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With this issue, *IMAGE* welcomes Mohsen Aliabadi as our new editor for Book Reviews. Mohsen has already been working hard on coordinating book reviews that we look forward to reading in upcoming issues of *IMAGE*.

For more information about ILAS, its journals, conferences, and how to join, visit `http://www.ilasic.org`.

FEATURE INTERVIEW

"I have learned from all my students"

Avi Berman Interviewed by Naomi Shaked-Monderer[1]

Abraham (Avi) Berman

N.S.-M. - Earlier this year, your 80th birthday was celebrated in a conference titled "Problem Solving," with talks about problems in mathematics and mathematics education, two areas close to your heart. It was accompanied by a booklet containing another favorite of yours: mathematical riddles. Shall we start with one such riddle?

A.B. - Here is a riddle on matrices. Let a *legal operation* on a real matrix be multiplying a row or a column (a *line*) by -1. Show that any real matrix can be transformed by a finite sequence of legal operations into a matrix with all line sums nonnegative.

N.S.-M. - Now let us go back to the beginning. How did you become a mathematician?

A.B. - Several years after I started to teach at the Technion, I met one of my primary school teachers, who told me that when we were asked what we wanted to be, I replied "professor of mathematics." I do not think that I knew, in primary school, what is a professor, but I was quite good at addition and multiplication. In high school, several of my math teachers were professors at the Technion, so I had the opportunity to see what a professor is and what is mathematics. I enjoyed solving math problems and was quite good at it, and I also learned not to feel too bad if I couldn't solve a problem. I also loved history, and still do, but my history teacher helped tilt things towards math and my math teachers influenced my decision to study mathematics at the Technion.

N.S.-M. - So how was the Technion?

A.B. - Great! We had good teachers, including visitors such as Menachem Max Schiffer and Paul Erdős. I studied with very bright people—two students of my class, the class of 1966, won the Nobel Prize. One of the students who took with me functional analysis was Avraham Lempel, who at that time was a Ph.D. student of Yaacov Ziv. In 1990, I went to the former Soviet Union to interview gifted students and convince them to come to the Technion. Before the interviews, I started a short talk by asking: "Who is familiar with the Ziv-Lempel algorithm?" Everybody knew about the algorithm, and I continued with: "Ziv is a professor of electrical engineering at the Technion. Lempel is a professor of computer science at the Technion. Come to the Technion!"

N.S.-M. - And how did you go into matrix theory?

A.B. - Three areas in mathematics that appealed to me were functional analysis, algebra, and graph theory. Matrix theory lies in the intersection of the three. I took a course in matrix theory from Benny Schwarz and was fascinated by the Perron-Frobenius Theorem. I recall that Rafi Loewy and Shmuel Friedland were in the same class. Shmuel wrote his doctorate under the supervision of Benny Schwarz; Rafi and I were master's students of David London (who was a Ph.D. student of Benny).

By the way, in 1974 I participated in the Gatlinburg conference that was organized by Fritz Bauer in Hopfen am See, a small resort at the foothills of the Bavarian Alps. I remember the conference because of the beauty of the mountains and because Oscar Perron came to visit. Despite his being 93 years old, he was in great shape, physically and mentally.

N.S.-M. - You wrote your Ph.D. under the supervision of Adi Ben-Israel, at Northwestern. How was Evanston?

A.B. - I did not like the weather, cold and windy in winter and hot in summer, but it made me concentrate on my studies. I did like the university and the lake and working with Adi, learning mathematical programming, generalized inverses, and linear inequalities.

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N.S.-M. - When you completed your Ph.D., you went to Montreal. How was it?

Vera Sós, Michel Deza, and Avi Berman in Montreal in 1971

A.B. - Also cold in winter and hot in summer, but it had beautiful parks and fantastic restaurants. I gave a course on optimization at McGill (one of the students was Henry Wolkowicz), where I worked with Sanjo Zlobec and George Styan. However, my main position was at the University of Montreal in the Centre de recherches mathématiques (CRM). At the CRM I worked with Gert Sabidussi and with Anton Kotzig, and with visitors like Vera Sós and Michel Deza.

