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About *IMAGE*

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Contributing editors: Adam Berliner (`berliner@stolaf.edu`), Sebastian Cioabă (`cioaba@udel.edu`), Anthony Cronin (`anthony.cronin@ucd.ie`), Colin Garnett (`Colin.Garnett@bhsu.edu`), Rajesh Pereira (`pereirar@uoguelph.ca`), and Amy Wehe (`awehe@fitchburgstate.edu`).

Acknowledgment: With this issue, *IMAGE* welcomes two new contributing editors. Adam Berliner is the new editor for interviews, beginning his tenure with the interview of Richard Brualdi on page 3. Anthony Cronin will serve as editor for education, in coordination with the rest of the ILAS Education Committee. We look forward to their contributions in the issues to come!

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FEATURE INTERVIEW

"The older I get, the better I was"

an interview with Richard Brualdi

Instead of a "formal" one-on-one interview, several of us joined in on a happy hour Zoom conversation with Richard A. Brualdi. The participants were Adam Berliner, Louis Deaett, Nancy Neudauer, Jennifer Quinn, Bryan Shader, and Michael Tsatsomeros. Richard began the chat with the title quote, inspired by a blues song by Big Joe Shelton & The Black Prairie Blues Ambassadors.

Richard Brualdi in Rio for the 2019 ILAS conference.

R.A.B. - Okay, so what do you want to know about me? I was once interviewed for the University of Wisconsin Archives and there are actually seven hours of tape if you want to listen to them.

J.Q. - What's your mathematical origin story?

R.A.B. - Both sides of my family come from the Pesaro, Italy area in the Marches provinces. They ended up in Connecticut, where there were a whole bunch of Italian people. When I was a kid, I thought everyone was Italian. I grew up in a blue collar family who instilled in me a very strong work ethic. Everybody worked. My grandfathers were both factory workers and so was my father. My grandmother, who did not read or speak good English, started a local catering business from her house for Italian weddings, banquets, and so forth.

I actually helped serve food at the Italian weddings she would cater. I also delivered newspapers for many years, and when I turned 16 and was still a senior in high school, I worked in the kitchen of the local hospital every day after school and every other weekend. After college at UConn [the University of Connecticut], I worked that summer for the New England Engineering company, where we cleaned and fixed machines at a plastics factory in Brooklyn, New York. We would work and then stay in a sleazy hotel in Manhattan during the week, before going back to Connecticut for the weekend.

Growing up, there were very few books in the house, and I essentially did not read anything I wasn't forced to read for English Literature class. (I have been playing catch-up ever since.) The standards at my school were not that high. I have one cousin and a brother who went to college before me. I applied to the University of Connecticut because my brother went there. At the time, I was automatically admitted because I was in the top 40% of my class; that was the only requirement at that time to get admitted.

When I started college, there was no placement test for mathematics that I was aware of and so my first mathematics class was trigonometry, even though I had had trigonometry in high school. That course was followed by a required course in college algebra. One of the topics was mathematical induction, and I just could not understand induction! I got a C in that course because of it.

But around the end of my third semester, I somehow just got this thing in me: I was just going to study and study and study. And that's what I did the rest of my time. I wanted to get an A in every course. And I took mathematics, I took business law, I took physics, mechanics, electricity and magnetism. And I took basically every mathematics course that was offered at Connecticut, including my last semester, when I took six courses, four of which were mathematics.

I graduated with highest honors and I was elected to Phi Beta Kappa on the basis of my record. I majored in mathematics. And I didn't know what a mathematician really did. When somebody asked me about it, I said I wanted to be an "applied mathematician." I had no idea what that meant, frankly.

I went to grad school at Syracuse because they gave the best offer – $2000 a year! In my third year I took a course from a brand new professor, Herb Ryser, and he ended up being my advisor. He had connections to Morris Newman at the National Bureau of Standards, and that connection led to my first job as a postdoc at NBS.

During that year, I applied for academic jobs. I applied to three universities and got three offers: Maryland, Michigan, and Wisconsin. It was easy to get a job in the 60s. I had never been to Wisconsin before, but I knew of several math faculty members there, my Ph.D. advisor got his Ph.D. there, and they gave me the best offer – $9000 a year – so I took it! Looking back, I don't know where I would be professionally if I had not accepted the job at Wisconsin.

M.T. - When I was in Madison, you were Chair of the department. What was that like?

R.A.B. - Frankly, those six years as chair were my best years as far as being in the department. I very much enjoyed being the chair. As I have said to many people, I enjoy being important. And, you know, I didn't have authority or power. But people came to me and asked me questions: "What are we going to do?" I viewed my job as taking care of the department so that everyone else could do their job. So those were really wonderful years. And I found it hard to give up.

N.N. - You also had ten Ph.D. students simultaneously while chair.

R.A.B. - It was a really wonderful time, though. So many others all over the country have had way more students and it was not too much for me. I was in the department every day from 7 AM to 6 PM.

A.B. - What was the weirdest question or request that you received as chair?

R.A.B. - That one is easy. One of the retired faculty had recently gotten really into rope climbing. He wanted to put a rope in the 12-floor stairwell of Van Vleck Hall [the home of the math department] hanging down from the 9th floor and then climb and descend using the rope instead of the stairs. I had to find a way to tell him he couldn't do that.

L.D. - Is it true that you never taught calculus while at Wisconsin?

R.A.B. - In my 42 1/2 years of teaching at UW, I never once taught the basic calculus sequence courses, although I did once teach Business Calculus. I am not sure why, but I was never assigned to teach them, and I guess I never volunteered. I probably cannot even solve a calculus problem now, since I have been working with discrete math and matrices for so long.

N.N. - What has eluded you mathematically? Is there something you've always wanted to prove?

R.A.B. - Oh, yes. One of the big problems from long ago is the "three transversal problem." Given three families of n sets, when is there a transversal which simultaneously works on all three sets? It probably doesn't have a nice solution. In my early life I spent a lot of time on transversals and systems of distinct representatives. I once thought I had a proof of the three transversal problem, but ... it was wrong.

I just start working on problems, and if it somehow takes me in a slightly different direction, that's fine! I let the problem lead me rather than trying to lead the problem. That's been my philosophy. If something else comes up, fine!

I've been very fortunate to be at Wisconsin and to have such a great group of students. I would not be where I am today without all the wonderful students of mine. They are the people I collaborated with at Wisconsin – I did not collaborate with other faculty at Wisconsin other than a couple of times with Hans Schneider.

So that's been my career – but *it's far from over!* I have three different projects with three different collaborators right now.

N.N. - So, Richard, what has retirement been like for you, then?

R.A.B. - It's quite good. Before COVID, I used to bike to campus at 8:00 in the morning and stay until 1:00 PM. Then I'd go running at The Shell [a campus gym/track]. These days, I stay home, but I get to have a quick lunch every day with my wife. Then I go back to my study and work until about 2:30 PM, and then go running (five days a week on a regular route of about 3 miles) or biking. I miss campus a lot, but I still do my mathematics and my editorial work.

Not much has changed since officially retiring from UW except I do not spend quite as much time with mathematics. But I still spend what some would consider to be a full-time job doing mathematics and related work. I want to keep doing what I am doing as long as I can, as long as I enjoy it, and as long as people will collaborate with me. I love coming up with ideas, and I have no problem doing that. I still can make valuable contributions.

M.T. - The real question is are you able to do what you want to do?

R.A.B. - The editorial work I do for *Linear Algebra and its Applications* (*LAA*) and *The Electronic Journal of Combinatorics* (*E-JC*) has been such a big part of my life and I cannot imagine not doing it. I know I will have to stop at some point, but it's just what I do! I have been an Editor-in-Chief for *E-JC* (there are 15 of us) for 12 years. And, can you believe I am now a CEO? CEO: Chief Editorial Officer. Well, it is a fancy title for a position with essentially no authority except to make sure that the journal with its editors-in-chief runs smoothly.

One of the things that had an important effect on my career occurred when Hans Schneider invited me to join him as an Editor-in-Chief of *LAA*. The number of submissions was increasing and he was looking for some help, and there I was in Madison. And all of a sudden I was sort of important. That had a profound effect on my career. It would have been (mathematically) totally different otherwise.

B.S. - How many papers do you think you've handled, especially during the time when they were delivered via hard copy up the Van Vleck stairs to your office?

R.A.B. - Oh...very, very many! Now *LAA* gets around 1400 and *E-JC* around 700 [per year].

N.N. - Have you ever thought of just stopping work and lying on the beach?

R.A.B. - Twenty minutes at the beach is enough for me. I enjoy reading, too, but I enjoy mathematics so much I don't think I can stop doing it.

M.T. - I know you're an avid runner. Do you "think math" when you run?

R.A.B. - I do! Usually I run with earphones and an iPod, listening to music or a country western radio station. Before I go, I think ahead about what I want to think about mathematically when I run. After a long morning of working on math, running really does wonders for me. It helps clear my mind.

A.B. - You've traveled to so many places as part of your work. Are there any countries you haven't been to that you want to go to?

R.A.B. - I really want to go to Norway. One of my biggest collaborators has been Geir Dahl of Oslo. He has invited me but somehow I have not gone yet! I went to

Richard Brualdi thinking about math during the 2019 Turkey Trot 5K race in Madison, WI. (Richard finished in 32:12, a pace of about 10 $^1/_4$ minutes/mile.)

Estonia when it was in the USSR, but I've never been to Russia, either. I've also never been to Africa, and I've been to every other continent, including Antarctica!

N.N. - What advice do you have for a new Ph.D.?

R.A.B. - The hurdles now are totally unlike those I faced. It's a much different time now and I know I was very lucky. I got tenure after 3 years at Madison. Did I put together a file? No! I got a call one day and they told me I was promoted to Associate Professor with tenure. And at the age of 31, they promoted me to full Professor.

Richard Brualdi and Nair Abreu at ILAS 2019.

I still tell people they should pursue what they enjoy and are truly interested in. Work on things that please you, that are important to you, that you feel that you can make a contribution to, that you like doing. Don't work on a paper just to work on a paper.

And, be kind and respectful to other people. That's very important.

R.A.B. - How did I become a vegetarian? Nobody asked me that!

R.A.B. - Hazel Perfect, whom I worked with during the year 1969–1970 in Sheffield, England, was a vegetarian all her life. Overnight, my (then) wife and I decided we would become vegetarians too; it was not very common then, and restaurants had little to offer vegetarians; I have been one now for 51 years. I also think I've become a pretty good vegetarian cook. I do often cook meat though for my wife Mona and children when they visit. Since I won't taste it I am never certain how it comes out. My favorite things to make are tofu and seitan stir fries, and vegetarian pasta and pizza. It relaxes me to cook although sometimes I think about a mathematics problem while I am cooking. I could have easily been a cook at a restaurant!

