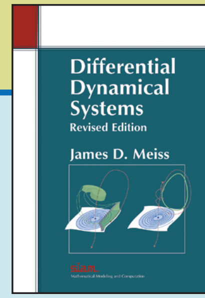

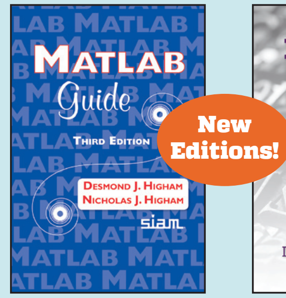

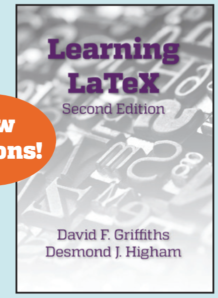

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Mark Asch, Marc Bocquet, and Maëlle Nodet
Fundamentals of Algorithms 11

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Mathematical Modeling and Computation 21

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M. Corless, C. King, R. Shorten, and F. Wirth
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This is the first comprehensive book on the AIMD algorithm, the most widely used method for allocating a limited resource among competing agents without centralized control. The original and best-known application of the algorithm is in the context of congestion control and resource allocation on the Internet, and readers will find details of several variants of the algorithm.

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3/17

JOURNAL ANNOUNCEMENT

Guidance for the *ELA* Refereeing Process

Dear ILAS community,

As one step to enhance the quality of papers appearing in *ELA*, and to make the refereeing process as efficient and informative as possible, the *ELA* board has prepared the document "Guidance for the *Electronic Journal of Linear Algebra* (*ELA*) Refereeing Process" given in its entirety below. Portions of the document were informed by the manuscript [1]. The document will be available on the *ELA* website `http://repository.uwyo.edu/ELA` and editorial board members will be encouraged to refer to it in their invitations to referees. We view this as a document that will evolve and improve over time, and thus welcome your comments and suggestions.

Sincerely,

Bryan Shader and Michael Tsatsomeros
Editors-in-Chief, *Electronic Journal of Linear Algebra*

Guidance for the *Electronic Journal of Linear Algebra* (*ELA*) Refereeing Process

The *ELA* Editorial Board believes that high standards for publication are of benefit to the authors, *ELA*, and the Linear Algebra community. As a referee, you can help uphold these high standards, as well as help the authors produce better papers in the future.

Judging the Quality of a Paper
ELA considers primarily original research papers, but also encourages survey papers. The latter, by their very nature, need not contain new results and should be evaluated according to their ability to provide a critical assessment and comparison of existing work, to stimulate new research and interest on a topic, as well as to summarize well-known and less known results for the benefit of future research.

The *ELA* Editorial Board asks referees to consider (some or all of) the following qualities in judging whether a research paper is publishable in *ELA* or not:

(i) Correctness of results and proofs
ELA adopts the Halmos standard: a referee is not required to certify the correctness of the results in the paper, but does need to indicate whether or not the results are plausible and seem correct. As a referee, you should spend some time in error-detection (e.g., identify misstated results or logical errors) and suggest minor fixes. You are not expected, however, to do the research intended by the author(s).

(ii) Significance, innovation, interest or timeliness

- Are the mathematical questions studied in the paper of some significance in your opinion? (Problems with obvious or simple solutions, or results that are minor improvements of known results will not be published.)
- Are the results original and innovative? (Simple extensions or combinations of known results will not be published.)
- Are the proof techniques new, or do they provide a novel use of known techniques?
- Does the paper provide motivation and put the results into a proper framework?
- Does the paper address a problem of recent or historical interest?
- Will the results of the paper provide new insights to ongoing research by others?

(iii) Succinctness and accessibility

- Do the proofs and the exposition provide an appropriate level of detail?

- · Is the material accessible for the typical International Linear Algebra Society (ILAS) member or *ELA* reader?
- · Are appropriate references provided to background material?

(iv) Readability, style and elegance

- · Will the reader be able to unpack the results in the paper with a reasonable amount of effort?
- · Is the exposition well-structured with proper use of scholarly vocabulary and grammar?
- · Is it evident that care has been taken by the author(s) in all aspects of the manuscript's preparation?

The Recommendation
Recommendations for papers generally fall into the following categories:

1. The subject is out of *ELA*'s scope.
2. Results are published elsewhere.
3. Results are not of high enough level of novelty, significance or difficulty.
4. The paper is tedious, or there is too much in the literature on the same subject.
5. The paper is thin in new results or does not represent enough progress.
6. The paper contains major errors.
7. The exposition of the material is poor.
8. The paper is acceptable, perhaps with some improvements or revisions.
9. The effort or results constitute a major contribution, perhaps with some improvements or revisions.

The Referee Report
Please prepare your report to the author(s) and editor so that it

- · clearly indicates your recommendation,
- · identifies specific ways that the paper can be improved, if any, and
- · provides comments that can benefit the author(s) by guiding them towards excellence.

Categories 2 and 5 do not apply to survey papers. Papers in categories 1-6 above will be rejected. If the substance of the paper is considered significant enough, then the author(s) of papers in categories 7-8 may be asked to provide a minor or major revision. We encourage referees to push authors to do better; so if you feel there is room for improvement, please do not put a paper in category 9. For papers in categories 8-9, help the author(s) and *ELA*'s managing editors team identify and correct typographical and linguistic errors to a reasonable extent.

Concluding Remarks
ELA editors expect referees to respect the guidelines above, but also to interpret and adjust them within reason and according to their professional experiences. For example, referees being consulted because of their expertise in a mathematical or scientific field other than linear algebra are expected to express their opinion on the impact of the results in their fields of expertise.

Please be timely in preparing your report. In the network of scholarly publication, the refereeing process lies upon the "critical path." Timely, thoughtful reports, allow good mathematics to be made available to the community sooner. If circumstances arise that prevent you from refereeing the paper by the promised deadline, then please let *ELA* know as soon as possible.

Your service through refereeing is greatly appreciated by *ELA* and ILAS, and helps make our community stronger and more highly respected.

References.

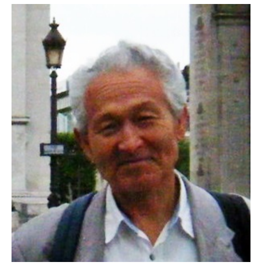
[1] Parberry, Richard. A Guide for New Referees in Theoretical Computer Science. *Journal of Information and Computation* 112 (1994) 96–116.

OBITUARY

Takayuki Furuta 1935–2016

Submitted by Takeaki Yamazaki

Takayuki Furuta, professor emeritus of Hirosaki University, passed away at home in Tokyo on June 27, 2016 at the age of 81. He spent the day safely at home; however, he passed away of cardiac infarction in his sleep.



Takayuki was born on January 8, 1935 in Nagoya, Aichi. In his childhood, he moved with his family to Iwaki, Fukushima. He obtained a B.A. from Tohoku University in 1958. Then he entered the graduate school of Tohoku University. However, the number of master course students in the Department of Mathematics was reduced at that time because of a policy of the national ministry of education. Professor Masanori Fukamiya, his adviser, found him a job teaching mathematics at a high school, then left before he graduated from Tohoku University.

Takayuki's first career was as a teacher of mathematics in Kitakata High School, Fukushima. But his passion for mathematics did not come to an end. He continued his research himself, and obtained a Ph.D. in 1972. His supervisor was Professor Tsuyoshi Ando. He moved around, to the National Institute of Technology Fukushima College, Ibaraki University, Hirosaki University in Aomori, and finally became a professor at Tokyo University of Science in Tokyo in 1992. While he was in Tokyo for 9 years, he supervised 5 Ph.D. students: Ritsuo Nakamoto, Eizabiro Kamei, Jun-Ichi Fujii, Masahiro Yanagida and Takeaki Yamazaki. In addition, he also supported Masatoshi Ito to obtain his Ph.D.

Co-editor of [2] and author of [3], one of Takayuki's most famous results is the Furuta inequality, obtained in 1987:

Theorem F (Furuta inequality [1])**.**

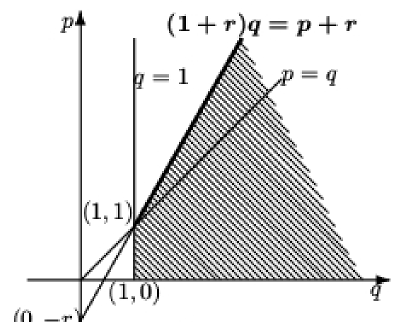
If $A \geq B \geq 0$*, then for each* $r \geq 0$*,*

(i) $$(B^{\frac{r}{2}} A^p B^{\frac{r}{2}})^{\frac{1}{q}} \geq (B^{\frac{r}{2}} B^p B^{\frac{r}{2}})^{\frac{1}{q}}$$

and

(ii) $$(A^{\frac{r}{2}} A^p A^{\frac{r}{2}})^{\frac{1}{q}} \geq (A^{\frac{r}{2}} B^p A^{\frac{r}{2}})^{\frac{1}{q}}$$

for $p \geq 0$ *and* $q \geq 1$ *with* $(1+r)q \geq p+r$*.*



Figure

It is a very beautiful result. Moreover, the domain of the parameters p, q and r in the inequalities attracts and at the same time confuses many mathematicians (see the figure). Some mathematicians asked him, "How did you get the figure?" He always answered, "It's a secret problem!"

Takayuki was a great sports player. He was a black belt holder in judo, and a good tennis player. For instance, he participated in the National Sports Festival in Japan as a representative tennis player of the Aomori prefecture. He learned swimming when he became 60 years old. Until that age, he could not swim at all! He first practiced holding his breath by putting his face into a washbowl. Then he went to a public pool two or three times a week. Finally, he could swim very well! He also enjoyed Karaoke with his neighborhood friends.

Takayuki and his wife Teiko have three daughters: Miho, Shiho and Mizuho.

His memorial ceremony was held at the Research Institute of Mathematical Science (RIMS), Kyoto University on November 11, 2016. His wife, his second daughter Shiho and many mathematicians mourned. They told of his great works and personality.

References.

[1] T. Furuta, $A \geq B \geq 0$ assures $(B^r A^p B^r)^{1/q} \geq B^{(p+2r)/q}$ for $r \geq 0$, $p \geq 0$, $q \geq 1$ with $(1+2r)q \geq p+2r$, *Proc. Amer. Math. Soc.*, 101 (1987) 85–88.

[2] T. Furuta, I. Gohberg, and T. Nakazi (Editors), *Contributions to Operator Theory and its Applications: The Tsuyoshi Ando Anniversary Volume*, Birkahäuser, 1993.