There is an interesting story concerning Anton. He was a well-known Slovak graph theorist who visited Canada in 1969 and could not return to Czechoslovakia due to the political situation. At that time there was mandatory retirement in Quebec at the age of 65. The mandatory retirement was cancelled when Anton was 65, so he wanted to continue to work. The university claimed that the change did not apply to him and brought the case to court. The judge ruled in Anton's favor, but accepted the university's request that he should teach basic courses, which he did not have to do as a research professor. Rumors are that the administration hoped that he would be a bad teacher and as such they would be able to fire him, but he was elected as the best teacher in the university!

In Montreal, I met Bob Plemmons for the first time, and we started to work together. On June 24th, 1971, we invited Bob and Mary Jo for dinner. It was a lovely evening, and after they left I took Dalia, my wife, to the hospital, where she gave birth (on June 25th) to Tali and Tami. June 25th is my lucky day. On June 25th, 1968, I defended my master's thesis. On June 25th, 1970, I defended my doctoral thesis, and on June 25th, 1971, I got the most important title: father of two! On September 8th, 1977, Bob had dinner with us, this time in Haifa. In September 9th, 1977, Dalia gave birth to Anat. After this happy occasion we decided to switch from dinners to lunches.

N.S.-M. - Good decision. Hearing the story about Kotzig, what do you do as an Emeritus?

A.B. - I do not have administrative duties, otherwise it is almost the same as before retirement. I travel less, but I read books, listen to music, and learn mathematics as before. I am particularly trying to understand how linear algebra is used in artificial intelligence and in quantum communication. I do not teach basic courses, because I hate grading tests, but last year I taught a graduate course on matrix theory. The course was given using the "problem solving before instruction" pedagogy. Each week, I presented and explained, but did not solve, problems that were due by the end of the week. In the beginning of the following week we discussed the solutions, and I gave new problems. I feel that this way of teaching is better than the traditional one. I also taught gifted students, and prepared a special course for teachers on problem solving and problem posing. I still work with doctoral and postdoc students. I am also a senior researcher in the Samuel Neeman Institute, a research institute focusing on policies in science, technology, education, environment, economy, society, and infrastructure. Most importantly, I enjoy my extended family.

N.S.-M. - What were your administrative duties before you retired? Did you enjoy them?

A.B. - I was head of the Interdepartmental Committee on Applied Mathematics, of the Center for Pre-University Studies, and of the Department of Science and Technology Education. I hope that I made some contribution to education in Israel. I was a member of the steering committee of the Technion senate and of the council of the Technion board of governors, as well as of the all-Technion promotion committee. Being in these committees enabled me to appreciate the important things done at the Technion, so I did like the administrative duties, but now I am happy that I am exempted from them.

N.S.-M. - Let us go back to Montreal. Why did you decide to return to Israel? Was it difficult to find a position?

A.B. - We wanted our daughters to grow up in Israel, and when they started to say words in Vietnamese—guess where did the babysitter come from?—we decided to return.

No, it was not difficult. I got offers from several universities. The Technion offered me the position of lecturer. Ben Gurion University, then a new university, offered me the position of a senior lecturer. There is a saying: "The work of the righteous is done by others." The BGU recruiting committee consisted of Technion professors and, without my doing anything, the Technion offer was upgraded, and in 1972 I joined the Technion as a senior lecturer.

Another case of "done by others" happened when I was a full professor and met with Technion supporters in Boston. At that time, John Kerry was running for senator and had a political meeting in the house next to where my meeting was held. By mistake he found himself in our meeting, and without getting confused he explained that the Technion is Israel's MIT and spoke about the special relations between Israel and Massachusetts, and between Haifa and Boston. My work was done by others.

N.S.-M. - How did you find the Technion, now as a faculty member?

A.B. - There was a lot of matrix theory activity at the Technion [see *IMAGE* 44, p. 22 and *IMAGE* 45, p. 19]. We had many visitors, including Henryk Minc, Hans Schneider, Tom Laffey, Wayne Barrett, and Charlie Johnson. Gene Golub was a member of the academic committee of the Board of Governors of the Technion, so he came to the Technion at least once every year. Many other matrix theorists participated in one or more of the sixteen Haifa Matrix Theory Conferences. The Department of Mathematics is not big and is very pleasant. Our graduates hold important positions in academia and industry. Among them are first-class mathematicians like Noga Alon, Maria Chudnovsky and Mark Braverman.

N.S.-M. - Over the years, you visited many mathematicians. Can you recall the early visits?