Send News for *IMAGE* Issue 67

IMAGE seeks to publish all news of interest to the linear algebra community. Issue 67 of *IMAGE* is due to appear online on December 1, 2021. Send your news for this issue to the appropriate editor by October 15, 2021. Photos are always welcome, as well as suggestions for improving the newsletter. Please send contributions directly to the appropriate editor:

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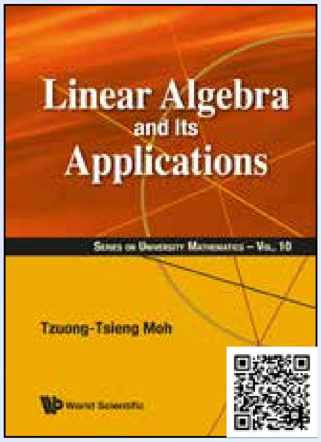

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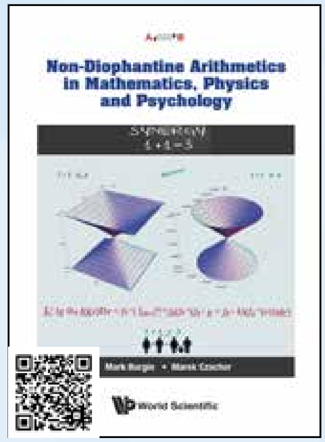

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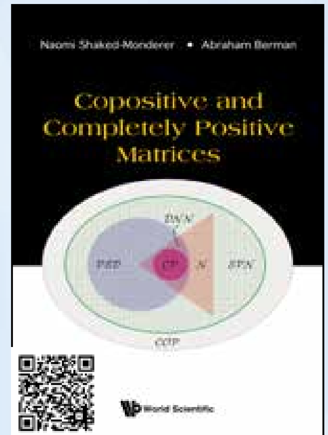

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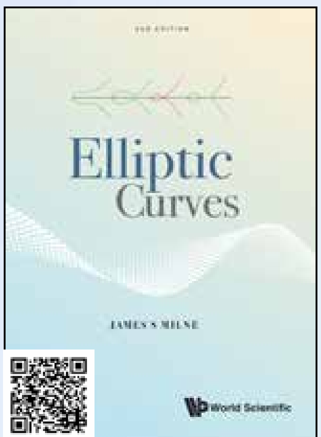

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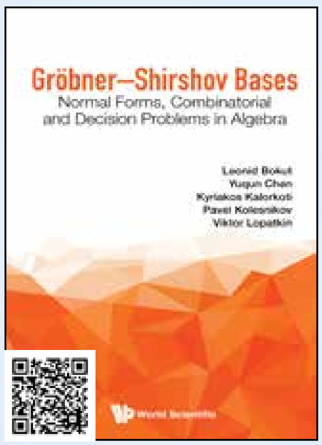

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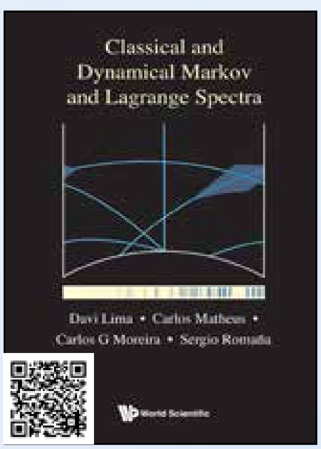

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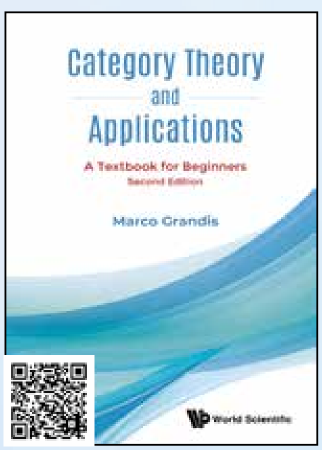

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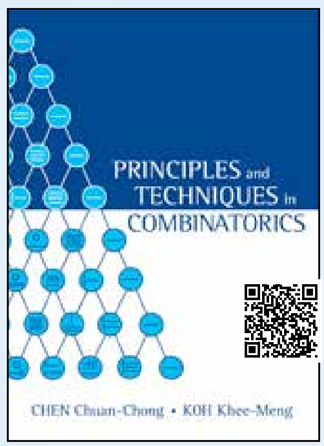
Classic Textbook



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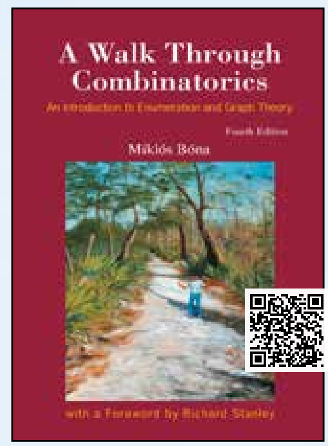

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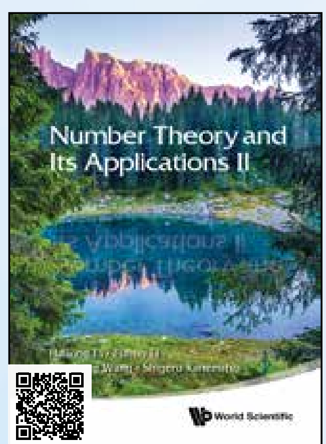

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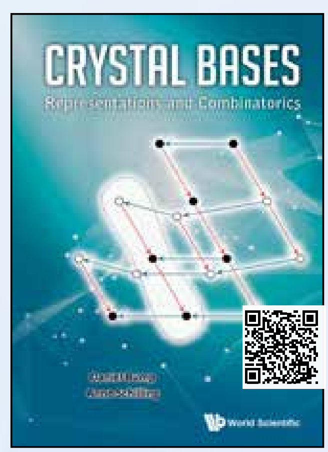

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FEATURE ARTICLE

Commuting Triples of Matrices versus the Computer

John Holbrook, University of Guelph, Guelph, ON, Canada, jholbroo@uoguelph.ca
and
Kevin C. O'Meara, University of Canterbury, Christchurch, New Zealand, staf198@uclive.ac.nz

1. Introduction. Triples A, B, C of commuting $n \times n$ matrices entail several mysteries that, while easy to state, have proved to be maddeningly difficult to resolve. The most accessible of these puzzles is the following.

Easily stated question. *Can the vector space (or algebra) of polynomials in commuting $n \times n$ matrices A, B, C over a field (including the identity I) have dimension greater than n?*

This question may be called the Gerstenhaber Problem (GP for short), because Gerstenhaber's 1961 paper [3] included a surprising result: If A and B are commuting $n \times n$ matrices (over any field), then the vector space (algebra) of polynomials in A and B (and I) has dimension at most n. The well-known Cayley–Hamilton Theorem ensures that the polynomials in a single $n \times n$ matrix A form a space spanned by $I, A, A^2, \ldots, A^{n-1}$, whence this space has dimension at most n, but Gerstenhaber's result is more subtle. On the other hand, there is a simple example (dating back at least to the time of Schur's formula for the maximal dimension of commutative matrix algebras; see [16]) that displays commuting 4×4 matrices A, B, C, D such that I, A, B, C, D are linearly independent. So it is *triples* that remain mysterious!

Over the past few years we have developed a computational strategy aimed at resolving the GP by constructing explicitly commuting $n \times n$ matrices A, B, C such that the corresponding polynomial algebra has dimension exceeding n. A detailed 23-page account of this strategy, including operating instructions for the MATLAB code, is posted at

`https://arxiv.org/abs/2006.08588`

and the suite of 40 MATLAB routines implementing the strategy can be freely obtained by contacting the authors via

`GPmfiles@gmail.com`

One purpose of this note is to draw attention to these resources, with the hope that others, including graduate students, may wish to join in our search, perhaps with better programming skills and more powerful computers than are currently available to us.

In essence, our approach is the "Monte Carlo Method": the use of repeated random sampling and statistical analysis to achieve a result (not that our reader need understand the analysis, only the linear algebra). The idea occurred first to Enrico Fermi in the 1930s, and was expanded by Stanislaw Ulam in the 1940s. But it took the first electronic computer ENIAC in 1945 and programming by the likes of John von Neumann to demonstrate the power of this new tool.

The following paragraphs serve as an introduction to the full treatment in [7].

2. Insights from Algebraic Geometry. Let us fix some notation and terminology. Let F be any field; it need not be algebraically closed, but it will often be convenient to think in terms of the algebraic closure of F or perhaps of some field extension containing the eigenvalues of the matrices in play. Let $F[A_1, A_2, \ldots, A_m]$ denote the unital algebra generated by matrices $A_1, A_2, \ldots, A_m$, assuming that they commute and have entries from F.

For us, a variety is a subset of F^N determined by a set of polynomial equations in the coordinates; for example, we have the variety $\mathcal{C}(m, n)$ of m-tuples of commuting $n \times n$ matrices with entries from F, interpreted as a subset of F^{mn^2}. A variety is called irreducible if it cannot be expressed as the union of two proper subvarieties; note that in the older literature (e.g., Gerstenhaber [3]) a "variety" was assumed to be irreducible.

Gerstenhaber established his result by first proving that $\mathcal{C}(2, n)$ is irreducible; this had also been proved somewhat earlier by Motzkin and Taussky [11], but it was Gerstenhaber who derived the result on dimension. Let $\mathcal{G}(m, n)$ denote the subset of $\mathcal{C}(m, n)$ consisting of tuples $A_1, \ldots, A_m$ such that $\dim F[A_1, \ldots, A_m] \leq n$; the following implication, which might be viewed as a generalized Gerstenhaber theorem, is quite accessible:

$$\mathcal{C}(m, n) \text{ is irreducible} \implies \mathcal{G}(m, n) = \mathcal{C}(m, n).$$

Indeed, $\mathcal{G}(m, n)$ is a subvariety of $\mathcal{C}(m, n)$: in view of the Cayley–Hamilton theorem, $F[A_1, \ldots, A_m]$ is spanned by the monomials $A_1^{k_1} \cdots A_m^{k_m}$ such that each $k_j < n$. Thus, $\dim F[A_1, \ldots, A_m] \leq n$ if and only if $\operatorname{rank} M \leq n$, where M is the

$n^2 \times n^m$ matrix whose columns are the monomials written as $n^2 \times 1$ vectors. This in turn is equivalent to the condition that each $(n+1) \times (n+1)$ subdeterminant of M is 0. Recall the notion of a regular (or nonderogatory) $n \times n$ matrix A and that two properties are each equivalent to the regularity of A: (1) Every B commuting with A is a polynomial in A, and (2) $I, A, A^2, \ldots, A^{n-1}$ are linearly independent. Let $\mathcal{R}(m,n)$ consist of the tuples in $\mathcal{C}(m,n)$ such that A_1 is regular. By (1), $\mathcal{R}(m,n) \subseteq \mathcal{G}(m,n)$ and, by (2), its complement $\mathcal{R}^c(m,n)$ consists of tuples with rank $[I, A_1, \ldots, A_1^{n-1}] < n$ and so is a proper subvariety; but $\mathcal{G}(m,n) \cup \mathcal{R}^c(m,n) = \mathcal{C}(m,n)$, so that if $\mathcal{C}(m,n)$ is irreducible, $\mathcal{G}(m,n)$ must be all of it.

In this way, commuting triples are the setting for a second challenging puzzle: When is $\mathcal{C}(3,n)$ irreducible? This too is unresolved, but certain facts are known. Šivic and others (see, for example, [18]) have, at considerable cost, shown that, if F has characteristic 0, then $\mathcal{C}(3,n)$ is irreducible if $n < 12$. On the other hand, Guralnick [4], in a paper apparently intended mainly to present a nice simplification of Motzkin and Taussky's proof that $\mathcal{C}(2,n)$ is irreducible, included a valuable bonus: a proof that $\mathcal{C}(3,n)$ is reducible for $n > 31$. Improvements in his method by Holbrook and Omladič [8] reveal that $\mathcal{C}(3,n)$ is reducible for $n > 28$.