[3] T. Furuta, *Invitation to Linear Operators: From Matrices to Bounded Linear Operators on a Hilbert Space*, CRC Press, 2001.

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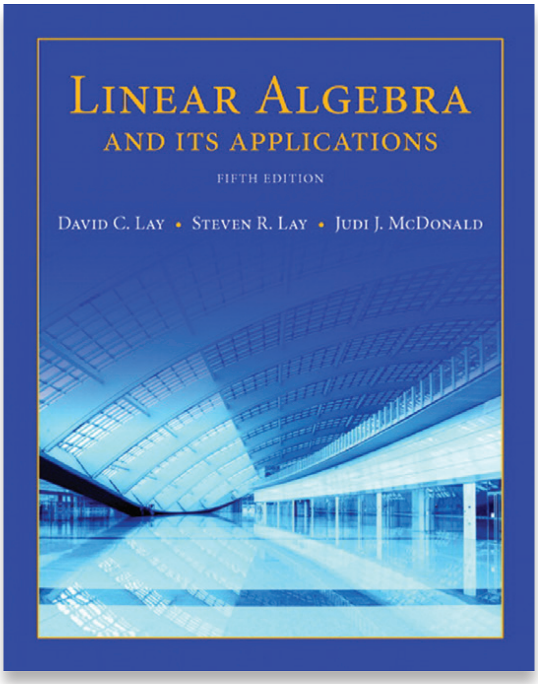

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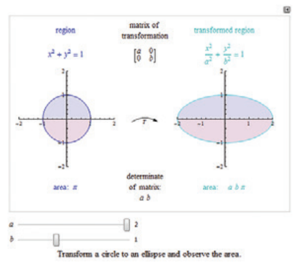

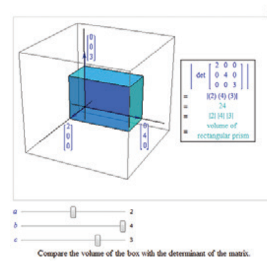

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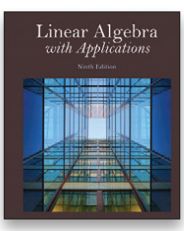

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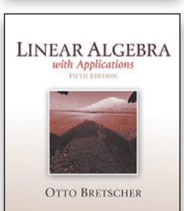

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UPCOMING CONFERENCES AND WORKSHOPS

The 8$^{\text{th}}$ Linear Algebra Workshop (LAW'17) Ljubljana, Slovenia, June 12–16, 2017

The 8$^{\text{th}}$ Linear Algebra Workshop (LAW'17) will be held at the Faculty of Mathematics and Physics, Ljubljana, Slovenia, June 12–16, 2017. The workshop has been endorsed by the International Linear Algebra Society (ILAS). Vladimir Müller was selected as the third Hans Schneider ILAS Lecturer.

The main theme of the workshop will be the interplay between operator theory and algebra. The workshop will follow the usual format. A few hours of talks will be scheduled for the morning sessions, while afternoons will be reserved for work in smaller groups.

Peer-reviewed original papers and survey papers related to the themes of the Workshop will be published in the topical issue of the international journal *Special Matrices*. The deadline for paper submission is December 1, 2017. The Editorial Board for the topical issue is the Scientific Committee of the Workshop.

The lectures will start on Monday, June 12, in the morning, and end on Friday, June 16, in the afternoon. The talks will be at the Department of Mathematics, Faculty of Mathematics and Physics, Jadranska 21.

For further details, see `http://www.law05.si/law17`.

The 6$^{\text{th}}$ International Conference on Matrix Analysis and Applications (ICMAA 2017) Da Nang City, Vietnam, June 15–18, 2017

The 6$^{\text{th}}$ International Conference on Matrix Analysis and Applications (ICMAA 2017) will be held at Duy Tan University, Da Nang City, Vietnam, June 15–18, 2017.

This meeting aims to stimulate research and interaction of mathematicians in all aspects of linear and multilinear algebra, matrix analysis, graph theory, and their applications and to provide an opportunity for researchers to exchange ideas and developments on these subjects. The previous conferences were held in China (Beijing, Hangzhou), United States (Nova Southeastern University), and Turkey (Selçuk University, Konya). Former keynote speakers are Roger Horn, Richard Brualdi, Chi-Kwong Li, Steve Kirkland, Alexander A. Klyachko (ILAS guest speaker), and Shmuel Friedland.

The keynote speaker of this ICMAA conference is Prof. Man-Duen Choi, University of Toronto, Canada.

The scientific organizing committee consists of Trung Hoa Dinh (Ho Chi Minh City University of Food Industry, Vietnam), Hiroyuki Osaka (Ritsumeikan University, Japan), Tin-Yau Tam (Auburn University, USA), Qing-Wen Wang (Shanghai University, China), and Fuzhen Zhang (Nova Southeastern University, USA).

The conference home page is `http://icmaa-2017.duytan.edu.vn` with mirror site `http://www.auburn.edu/~tamtiny/2017ICMAA.html`.

Contact: Fuzhen Zhang at zhang@nova.edu or any committee member.

The Householder Symposium XX on Numerical Linear Algebra Blacksburg, Virginia, USA, June 18–23, 2017

The next Householder Symposium will be held June 18–23, 2017 at the Inn at Virginia Tech in Blacksburg, Virginia. This meeting is the twentieth in a series, previously called the Gatlinburg Symposia, but now named in honor of its founder, Alston S. Householder, a pioneer of numerical linear algebra. As envisioned by Householder, the meeting is informal, emphasizing an intermingling of young and established researchers. The sixteenth Householder Prize for the best Ph.D. thesis in numerical linear algebra since January 1, 2014 will be presented.

The Householder Symposium takes place in cooperation with the Society for Industrial and Applied Mathematics (SIAM) and the SIAM Activity Group on Linear Algebra.

For further details, please visit the conference webpage: `http://www.math.vt.edu/HHXX`.

Preservers Everywhere
Szeged, Hungary, June 19–23, 2017

Preservers Everywhere is an international conference on the dynamically developing topic commonly referred to as "Preserver Problems." A *preserver* is a transformation between mathematical structures that leaves a certain quantity/operation/relation/set/etc. invariant. The usual goal is to give a complete description of these transformations. Typical examples include homomorphisms of groups (i.e., maps which respect group operations) and isometries of metric spaces (i.e., maps preserving distances). The purpose of this meeting is to bring together mathematicians from various areas who are either working on or interested in such problems. Amongst others, we will have talks on preserver problems related to Algebra, Analysis, Functional Analysis, Geometry, Linear Algebra, Mathematical Physics and Operator Theory. We encourage everybody interested in this topic to participate in the meeting.

The venue is the Bolyai Institute of the University of Szeged in Szeged, Hungary. The organizers are Lajos Molnár and György Pál Gehér (University of Szeged). If you have any questions or queries please contact György Pál Gehér (gehergyuri@gmail.com).

The conference website is `http://www.math.u-szeged.hu/~gehergy/conference.html`.

The Second Malta Conference in Graph Theory and Combinatorics (2MCGTC 2017)
University of Malta, Malta, June 26–30, 2017

The Department of Mathematics, within the Faculty of Science, of the University of Malta is pleased to announce "The Second Malta Conference in Graph Theory and Combinatorics." This conference will commemorate the 75$^{\text{th}}$ birthday of Professor Stanley Fiorini, who was the first to introduce graph theory and combinatorics at the University of Malta.

The Conference will be held in the seaside resort of Qawra in the north of the island, and will run from Monday 26 to Friday 30 June 2017. The aim of this conference is to bring together experts and researchers in all the areas of graph theory and combinatorics from across the world to share their research findings, and to enhance collaboration between researchers who are in different stages of their careers.

The Plenary Speakers are: Richard A. Brualdi (University of Wisconsin–Madison, USA), Yair Caro (University of Haifa-Oranim, Israel), Peter Dankelmann (University of Johannesburg, South Africa), Patrick W. Fowler, F.R.S. (University of Sheffield, United Kingdom), Chris Godsil (University of Waterloo, Canada), Wilfried Imrich (Montanuniversität Leoben, Austria), Gyula O. H. Katona (Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary), Sandi Klavžar (University of Ljubljana, Slovenia), Mikhail Klin (Ben-Gurion University of the Negev, Israel), Imre Leader (University of Cambridge, United Kingdom), Brendan McKay (Australian National University, Australia), Karen Meagher (University of Regina, Canada), Raffaele Scapellato (Politecnico di Milano, Italy), and Andrew Thomason (University of Cambridge, United Kingdom).

A number of parallel sessions for talks delivered by participants will be held. A special issue of *Discrete Applied Mathematics*, containing selected full-length papers, refereed according to the high standards of the journal, will be dedicated to the Conference.

The Organising Committee consists of Peter Borg, John Baptist Gauci, Josef Lauri, and Irene Sciriha.

Further information can be found at `http://www.um.edu.mt/events/2mcgtc2017`; e-mail: 2mcgtc2017@um.edu.mt.

Special Western Canada Linear Algebra Meeting honouring Peter Lancaster
Banff, Alberta, Canada, July 7–9, 2017

The Banff International Research Station will host the "Special Western Canada Linear Algebra meeting honouring Peter Lancaster" workshop in Banff from July 7 to July 9, 2017.

This workshop will bring together researchers working in various areas that include matrix analysis, combinatorial matrix theory, applied and numerical linear algebra, combinatorics, operator theory and operator algebras. Further, because of linear algebra's ubiquity and utility, the discipline enjoys a continuing dialogue with numerous areas of application which include transportation, communication, optimizations, and many areas of modern mathematics. The highlight of the meeting is the recognition of the outstanding contributions to the areas of research by Professor Peter Lancaster, the founding member of the series meeting, at his 88$^{\text{th}}$ birthday.

The workshop organizers are Pauline van den Driessche (University of Victoria), Shaun Fallat (University of Regina) and Hadi Kharaghani (University of Lethbridge).

Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics (GRWC 2017)
Denver, Colorado, USA, July 9–22, 2017

The Rocky Mountain–Great Plains Graduate Research Workshop in Combinatorics 2017 (GRWC 2017) will be held July 9–22, 2017, and will be hosted by the University of Colorado Denver and the University of Denver.

The GRWC is a 2-week collaborative research workshop for experienced graduate students and postdocs from all areas of combinatorics. Participants will work in collaborative groups with faculty and postdocs on research problems from across the discipline. The workshop will also host a variety of professional development workshops to prepare students and postdocs for industrial and academic careers.