A.B. - One reason to travel is to visit friends. A good reason to have friends is that you can travel. We visited Caltech and admired Olga Taussky's cacti garden. I visited Kent State and Dick Varga taught me how to play ping-pong. Alan Hoffman told me how to decide if a paper is ready to be submitted (be your own judge), and in Prague I worked on M-matrices with Mirek Fiedler. We also enjoyed the return visits of Olga, Richard, Alan and Mirek.

N.S.-M. - Can you recall any special talks that you gave?

A.B. - My talk at the University of the South Pacific in Suva, Fiji, was unique as I and Dalia were the only participants that did not wear swimming suits.

My first talk at a professional meeting was at the Auburn Matrix Theory Conference in June 1970. I had 30 minutes for the talk and I very carefully prepared every minute. After I answered the questions, Wallace Givens, who was the chair of the session, said that the next speaker, Theodore Motzkin, had had to cancel his talk, and offered me 30 more minutes!

N.S.-M. - Can you describe some of the international research projects in which you were involved?

A.B. - I was an international investigator for an Irish Science Foundation grant that Bob Shorten had, and this allowed me to visit the Hamilton Institute and work with Bob and with Tom Laffey. You and I had a German-Israel Foundation grant with Mirjam Dür, so we visited Trier several times. With Mirjam and Immanuel Bomze, we organized a workshop in Oberwolfach on matrices and optimization. We also had an ISF-NSFC grant with Xiao-Dong Zhang that brought me several times to Shanghai.

N.S.-M. - You have written several books. Any favorites?

A.B. - Of course, our books on completely positive and copositive matrices! The book with Bob on nonnegative matrices is highly quoted and it became a classic thanks to SIAM. Another favorite is "A Problem Based Journey from Elementary Number Theory to an Introduction to Matrix Theory." It is based on the lecture notes of a course that I gave to ninth-grade highly gifted students in a program preparing "the scientists of the future" that introduces them to number theory, set theory, group theory, graph theory and matrix theory, all through problem solving.

N.S.-M. - Speaking of favorites, can you mention one of your favorite results?

A.B. - "Every doubly nonnegative matrix realization of a graph G is completely positive if and only if the line graph of G is perfect." [1] I like this result because it connects matrix theory and graph theory and positivity with perfectness.

Avi Berman (center) with Dafna Shasha (left) and Noam Krupnik (right) at the "Problem Solving" conference held in honor of Avi's 80th birthday.

N.S.-M. - You had many doctoral and master's students, and guided the research projects of young students. Do you enjoy mentoring?

A.B. - Yes. Very much. All my students were very good, and I am proud of all of them. I feel like Rabbi Hanina of the Babylonian Talmud: "I have learned from all my teachers and much more from my students."

My first doctoral students were Ron Aharoni, Daniel Hershkowitz, Dan Shemesh and Dafna Shasha. An interesting anecdote: Dafna's grandson, Noam Krupnik, studies in a special program for excellent students. In this program they have to write a research paper, and I mentored his work in spectral graph theory.

N.S.-M. - In addition to your work in mathematics, you also worked on math education. How did that start? What were the topics you worked on in this area?

A.B. - I edited, with Joe Gillis of the Weizmann Institute, a mathematics journal for high school students, *Etgar - Gilyonot le-matematika*. This aroused my interest in math education. My main interests in math education are problem solving, problem posing, creativity, and giftedness. Here too I learned a lot from my students, particularly from Roza Leikin, who is the dean of the faculty of education at the University of Haifa, and from Boris Koichu, who heads the math education group of Weizmann.

Naturally, I was also interested in teaching linear algebra.

N.S.-M. - When did you join ILAS?

A.B. - I have been a member of ILAS since it was founded. In 1987, I participated in the Combinatorial Matrix Analysis Conference, organized by Dale Olesky and Pauline van den Driessche in Victoria. At this conference, the International Matrix Group (IMG) was founded, and an international advisory committee was chosen to support the organizers of IMG. I was honored to represent Israel on the advisory committee. Two years later, in 1989, IMG became ILAS.

N.S.-M. - What is your advice to young people on their first steps in mathematics?

A.B. - Learn as much as you can. Study with friends. Go to seminars, you can always learn something. Enjoy mathematics, but don't forget to enjoy life.

N.S.-M. - Finally, can you give a solution to the riddle we began with?