There are, perhaps, limits to what algebraic geometry can offer in resolving the GP. However, there exist proofs of Gerstenhaber's $m = 2$ result that do not involve algebraic geometry. Barría and Halmos [1] and, independently, Laffey and Lazarus [10] construct, given commuting $n \times n$ matrices A and B, a set of n monomials in A and B that span $F[A,B]$. The choice of monomials depends on the Jordan structure of A, but a similar approach to $F[A,B,C]$ seems ruled out by the examples presented in [12].

In summary, the way seems open to finding examples answering '*nay*' to the GP. In fact, while such examples may be elusive, other arguments suggest that they are out there somewhere! See, for example, the authors' [6].

3. Key Elements of the Computational Strategy. Like the Jordan form, the Weyr form is a matrix canonical form with respect to similarity, but it has significant advantages over the JCF when dealing with commuting matrices. Introduced by Eduard Weyr in 1885, the Weyr form was then largely forgotten; it has been "rediscovered" several times. Readers of *IMAGE* may have seen the interview [19] with Helene Shapiro, which includes an account of her adventures with the Weyr form. The book [14] discusses this device and its applications in detail. The Weyr form is also covered in Chapter 3 of the 2$^{\text{nd}}$ edition (2013) of Horn and Johnson [9].

By the generalized eigenspace decomposition, the GP reduces to the case of unispectral matrices, which we may as well take to be nilpotent. Thus it will suffice here to describe the Weyr form of an $n \times n$ nilpotent A. Essentially, the Weyr form W is a block matrix version of a basic (only 1 diagonal block) nilpotent Jordan matrix. Each nilpotent A has a unique Weyr structure $[n_1, \ldots, n_r]$, where $n_1 \geq n_2 \geq \cdots \geq n_r \geq 1$ and the n_k sum to n. A is similar to its Weyr form W, a block matrix with $n_k \times n_j$ blocks. Every block holds only zeros, except for the superdiagonal blocks:

$$W_{k,k+1} = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

where I is the $n_{k+1} \times n_{k+1}$ identity matrix and 0 is the $(n_k - n_{k+1}) \times n_{k+1}$ zero matrix. For example, the nilpotent Weyr matrix of structure $[3,2,1]$ is

$$W = \left[\begin{array}{ccc|cc|c} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline & & & 0 & 0 & 1 \\ & & & 0 & 0 & 0 \\ \cline{4-6} & & & & & 0 \end{array}\right].$$

A crucial advantage of the Weyr form W is that a matrix K commuting with W has an especially simple structure: K must have the same block structure as W and be block upper–triangular, and the lower blocks are all determined by the blocks in the top row, reminiscent of the form of Toeplitz matrices that commute with a basic nilpotent Jordan matrix. We illustrate this in the case of the Weyr matrix W above of structure $[3,2,1]$: K must look like

$$K = \left[\begin{array}{ccc|cc|c} a & b & d & g & i & l \\ 0 & c & e & h & j & m \\ 0 & 0 & f & 0 & k & n \\ \hline & & & a & b & g \\ & & & 0 & c & h \\ \cline{4-6} & & & & & a \end{array}\right].$$

The top row blocks are not arbitrary: they must be zero below certain staircase patterns determined by the Weyr structure. Our procedure for constructing triples W, K, L is to first choose a Weyr structure, then choose the top row blocks of K and L, following the pattern of zeros, that ensure that, when completed, K and L automatically commute with W. The top row blocks of K and L are determined recursively, and in tandem, by linear systems where free variables are assigned randomly (subject to sparsity constraints). The aim is to ensure that the partially constructed K and L also commute with one another. It is sometimes impossible to complete this process due to earlier choices. In this case we get only a commuting triple of smaller-sized matrices. The parameters governing sparsity constraints must be adjusted carefully to facilitate the construction of non-trivial triples with the full desired Weyr structure.

In 2006, O'Meara and Vinsonhaler [13] provided an important ingredient for our strategy: Commuting matrices can be simultaneously triangularized (by a similarity) while one of them is put into Weyr form (not possible for the Jordan form). Thus we can work with W, K, L where all are upper-triangular (not just block upper-triangular). Even more hopeful for a resolution of the 60-year-old GP are the results of O'Meara [12] telling us, in particular, that if an "excessive" dimension occurs it will occur when F is the finite field of integers modulo some prime p. Even for rather large p, exact calculations modulo p are delightfully efficient, whereas they may become prohibitively slow and unreliable when using other fields.

Even for rather complex Weyr structures, we can obtain a rich harvest of commuting test triples. For each such triple, we then determine if $\dim F[W, K, L] > n$. We have followed this experimental program rather vigorously for smallish matrices (up to 50×50), but until now no excessive dimensions have been observed – no matter what the field F. Perhaps with your help...

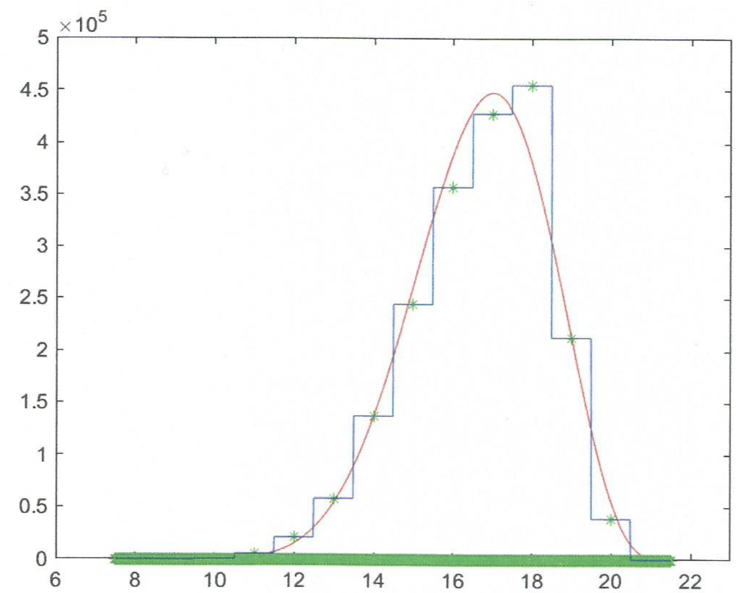

Figure 1: Beta fit for $n = 20$ and Weyr structure $[7, 5, 5, 2, 1]$. X-axis: dimension; Y-axis: number of cases

Figure 1 illustrates the sort of information provided by our routines. The histogram shows the distribution of algebra dimensions from about two million trials with the indicated Weyr structure. The red curve traces a beta distribution fitted to the experimental data. This may be used to estimate the number of trials needed for a "Eureka" (dimension greater than n) if such exists for our particular setting of parameters.

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LINEAR ALGEBRA EDUCATION

Reflections on Linear Algebra Education

Sepideh Stewart[1]

> *"Chapter 4* [on vector spaces] *was like a very juicy peach on a high branch just outside of my reach. I understood the concepts as they were explained, but the entirety of the connections to each other and previous concepts remained elusive. The concept map activity made me further realize how jumbled my brain is with regards to what we've learned."*
>
> *Linear Algebra Student-Math Major [8, p. 223]*

Linear algebra is a core subject for mathematics students and is required or recommended for many STEM majors. However, research shows that the transition to an abstract and formal presentation of concepts through definitions appears to create difficulties for many students who are still coping with the procedural aspects of the subject. For example, in a study by Stewart and Thomas [12], one mathematics student stated that he did not know how to explain the definitions of the given concepts and could only calculate (see Figure 1).

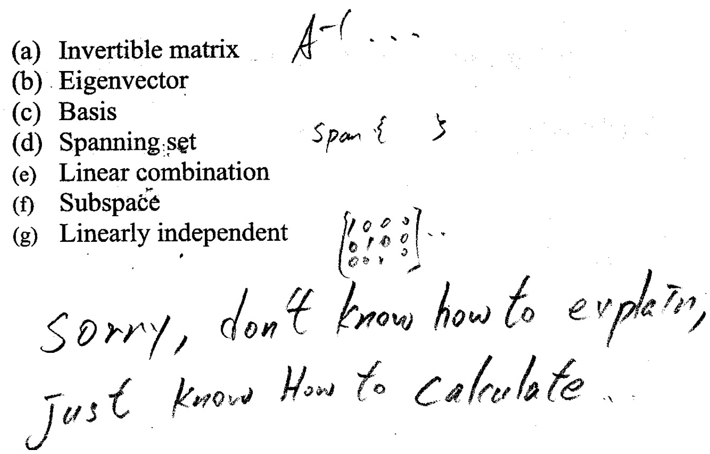

Figure 1: A Linear Algebra student's response [12].

Research on students' conceptual difficulties with linear algebra emerged in the late 80s and focused on student difficulties related specifically to first-year linear algebra courses, see, e.g., [3, 4, 5, 6]. Linear algebra education research over the past decade has also concentrated on the nature of student difficulties and thought processes, e.g., [7, 13, 14, 15].

We now have a large amount of evidence that these problems do exist, but, despite this, "still the fog rolls in, and students feel as though they have been taken to a new world" [1, p. 963].

Although there has been an impressive amount of research published on the teaching and learning of linear algebra, there is an enormous need to collaborate and unify our voices to serve our community and especially our students better. In Summer 2016, a *Discussion Group on Teaching Linear Algebra* was held at the 13$^{\text{th}}$ International Congress on Mathematical Education (ICME-13) in Hamburg. The key questions and issues discussed were [9, p. ix]:

(a) How can applications of linear algebra be used as motivation for studying the topic?

(b) What are the advantages of proving results in linear algebra in different ways?

(c) In what ways can a linear algebra course be adapted to meet the needs of students from other disciplines, such as engineering, physics, and computer science?

(d) How can challenging problems be used in teaching linear algebra?

[1]Department of Mathematics, University of Oklahoma, Norman, OK, 73019, USA, sepidehstewart@ou.edu.

(e) In what way should technology be used in teaching linear algebra?

(f) What is the role of visualization in learning linear algebra?

(g) In what order (pictures, symbols, definitions and theorems) should we teach linear algebra concepts?

(h) How can we educate students to appreciate the importance of a deep understanding of linear algebra concepts?

This effort resulted in an edited book [9] whose 18 chapters represent work from nine countries (Austria, Germany, Israel, Ireland, Mexico, Slovenia, Turkey, the USA, and Zimbabwe) and share a thread of commonality in their focus on the use of challenging problems or tasks that are supportive of student learning. This year, a discussion group on teaching linear algebra will be held virtually at the ICME-14 (`https://www.icme14.org/static/en/index.html`) meeting to be held July 11 – 18.

LACSG 1.0 and LACSG 2.0

In response to growing concerns regarding the linear algebra curriculum, in the 1990s a group of mathematicians formed the Linear Algebra Curriculum Study Group (LACSG). One outcome was a set of recommendations, published in 1993, for the first-year linear algebra course [2].

This original Linear Algebra Curriculum Study Group (LACSG 1.0) provided important contributions in changing the curriculum by putting linear algebra on the agenda. The recommendations were an essential first step in modifying the curriculum and significantly impacted linear algebra education, both within the United States and internationally. However, after a quarter century, it was clearly important that these recommendations be revisited and reevaluated, and, moreover, validated through research.