The workshop will be centered on problems developed and presented by students and postdocs, who will work together with a faculty mentor from the organizing committee. We anticipate funding (travel, meal stipend and lodging) for up to 20 graduate students and postdocs from outside of the organizing institutions. Applications are encouraged from students at institutions not in the United States, but funding is extremely limited for international participants.

For application information and further details about the workshop, please visit `https://sites.google.com/site/rmgpgrwc`, and for inquiries, please e-mail grwc2016@gmail.com.

Graduate Student Modeling Workshop (IMSM 2017)
North Carolina, USA, July 16–26, 2017

The 23rd Industrial Mathematical & Statistical Modeling (IMSM) Workshop for Graduate Students will take place at North Carolina State University, July 16–26, 2017. The workshop is sponsored by the Statistical and Applied Mathematical Sciences Institute (SAMSI) together with the Center for Research in Scientific Computation (CRSC) and the Department of Mathematics at North Carolina State University.

The IMSM workshop exposes graduate students in mathematics, engineering, and statistics to exciting real-world problems from industry and government. The workshop provides students with experience in a research team environment and exposure to possible career opportunities. On the first day, a Software Carpentry bootcamp will bring students up-to-date on their programming skills in Python/Matlab and R, and introduce them to version control systems and software repositories.

Local expenses and travel expenses will be covered for students at US institutions. Information is available at `https://www.samsi.info/programs-and-activities/2016-2017-education-and-outreach-programs-and-workshops/2017-industrial-mathstat-modeling-workshop-graduate-student-july-16-26-2017` and questions can be directed to grad@samsi.info.

ILAS 2017: Connections
30th Birthday of ILAS
Ames, Iowa, USA, July 24–28, 2017

The 21st conference of the International Linear Algebra Society, ILAS 2017: Connections, will be held July 24–28, 2017 at Iowa State University in Ames, Iowa, USA. The theme of the conference is connections between linear algebra and other areas of mathematics, science, and engineering, and talks about such connections as well as within linear algebra itself are welcome. At this conference we will celebrate the 30th birthday of ILAS, which was founded in 1987 as the International Matrix Group.

Invited Plenary Speakers:

Rajendra Bhatia, Indian Statistical Institute, Hans Schneider Prize Speaker, *Another metric, another mean*

Hal Caswell, Woods Hole Oceanographic Institute, *Matrix population models: Connecting individuals and populations*

Chris Godsil, University of Waterloo, *Graph invariants from quantum walks*

Stefan Güttel, The University of Manchester, SIAG-LA Lecturer, *The Nonlinear Eigenvalue Problem*

Willem Haemers, Tilburg University, *Spectral characterizations of graphs*

Tamara G. Kolda, Sandia National Laboratories, *Tensor Decomposition: A Mathematical Tool for Data Analysis and Compression*

Miklos Palfia, Kyoto University, Taussky-Todd Lecturer, *On the recent advances in the multivariable theory of operator monotone functions and means*

Vern Paulsen, University of Waterloo, *Quantum Chromatic Numbers*

Helena Šmigoc, University College Dublin, LAMA Lecturer, *From positive matrices to negative polynomial coefficients*

Raf Vandebril, KU Leuven, ILAS 30$^{\text{th}}$ Anniversary LAA Lecturer, *The QR Algorithm Revisited – Core Chasing Algorithms*

Van Vu, Yale University, *Perturbation with random noise*

Rachel Ward, University of Texas, *Learning dynamical systems from highly corrupted measurements*

Mini-Symposia (with organizers):

MS-1 Linear Algebra Aspects of Association Schemes: Allen Herman and Bangteng Xu

MS-2 Combinatorial Matrix Theory: Minerva Catral, Louis Deaett

MS-3 Compressed sensing and matrix completion: Simon Foucart, Namrata Vaswani

MS-4 Distance problems in linear algebra, dynamical systems and control: Elias Jarlebring, Wim Michiels

MS-5 Distances on networks and its applications: Angeles Carmona, Andres M. Encinas, and Margarida Mitjana

MS-6 Linear Algebra Education: Rachel Quinlan and Megan Wawro

MS-7 Linear Algebra and Geometry (Geometry): Gabriel Larotonda and Alejandro Varela

MS-8 Krylov and filtering methods for eigenvalue problems: Jared Aurentz and Karl Meerbergen

MS-9 Linear Algebra and Mathematical Biology (Mathematical Biology): Julien Arino and Natalia Komarova

MS-10 Matrix Analysis: Inequalities, Means, and Majorization: Fumio Hiai and Yongdo Lim

MS-11 Matrix Polynomials: Froilán Dopico and Paul Van Dooren

MS-12 The Nonnegative Inverse Eigenvalue Problem: Charles R. Johnson and Pietro Paparella

MS-13 Recent Advancements in Numerical Methods for Eigenvalue Computation: James Vogel, Xin Ye, and Jianlin Xia

MS-14 Numerical Ranges: Patrick X. Rault and Ilya Spitkovsky

MS-15 Matrix techniques in operator algebra theory: Vern Paulsen and Hugo Woerdeman

MS-16 Linear Algebra and Positivity with Applications to Data Science: Dominique Guillot, Apoorva Khare, and Bala Rajaratnam

MS-17 Linear Algebra and Quantum Information Science: Chi-Kwong Li, Yiu Tung Poon, and Raymond Nung-Sing Sze

MS-18 Random matrix theory for networks: Dustin Mixon and Rachel Ward

MS-19 Representation Theory: Jonas Hartwig

MS-20 Solving Matrix Equations: Qing-Wen Wang and Yang Zhang

MS-21 Spectral Graph Theory: Nair Abreu and Leonardo de Lima

MS-22 Matrices, Tensors and Manifold Optimization: Daniel Kressner and Bart Vandereycken

MS-23 Toeplitz Matrices and Riemann Hilbert Problems: Jani Virtanen and György Pal Gehér

MS-24 Zero Forcing: Its Variations and Applications: Daniela Ferrero, Mary Flagg, and Michael Young

LAA Early Career Speakers

Mahya Ghandehari, University of Delaware, in MS-16 Linear Algebra and Positivity with Applications to Data Science

Nathaniel Johnston, Mount Allison University, in MS-17 Linear Algebra and Quantum Information Science

Andrii Dmytryshyn, Umeå University, in MS-11 Matrix Polynomials

Thomas Mach, Nazarbayev University, in MS-8 Krylov and filtering methods for eigenvalue problems

Gabriel Coutinho, Universidade de São Paulo, in MS-21 Spectral Graph Theory

The Scientific Organizing Committee consists of: Leslie Hogben (chair), Fan Chung, Mark Embree, Stephen Kirkland, Wolfgang Kliemann, Bala Rajaratnam, Joachim Rosenthal, Peter Šemrl, Bryan Shader, Tatjana Stykel, Jared Tanner, Raf Vandebril and Pauline van den Driessche. **The Local Organizing Committee consists of**: Leslie Hogben (chair), Cliff Bergman, Steve Butler, Stephen Kirkland, Wolfgang Kliemann, Ryan Martin, Yiu Tung Poon, Bryan Shader, Sung-Yell Song and Michael Young.

For further details, please visit the conference website at `https://ilas2017.math.iastate.edu`, or send e-mail to ilas2017@iastate.edu.

Preconditioning 2017 – International Conference on Preconditioning Techniques for Scientific and Industrial Applications
Vancouver, Canada, July 31–August 2, 2017

The 10$^{\text{th}}$ International Conference on Preconditioning Techniques for Scientific and Industrial Applications will take place July 31 - August 2, 2017 at the University of British Columbia, Vancouver, Canada. The goal of this series of conferences is to address the complex issues related to the solution of general sparse matrix problems in large-scale applications and in industrial settings.

Confirmed plenary speakers: Michele Benzi (Emory University, USA), Jed Brown (University of Colorado at Boulder, USA), Jie Chen (IBM Research Center, USA), Eric Darve (Stanford University, USA), Tom Jönsthövel (Schlumberger Abingdon Technology Centre, UK), Alison Ramage (Strathclyde University, UK), Sander Rhebergen (University of Waterloo, Canada), and Nicole Spillane (École Polytechnique, France).

The Organizing Committee: Chen Greif (University of British Columbia, Canada), Esmond Ng (Lawrence Berkeley National Laboratory, USA), Yousef Saad (University of Minnesota, USA) and Andy Wathen (University of Oxford, United Kingdom).

Registration is now open. Please go to `http://www.cs.ubc.ca/~greif/precon17` for further details.

Algebraic and Extremal Graph Theory Conference
Delaware, USA, August 7–10, 2017

The Algebraic and Extremal Graph Theory Conference will be held August 7–10, 2017 (Monday-Thursday) at the University of Delaware, in Gore Hall 103. There will be contributed presentations by conference participants as well as invited talks by the following invited speakers: Aida Abiad (Maastricht University), Camino Balbuena (Universitat Politecnica de Catalunyia, Barcelona), Misha Klin (Ben-Gurion University of the Negev), Felix Lazebnik (University of Delaware), Edwin van Dam (Tilburg University), Misha Muzychuk (Netanya Academic College), Chris Godsil (University of Waterloo), Andrew Thomason (University of Cambridge), Willem Haemers (Tilburg University), Vasil Ustimenko (Maria Curie-Sklodowska University in Lublin), Jack Koolen (University of Science and Technology, China, ILAS Speaker), Ron Solomon (Ohio State University), William M. Kantor (University of Oregon) and Andrew Woldar (Villanova University).

The Organizing Committee consists of Sebastian Cioabă (University of Delaware), Robert Coulter (University of Delaware), Qing Xiang (University of Delaware), and Gene Fiorini (Muhlenberg College).

The conference is sponsored by the National Science Foundation, Department of Mathematical Sciences, the University of Delaware, Villanova University, Muhlenberg College, the Center for Discrete Mathematics and Theoretical Computer Science of Rutgers University, the International Linear Algebra Society, and the Institute for Mathematics and its Applications.

Further information can be found at `https://www.mathsci.udel.edu/events/conferences/aegt`.

Matrix Analysis and its Applications – A Special Session for PRIMA 2017
Oaxaca, Mexico, August 14–18, 2017

The 3$^{\text{rd}}$ Pacific Rim Mathematical Association (PRIMA 2017) Congress will take place in Oaxaca, Mexico, August 14–18, 2017. PRIMA is an association of mathematical sciences institutes, departments and societies from around the Pacific Rim. The previous PRIMA conferences were held in Sydney (2009) and in Shanghai (2013).