A.B. - First I want to apologize to my many friends that are not mentioned in the interview. Now, the solution. Define a function f on all real matrices, mapping a matrix to the sum of all its elements. Given any real matrix A, which has a negative line sum, let A' be the matrix obtained from A by multiplying that line by -1. Then $f(A') > f(A)$. Repeat the process as long as the matrices obtained have a negative line sum. As f strictly increases at each step, and the set of matrices obtained from A by legal operations is finite (as each matrix in this set has a_{ij} or $-a_{ij}$ in position i, j), the process must end. So does the interview. Thank you very very much!

References.

[1] N. Kogan and A. Berman. Characterization of completely positive graphs. *Discrete Math.*, 114(1-3):297–304, 1993.

LINEAR ALGEBRA EDUCATION

Mastery-Based Testing in Linear Algebra

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and
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1. Introduction. Instructors from across higher education have increasingly begun to rethink approaches to assessment and grading, moving away from the limitations of traditional points-based grading and high-stakes exams. The past decade has seen an acceleration in the development and adoption of a variety of systems that can replace traditional grading in many course contexts, ranging from "contract grading" and "standards-based grading" to "ungrading." These are collectively known as *alternative grading systems*. Mathematics educators face both a challenge and an opportunity in redesigning key courses from our curriculum like Linear Algebra with an alternative grading system, as there are numerous stakeholders to consider as well as significant potential benefits. We believe that the system known as *mastery-based testing* [1] (or *standards-based testing*) delivers the key benefits of alternative grading for students and instructors while also minimizing the startup costs and the risks associated with redesigning foundational courses like Linear Algebra.

Desiring a better alignment of our grading practices with course learning outcomes, organic learning processes, and the support of student growth, in Fall 2022 we saw an opportunity to implement an alternative grading system in Linear Algebra at the Colorado School of Mines ("Mines") and set to work planning. Mines is a public engineering university of about 5,500 undergraduates that offers an upper-level linear algebra course that is required by several major programs, including Computer Science, Electrical Engineering, Physics, and Mathematics. In a typical 16-week semester, our department offers eight or nine uncoordinated sections of Linear Algebra, each with 30–40 students. We first implemented mastery-based testing (MBT) in three of these sections in Spring 2023 and, based on our initial success, are continuing to expand to more sections over the 2023–24 academic year.

2. Mastery-based testing. In our Linear Algebra implementation of MBT, students are given a list of 25 learning outcomes, corresponding to problem-types that the course prepares students to be able to do. Over the semester, the students are given multiple opportunities to demonstrate mastery over these learning outcomes in a proctored, individual test setting, and they receive targeted feedback for each attempt which does not meet the criteria for mastery. Once a student has mastered a given outcome, they no longer need to attempt the outcome on future testing days.

At the end of the semester, what would be the "Exams" portion of a student's grade is replaced by a "Mastery Testing" grade, calculated as simply the number of outcomes for which the student has demonstrated mastery divided by the total number of course outcomes. Thus, a student's grade for each outcome ends up as effectively pass/no credit, though other course components may be points-based. On MBT assessments during the semester, students can receive one of three possible marks.

- **M: Mastery.** This is earned by completing the problem correctly and communicating work fluently by using proper terminology and notation.
- **MC: Communication Error.** This is earned by completing the problem correctly, but the work contains terminology or notation errors.
- **P: Progressing.** This is earned by submitting a problem that contains errors.

A student that earns a **P** on a given problem will be expected to reattempt that outcome on a future assessment. The **MC** mark allows students to complete a revision and reflection assignment to earn an **M** for that outcome, though they will not receive credit for this outcome until their revision is satisfactory. The number of attempts available for each outcome ranges from two to six, with more attempts available for early-semester material. A detailed testing schedule appears in Figure 1.

3. Why Linear Algebra is a good place to try MBT. Many linear algebra courses deliver lots of new terminology and unfamiliar concepts to students fairly quickly. It can take students time to assimilate this information and learn to properly assemble the language of linear algebra, despite possessing computational or conceptual understanding. MBT, especially with the use of a mark like our **MC** category, forces students to address these communication issues early in the course, leading to greater eventual mathematical fluency.