In 2018, funded by the NSF (the National Science Foundation of the United States), a workshop on linear algebra education, *Pedagogical Initiatives in Linear Algebra*, was organized to reexamine those recommendations.[2] The consensus from this workshop was that it is time to reevaluate the important work begun by LACSG 1.0 and create a new set of recommendations that address current and future needs. Hence, a second Linear Algebra Curriculum Study Group (LACSG 2.0) was formed. This newly-formed committee met in subsequent workshops and produced a new set of recommendations [11]. Some outcomes from this collaboration were presented in a panel discussion as part of the MAA's Virtual Programming offerings, as well as at the SIAM Conference on Applied Linear Algebra, both in May 2021.

The field of linear algebra education needs to continuously reflect on what has been achieved so far and where the field is heading. We anticipate that research will positively influence how we teach linear algebra. In a 2019 survey paper [10] by Stewart, Andrews-Larson, and Zandieh, the authors noted advances in linear algebra education in some areas (e.g., span, linear independence, eigenvectors and eigenvalues). However, the authors also identified areas needing more research (e.g., linear transformations) and several gaps in the literature, such as examining how students make sense of linear algebra proofs and how to teach proofs specific to linear algebra. In addition, research into the teaching and learning of topics in second courses of linear algebra, which contain more abstract content, is desperately needed.

We are learning that the industry requires individuals skilled in more advanced linear algebra. For example, the job market is in need of data scientists, which, as we know, require a deep appreciation of linear algebra [11].

As we advance, some critical questions to ponder are:

(a) What are the current research areas in linear algebra education, and what have their impacts been so far?

(b) What would be helpful in future research?

(c) What are the content and curriculum issues of the 21$^{\text{st}}$ century?

Reading through past *IMAGE* articles, I noted the tremendous effort by ILAS education committees over the past years. It is with great privilege and honor that the new education committee follows their legacy and continues this work.

I would especially like to thank David Strong, who served as the chair of the education committee for many years. I also would like to express my appreciation to Avi Berman, Frank Uhlig, and Steve Leon for their continuous support over the years.

[2] *Editor's note:* See the report on this meeting, also written by Sepideh Stewart, in the Education section of *IMAGE* 61.

Linear algebra is a vital and beautiful subject, and we need to find ways to communicate linear algebra ideas to our students and make sure they are equipped with the knowledge and skills to be successful. We would like to invite the whole ILAS community to be part of the education committee's endeavors and hope to engage and include everyone in our upcoming events and activities.

Sepideh Stewart
on behalf of the ILAS Education Committee:
Anthony Cronin, Judi McDonald, Rachel Quinlan, Sepideh Stewart (chair) and David Strong

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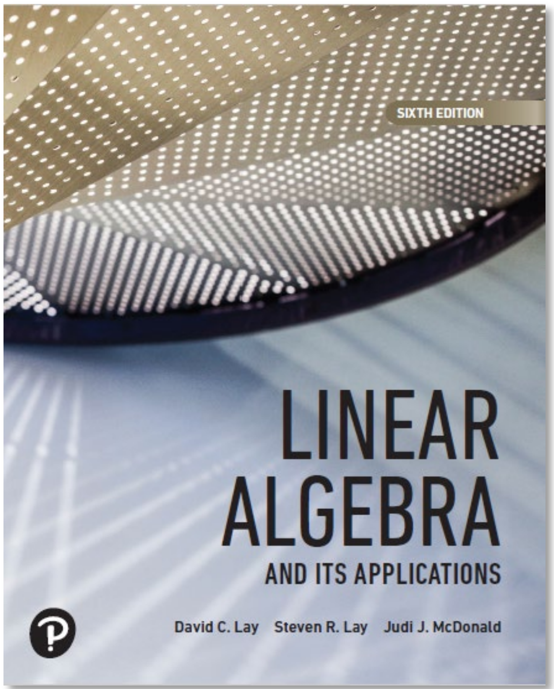
Lay's *Linear Algebra*, Sixth Edition



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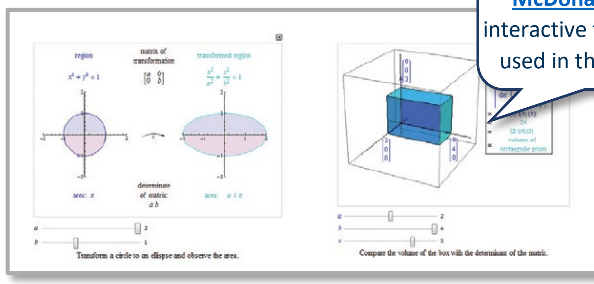

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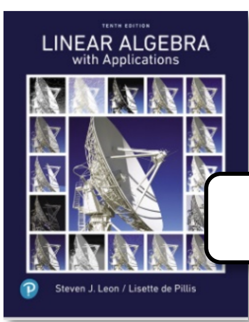

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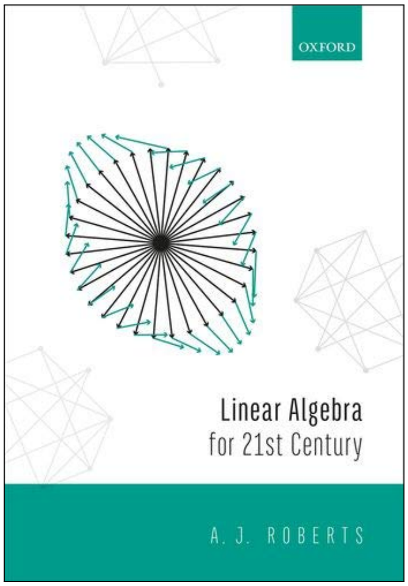
BOOK REVIEW

Linear Algebra for the 21st Century
by A. J. Roberts

Oxford University Press, 2020, ISBN 978-0-19-885640-5, xx+660 pages.
Reviewed by Colin Garnett, Black Hills State University, colin.garnett@bhsu.edu



The stated purpose of this book is to give a method for teaching the topics in Linear Algebra using the singular value decomposition as the driving force. The author acknowledges the difficulty of using the singular value decomposition when all of the calcuations must be done by hand. To alleviate this problem, computational code is given throughout the book that can be implemented using either Matlab or Octave. The code given is clean and easy for the student to reproduce. Ideally, this will give the instructor more time to talk about the theory surrounding each of the topics, but, depending on the preparedness of the students, it may be necessary to spend a lot of time introducing the software, and so this time will have to be built into the course.

At my own institution, many of our students enter Linear Algebra from Calculus III, so I noted that this book does a very nice job of starting with a discussion of vector operations, including the dot product and cross product. This differs from many of the books out there, but it seems that many of our students would benefit from more discussion of these important and familiar topics early in the course. Moreover, the author is able to use these topics to present interesting applications at an earlier stage.

While this book is most suitable for an Applied Linear Algebra course, it could also serve as a supplemental text for any first Linear Algebra course. There are great examples throughout; one relating to the Global Positioning System in particular is developed along with the topics in several sections. This serves as a great motivator in these sections for the students and the instructor. The author motivates many of the topics with a discussion of geometry. This leads to a bit of switching between the row and column forms of vectors which requires the students to be careful as they read through the text and may lead to some confusion. The author introduces singular value decomposition (SVD) much earlier than other books, which allows him to work through more real-world applications. In particular, he is able to spend a section using projections to approximate solutions to an inconsistent system of equations through the use of the SVD. This and many other applications are directly applicable to data science, engineering, and the physical and biological sciences.

Overall, this book has many nice examples, and develops difficult topics well. Use of this book will require careful consideration of the topics involved and class time set aside for as much discussion as necessary of either Matlab or Octave. The many applications to science and engineering included in the book give students a strong sense of what can be done by applying linear algebra to these important areas of study. As with all linear algebra textbooks, there is not enough time in a single-semester course to cover all of the material this book contains, making it important to plan and decide what topics students will need to move forward in their programs of study.

I feel that this book would be a great addition as a supplement to my Linear Algebra course, and a great main textbook if I had an Applied Linear Algebra course.

JOURNAL ANNOUNCEMENTS

Electronic Journal of Linear Algebra (*ELA*) moves to professional copyediting

Contributed announcement from Michael Tsatsomeros

The *Electronic Journal of Linear Algebra (ELA)* has transitioned to a professional copyediting service starting with Volume 37 (2021). *ELA* and ILAS wish to express their deepest gratitude to Panos Psarrakos and Sarah Carnochan Naqvi for their tremendous contributions in performing this editorial service for more than a decade. Their tireless work was marked by high-quality professionalism and it exemplified the volunteerism that benefits our society.

Special issue of *Computational and Mathematical Methods* on Linear Algebra, Matrix Analysis and Applications

Contributed announcement from Fernando De Terán

The journal *Computational and Mathematical Methods (CMM)* will publish a special issue on on Linear Algebra, Matrix Analysis and Applications.

From the journal web site (`https://onlinelibrary.wiley.com/journal/25777408`): "*Computational and Mathematical Methods* is an interdisciplinary journal dedicated to publishing the world's top research in the expanding area of computational mathematics, science and engineering. The journal connects methods in business, economics, engineering, mathematics and computer science in both academia and industry."

The description of this special issue is as follows.

> "Mathematical modeling of problems arising in engineering, physics, mechanics, etc., leads in many cases, directly or after a discretization process, to solving systems of linear equations of finite dimension. Besides, other problems of numerical analysis, such as approximation, interpolation, nonlinear systems, computation of eigenvalues, etc., lead to the resolution of large systems. Thus, it is essential to develop efficient numerical linear algebra methods, in both sequential and parallel environments, to solve them. The main purpose of this special issue is to collect the most recent methods for solving such problems and applications from a multidisciplinary approach."

The guest editors responsible for this special issue are:

- Luca Bergamaschi, Department of Civil Environmental and Architectural Engineering, University of Padua, Italy (`berga@dmsa.unipd.it`)
- Fernando de Terán, Departamento de Matemáticas, Universidad Carlos III, Spain (`fteran@math.uc3m.es`)
- Pedro Alonso, Departamento de Matemáticas, Universidad de Oviedo, Spain (`palonso@uniovi.es`)

All papers must be submitted via the online system `https://mc.manuscriptcentral.com/compandmathmethods`. Authors need to make sure that they specify that the paper is a contribution for "Special Issue on Linear Algebra, Matrix Analysis and Applications" and select the article type, when prompted. All papers will be peer reviewed according to the high standards of *CMM*.

The deadline for submissions is June 30, 2021.

ILAS NEWS

ILAS Election Results

Minerva Catral has been elected to a three-year term as ILAS Secretary/Treasurer, beginning March 1, 2021.

Melina Freitag and Apoorva Khare were elected to three-year terms as members of the ILAS Board of Directors. They will start their terms on March 1, 2021.

ILAS Web Site Completely Revamped

Contributed announcement from Daniel Szyld, ILAS President

ILAS has a newly revamped website. Check it out at the usual address: `https://ilasic.org`.

The new web site continuously lists the latest five *ELA* papers (titles only), as well as the latest five postings from ILAS-NET.

In addition, concurrent to the launch of the new web site, ILAS now accepts PayPal (and credit cards through PayPal) for membership dues and donations.