A special session on “Matrix Analysis and its Applications” will be held for PRIMA 2017 that will feature up to 16 invited talks by the experts and leaders in the field. The purpose of this special session of PRIMA is to present and showcase the current trends and developments in matrix analysis and various aspects of linear and multilinear algebra and applications. Topics include matrices and graphs, matrix equations and matrix inequalities, matrix computation, and applications of matrix theory in many other fields such as quantum information theory.

The participants (by invitation only) are: Richard Brualdi (University of Wisconsin–Madison, USA), Changjiang Bu (Harbin Engineering University, China), Jianlong Chen (Southeast University, China), Fumio Hiai (Tohoku University, Japan), Steve Kirkland (University of Manitoba, Canada), Wen Li (South China Normal University, China), Miklos Palfia (Sungkyunkwan University, Korea), Sarah Plosker (Brandon University, Canada), Tin-Yau Tam (Auburn University, USA), Luis Verde-Star (Universidad Autonoma Metropolitana, Mexico), Xiao-Dong Zhang (Shanghai Jiaotong University, China), Xiankun Du (Jilin University, China). The organizers of the special session are: Qing-Wen Wang (Shanghai University, China), Fuzhen Zhang (Nova Southeastern University, USA), and Yang Zhang (University of Manitoba, Canada).

To know more about PRIMA 2017, please visit `http://www.primath.org/congress/2017` (see also `http://prima2017.math.unam.mx/`). Contact: Fuzhen Zhang (zhang@nova.edu).

MAT-TRIAD'2017 - International Conference on Matrix Analysis and its Applications Będlewo, Poland, September 25–29, 2017

MatTriad'2017 (the 7$^{\text{th}}$ conference from the MatTriad series) will be held September 25–29, 2017 at the Research and Conference Center of the Polish Academy of Sciences in Będlewo near Poznań, Poland. The conference provides an opportunity to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications to other areas of science. Researchers and graduate students interested in recent developments in matrix and operator theory and computation, spectral problems, applications of linear algebra in statistics, statistical models, matrices and graphs, as well as combinatorial matrix theory are particularly encouraged to attend.

The format of the meeting will involve plenary sessions, sessions with contributed talks, and a poster session. Special sessions are welcome.

A special issue of *ELA* (*The Electronic Journal of Linear Algebra*) will be published with the papers strongly correlated to the talks presented during the conference.

The conference will include two invited lectures by Lynn Roy LaMotte and Fuzhen Zhang, as well as invited talks by Narayanaswamy Balakrishnan, Andreas Frommer, Volker Mehrmann, K. Manjunatha Prasad, and Miroslav Rozloznik, and by the winners of the Young Scientists Awards of MatTriad'2015, Jolanta Pielaszkiewicz and Ernest Šanca.

For more information, please visit `https://mattriad.wmi.amu.edu.pl`.

International Conference on Linear Algebra and its Applications Manipal, India, December 11–15, 2017

The International Conference on Linear Algebra and its Applications (ICLAA 2017) will be held December 11–15, 2017 at Manipal University, Manipal, India (`http://manipal.edu`). This ILAS-endorsed conference will be held in sequel to the earlier conferences CMTGIM 2012 and ICLAA 2014 on the same theme at Manipal University in January 2012 and December 2014, respectively. The theme of the conference will focus on (i) Classical Matrix Theory covering different aspects of Linear Algebra (ii) Matrices and Graphs (iii) Combinatorial Matrix Theory (iv) Matrix and Graph Methods in Statistics, and (v) Covariance Analysis and its Applications. The scientific advisory committee consists of Ravindra B. Bapat, Steve Kirkland, K. Manjunatha Prasad, and Simo Puntanen. The present conference shall provide a platform for leading mathematicians, statisticians, and applied mathematicians working around the globe in the theme area to discuss several research issues on the topic and to introduce new innovations. The fourth DAE-BRNS Meeting on the generation and use of covariance matrices in the applications of nuclear data will start as a preconference meeting and will continue in parallel to the conference as part of Covariance Analysis and its Applications in ICLAA 2017.

Original research articles in the theme area of (i) Linear Algebra and Matrix Theory, (ii) Combinatorial Matrix Theory, (iii) Matrices and Graphs, and (iv) Matrix and Graph Methods in Statistics may be submitted to a special issue in *Special Matrices* (`https://www.degruyter.com/view/j/spma`) dedicated to the conference. All the articles submitted to the special issue will go through the regular review process of the journal and only the articles which meet a high standard of quality will be accepted for publication in the special issue.

For more details and registration, please visit `http://iclaa2017.com`, or contact the organizing secretary at iclaa2017@gmail.com or km.prasad@manipal.edu.

SIAM Conference on Applied Linear Algebra (SIAM-ALA18) Hong Kong Baptist University, China, May 4–8, 2018

Important Dates

June 1, 2017	–	September 30, 2017:	Minitutorial proposals
June 1, 2017	–	September 30, 2017:	Mini-symposium proposals
June 1, 2017	–	October 30, 2017:	Abstracts for contributed and mini-symposium speakers
June 1, 2017	–	October 30, 2017:	Abstracts for posters
December 1, 2017	–	January 30, 2018:	Pre-registration
		May 4 – 8, 2018:	SIAM ALA18 Conference

The SIAM Conferences on Applied Linear Algebra, organized by SIAM every three years, are the premier international conferences on applied linear algebra, bringing together diverse researchers and practitioners from academia, research laboratories, and industries all over the world to present and discuss their latest work and results on applied linear algebra.

The Scientific Committee consists of Co-Chairs David Bindel (Cornell University, USA), Raymond Chan (The Chinese University of Hong Kong, China), and Michael Ng (Hong Kong Baptist University, China), and Members Mario Arioli (University of Wuppertal, Germany), Haim Avron (Tel Aviv University, Israel), Zlatko Drmac (University of Zagreb, Croatia), Vladimir Druskin (Schlumberger Doll Research, USA), Melina Freitag (University of Bath, UK), Haesun Park (Georgia Tech, USA), Hans de Sterck (Monash University, Australia), Eugene Tyrtyshnikov (Russian Academy of Science, Russia), Tetsuya Sakurai (University of Tsukuba, Japan), and Chao Yang (Lawrence Berkeley National Laboratory, USA).

Further information can be found at the conference website: `http://www.math.hkbu.edu.hk/SIAM-ALA18/index.html`.

ILAS NEWS

Honors for ILAS Members

The Society for Industrial and Applied Mathematics (SIAM) announced its 2017 SIAM Fellows, and among those named are (present or past) ILAS and/or SIAG/LA members:

Zhaojun Bai	Peter Benner
Angelika Bunse-Gerstner	Lieven De Lathauwer
Bart De Moor	Michael Kwok-Po Ng
Daniel B. Szyld	

Zhaojun was recognized for contributions to numerical linear algebra, especially eigenvalue computation, and applications to computational science and engineering, Peter for contributions to numerical methods for optimal control and model reduction, Angelika for contributions in numerical linear algebra, control theory, and model reduction, Lieven for fundamental contributions to theory, computation, and application of tensor decompositions, Bart for contributions to concepts and algorithms in numerical multilinear algebra and applications in engineering, Michael for fundamental contributions to algorithms for structured linear systems and image processing, and Daniel for contributions to numerical algebra and matrix theory. For more information, see `https://sinews.siam.org/Details-Page/siam-announces-class-of-2017-fellows`.

ILAS Election Results

Peter Šemrl was re-elected to the office of ILAS president; James Nagy and Rachel Quinlan were elected to the ILAS Board for three-year terms beginning on March 1, 2017.

ILAS President/Vice President Annual Report: March 31, 2017

Respectfully submitted by Peter Šemrl, ILAS President, peter.semrl@fmf.uni-lj.si
and Hugo Woerdeman, ILAS Vice-President, hugo@math.drexel.edu

1. Board approved actions since the last report include:

   - Final approval was given by the Board for ILAS to support each ILAS conference with up to $2000.
   - Guidelines for a newly acquired credit card were developed by the Executive Board and approved by the Board.
   - The 2017 ILAS Conference will include a celebration of volunteers who have helped ILAS throughout the years. In particular, longtime ILAS volunteers Jim and Judi Weaver will be recognized. The Board approved funds to support these activities.
   - The Board approved the appointment of Steve Butler to be the Assistant Secretary/Treasurer.

- The Board reappointed Bryan Shader as Editor-in-Chief of *ELA* for another three year term starting on November 1, 2016.
- The proposal to have ILAS 2019 in Rio de Janeiro was approved by the Board.
- The Board approved two changes of the by-laws. Details can be found in the minutes of the 2016 ILAS business meeting.
- The Board approved a three year renewable agreement with Elsevier Inc., where ILAS recognizes *Linear Algebra and its Applications* (*LAA*) as an affiliated journal and allows Elsevier Inc. to engage in associated promotion activities, and where Elsevier Inc. provides speaker support for ILAS conferences as well as reduced *LAA* subscription rates for ILAS members.
- The Board approved The Israel Gohberg ILAS-IWOTA Lecture series, in collaboration with the International Workshop on Operator Theory and its Applications (IWOTA). This series of lectures, funded via donations, will be delivered at both IWOTA and ILAS Conferences, in different years, in the approximate ratio two-thirds at IWOTA and one-third at ILAS.

2. ILAS elections ran from January 12, 2017 to February 16, 2017 and proceeded via electronic voting. The following were (re-)elected to offices with three-year terms that began on 1 March 2017:

   - President: Peter Šemrl
   - Board of Directors: James Nagy and Rachel Quinlan.

   The following continue in the ILAS offices which they currently hold:

   - Vice President: Hugo Woerdeman (term ends 28 February 2019)
   - Second Vice President (for ILAS conferences): Steve Kirkland (reappointed by ILAS President for a second term, ending 29 February 2020)
   - Secretary/Treasurer: Leslie Hogben (term ends 28 February 2018)
   - Board of Directors: Ravindra Bapat (term ends 28 February 2019), Beatrice Meini (term ends 28 February 2018), Helena Šmigoc (term ends 28 February 2019), and Henry Wolkowicz (term ends 28 February 2018).

   Froilán Dopico and Michael Overton completed their terms on the ILAS Board of Directors on 28 February 2017. We thank them for their valuable contributions as Board members; their service to ILAS is most appreciated. We also thank the members of the Nominating Committee – Heike Faßbender (chair), Shmuel Friedland, Sang-Gu Lee, Chi-Kwong Li, Luis Verde Star – for their work on behalf of ILAS, and also extend gratitude to all candidates that agreed to have their names stand for the elections.