In addition, Linear Algebra content continually builds on itself. So, by having students revisit older material until they have demonstrated a high level of understanding, we support their learning of newer material. We feel that the degree of interrelation of course content also averts the need to complicate the assessment design by, for instance, creating a hierarchy of outcomes, forcing multiple demonstrations of mastery over certain topics, or "bundling" outcomes to unlock different final grades (common features of other alternative grading systems). Finally, at many institutions (including

			UNIT 1			UNIT 2			UNIT 3	
WEEK	TIME PROVIDED	ASSESSMENT	Outcomes 1-3	Outcomes 4-6	Outcomes 7-9	Outcomes 10-12	Outcomes 13-14	Outcomes 15-18	Outcomes 19-22	Outcomes 23-25
WEEK 4	20 minutes	QUIZ 1	✓							
WEEK 5	30 minutes	QUIZ 2	✓	✓						
WEEK 7	90 minutes	EXAM 1	✓	✓	✓					
WEEK 9	20 minutes	QUIZ 3				✓				
WEEK 11	30 minutes	QUIZ 4				✓	✓			
WEEK 12	90 minutes	EXAM 2	✓	✓	✓	✓	✓	✓		
WEEK 14	25 minutes	QUIZ 5							✓	
WEEK 15	50 minutes	RETEST DAY	✓	✓	✓	✓	✓	✓	✓	✓
FINAL EXAM PERIOD	120 minutes	FINAL EXAM	✓	✓	✓	✓	✓	✓	✓	✓
TOTAL NUMBER OF ATTEMPTS GIVEN			6	5	4	5	4	3	3	2

Figure 1: MBT assessment schedule

Mines), Linear Algebra is a course for advanced-standing students that have already taken other college mathematics courses. We do not believe this is necessary given the relative simplicity of MBT, but it can feel safer to try new pedagogy with more experienced students, as there is less of a risk of students feeling overwhelmed by the need to learn how to navigate several new systems at once. Due to its place in our curriculum, Mines students regard Linear Algebra as a challenging course. The reattempt structure in MBT supports students in their efforts to succeed by offering them time and flexibility as they learn the material.

4. How it went. Our first foray into alternative grading was a positive experience. In fact, returning to a traditional grading system would now be a challenge! The foundation of this success was building trust and buy-in with our students. On day one of the semester, we discuss with students both the system and our reasons for using it. While many students immediately saw the benefits, others were understandably apprehensive given the weight grades carry. We continued to discuss the system and answer questions about it as the semester progressed, and many students that were initially hesitant discovered MBT's benefits. While you can explore data related to the students' experience in the web resource [2], here we want to focus on our own observations as instructors of mathematics:

- *Students were excited!* Throughout the semester, students would stay after class or come to office hours to discuss the system. Many expressed thanks for implementing MBT. Others said it was the first time they had had fun in a math class and asked if the other math classes were moving to this grading system.
- *It was easier to track student progress.* Since we knew which outcomes each student had successfully completed, we knew both which individual students were struggling and which Linear Algebra content was challenging for them.
- *Conversations were better.* Students also knew which outcomes they were having trouble with. This meant that when they came to office hours, conversations were about course content instead of grades or marks. So, instead of hearing "What do I need to do on the next exam to get an A in the course?" we had questions along the lines of "I'm really struggling with how to determine if this transformation is one-to-one. Can you help me?"
- *Writing was better, too!* Because we had an **MC** mark, students were required to correct their work and learn proper usage in order to get their **M**. This meant that students made terminology and notation errors much less often as the semester progressed.
- *Students seemed less stressed.* While some students directly told us they were less stressed, in their survey responses to the statements in Figure 2 about making mistakes on exams and feeling anxious before exams, students had very similar pre- and post-course responses in regards to their experiences in other math courses. But when asked to apply the statement to their Linear Algebra class at the end of the semester, the number of people agreeing or strongly agreeing went down significantly. As one student said:

> The mastery-based grading system in this course gives students a much lower-stress experience within the class. People are able to take their time and work on actually understanding the concepts at their own pace, rather than simply memorizing specific things for a few days so they will pass one exam.