The creation of this new web site has been a big effort over several months, and thanks are due to those involved, many putting in an inordinate amount of time:

- Dominique Guillot, a member volunteer, did the technical work of the whole redesign.
- Sarah Carnochan Naqvi, our ILAS-NET and ILAS Website Manager, worked with Dominique throughout the process.
- Minnie Catral, our Secretary/Treasurer, worked out the details with PayPal.
- Hugo Woerdeman, our Vice President, oversaw the whole enterprise and provided design ideas.

New Membership of the ILAS Education Committee

Contributed announcement from Daniel Szyld, ILAS President

The ILAS Education Committee has been newly constituted; its membership for the next two years will be as follows:

- Anthony Cronin (University College Dublin)
- Judith McDonald (Washington State University)
- Sepideh Stewart (University of Oklahoma, Chair)
- David Strong (Pepperdine University)
- Rachel Quinlan (National University of Ireland Galway)

Creation of the ILAS Richard A. Brualdi Early Career Prize

Contributed announcement from Daniel Szyld, ILAS President

The ILAS Board has approved the creation of the ILAS Richard A. Brualdi Early Career Prize.

This prize will be awarded every three years at an ILAS Conference to an outstanding early career researcher in linear algebra for distinguished contributions to the field within seven years of receiving the Ph.D. or equivalent degree. For complete guidelines, see `https://ilasic.org/brualdi-guidelines`.

The prize is named for Richard A. Brualdi, who has been active in ILAS since its inception and who has been and continues to be instrumental to its success. Indeed, in 1987 he was a committee member when the International Matrix Group/International Linear Algebra Community was established, which soon thereafter became ILAS. He has since served ILAS in several capacities including as Vice President (1992–1995) and President (1996–2002). The first ILAS Richard A. Brualdi Early Career Prize will be awarded at the 24^{th} ILAS Conference in Madrid in 2023, where the awardee will also present a plenary lecture. An official call for nominations will be issued in 2022; nominees must have received a doctoral degree no earlier than October 1, 2015.

To contribute to the ILAS Richard A. Brualdi Early Career Prize fund, please visit `https://ilasic.org/awards`.

Nominations Sought for the 2022 Hans Schneider Prize

Contributed announcement from Paul Van Dooren

The 2022 Hans Schneider Prize in Linear Algebra Committee has been appointed by the ILAS President upon the advice of the ILAS Executive Board. It consists of:

- Rajendra Bhatia
- Anne Greenbaum
- Leslie Hogben
- Martha Takane
- Paul Van Dooren (chair)
- and Daniel Szyld (ILAS president - ex-officio member)

It is the responsibility of the committee to solicit nominations and to make a recommendation to the ILAS Executive Board for this prize, to be awarded at the ILAS 2022 meeting in Galway.

Nominations of distinguished individuals judged worthy of consideration for the Prize are now being invited from members of ILAS and the linear algebra community in general. In nominating an individual, the nominator should include:

(1) a brief biographical sketch, and

(2) a statement explaining why the nominee is considered worthy of the prize, including references to publications or other contributions of the nominee which are considered most significant in making this assessment.

The prize guidelines can be found at `https://ilasic.org/hs-guidelines` and the list of all Hans Schneider Prize winners at `https://ilasic.org/hs-prize-winners`. Nominations are open until September 30, 2021 and should be sent by e-mail to the chair, Paul Van Dooren (`paul.vandooren@uclouvain.be`). The committee may ask the nominator to supply additional information.

Shmuel Friedland Selected as 2021 SIAM Fellow

Longtime ILAS member Shmuel Friedland, of the University of Illinois at Chicago, has been selected as a 2021 SIAM Fellow. SIAM Fellows are chosen by SIAM based on their significant contributions to the field. In this case, Shmuel was recognized "for deep and varied contributions to mathematics, especially linear algebra, matrix theory, and matrix computations."

The complete 2021 class of fellows can be viewed at:

`https://www.siam.org/prizes-recognition/fellows-program/all-siam-fellows/class-of-2021`

Shmuel Friedland

Pauline van den Driessche Selected to be First ILAS Lecturer at the Joint Mathematics Meetings (JMM)

Pauline van den Driessche

Pauline van den Driessche has been selected by the ILAS Board to be the first ILAS Lecturer at the JMM. She will deliver her lecture at the Joint Mathematics Meetings (JMM) to be held in Seattle, WA, January 5–8, 2022.

ILAS President/Vice President Annual Report: May 1, 2021

Respectfully submitted by Daniel B. Szyld, ILAS President, szyld@temple.edu and Hugo J. Woerdeman, ILAS Vice President, hugo@math.drexel.edu

1. Board-approved actions since the last report include:

   - The Board decided to allow for endorsements of seminar series during the ongoing pandemic only.
   - The Board approved the proposal that ILAS be a partner at the Joint Mathematics Meetings (JMM). This means that ILAS will provide a principal speaker at the JMM as well as organize several special sessions. This will increase the visibility of ILAS within the mathematical community. For JMM 2022, the Board chose Pauline van den Driessche to be the first ILAS Lecturer at the JMM. For subsequent years, a committee of four members plus the ILAS President will make a recommendation to the Executive Board of whom to choose as the ILAS Lecturer at the JMM.
   - The Board approved a proposal for *ELA* to use a private company to do its copyediting. *ELA* will start suggesting to authors to make donations to *ELA* (or use their institutions' or grants' budgets for page charges) and also to become ILAS members. These suggestions are sent when authors are informed of technical details after having earlier received an acceptance letter.
   - The Board approved the institution of "The ILAS Richard A. Brualdi Early Career Prize," and that it will be awarded every three years at an ILAS meeting (this prize and the LAA Lecture will alternate) to someone within 7 years of their Ph.D. Nomination for the award will require a nomination letter, and two supporting letters (from different research groups).
   - The Board approved the proposal to change the Taussky-Todd Lecture to the ILAS Taussky-Todd Prize. Together with the ILAS Richard A. Brualdi Early Career Prize and the Hans Schneider Prize, ILAS will have a triplet of prizes for early career, mid-career, and full career recognition. The committee in charge of the ILAS Lecturer at the JMM will also be tasked to recommend the ITT Prize winner to the Executive Board.
   - The Board approved the proposal for term limits for *ELA* editors that was put forth by the ILAS Journals Committee. The details include (i) Editors-in-Chief are limited to two consecutive three-year terms; (ii) Associate Editors are limited to two consecutive three-year terms; and (iii) Advisory Editors are limited to three consecutive three-year terms. In the latter two cases, the Editor(s)-in-Chief can ask the Journals Committee for exceptions, with proper justification.
   - The Board approved the amended bylaws that an ad hoc committee (with members Leslie Hogben, Peter Šemrl, and Hugo Woerdeman) prepared. The amendments included the newly approved term limits for *ELA* editors, as well as other updates and clarifications.

   Other news:

   - The ILAS website was redesigned. Member volunteer Dominique Guillot (University of Delaware) did a fantastic job of designing the new web pages with only a very broad outline of ideas provided by us. Dominique is still active in attending to technical details even after our rollout on March 31, 2021, and he has agreed to stay on as Assistant Manager for the website and ILAS-NET. We very much want to express our gratitude for all his hard and excellent work from which our society will surely benefit for many years to come.
   - The Membership and Outreach Committee established a Facebook page, and a Twitter account. In addition, an e-mail campaign was initiated last summer to encourage linear algebraists to become ILAS members.
   - The payment options now include PayPal and credit card payments via PayPal, with the advantage of reduced fees.

2. ILAS elections ran December 17, 2020 – January 31, 2021, and proceeded via electronic voting. The following were elected to offices with three-year terms that began on March 1, 2021:

   - Secretary/Treasurer: Minerva Catral
   - Board of Directors: Melina Freitag and Apoorva Khare.

   The following continue in the ILAS offices which they currently hold:

   - President: Daniel Szyld (term ends February 28, 2023)
   - Vice President: Hugo Woerdeman (term ends February 28, 2022)
   - Second Vice President (for ILAS conferences): Raf Vandebril (term ends June 30, 2023)

- Board of Directors: Sebastian Cioabă (term ends February 28, 2023), Dragana Cvetković Ilić (term ends February 28, 2023), Valeria Simoncini (term ends February 28, 2022), and Michael Tsatsomeros (term ends February 28, 2022).

On February 28, 2021, Leslie Hogben completed four consecutive terms as ILAS Secretary/Treasurer, for a total of twelve years on the ILAS Executive Board. Leslie Hogben will now serve a one-year term on the Board of Directors until February 28, 2021. We extend sincere thanks to Leslie for her dedicated service to the Society.

Maria Isabel Bueno and Vilmar Trevisan completed their terms on the ILAS Board of Directors on February 28, 2021. We thank them for their valuable contributions as Board members; their service to ILAS is most appreciated.

We also thank the members of the Nominating Committee – Michael Overton (chair), Raphi Loewy, Jennifer Pestana, Rachel Quinlan, and Alison Ramage – for their efforts on behalf of ILAS, and all of the nominees for their participation in the elections.

3. New appointments and reappointments:

Outreach Director:

David Watkins

Education Committee:

Anthony Cronin
Judith McDonald
Rachel Quinlan
Sepideh Stewart (chair)
David Strong

Hans Schneider Prize 2022 Committee:

Rajendra Bhatia
Anne Greenbaum
Leslie Hogben
Daniel Szyld
Martha Takane
Paul Van Dooren (chair)

***LAA* Lecture 2022 Committee:**

Sebastian Cioabă
Peter Šemrl (chair)
Françoise Tisseur

JMM 2023–24 and ITT Prize 2023 Committee:

Nair Abreu
Ilse Ipsen (chair)
Joseph Landsberg
Daniel Szyld
Pauline van den Driessche

ILAS-NET and ILAS Website Assistant Manager:

Dominique Guillot
Pietro Paparella

Scientific Organizing Committee for ILAS 25 in Madrid:

Raymond Chan
Fernando De Terán (chair)
Gianna M. Del Corso
Shaun Fallat
Heike Faßbender
Elias Jarlebring
Linda Patton
Jennifer Pestana
João Queiró
Naomi Shaked-Monderer
Zdeněk Strakoš
Daniel Szyld
Raf Vandebril

Israel Gohberg ILAS-IWOTA Lecture 2022 Committee:

Joseph Ball (IWOTA)
André Ran (ILAS; chair)
Ilya Spitskovsky (ILAS)
Nikolai Vasilevski (IWOTA)

4. Due to the pandemic, no ILAS-endorsed meetings have taken place since our last report.

5. ILAS endorsed the following conferences and seminar series of interest to ILAS members.

- Online seminar series on Numerical Linear Algebra, alternating Wednesdays at 4:00 p.m. CET (Central European Time). `https://sites.google.com/view/e-nla/home`
- Online seminar series "Communications in Numerical Linear Algebra", weekly seminar series, Mondays at 3:00 p.m. CET (Central European Time). `https://sites.google.com/view/commnla/home`
- XXI Householder Symposium Selva di Fasano, Italy, June 12–17, 2022. `https://users.ba.cnr.it//iac/irmanm21/HHXXI/index.html`