3. The following ILAS-endorsed meetings have taken place since our last report:

   - Western Canadian Linear Algebra Meeting, University of Manitoba, Winnipeg, Canada, 14–15 May, 2016, Kevin Vander Meulen was the ILAS lecturer
   - The International Workshop on Operator Theory and Applications, Washington University, St. Louis, USA, 18–22 July, 2016, Gitta Kutyniok was the Hans Schneider ILAS lecturer
   - The Twelfth International Conference on Matrix Theory and Applications, Lanzhou University, Lanzhou City, China, 22–26 July, 2016, Richard Brualdi was the ILAS lecturer.

4. ILAS has endorsed the following conferences of interest to ILAS members:

   - 8^{th} Linear Algebra Workshop, Ljubljana, Slovenia, June 12–16, 2017. Vladimir Muller will be a Hans Schneider ILAS lecturer. `www.law05.si/law17`
   - 6^{th} International Conference on Matrix Analysis and Applications, Duy Tan University, Da Nang City, Vietnam, June 15–18, 2017 `http://icmaa-2017.duytan.edu.vn/`
   - Householder Symposium XX, Virginia Tech, Blacksburg, VA, USA, June 18–23, 2017. Paul van Dooren will be an ILAS lecturer. `http://www.math.vt.edu/HHXX`
   - The Second Malta Conference in Graph Theory and Combinatorics, Qawra, Malta, June 26–30, 2017 `http://www.um.edu.mt/events/2mcgtc2017/`
   - Rocky Mountain-Great Plains Graduate Research Workshop in Combinatorics Denver, CO, USA, July 9–22, 2017 `https://sites.google.com/site/rmgpgrwc/`
   - Algebraic and Extremal Graph Theory Conference, University of Delaware, Newark, DE, USA, August 7–10, 2017. Jack Koolen will be an ILAS lecturer. `https://www.mathsci.udel.edu/events/conferences/aegt`

- MatTriad-2017, Będlewo, Poland, September 25–29, 2017 `https://mattriad.wmi.amu.edu.pl/`
- International Conference on Linear Algebra and its Applications, Manipal University, Manipal, India, December 11–15, 2017 `http://iclaa2017.com/`
- SIAM Conference on Applied Linear Algebra (SIAM-ALA18), Hong Kong Baptist University, Hong Kong, May 4–8 2018. Mark Embree and Valeria Simoncini will be ILAS lecturers. `http://www.math.hkbu.edu.hk/siam-ala18/`

5. The following ILAS conferences are scheduled:
   - 21st ILAS Conference: Connections, 24–28 July 2017, Ames, Iowa, USA. The chair of the organizing committee is Leslie Hogben. For more information see `https://ilas2017.math.iastate.edu/program/`.
   - 22nd ILAS Conference: Linear Algebra Without Borders, 8–12 July, 2019, Rio de Janeiro, Brazil. The chair of the organizing committee is Nair Abreu.

6. Rajendra Bhatia and Paul Van Dooren are 2016 Hans Schneider Prize recipients for lifetime contributions to linear algebra. Paul Van Dooren was awarded the Prize at the ILAS Conference in Leuven in 2016, and Rajendra Bhatia will be awarded the Prize at the ILAS Conference in Ames in 2017. The 2016 Hans Schneider Prize Committee consisted of Richard Brualdi (chair), Shmuel Friedland, Ilse Ipsen, Thomas Laffey, Peter Šemrl (ex-officio member) and Xingzhi Zhan.

7. The *Electronic Journal of Linear Algebra* (*ELA*) is now in its 32nd volume. *ELA*'s url is `http://repository.uwyo.edu/ela/`. In 2016 *ELA* published 56 papers in volume 31.

   Bryan Shader (University of Wyoming) and Michael Tsatsomeros (Washington State University) continue as the Editors-in-Chief. Christian Mehl and Harm Bart concluded their many years of excellent service as *ELA* Editorial Board members.

   The basic format of *ELA* papers has changed to include the ILAS Logo, and adapt to the new SIAM Latex2e styles. Before submitting to *ELA* please read the new formatting requirements at `http://repository.uwyo.edu/ela/policies.html#formatting`.

   There will be a special issue of *ELA* devoted to the International Conference on Matrix Analysis and Its Applications – MatTriad-2017. The editors of the special issue are: Oskar Maria Baksalary, Natalia Bebiano, Ljiljana Cvetković, Heike Faßbender, and Simo Puntanen.

8. *IMAGE* is the semi-annual bulletin for ILAS available online at `http://ilasic.org/IMAGE/`. The Editor-in-Chief is Kevin N. Vander Meulen. After the spring issue 2017, Michael Cavers will be finishing his term as a contributing editor. We thank him for his excellent service, including his help to set up LaTeX structures for *IMAGE* in 2012.

9. ILAS-NET is a moderated newsletter for mathematicians worldwide, with a focus on linear algebra; it is managed by Sarah Carnochan Naqvi.

   An archive of ILAS-NET messages is available at `http://www.ilasic.org/ilas-net/`. To send a message to ILAS-NET, please send the message (preferably in text format) in an email to `ilasic@uregina.ca` indicating that you would like it to be posted on ILAS-NET. If the message is approved, it will be posted soon afterwards. To subscribe to ILAS-NET, please complete the form at `http://ilasic.us10.list-manage.com/subscribe?u=6f8674f5d780d2dc591d397c9&id=dbda1af1a5`.

   ILAS' website, known as the ILAS Information Centre (IIC), is located at `http://www.ilasic.org` and provides general information about ILAS (e.g. ILAS officers, By-laws, Special Lecturers), as well as links to pages of interest to the ILAS community.

Respectfully submitted,

Peter Šemrl, ILAS President (peter.semrl@fmf.uni-lj.si); and
Hugo J. Woerdeman, ILAS Vice-President (hugo@math.drexel.edu).

ILAS 2016–2017 Treasurer's Report

April 1, 2016 – March 31, 2017
By Leslie Hogben

Net Account Balance on March 31, 2016			
Vanguard (ST Fed Bond Fund Admiral 7876.686 Shares)		$ 85,225.74	
Checking Account - Great Western		$ 49,112.70	
Certificate of Deposit		$ 45,450.00	
Account's Payable		$ (72.76)	
			$ 179,715.68
General Fund		$ 92,417.82	
Conference Fund		$ 9,864.55	
Olga Taussky Todd/John Todd Fund		$ 12,151.68	
Hans Schneider Lecture Fund		$ 14,677.54	
Frank Uhlig Education Fund		$ 5,357.31	
Hans Schneider Prize Fund		$ 29,333.11	
ELA Fund		$ 322.54	
ILAS/*LAA* Fund		$ 15,591.11	
			$ 179,715.68
INCOME:			
Dues	$ 11,268.28		
Israel Gohberg ILAS-IWOTA Lecture	$ 3,500.00		
General Fund	$ 3,030.00		
Conference Fund	$ 10.00		
Taussky-Todd Fund	$ 180.00		
Hans Schneider Lecture Fund	$ 50.00		
Uhlig Education Fund	$ -		
Schneider Prize Fund	$ 326.00		
ELA	$ 20.00		
Misc Income	$ 679.94		
Interest - Great Western	$ 97.85		
Interest on Great Western Certificate of Deposit (#1)	$ 455.27		
Vanguard - Dividend Income	$ 1,193.90		
- Short Term Capital Gains	$ 214.37		
- Long Term Capital Gains	$ 50.29		
Variances in the Market	$ (951.72)		
Total Income		$ 20,124.18	
EXPENSES:			
First Data - Credit Card Processing Fees	$ 1,060.32		
Speaker Fees	$ 706.08		
ELA	$ 2479.26		
Hans Schneider Lecture	$ 1,500.00		
Treasurer's Assistant	$ 0.00		
Conference Expenses	$ 1,665.95		
2015 ILAS Conference - Elsevier Flowthru	$ 2,500.00		
Business License	$ 61.25		
IMAGE Costs	$ 1,161.19		
Ballot Costs	$ 207.38		
Web Hosting & Online Membership Forms	$ 282.78		
Misc Expenses	$ 230.40		
Total Expenses		$ 11,854.61	
Net Account Balance on March 31, 2017			
Vanguard (ST Fed Bond Fund Admiral 7998.604 Shares)		$ 85,505.08	
Checking Account - Great Western		$ 56,420.16	
Certificate of Deposit #1		$ 45,905.27	
Accounts Payable (includes memberships paid through ILAS 2017 not yet received)		$ 154.73	
			$ 187,985.24
General Fund		$ 98,101.40	
Israel Gohberg ILAS-IWOTA Lecture		$ 3,500.00	
Conference Fund		$ 9,874.55	
Olga Taussky Todd/John Todd Fund		$ 12,331.68	
Hans Schneider Lecture Fund		$ 13,227.54	
Frank Uhlig Education Fund		$ 5,357.31	
Hans Schneider Prize Fund		$ 29,659.11	
ELA Fund		$ 342.54	
ILAS/*LAA* Fund		$ 15,591.11	
			$ 187,985.24

IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to Problems 56-2, 56-5, 56-6, 57-1, 57-3, 57-5 and 57-6. Solutions are invited to Problems 57-2 and 57-4, and for all of the problems of issue 58.

Problem 56-2: Determinant of a Sum of Squares

Proposed by Gérald Bourgeois, *Université de la Polynésie Française, BP 6570, 98702 FAA'A, Tahiti, Polynésie Française*, bourgeois.gerald@gmail.com

Let $(A_i)_{1\le i\le p}$ be n-by-n real matrices that are simultaneously triangularizable over $\mathbb{C}$. Show that $\det(\sum_{i=1}^p A_i^2)\ge 0$.

Solution 56-2.1 by Meiyue Shao, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

Our proof is based on the following theorem.

Theorem (McCoy). *For $A_1, \ldots, A_p\in\mathbb{C}^{n\times n}$, there is a nonsingular matrix $U\in\mathbb{C}^{n\times n}$ such that $U^{-1}A_iU$ is upper triangular for each $i\in\{1,\ldots,p\}$ if and only if*

$$\big[(A_jA_k-A_kA_j)F(A_1,\ldots,A_p)\big]^n=0$$

holds for every polynomial $F(t_1,\ldots,t_p)$ in p noncommuting variables and every j, $k\in\{1,\ldots,p\}$.

For a proof of McCoy's theorem, we refer to, e.g., [1, Theorem 2.4.8.7, p. 119].

We first show that the real matrices $(A_i)_{i=1}^p$ are simultaneously quasi upper triangularizable over $\mathbb{R}$. By quasi upper triangular, we mean block upper triangular with block size at most 2×2.

Let $U\in\mathbb{C}^{n\times n}$ be a nonsingular matrix that upper triangularizes all A_i, and $u=Ue_1$ the first column of U.

(1) If u and $\bar u$ are linearly dependent, then u can be rescaled to a real vector $q=\alpha u\in\mathbb{R}^n$. Let Q be a real nonsingular matrix whose first column is q, i.e., $Qe_1=q$. Then $T_i=Q^{-1}A_iQ$ is of the form

$$T_i=\begin{bmatrix}\lambda_i & *\\ 0 & \hat T_i\end{bmatrix}.$$

From McCoy's theorem that

$$\big[(T_jT_k-T_kT_j)F(T_1,\ldots,T_p)\big]^n=Q^{-1}\big[(A_jA_k-A_kA_j)F(A_1,\ldots,A_p)\big]^nQ=0,$$

we obtain $(\hat T_j\hat T_k-\hat T_k\hat T_j)F(\hat T_1,\ldots,\hat T_p)]$ is nilpotent for any j, k, and all complex polynomials F. Thus $(\hat T_i)_{i=1}^p$ are simultaneously triangularizable over $\mathbb{C}$.

(2) If u and $\bar u$ are linear independent, we let $u=x+\mathrm{i}y$, where x, $y\in\mathbb{R}^n$. Then x and y are also linearly independent. We choose a nonsingular matrix $Q\in\mathbb{R}^{n\times n}$ whose first two columns are x and y. Then $T_i=Q^{-1}A_iQ$ is block upper triangular for all i. Partition T_i as

$$T_i=\begin{bmatrix}C_i & *\\ 0 & \hat T_i\end{bmatrix},$$

where $C_i\in\mathbb{R}^{2\times 2}$. Again, from McCoy's theorem, we obtain that $(\hat T_j\hat T_k-\hat T_k\hat T_j)F(\hat T_1,\ldots,\hat T_p)$ is nilpotent for any j, k, and all complex polynomials F, and hence $(\hat T_i)_{i=1}^p$ are simultaneously triangularizable over $\mathbb{C}$.

In either case, we have reduced the problem to a set of matrices $(\hat T_i)_{i=1}^p$ whose dimensions are smaller than n. Repeating this process eventually yields the trivial case $n=1$ or $n=2$.

Since we have shown that $(A_i)_{i=1}^p$ are simultaneously quasi upper triangularizable over $\mathbb{R}$, we choose a real nonsingular matrix U from the procedure described above such that $U^{-1}A_iU$ is quasi upper triangular for all i. Then the problem reduces to showing that the result holds for all diagonal blocks of these quasi upper triangular matrices.

(1) For 1×1 diagonal blocks, the result holds trivially.

(2) For 2×2 diagonal blocks, denoted by $(D_i)_{i=1}^p$, from the analysis above we know that at least one of the D_i's has nonreal eigenvalues and eigenvectors. Also, by McCoy's theorem, $(D_i)_{i=1}^p$ are simultaneously triangularizable over $\mathbb{C}$ because $(D_jD_k-D_kD_j)F(D_1,\ldots,D_p)$ from the procedure above. Let $Q=[q_1,q_2]\in\mathbb{C}^{2\times 2}$ be a nonsingular matrix such that $Q^{-1}D_iQ$ is upper triangular for all i. Note that both q_1 and $\bar q_1$ are eigenvectors of D_i. Therefore, for each D_i, its eigenvalues appear in pairs of conjugate complex numbers. In addition, $\hat Q=[q_1,\bar q_1]$ also upper triangularizes all D_i. Hence, $\hat Q^{-1}(\sum_{i=1}^p D_i^2)\hat Q$ is upper triangular, and its diagonal elements consist of a pair of conjugate complex numbers. The inequality $\det(\sum_{i=1}^p D_i^2)=\det\big(Q^{-1}(\sum_{i=1}^p D_i^2)Q\big)\ge 0$ holds naturally for this case.

Reference

[1] R. A. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge University Press, Cambridge, 2013.

Problem 56-5: Probability That a Large $\mathbb{F}_q$-Matrix is Singular

Proposed by Denis SERRE, *École Normale Supérieure de Lyon, France*, denis.serre@ens-lyon.fr

Let $\mathbb{F}_q$ be a finite field with q elements. If matrices inside $\mathbf{M}_n(\mathbb{F}_q)$ are chosen randomly with the entries independent and uniformly distributed over $\mathbb{F}_q$, find a closed formula for the probability $p_n(q)$ that $\det M = 1$ for $M \in \mathbf{M}_n(\mathbb{F}_q)$. Verify that $p_n(q) < \frac{1}{q}$ when $n > 1$ (hence $\det M = 0$ is more likely). Show that $n \mapsto p_n(q)$ is decreasing and admits a limit $p(q) \in (0, \frac{1}{q})$. (Underlining indicates where the phrasing of the question has been slightly modified for accuracy and clarity.)

Solution 56-5.1 by Meiyue SHAO, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

We use pivoted Gaussian elimination to evaluate $\det M$. The pivoting strategy is simply choosing the first nonzero entry from the first column of the Schur complement in each step of Gaussian elimination. Then $\det M = 0$ if and only if Gaussian elimination breaks down when a pivot cannot be found. Clearly for any $x_1, x_2 \in \mathbb{F}_q \backslash \{0\}$, we have

$$\text{Prob}\{\det M = x_1\} = \text{Prob}\{\det M = x_2\},$$

because we can scale the first row of M to change its determinant. Therefore,

$$p_n(q) = \text{Prob}\{\det M = 1\} = \frac{1}{q-1}\text{Prob}\{\det M \neq 0\}.$$

For any given $k \in \mathbb{F}_q$, if c and d are randomly and independently chosen from the uniform distribution over $\mathbb{F}_q$, then

$$\text{Prob}\{d - ck = x_1\} = \text{Prob}\{d - ck = x_2\} \qquad (\forall x_1, x_2 \in \mathbb{F}_q).$$

Furthermore, if c, d_1 and d_2 are randomly and independently chosen from the uniform distribution over $\mathbb{F}_q$, then

$$\text{Prob}\{d_1 - ck = x_1,\ d_2 - ck = x_2\} = \text{Prob}\{d_1 - ck = x_1\}\text{Prob}\{d_2 - ck = x_2\} \qquad (\forall x_1, x_2 \in \mathbb{F}_q).$$

Hence, in each step of Gaussian elimination, entries of the Schur complement still independently follow the uniform distribution over $\mathbb{F}_q$. Then it is easy to verify that

$$\text{Prob}\{\det M \neq 0 | M \in \mathbf{M}_n(\mathbb{F}_q)\} = \Big(1 - \frac{1}{q^n}\Big)\text{Prob}\{\det S \neq 0 | S \in \mathbf{M}_{n-1}(\mathbb{F}_q)\},$$

where S is the $n-1$ by $n-1$ Schur complement submatrix formed from Gaussian elimination. By induction, we obtain

$$\text{Prob}\{\det M \neq 0 | M \in \mathbf{M}_n(\mathbb{F}_q)\} = \prod_{j=1}^{n}\Big(1 - \frac{1}{q^j}\Big).$$

Hence,

$$p_n(q) = \frac{1}{q-1}\prod_{j=1}^{n}\Big(1 - \frac{1}{q^j}\Big) = \frac{1}{q}\prod_{j=2}^{n}\Big(1 - \frac{1}{q^j}\Big).$$

Clearly, $p_n(q)$ is strictly decreasing with respect to n, and we have $p_n(q) < p_1(q) = 1/q$ when $n > 1$. As $n \to \infty$, $p_n(q)$ has a nonzero limit

$$p(q) = \lim_{n\to\infty} p_n(q) = \frac{1}{q}\prod_{j=2}^{\infty}\Big(1 - \frac{1}{q^j}\Big) > 0,$$

because the series

$$\sum_{j=2}^{\infty} \ln\Big(1 - \frac{1}{q^j}\Big)$$

converges absolutely. Thus, $p(q) \in (0, 1/q)$.

Problem 56-6: An Asymptotic Moment Property for Unitary Matrices

Proposed by Stephen Rush, *University of Guelph, Guelph, Canada*, srush01@uoguelph.ca

Suppose U, V are unitary matrices (not necessarily of the same dimension) with $\lim_{k\to\infty} \left|\mathrm{Tr}(U^k) - \mathrm{Tr}(V^k)\right| = 0$. Show that U and V are unitarily similar.

Solution 56-6.1 by Meiyue Shao, *Lawrence Berkeley National Laboratory, Berkeley, California, USA*, myshao@lbl.gov

Let $\lambda_1 = \exp(2\pi\mathrm{i}\alpha_1), \ldots, \lambda_m = \exp(2\pi\mathrm{i}\alpha_m)$ and $\mu_1 = \exp(2\pi\mathrm{i}\beta_1), \ldots, \mu_n = \exp(2\pi\mathrm{i}\beta_n)$, respectively, be the eigenvalues of U and V (counting multiplicity), where $\alpha_1, \ldots, \alpha_m, \beta_1, \ldots, \beta_n \in \mathbb{R}$. We prove the result in three steps:

(1) $m = n$,
(2) $\mathrm{Tr}(U^k) = \mathrm{Tr}(V^k)$ for all $k \in \mathbb{N}$,
(3) U and V have the same eigenvalues.

The proof is based on a weaker version of Dirichlet's approximation theorem: *Given $\epsilon > 0$ and $\theta_1, \ldots, \theta_d \in \mathbb{R}$, there exist $q, p_1, \ldots, p_d \in \mathbb{Z}$ such that $|q\theta_i - p_i| < \epsilon$ holds for $i = 1, \ldots, d$.*

(1) By Dirichlet's approximation theorem, for any $\epsilon > 0$ there are infinitely many integers q_k and $p_{k,i}$ ($1 \le i \le m+n$, $k = 1, 2, \ldots$), such that

$$|q_k\alpha_i - p_{k,i}| < \epsilon \qquad |q_k\beta_j - p_{k,m+j}| < \epsilon, \qquad (1 \le i \le m,\ 1 \le j \le n).$$

Obviously we can further require all q_k's to be positive. Then we have

$$\begin{aligned}\left|\lambda_i^{q_k} - 1\right| &= \left|\exp\bigl(2\pi\mathrm{i}(q_k\alpha_i - p_{k,i})\bigr) - 1\right| \le \left|2\pi(q_k\alpha_i - p_{k,i})\right| < 2\pi\epsilon,\\ \left|\mu_i^{q_k} - 1\right| &= \left|\exp\bigl(2\pi\mathrm{i}(q_k\beta_j - p_{k,m+j})\bigr) - 1\right| \le \left|2\pi(q_k\beta_i - p_{k,m+j})\right| < 2\pi\epsilon.\end{aligned}$$

Consequently, $\|U^{q_k} - I_m\|_2 < 2\pi\epsilon$ and $\|V^{q_k} - I_n\|_2 < 2\pi\epsilon$.