- International Conference on Linear Algebra and its Applications (ICLAA 2021) CARAMS, MAHE, Manipal, India, December 15–17, 2021. Stephen Kirkland will be a Hans Schneider ILAS Lecturer. `https://carams.in/events/iclaa2021`
- Matrix Equations and Tensor Techniques Workshop, Perugia, Italy, September 9–10, 2021. `https://indico.cs.dm.unipi.it/event/7/`
- International Workshop on Operator Theory and its Applications (IWOTA 2021) Lancaster, England, UK, August 16–20, 2021. Vern Paulsen will be an Israel Gohberg ILAS-IWOTA Lecturer. `https://www.lancaster.ac.uk/maths/iwotauk2021`
- Applied Matrix Positivity, to be held virtually. International Centre for Mathematical Science, Edinburgh, Scotland, July 19–23, 2021. `https://www.icms.org.uk/events/workshops/amp`
- Western Canada Linear Algebra Meeting, to be held virtually. Brandon University, Manitoba, Canada, May 29–30, 2021. `https://www.brandonu.ca/wclam`
- SIAM Conference on Applied Linear Algebra (LA21), to be held virtually, originally scheduled in New Orleans, LA, USA, May 17–21, 2021. The ILAS 2021 conference is embedded in this conference. Bryan Shader will be the ILAS-*ELA* 25th Anniversary Lecturer, Paola Boito will be an ILAS Lecturer, Raf Vandebril will be the ILAS Taussky-Todd Prize lecturer, and Lek-Heng Lim will be a Hans Schneider Prize Lecturer. `https://www.siam.org/conferences/cm/conference/la21`

6. The following ILAS conferences are scheduled:
   - The 23$^{\text{rd}}$ ILAS Conference will be embedded in the SIAM Conference on Applied Linear Algebra (SIAM-LA21) scheduled for May 17–21, 2021 to be held virtually. More information on the conference is given above.
   - The 24$^{\text{th}}$ ILAS Conference is scheduled to be held at National University of Ireland, Galway, June 20–24, 2022. The cochairs of the organizing committee are Rachel Quinlan and Helena Šmigoc. Ilse Ipsen was chosen to deliver the LAA Lecture. Other special lectures will be announced in due course.
   - The 25$^{\text{th}}$ ILAS Conference is scheduled to be held at Universidad Nacional de Educación a Distancia (UNED), Madrid, Spain, June 5–9, 2023. The chair of the organizing committee is Fernando De Terán.

7. The Electronic Journal of Linear Algebra (*ELA*) is now in its 37$^{\text{th}}$ volume. *ELA*'s URL is `https://journals.uwyo.edu/index.php/ela`.

   Volume 36 was published in 2020 and contains 64 papers. *ELA* has surpassed 1,000 papers published since its inaugural volume in 1996, i.e., 25 years ago. Paper number 1,000 was published on October 11, 2019. The total number of published papers as of March 3, 2021 was 1,092. *ELA* receives over 200 papers every year. Its acceptance rate is less than 30%.

   Froilán M. Dopico (Universidad Carlos III de Madrid) and Michael Tsatsomeros (Washington State University) are the Editors-in-Chief.

   ILAS members are strongly encouraged to submit their work to *ELA*, the flagship research journal of the Society.

8. *IMAGE* is the semiannual bulletin of ILAS, available online at `http://ilasic.org/IMAGE`. The Editor-in-Chief is Louis Deaett (Quinnipiac University).

9. ILAS-NET is a moderated electronic newsletter for mathematicians worldwide, with a focus on linear algebra; it is managed by Sarah Carnochan Naqvi.

   An archive of ILAS-NET messages is available at `http://www.ilasic.org/ilas-net`. To send a message to ILAS-NET, please send the message (preferably in text format) in an e-mail to `ilasic@uregina.ca` indicating that you would like it to be posted on ILAS-NET. If the message is approved, it will be posted soon afterwards. To subscribe to ILAS-NET, please go to `http://www.ilasic.org/ilas-net`.

10. The website for ILAS is located at `http://www.ilasic.org` and highlights the main activities of ILAS: the Electronic Journal of Linear Algebra (*ELA*), the conferences, *IMAGE*, ILAS-NET, and other activities. In addition, the website provides general information about ILAS (e.g., ILAS officers, bylaws, special lecturers) as well as links to pages of interest to the ILAS community.

Finally, we want to express our great gratitude to all the officers of ILAS who all show wonderful dedication to the society. Without any of them ILAS would not be what it is today.

Respectfully submitted,

Daniel B. Szyld, ILAS President (`szyld@temple.edu`); and
Hugo J. Woerdeman, ILAS Vice President (`hugo@math.drexel.edu`).

ILAS 2020–2021 Treasurer's Report

April 1, 2020 – March 31, 2021

by Leslie Hogben, outgoing ILAS Secretary/Treasurer and Minerva Catral, incoming ILAS Secretary/Treasurer

Net Account Balance on March 31, 2020			
Checking Account - Great Western		$ 130,259.54	
Certificate of Deposit 2019		$ 60,000.00	
Accounts Receivable		$ 8,901.56	
			$ 199,161.10
General Fund		$ 117,861.91	
Israel Gohberg ILAS-IWOTA Lecture		$ 3,770.00	
Conference Fund		$ 9,282.29	
Olga Taussky-Todd/John Todd Fund		$ 11,091.68	
Hans Schneider Lecture Fund		$ 10,685.89	
Frank Uhlig Education Fund		$ 5,367.31	
Hans Schneider Prize Fund		$ 24,918.37	
ELA Fund		$ 592.54	
ILAS/*LAA* Fund		$ 15,591.11	
			199.161.10
INCOME:			
Dues	$ 10,120.00		
Israel Gohberg ILAS-IWOTA Lecture Fund Donations	$ 150.00		
General Fund Donations	$ 3,351.00		
Conference Fund Donations	$ 70.00		
Taussky-Todd Fund Donations	$ 30.00		
Hans Schneider Lecture Fund Donations	$ 50.00		
Uhlig Education Fund Donations	$ 0.00		
Schneider Prize Fund Donations	$ 30.00		
ELA Fund Donations	$ 0.00		
Corporate Dues Income	$ 550.00		
Interest – Great Western	$ 4.69		
Interest on expired Great Western Certificates of Deposit	$ 1,145.23		
Elsevier flow-through	$ 0.00		
Misc Income	$ 404.44		
Total Income		$ 15,905.36	
EXPENSES:			
ILAS Conference Expenses	$ 0.00		
ELA	$ 1,047.96		
IMAGE	$ 0.00		
IWOTA	$ 0.00		
Credit Card Processing & Bank Fees	$ 935.84		
Non-ILAS Conferences	$ 0.00		
Hans Schneider Lecture	$ 0.00		
Hans Schneider Prize	$ 0.00		
Taussky-Todd Lecture	$ 0.00		
Business License	$ 61.25		
Election Costs	$ 331.80		
Web Hosting & Online Membership Forms	$ 1,434.99		
Accounts payable (ELA)	$ 738.00		
Elsevier flow-through	$ 0.00		
Misc Expenses	$ 0.00		
Total Expenses		$ 4,549.84	
Net Account Balance on March 31, 2021			
Checking Account – Great Western		$ 39,710.39	
Certificate of Deposit 1 created 2020		$ 60,962.22	
Certificate of Deposit 1 created 2021		$ 100,183.01	
Accounts Payable		-$ 738.00	
Accounts Receivable		$ 10,399.00	
			$ 210,516.62
General Fund		$ 128,887.43	
Israel Gohberg ILAS-IWOTA Lecture Fund		$ 3,920.00	
Conference Fund		$ 9,352.29	
Olga Taussky-Todd/John Todd Fund		$ 11,121.68	
Hans Schneider Lecture Fund		$ 10,735.89	
Frank Uhlig Education Fund		$ 5,367.31	
Hans Schneider Prize Fund		$ 24,948.37	
ELA Fund		$ 592.54	
ILAS/*LAA* Fund		$ 15,591.11	
			$ 210,516.62

OBITUARY NOTICE

Elizabeth Meckes, 1980–2020

Elizabeth Meckes, Professor of Mathematics at Case Western Reserve University, passed away on December 16, 2020, at the age of 40. She worked in the area of probability, completing her Ph.D. under Persi Diaconis in 2006, and her recent research focused on random matrix theory. (Her monograph, *The Random Matrix Theory of the Classical Compact Groups*, was published in 2019 by Cambridge University Press.)

Naturally, Elizabeth had a great appreciation for linear algebra. She was a plenary speaker at the 2016 ILAS conference in Leuven, Belgium, where she gave a talk entitled "Random orthogonal and unitary matrices." With her husband, Mark Meckes (also a professor at Case), she coauthored an undergraduate linear algebra textbook, again published by Cambridge. (A review of this textbook appeared in the Fall 2019 issue of *IMAGE*.) Elizabeth was the author of the feature article "The Eigenvalues of Random Matrices" in the Fall 2020 issue of *IMAGE*.

A detailed obituary that appeared in the bulletin of the Institute for Mathematical Statistics can be read at:

`https://imstat.org/2021/02/23/obituary-elizabeth-meckes-1980-2020`

V.V. Sergeichuk, 1949–2021

Submitted by Andrii Dmytryshyn

Professor Vladimir Sergeichuk, Leading Researcher at the Institute of Mathematics of the National Academy of Sciences of Ukraine, passed away on February 13, 2021 in Kyiv, Ukraine. He was 71 years old.

Vladimir was an active member of the community: He was a plenary speaker at the 19$^{\text{th}}$ ILAS conference in Seoul and a Senior Editor of *Linear Algebra and its Applications*. Sergeichuk is well-known for his contributions to many areas of linear algebra, in particular for his results about canonical matrices and classification problems. In 2019, a special issue of *Linear Algebra and its Applications* was dedicated to him on the occasion of his 70$^{\text{th}}$ birthday:

`https://www.sciencedirect.com/journal/linear-algebra-and-its-applications/vol/568/suppl/C`

He is and will be greatly missed by his family, students, friends, and colleagues!

CONFERENCE REPORTS

Special session on Graphs and Matrices *at the* Fall 2020 AMS Western Sectional Meeting (held online) October 24–25, 2020

Report by Mark Kempton

A special session on "Graphs and Matrices" was held at the Fall 2020 AMS Western Sectional Meeting on October 24–25, 2020. The meeting itself was originally scheduled to be held at the University of Utah, and was subsequently moved online. The special session was organized by Emily Evans, Mark Kempton, and Ben Webb, all of Brigham Young University.

The purpose of this session was to highlight the important interplay between graphs and matrices, and to showcase some of the very rich areas of research in this field. Spectral properties of matrices reveal critical pieces of information about graphs, such as bounds on connectivity, mixing of random walks, dynamics on graphs, quantum information in graphs, and many others. In addition, graph theoretic ideas have provided important tools in studying matrices, as with zero forcing and the study of inverse eigenvalue problems.

Talks at this session addressed a variety of research areas such as zero forcing, power domination, random walks on graphs, extremal spectral graph theory, matrices associated with graphs and hypergraphs, and other related topics. The full program for the session, including links to abstracts, can be found at `http://www.ams.org/meetings/sectional/2279_program_ss12.html#title`.

Presenters included:

- Sinan Aksoy (Pacific Northwest National Laboratory)
- Jane Breen (Ontario Tech University)
- Steve Butler (Iowa State University)
- Sebastian Cioabă (University of Delaware)
- Shaun Fallat (University of Regina)
- Cory Glover (Brigham Young University)
- Leslie Hogben (Iowa State University)
- Franklin Kenter (United States Naval Academy)
- Kate Lorenzen (Iowa State University)
- Bryan Shader (University of Wyoming)
- Michael Tait (Villanova University)
- Andrew Tawfeek (Amherst College)
- Violeta Vasilevska (Utah Valley University)
- Stephen J. Young (Pacific Northwest National Laboratory)

Special session on **Graphs and Matrices** ***at the*** **2021 Joint Mathematics Meetings (held online) January 7, 2021**

Report by Bryan Shader

A special session on "The Inverse Eigenvalue Problem for Graphs, Zero Forcing, and Related Topics" was held as part of the Joint Mathematics Meetings in January 2021. The session was held virtually, and was organized by Leslie Hogben (Iowa State University and the American Institute of Mathematics) and Bryan Shader (University of Wyoming). Average attendance for the talks was around 40. The keynote speakers were Hein van der Holst (Georgia State University), who gave a talk on "Some signed graphs with $\nu \leq 3$," and Daniela Ferrero (Texas State University), who talked on "Product throttling for power domination." Other talks included:

The damage throttling number of a graph (Carolyn Reinhart, Iowa State University)

Computational methods and heuristics for zero forcing (Boris Brimkov, Slippery Rock University)

Forbidden induced subgraph characterizations for throttling (Juergen Kritschgau, Iowa State University)

Comparing the domination number and the k-power domination number in hypergraphs a preliminary report (Joseph Alameda, Iowa State University)

Rigid linkage forcing and eigenvalue multiplicities (Mary K. Flagg, University of St. Thomas)

The Boustrophedon Conjecture: Using vertex orderings to strengthen the Graph Complement Conjecture for maximum nullity of a graph (H. Tracy Hall, Hall Labs LLC)

The strong spectral property for graphs (Jephian C.-H. Lin, National Sun Yat-sen University)

On graphs admitting certain multiplicity partitions (Shaun M. Fallat, University of Regina)

Extremal matrices with the strong inner product property (Bryan A. Curtis, Sungkyunkwon University)

The web page at `https://jointmathematicsmeetings.org/meetings/national/jmm2021/2247_program_ss37.html` includes titles, coauthors, and abstracts.

In addition to the talks, there was a social event using GatherTown, and a Question and Answer session.

UPCOMING CONFERENCES AND WORKSHOPS

Applied Matrix Positivity (to be held online) July 19–23, 2021

A workshop entitled "Applied Matrix Positivity" will take place at the International Centre for Mathematical Sciences, Edinburgh, UK during the week of July 19–23, 2021. The web site for the meeting is available here:

`https://www.icms.org.uk/events/workshops/amp`

Due to the current restrictions the meeting will take place online, via the Sococo platform (`https://www.sococo.com`). Four topics related to matrix positivity will be explored: positivity transforms, the construction of tight wavelet frames, spatial statistics, and quantum information/algebraic quantum theory. Half days or full days of the workshop will be devoted to each theme, with a few talks well separated by ad hoc discussions.

Before technical presentations, during the first two days some preliminary lectures aimed at comprehensive overviews of introductory material will be delivered by speakers including Christian Berg (Copenhagen), Rajendra Bhatia (Ashoka), Maria Charina (Vienna), Apoorva Khare (Bangalore) and Adam Skalski (Warsaw).

Details regarding the programme, format of the workshop and participation will appear on the ICMS website.

International Workshop on Operator Theory and its Applications (IWOTA) Lancaster, UK, August 16–20, 2021

The International Workshop on Operator Theory and its Applications meeting will be hosted at Lancaster in the UK from August 16 to 20, 2021. The meeting will present several plenary and semiplenary lectures by leading international researchers in operator theory and related areas of analysis. In conjunction with the International Linear Algebra Society, the meeting will feature the inaugural Israel Gohberg Lecture by Vern Paulsen. The lecture is named in honour of the scientific contributions of Israel Gohberg, who was a visionary and driving force of IWOTA. There will be several special sessions, covering many aspects of operator theory and its applications.

In view of the ongoing travel restrictions, the organizers intend to hold IWOTA Lancaster UK 2021 both online and in person. The main talks will be available to workshop participants online via Teams. The special sessions will use Teams and Zoom for talks and interactive discussions. These facilities will be open to registered participants once the formal registration opens.

Early career researchers are welcome to attend IWOTA and are eligible to apply to the organizers for financial support, which has been generously provided by EPSRC and the US National Science Foundation. The IWOTA meeting will be preceded by the Young Functional Analysis Workshop at Lancaster, August 12–14, 2021.

For more details and up-to-date information, see the conference web site at:

`https://www.lancaster.ac.uk/maths/iwotauk2021`

Workshop on Matrix Equations and Tensor Techniques (METTIX) Perugia, Italy, September 9–10, 2021

The ninth edition of the Matrix Equations and Tensor Techniques Workshop (METTIX) will be hosted on Thursday and Friday, September 9–10, 2021 at the Department of Mathematics and Computer Science of the University of Perugia, Italy. As in previous meetings, the focus will be on recent theoretical and computational advances in the solution of tensor problems and linear and nonlinear matrix equations.

The workshop will feature two keynote lectures, delivered by Dario A. Bini (University of Pisa, Italy) and Lieven De Lathauwer (KU Leuven, Belgium). Registration is free of charge and attendees are invited to contribute a talk. Additional information on the conference, including deadlines and abstract submission procedures, is available on the conference web site at `http://mettconference.cloud`.

We are currently planning for the meeting to be held in a hybrid format (virtual and in-person). All participants, including those presenting a contribution, will be given the option to take part in the conference virtually if they so prefer, but we strongly encourage in-person attendance whenever possible. Should travel restrictions make the journey to Perugia impractical, the conference will switch to a fully virtual event. A final decision on the format will be announced by August 8, 2021.

28th International Workshop on Matrices and Statistics (IWMS 2021) Manipal, India, December 13–15, 2021

The 28th International Workshop on Matrices and Statistics (IWMS 2021) will be held December 13–15, 2021 at the Center for Advanced Research in Applied Mathematics and Statistics (CARAMS), Manipal Academy of Higher Education (MAHE), Manipal, Karnataka, India.

The purpose of the Workshop is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. The Workshop will provide a forum through which statisticians may be better informed of the latest developments and newest techniques in linear algebra and matrix theory and may exchange ideas with researchers from a wide variety of countries. In addition to featuring a range of plenary speakers, the meeting will strengthen the interactions between participants through a range of mini-symposia in various areas of specialization.

Themes of the workshop will include: matrix analysis, projectors in linear models & multivariate analysis, growth curve models, linear regression models, linear statistical inference, modelling covariance structures, multivariate and mixed linear models, and statistics in big data analysis.

The Scientific Committee consists of Ravindra B. Bapat, Manjunatha Prasad Karantha, Steve Kirkland, and Simo Puntanen. The Organizing Committee consists of Narayana Sabhahit (Chairman, Registrar, MAHE) and Manjunatha Prasad Karantha (Organizing Secretary, Coordinator, CARAMS, MAHE).

CRR Day on December 15th, 2021: IWMS 2021 will be held alongside ICLAA 2021 (December 15–17, 2021) and CRR Day will be held on December 15th, the day common to these events, to celebrate 100 years of C. R. Rao, who is among the greatest statisticians and matrix theorists India has ever produced.

Please visit `https://carams.in/events/international-workshop-on-matrices-and-statistics` for more details and registration.

International Conference on Linear Algebra and its Applications (ICLAA 2021) Manipal, India, December 15–17, 2021

The next meeting of the International Conference on Linear Algebra and its Applications (ICLAA) conference series, previously scheduled for December 17–19, 2020, has been moved to December 2021 due to the COVID-19 pandemic. ICLAA 2021 will be held at the Center for Advanced Research in Applied Mathematics and Statistics (CARAMS), Manipal Academy of Higher Education (MAHE), Manipal, Karnataka, India.

The themes of the conference shall focus on classical matrix theory, nonnegative matrices and special matrices, matrices and graphs, combinatorial matrix theory, matrix and graph methods in statistics and biological science, and matrices in error analysis and its applications.

The Scientific Committee consists of Ravindra B. Bapat, Manjunatha Prasad Karantha, Steve Kirkland, and Simo Puntanen. The Organizing Committee consists of Narayana Sabhahit (Chairman, Registrar, MAHE) and Manjunatha Prasad Karantha (Organizing Secretary, Coordinator, CARAMS, MAHE).

ICLAA 2021 has been endorsed by ILAS, which will support Steve Kirkland to deliver the Hans-Schneider Lecture at the meeting. In addition, the *Electronic Journal of Linear Algebra (ELA)* will publish a special issue dedicated to ICLAA 2021, in honor of Professor C. R. Rao.

CRR Day on December 15, 2021: ICLAA 2021 will be held alongside IWMS 2021 (December 13–15, 2021) and CRR Day will be held on December 15, the day common to these events, to celebrate 100 years of C. R. Rao, who is among the greatest statisticians and matrix theorists India has ever produced.

Please visit `https://carams.in/events/iclaa2021` for more details and registration.

ILAS 2022: Classical Connections
Galway, Ireland, June 20–25, 2022

The 24th meeting of the International Linear Algebra Society, ILAS 2022: Classical Connections, will be hosted by the School of Mathematics at the National University of Ireland, Galway, June 20–25, 2022. The venue will be the beautiful riverside campus of the National University of Ireland, Galway.

Ilse Ipsen (North Carolina State University) will deliver the *LAA* Lecture.

This meeting, initially scheduled for 2020, was cancelled due to the COVID-19 pandemic. The scientific programme will be refreshed from the one planned for 2020, and contributions on all aspects of linear algebra and its applications are welcome. The conference theme is "Classical Connections." This will be reflected in the plenary programme and mini-symposia, and all participants will be encouraged to think about relating their themes to their historical roots.

Ongoing updates and more information about the conference will be posted at `http://ilas2020.ie`.

The scientific organizing committee includes: Nair Abreu, Peter Cameron, Mirjam Dür, Ernesto Estrada, Vyacheslav Futorny, Stephen Kirkland, Yongdo Lim, Rachel Quinlan, Peter Šemrl, Helena Šmigoc, Françoise Tisseur, and Paul Van Dooren.

The local organizing committee includes: Paul Barry, Jane Breen, Anthony Cronin, Richard Ellard, Kevin Jennings, Thomas Laffey, Niall Madden, Oliver Mason, Collette McLoughlin, Rachel Quinlan, Helena Šmigoc, and Kirk Soodhalter.

6th Workshop on Algebraic Designs, Hadamard Matrices & Quanta
Kraków, Poland, June 27–July 1, 2022

The 6th Workshop on Algebraic Designs, Hadamard Matrices & Quanta will be held at Jagiellonian University, as well as at the Institute of Mathematics, in Kraków, Poland.