If $m \ne n$, we choose $\epsilon = \left[4\pi(m+n)\right]^{-1}$. It follows from

$$|\mathrm{Tr}(U^{q_k}) - m| < 2\pi m\epsilon, \qquad |\mathrm{Tr}(V^{q_k}) - n| < 2\pi n\epsilon$$

that

$$\begin{aligned}|\mathrm{Tr}(U^{q_k}) - \mathrm{Tr}(V^{q_k})| &= |(m-n) + (\mathrm{Tr}(U^{q_k}) - m) - (\mathrm{Tr}(V^{q_k}) - n)|\\ &\ge |m-n| - |\mathrm{Tr}(U^{q_k}) - m| - |\mathrm{Tr}(V^{q_k}) - n|\\ &\ge 1 - 2\pi(m+n)\epsilon\\ &> \frac{1}{2}.\end{aligned}$$

This contradicts the condition that $\lim_{k\to\infty} |\mathrm{Tr}(U^k) - \mathrm{Tr}(V^k)| = 0$. Therefore, we have $m = n$.

(2) Suppose that $\mathrm{Tr}(U^a) \ne \mathrm{Tr}(V^a)$ for some $a \in \mathbb{N}$. We choose $\epsilon = |\mathrm{Tr}(U^a) - \mathrm{Tr}(V^a)|/(8\pi n)$. It can be verified that

$$|\mathrm{Tr}(U^{a+q_k}) - \mathrm{Tr}(U^a)| \le n\|U^{a+q_k} - U^a\|_2 = n\|U^{q_k} - I_n\|_2 < 2\pi n\epsilon.$$

(Note, the sequence $\{q_k\}$ here is different from that in step (1), as it corresponds to a different value of ϵ.) Similarly,

$$|\mathrm{Tr}(V^{a+q_k}) - \mathrm{Tr}(V^a)| \le n\|V^{a+q_k} - V^a\|_2 = n\|V^{q_k} - I_n\|_2 < 2\pi n\epsilon.$$

We obtain

$$\begin{aligned}|\mathrm{Tr}(U^{a+q_k}) - \mathrm{Tr}(V^{a+q_k})| &\ge |\mathrm{Tr}(U^a) - \mathrm{Tr}(V^a)| - |\mathrm{Tr}(U^{a+q_k}) - \mathrm{Tr}(U^a)| - |\mathrm{Tr}(V^{a+q_k}) - \mathrm{Tr}(V^a)|\\ &\ge |\mathrm{Tr}(U^a) - \mathrm{Tr}(V^a)| - 4\pi n\epsilon\\ &\ge \frac{1}{2}|\mathrm{Tr}(U^a) - \mathrm{Tr}(V^a)|\\ &> 0,\end{aligned}$$

which contradicts the condition $\lim_{k\to\infty} |\mathrm{Tr}(U^k) - \mathrm{Tr}(V^k)| = 0$. Thus $\mathrm{Tr}(U^k) = \mathrm{Tr}(V^k)$ holds for all $k \in \mathbb{N}$.

(3) By Vieta's formulas, $\lambda_1, \ldots, \lambda_n$ are uniquely determined by their elementary symmetric polynomials, which are uniquely determined by $\sum_{i=1}^n \lambda_i^k$ with $k = 1, \ldots, n$ according to the Newton–Girard formulas. Therefore, the eigenvalues

of U are uniquely determined by $\mathrm{Tr}(U^k)$ with $k = 1, \ldots, n$. It then follows from step (2) that U and V have the same eigenvalues, and are hence similar.

Solution 56-6.2 by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Suppose $\lim_{k\to\infty} |\mathrm{Tr}(U^k) - \mathrm{Tr}(V^k)| = 0$. Let the multiplicities of 1 as an eigenvalue of U and V be m_u and m_v, respectively. (These could be zero if 1 is not an eigenvalue.) Now choose n large so that $|\mathrm{Tr}(U^k) - \mathrm{Tr}(V^k)| < \frac{1}{2}$ if $k \geq n$ and so that no nth root of unity except perhaps one is in the spectrum of either U or V. Let P_U and P_V be the orthogonal projections onto the eigenspaces of U^n and V^n, respectively, corresponding to the eigenvalue 1. (These are also the eigenspaces of U and V, respectively, corresponding to the eigenvalue 1, since no nth root of unity except perhaps one is in the spectrum of either U or V.) We take P_U (resp. P_V) to be zero if 1 is not an eigenvalue of U (resp. V). Then by Von Neumann's ergodic theorem, $\lim_{N\to\infty} \frac{1}{N}\sum_{j=0}^{N-1} \mathrm{Tr}(U^{jn}) = \mathrm{Tr}(P_U) = m_u$ and $\lim_{N\to\infty} \frac{1}{N}\sum_{j=0}^{N-1} \mathrm{Tr}(V^{jn}) = \mathrm{Tr}(P_V) = m_v$.

Hence $|m_u - m_v| = |\lim_{N\to\infty} \frac{1}{N}[\sum_{j=0}^{N-1} \mathrm{Tr}(U^{jn}) - \sum_{j=0}^{N-1} \mathrm{Tr}(V^{jn})]| \leq \lim_{N\to\infty} \frac{1}{N}\sum_{j=0}^{N-1} |\mathrm{Tr}(U^{jn}) - \mathrm{Tr}(V^{jn})| \leq \frac{1}{2}$. This means that $m_u = m_v$. Applying this result to ωU and ωV for every ω of modulus one shows us that U and V have the exact same eigenvalues with the same multiplicities. Therefore U and V are unitarily similar.

Problem 57-1: Perfect Condition Number

Proposed by Bojan KUZMA, *University of Primorska, Slovenia*, bojan.kuzma@famnit.upr.si

Recall that the *condition number* of an invertible matrix A, relative to a norm $\|\cdot\|$ is $\operatorname{cond} A := \|A\| \cdot \|A^{-1}\|$. Show that invertible matrices with condition number equal to one with respect to some multiplicative norm are diagonalizable.

Solution 57-1.1 by Edward POON, *Embry-Riddle Aeronautical University, Prescott, AZ, USA*, edward.poon@erau.edu

Suppose $A \in M_n$ has condition number equal to one. Then, for all $X \in M_n$,

$$\|X\| = \|A^{-1}(AXA^{-1})A\| \leq \|A^{-1}\|\|AXA^{-1}\|\|A\| = \|AXA^{-1}\| \leq \|A\|\|X\|\|A^{-1}\| = \|X\|,$$

so the map $\phi(X) = AXA^{-1}$ is an isometry. By Auerbach's theorem [1, Exercise 5.4P11, p. 334], ϕ is similar to a unitary operator from M_n to M_n, and hence diagonalizable. Now if A were not diagonalizable, we may, by rescaling A if necessary, assume that 1 is an eigenvalue of A and choose $v \neq 0$ satisfying $(A - I)^2 v = 0$ and $(A - I)v \neq 0$. Choose $w \neq 0$ so that $(A^{-1})^T w = w$ and let $X = vw^T$. Then $(\phi - id)^2 X = 0$ and $(\phi - id)X \neq 0$, implying that ϕ is not diagonalizable, a contradiction.

Reference

[1] R. A. Horn and C. R. Johnson. *Matrix Analysis*. Cambridge University Press, Cambridge, 2013.

Solution 57-1.2 by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

We claim that, for all matrices M of the same size as A, $\|AM\| = \|A\| \cdot \|M\|$. For if $\|AM\| < \|A\| \cdot \|M\|$ for some M, then

$$\|AM\| < \|A\| \cdot \|A^{-1}AM\| \leq \|A\| \cdot \|A^{-1}\| \cdot \|AM\| = \|AM\|$$

which is a contradiction. Suppose A is not diagonalizable. Then there is an eigenvalue λ_0 of A whose geometric multiplicity is smaller than its algebraic multiplicity. Hence, for any corresponding eigenvector v,

$$\|A\| \cdot \|vv^T\| = \|Avv^T\| = \|\lambda_0 vv^T\| = |\lambda_0| \cdot \|vv^T\|$$

and so $|\lambda_0| = \|A\|$. Therefore, if we define $B = \frac{1}{\lambda_0}A$, then $\|B\| = 1$, $\|BM\| = \|B\| \cdot \|M\|$ for all matrices M of the same size as B, and 1 is an eigenvalue of B whose geometric multiplicity is smaller than its algebraic multiplicity. From this last property of B, we may infer that there are linearly independent vectors x and y such that $Bx = x$ and $By = x + y$. Thus, for all real numbers r, we have

$$\|rxx^T + yx^T\| = \|B(rxx^T + yx^T)\| = \|(r+1)xx^T + yx^T\|.$$

This is a contradiction, which can be seen as follows. By the above identity, for all integers n we have

$$\|rxx^T + yx^T\| = \|(r+n)xx^T + yx^T\| \geq |r+n|\|xx^T\| - \|yx^T\| \to \infty \text{ as } n \to \infty.$$

Also solved by Gérald BOURGEOIS.

Problem 57-3: A PSD Matrix Decomposition

Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let $A \in M_n(\mathbb{C})$ be a positive semidefinite matrix and let $\|\cdot\|$ be the Euclidean norm on $\mathbb{C}^n$. Show that there exist n not necessarily distinct vectors $\{v_k\}_{k=1}^n$ in $\mathbb{C}^n$ such that $\|v_k\|^2 = a_{kk}$ for all k with $1 \leq k \leq n$ and

$$A = \sum_{k=1}^{n} v_k v_k^*.$$

Solution 57-3.1 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

The matrix A has a positive semidefinite square root B; that is, $B^* = B$ and $A = B^2$. Write $B = [b_{ij}]_{i,j=1}^n$, and let

$$v_k = [b_{1k}, \dots, b_{nk}]^T, \quad k = 1, \dots, n.$$

Then $(v_k v_k^*)_{ij} = b_{ik}\overline{b_{jk}} = b_{ik}b_{kj}$, and so

$$\sum_{k=1}^{n} (v_k v_k^*)_{ij} = \sum_{k=1}^{n} b_{ik}b_{kj},$$

which is the (i, j)-entry of B^2 and hence of A. Furthermore,

$$\|v_k\|^2 = v_k^* v_k = \sum_{j=1}^{n} \overline{b_{jk}} b_{jk} = \sum_{j=1}^{n} b_{kj}b_{jk}$$

which is the (k, k)-entry of B^2 and hence of A.