The list of confirmed invited speakers includes:

- Ingemar Bengtsson (Stockholm, Sweden),
- Robert Craigen (Winnipeg, Canada),
- Dane Flannery (Galway, Ireland),
- Shmuel Friedland (Chicago, USA),
- Dardo Goyeneche (Antofagasta, Chile),
- Markus Grassl (Gdańsk, Poland),
- Hadi Kharaghani (Lethbridge, Canada),
- Ilias Kotsireas (Waterloo, Canada),
- Máté Matolcsi (Budapest, Hungary),
- Koji Momihara (Kumamoto, Japan),
- Akihiro Munemasa (Tōhoku, Japan),
- Ion Nechita (Toulouse, France),
- Padraig Ó Catháin (Worcester, USA),
- Eric Swartz (William & Mary, USA),
- Behruz Tayfeh-Rezaie (Tehran, Iran),
- Mihály Weiner (Budapest, Hungary),
- Qing Xiang (Newark, USA), and
- Danylo Yakymenko (Kyiv, Ukraine).

Early conference registration is due by March 1, 2022. Further information can be found at `https://chaos.if.uj.edu.pl/hadamard2020`.

New SIAM Titles

Mathematics of Data Science
A Computational Approach to Clustering and Classification

Daniela Calvetti and Erkki Somersalo

This self-contained textbook provides a solid mathematical basis for understanding popular data science algorithms for clustering and classification and shows that an in-depth understanding of the mathematics powering these algorithms gives insight into the underlying data. It presents a step-by-step derivation of these algorithms, outlining their implementation from scratch in a computationally sound way. The book proposes different ways of visualizing high-dimensional data to unveil hidden internal structures and includes graphical explanations and computed examples using publicly available data sets.

2020 • x + 189 pages • Softcover • 978-1-611976-36-6
List $64.00 • SIAM Member $44.80 • DI01

Nonnegative Matrix Factorization

Nicolas Gillis

Nonnegative matrix factorization (NMF) in its modern form has become a standard tool in the analysis of high-dimensional data sets. This book provides a comprehensive and up-to-date account of the most important aspects of the NMF problem and is the first book to cover in detail the theoretical aspects of NMF, including geometric interpretation, nonnegative rank, complexity, and uniqueness. It explains why understanding these theoretical insights is key to using this computational tool effectively and meaningfully. *Nonnegative Matrix Factorization* is accessible to a wide audience and is ideal for anyone interested in the workings of NMF. It discusses new results on identifiability and complexity and the separable NMF and contains MATLAB codes for readers to run numerical examples.

2020 • xxvi + 350 pages • Softcover • 978-1-611976-40-3
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Location Estimation from the Ground Up

Sivan Toledo

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Second Edition

Lars Elden

This thoroughly revised second edition provides an updated treatment of numerical linear algebra techniques for solving problems in data mining and pattern recognition. Adopting an application-oriented approach, the author introduces matrix theory and decompositions, describes how modern matrix methods can be applied in real life scenarios, and provides a set of tools that students can modify for a particular application. It adds a new chapter on graphs and matrices, and a new chapter on spectral graph partitioning applied to social networks and text classification.

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Gilbert Strang

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Fast Direct Solvers for Elliptic PDEs

Gunnar Martinsson

This is the first textbook to detail the active field of fast direct solvers, presenting fast solvers from the point of view of integral equation formulations, which enable unparalleled accuracy and speed in many applications. It provides an accessible introduction to boundary integral equation methods, the fast multipole method, fast direct solvers for both dense and sparse linear systems, and randomized algorithms for factorizing matrices. Written with an emphasis on mathematical intuition rather than theoretical details, the text is richly illustrated and provides pseudocode for all key techniques.

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M. Vidyasagar

Compressed sensing is a relatively recent area of research that has applications to signal/image processing and computer algorithms, and it draws from a variety of mathematical techniques such as graph theory, probability theory, linear algebra, and optimization. The author presents significant concepts never before discussed as well as new advances in the theory, providing an in-depth initiation to the field of compressed sensing. The text contains substantial material on graph theory and the design of binary measurement matrices, is the only book to thoroughly study the problem of matrix recovery, and supplies relevant results alongside their proofs in a compact and streamlined presentation that is easy to navigate.

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Extended Edition

Lloyd N. Trefethen

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IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to Problem 60-2 and Problem 61-3. Solutions are invited to Problems 63-1, 63-3 and for all of the problems of both issues 65 and 66.

Problem 60-2: A System of Matrix Equations

Proposed by Gérald Bourgeois, *Université de la Polynésie française, FAA'A, Tahiti, Polynésie française*, bourgeois.gerald@gmail.com

We consider the system of matrix equations

$$\mathcal{S}: \quad A^3+B^3=0_3, \quad AB-B^2A^2=I_3$$

where the unknowns A and B are 3×3 complex matrices.

1. Find an explicit solution (A,B) of $\mathcal{S}$ such that A and B have no common invariant proper subspace.

2. Find an explicit solution (A,B) of $\mathcal{S}$ such that A and B have at least one common invariant proper subspace and A^6 is not a scalar matrix.

Solution 60-2 by the proposer

1. We seek a particular solution in the form of matrices

$$A=\operatorname{diag}(a,b,c) \quad \text{and} \quad B=\begin{pmatrix}0 & p & q\\ p & 0 & s\\ q & s & 0\end{pmatrix}$$

with a, b and c distinct nonzero complex numbers and p, q and s nonzero complex numbers. Since the invariant proper subspaces of A are $\operatorname{span}(e_i)$ for $1\le i\le 3$ and $\operatorname{span}(e_i,e_j)$ for $1\le i<j\le 3$, we may observe that A and B have no common invariant proper subspaces.

The equation $A^3+B^3=0$ is equivalent to $a^3=b^3=c^3=-2qsp$ and $p^2+q^2+s^2=0$. Hence, a, b and c must have the same modulus.

Now $AB-B^2A^2=I$ implies that $a^2=\dfrac{-1}{p^2+q^2}=\dfrac{1}{s^2}$ and, in the same way, that $b^2=\dfrac{1}{q^2}$ and $c^2=\dfrac{1}{p^2}$.

It follows that p, q and s have the same modulus and, consequently, p^2, q^2 and s^2 are proportional to (up to reordering) 1, ω and ω^2, where $\omega=e^{2i\pi/3}$. Finally, p, q and s (or, equivalently, $\frac{1}{p}$, $\frac{1}{q}$ and $\frac{1}{s}$) are proportional to (up to reordering) ±1, $\pm\omega$ and $\pm\omega^2$.

We choose $p=\dfrac{1}{c}$, $q=\dfrac{1}{b}$ and $s=\dfrac{1}{a}$, along with $a=\delta$, $b=\delta\omega$ and $c=\delta\omega^2$, where $\delta\neq 0$ is to be determined.

We then obtain $A^3+B^3=\dfrac{\delta^6+2}{\delta^3}I_3$ and $AB-B^2A^2=I_3$. We choose δ such that $\delta^6=-2$ and we are done. Note that this solution has $A^6=B^6=-2I_3$.

2. Consider a solution where A and B are upper triangular; let (λ_i) and (μ_i) be the ordered diagonals of A and B, respectively. We easily see that for every i, $\lambda_i^3=-\mu_i^3=\pm1$ and $\lambda_i\mu_i\in\{-\omega,-\omega^2\}$. That implies $\lambda_i,\mu_i\in\{\pm1,\pm\omega,\pm\omega^2\}$. Then $A^6=I+N$, where N is a nonzero nilpotent matrix and A is not diagonalizable.

For example, we may choose $A=\begin{pmatrix}\omega & p & q\\ 0 & 1 & r\\ 0 & 0 & \omega\end{pmatrix}$ and $B=\begin{pmatrix}-1 & a & b\\ 0 & -\omega^2 & c\\ 0 & 0 & -1\end{pmatrix}$, where p,q,r,a,b,c are to be determined.

In particular, we can choose $r=0$, $q=1$, $b=0$, $c=1$, and therefore $p=3\omega$ and $a=1+2\omega^2$.

Problem 61-3: Square Roots of Matrix Conjugation
Proposed by Dijana ILIŠEVIĆ, *University of Zagreb, Croatia*, ilisevic@math.hr
and Bojan KUZMA, *University of Primorska, Slovenia*, bojan.kuzma@famnit.upr.si

Entrywise complex conjugation on $M_n(\mathbb{C})$ is a real-linear operator. Show it has a real-linear square root if and only if n is even. Find an explicit formula for at least one of its real-linear square roots.

Solution 61-3 by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

As a real-linear operator, entrywise complex conjugation on $M_n(\mathbb{C})$ has two eigenvalues, 1 and -1. The real matrices are the eigenspace corresponding to the eigenvalue 1 and the matrices with all pure imaginary entries are the eigenspace corresponding to the eigenvalue -1. Both of these eigenspaces have real dimension n^2. Any square root of entrywise complex conjugation would have to have eigenvalues $1, -1, i, -i$. Let m_i and m_{-i} be the multiplicities of i and $-i$, respectively. Then $m_i + m_{-i} = n^2$. If, further, the square root is a real operator, then its complex eigenvalues come in conjugate pairs, and hence $m_i = m_{-i}$, which is impossible if n is odd.

Now let n be even and let R be any real $n \times n$ matrix satisfying $R^2 = -I$; then the operator which maps $A + Bi$ to $A + RBi$ for all $A, B \in M_n(\mathbb{R})$ is a real-linear square root of the entrywise complex conjugation operator on $M_n(\mathbb{C})$. One of the many choices for the matrix R is the block diagonal matrix consisting of $\frac{n}{2}$ diagonal blocks all of the form $\left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$.

Also solved by the proposers.

IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce four new problems in this issue and invite readers to submit solutions for publication in *IMAGE*.
Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Rajesh Pereira (pereirar@uoguelph.ca).

NEW PROBLEMS:

Problem 66-1: A Special Property of some Special Linear Groups
Proposed by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let $n \geq 2$ and $\mathbb{F}_2$ be the two-element field. Show that $SL_n(\mathbb{F}_2)$, the group of $n \times n$ invertible matrices with entries in $\mathbb{F}_2$, has a generating set all of whose elements lie in the same conjugacy class of $SL_n(\mathbb{F}_2)$.

Problem 66-2: The Cardinality of Hadamard Matrices
Proposed by Richard William FAREBROTHER, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@hotmail.com

Let n be an even number. An $n \times n$ matrix H is called a Hadamard matrix if its entries lie in the set $\{-1, 1\}$ and it satisfies $HH^T = nI_n$.

(a) Show that the number of $n \times n$ Hadamard matrices is divisible by $2^{2n-1}(n-1)!$.

(b) Show that the number of $n \times n$ Hadamard matrices is exactly $2^{2n-1}(n-1)!$ if and only if $n = 2$ or 4.

(c) Show that the number of 4×4 symmetric Hadamard matrices is exactly 32.

Problem 66-3: A Set of Circles that Form an Ellipse
Proposed by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let k be a positive real number and, for any fixed $t \in [0, 1]$, let C_t be the circle $(x - t)^2 + y^2 = kt - kt^2$. Show that $\bigcup_{t \in [0,1]} C_t$ is a solid ellipse.

Problem 66-4: A Random Walk on an Abelian Group
Proposed by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let G under the operation $+$ be a finite abelian group of odd order. Let P be an arbitrary probability distribution on G. Consider the discrete random walk generated by this probability distribution (i.e., for all $g, h \in G$, if you are at g, then you have a probability $P(\{h\})$ of being at $g + h$ at the next time step). Show that the determinant of the transition matrix of this random walk lies in the interval $[0, 1]$.

Solutions to Problems 60-2 and 61-3 are on pages 32–33.