Also solved by Dario Fasino *and Pan Shun* Lau.

Problem 57-5: Eisenstein Circulant Matrices

Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

An Eisenstein integer is a complex number of the form $a + b\omega$ where a and b are integers and $\omega = \frac{-1+\sqrt{3}i}{2}$. The Eisenstein integers form a commutative ring with identity. Let A be a three by three invertible Hermitian circulant matrix all of whose entries are Eisenstein integers. Show that $\mathrm{Tr}(A^n) \neq 0$ for all natural numbers $n \geq 2$.

Solution 57-5.1 by Tamás F. Görbe, *University of Szeged, Hungary*, tfgorbe@physx.u-szeged.hu

An arbitrary 3×3 circulant matrix with Eisenstein integers for entries has the form

$$A = \begin{bmatrix} e_0 & e_2 & e_1 \\ e_1 & e_0 & e_2 \\ e_2 & e_1 & e_0 \end{bmatrix}, \quad e_k = a_k + b_k\omega,\ a_k, b_k \in \mathbb{Z} \quad (k = 0, 1, 2).$$

Notice that $\omega = e^{2\pi i/3}$, thus $\bar{\omega} = \omega^{-1} = e^{-2\pi i/3} = -\frac{1+\sqrt{3}i}{2} = -(1 + \omega)$. The conjugate transpose of A is

$$A^* = \begin{bmatrix} \bar{e}_0 & \bar{e}_1 & \bar{e}_2 \\ \bar{e}_2 & \bar{e}_0 & \bar{e}_1 \\ \bar{e}_1 & \bar{e}_2 & \bar{e}_0 \end{bmatrix},$$

therefore A is Hermitian ($A = A^*$) if and only if $e_0 = \bar{e}_0$ and $e_2 = \bar{e}_1$, or equivalently

$$e_0 = a_0, \quad e_1 = a_1 + b_1\omega, \quad e_2 = a_1 - b_1 - b_1\omega \quad (a_0, a_1, b_1 \in \mathbb{Z}).$$

It can readily checked that the eigenvalues and corresponding eigenvectors of A are

$$\lambda_k = e_0 + e_2\omega^k + e_1\bar{\omega}^k, \quad v_k = \begin{bmatrix} 1 \\ \omega^k \\ \bar{\omega}^k \end{bmatrix} \quad (k = 0, 1, 2).$$

Expressing the eigenvalues in terms of the integers a_0, a_1, b_1 shows that $\lambda_0, \lambda_1, \lambda_2$ are also integers, namely

$$\lambda_0 = a_0 + 2a_1 - b_1, \quad \lambda_1 = a_0 - a_1 + 2b_1, \quad \lambda_2 = a_0 - a_1 - b_1.$$

The matrix A is invertible if and only if $\det(A) = \lambda_0\lambda_1\lambda_2 \neq 0$, which implies that none of the eigenvalues vanish. Next, we recall that the trace of the n-th power of a matrix is the n-th power sum of its eigenvalues. In our case,

$$\mathrm{Tr}(A^n) = \lambda_0^n + \lambda_1^n + \lambda_2^n.$$

If n is even ($n = 2k$ for some $k \in \{1, 2, 3, \dots\}$), then $\mathrm{Tr}(A^n)$ is a sum of (nonzero) squares, therefore it is positive, i.e.

$$\mathrm{Tr}(A^n) = (\lambda_0^k)^2 + (\lambda_1^k)^2 + (\lambda_2^k)^2 > 0.$$

If n is odd ($n = 2k + 1$ for some $k \in \{1, 2, 3, \dots\}$), then $\mathrm{Tr}(A^n)$ is certainly not zero if all three eigenvalues have the same sign. The only remaining case is if one of the eigenvalues has a sign different from the other two. Without loss of generality, we can assume that λ_0 and λ_1 are positive integers and λ_2 is a negative integer[1], then

$$\mathrm{Tr}(A^n) = 0 \qquad \Longleftrightarrow \qquad \lambda_0^n + \lambda_1^n = (-\lambda_2)^n, \quad \text{where } \lambda_0, \lambda_1, -\lambda_2 \text{ are positive integers and } n \geq 3.$$

This would contradict Fermat's Last Theorem, thus $\mathrm{Tr}(A^n) \neq 0$.

Problem 57-6: A Commuting Matrix Problem

Proposed by Roger A. Horn, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu

Let A and B be $n \times n$ complex matrices. Suppose that (a) B is nonderogatory, and (b) $Ax = \lambda x$ whenever $Bx = \lambda x$. Show that if A and B commute, then they have the same eigenvalues with the same algebraic multiplicities, that is, they have the same characteristic polynomial.

Solution 57-6.1 by Pan Shun Lau, *The Hong Kong Polytechnic University, Hong Kong*, panlau@connect.hku.hk

We assume without loss of generality that $B = J_{n_1}(\lambda_1) \oplus \cdots \oplus J_{n_k}(\lambda_k)$ where $J_m(\lambda)$ is an $m \times m$ Jordan block and $\lambda_1, \dots, \lambda_k$ are distinct eigenvalues of B with algebraic multiplicities $n_1, \dots, n_k$ respectively. As B is nonderogatory and $AB = BA$, there exists a polynomial $p(t)$ such that

$$A = p(B) = p(J_{n_1}(\lambda_1)) \oplus \cdots \oplus p(J_{n_k}(\lambda_k))$$

(see, e.g., Theorem 3.2.4.2 of Horn and Johnson's *Matrix Analysis*, 2013). Let $e_1, \dots, e_n$ be the standard basis of $\mathbb{C}^n$. Noting that $Be_{n_1+\cdots+n_j+1} = \lambda_{j+1}e_{n_1+\cdots+n_j+1}$ and $Ae_{n_1+\cdots+n_j+1} = p(\lambda_{j+1})e_{n_1+\cdots+n_j+1}$. Therefore, by (b), $p(\lambda_i) = \lambda_i$ are eigenvalues of A with algebraic multiplicities n_i, $i = 1, \dots, k$.

Also solved by Gérald Bourgeois*, Daryl Q.* Granario *and Eugene A.* Herman.

[1]All other cases can be obtained by permuting the indices or multiplying by an overall factor of -1.

IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce 4 new problems in this issue and invite readers to submit solutions for publication in *IMAGE*. Solutions: We present solutions to all three remaining problems in the issue [*IMAGE* 56 (Spring 2016)] as well as four problems from the previous issue [*IMAGE* 57 (Fall 2016)]. Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Rajesh Pereira (`pereirar@uoguelph.ca`).

NEW PROBLEMS:

Problem 58-1: Singular Value Decomposition of Involutory and Skew-Involutory Matrices

Proposed by Richard William FAREBROTHER, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@hotmail.com

A complex $n \times n$ matrix A is involutory if it satisfies $A^2 = I_n$ and skew-involutory if it satisfies $A^2 = -I_n$. Let A be a matrix and UDV be its singular value decomposition. Show that if there exists a complex number s with $|s| = 1$ such that $A^2 = sI_n$, then $(VU)^2 = sI_n$. In particular, if A is involutory (resp. skew involutory) then VU is also involutory (resp. skew involutory).

Editor's note: Problem Corner readers may be interested in Dr. Farebrother's article on Lewis Carroll, Graeco-Latin Squares and Magic Squares in Acta et Commentationes Universitatis Tartuensis de Mathematica available online at `http://acutm.math.ut.ee/index.php/acutm/article/download/ACUTM.2015.19.09/47`.

Problem 58-2: A Matrix Exponential Problem

Proposed by Moubinool OMARJEE, *Lycée Henri IV, Paris, France*, ommou@yahoo.com

Let A be an n by n matrix with complex entries. The matrix exponential e^A is defined using the usual power series

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}.$$

We say that an n by n matrix A has the entrywise exponential property if the (i, j)-entry of e^A is $e^{a_{ij}}$ for all $1 \leq i, j \leq n$.

(a) Show that an n by n skew-Hermitian matrix has the entrywise exponential property only if $n = 1$.

(b) Show that an n by n Hermitian matrix has the entrywise exponential property only if $n = 1$.

(c) Show that no two by two matrix has the entrywise exponential property.

Editor's note: The conjecture that an n by n matrix has the entrywise exponential property only if $n = 1$ is an open problem.

Problem 58-3: Upper and Lower Bounds of Ranks of Matrices

Proposed by Yongge TIAN, *Central University of Finance and Economics, Beijing, China*, yongge.tian@gmail.com

(a) Let $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$, where X_{11}, X_{12}, X_{21}, and X_{22} are $m_1 \times n_1$, $m_1 \times n_2$, $m_2 \times n_1$, and $m_2 \times n_2$ matrices, respectively, and assume that $\operatorname{rank}(X) = t$. Show that the rank of X_{11} satisfies the inequalities

$$\max\{0, t - m_2 - n_2\} \leq \operatorname{rank}(X_{11}) \leq \min\{m_1, n_1, t\}$$

and also show that these upper and lower bounds are best possible for all values of m_1, m_2, n_1, n_2, and all t with $0 \leq t \leq \min\{m_1 + m_2, n_1 + n_2\}$.

(b) Let X_1, X_2, and X_3 be $m_1 \times m_2$, $m_2 \times m_3$, and $m_3 \times m_4$ matrices with $\operatorname{rank}(X_1) = t_1$, $\operatorname{rank}(X_2) = t_2$, and $\operatorname{rank}(X_3) = t_3$. Show that

$$\max\{0, t_1 + t_2 + t_3 - m_2 - m_3\} \leq \operatorname{rank}(X_1X_2X_3) \leq \min\{t_1, t_2, t_3\},$$

and also show that the upper and lower bounds are best possible for all appropriate values of m_1, m_2, m_3, m_4, t_1, t_2, t_3.

Problem 58-4: Characteristic Polynomials of 3 by 3 Real Correlation Matrices

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A real correlation matrix is a positive semidefinite matrix all of whose entries are real and all of whose main diagonal entries are equal to one. Let A be a three by three real correlation matrix and let $p(x)$ be the characteristic polynomial of A. Show that if A is a convex combination of real rank one correlation matrices then there exists $a \geq 0$ such that the polynomial $q(x) = \int_a^x p(t)dt$ has all four of its roots real and nonnegative.

Solutions to Problems 56-2, 56-5, 56-6, 57-1, 57-3, 57-5 and 57-6 are on pages 37–42.