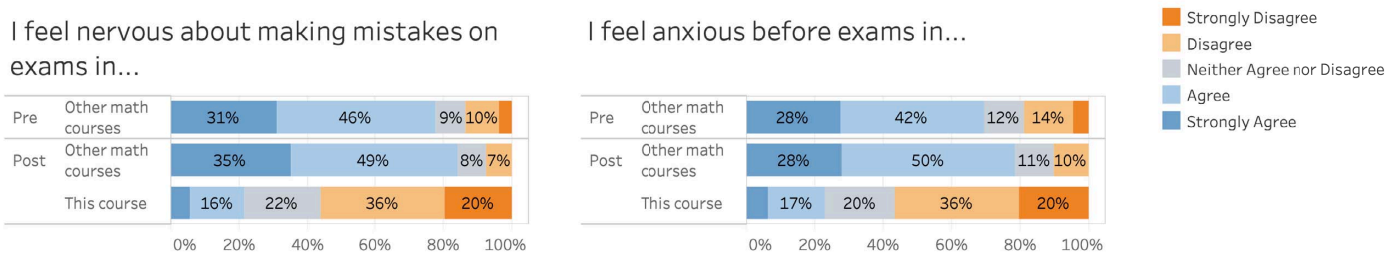

Figure 2: Student survey responses

5. Still unsure? In case you are still feeling skeptical about implementing this in your Linear Algebra class, we created a list of frequently asked questions, based upon our own conversations with colleagues, to address potential concerns.

1. **Isn't there a lot of work to get started?** Any time we try something new in our classes, there is an associated startup cost. In this case, determining the outcomes, planning the schedule, and working out the details with our university's learning management system took careful consideration. However, now that we've done it, the workload in implementing MBT in a second semester is comparable to that of previous semesters. You can see how we worked out these details and browse further references in [2], so you don't have to start from scratch!

2. **Will I be writing and grading more quiz/exam problems?** Although we were writing more problems, Linear Algebra lends itself well to an MBT system. For most of the outcomes, there were many variations we could quickly and easily find to create suitable problems. We found that the grading load was comparable to a traditional semester, but distributed across the semester more evenly. Grading goes much more quickly when you aren't deciding on point values. When the goal is giving feedback that students will actually use to improve their learning, we found that grading is more rewarding.

3. **Won't a new system be confusing or stressful for students?** In our experience, no. On our preliminary survey we had some students express anxiety, but by the end of the semester, they reported feeling much less anxious than they did in their other classes. Among alternative grading systems, this is one of the simplest, as the exam score is defined as just the number of outcomes mastered divided by the total number of outcomes.

4. **Won't students be unhappy that there is no partial credit?** Some were: 22% responded to a question about drawbacks of the system by referencing the lack of partial credit. This is understandable, given that our students were accustomed to receiving partial credit for their work in a math class. However, we find that when students do not have the opportunity or incentive to learn from or fix their mistakes, they don't. In MBT, they benefited by correcting their understanding so that they would be successful on a subsequent attempt. As one student said:

   > [A drawback was] the all or nothingness of it. I live on partial credit and the lack of it in this class was hard to get used to. With that said, because I had to get everything right, I feel like I had a better overall understanding of the content.

5. **If students don't have to redo outcomes they've mastered, won't they forget how to do them?** As indicated above, early ideas in Linear Algebra are reinforced throughout the course, so this was not a concern for us. The following student quote supports this:

   > I expected that a potential drawback of this system would be lack of review of content that was covered early in the semester, but I didn't feel that this was a substantial problem, because of how interconnected everything was.

6. **Won't this cause grade inflation?** We don't believe grades are a scarce commodity! In this system, a top grade clearly connects to a student being able to correctly complete almost all of the course outcomes. Grade inflation refers to an increase in grades due to standards being lowered, which is not what is happening in this system. We believe we can support more students in truly learning, and that is our idea of success.

References.

[1] J. B. Collins, A. Harsy, J. Hart, K. A. Haymaker, A. M. A. Hoofnagle, M. K. Janssen, J. S. Kelly, A. T. Mohr, and J. OShaughnessy. Mastery-based testing in undergraduate mathematics courses. *PRIMUS*, 29(5):441–460, 2019.

[2] Web resource: `https://trefnycenter.mines.edu/resources/masterygrading`

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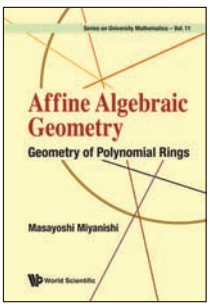

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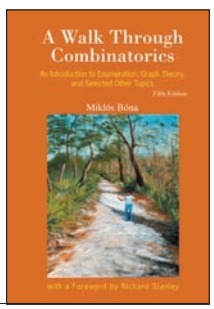

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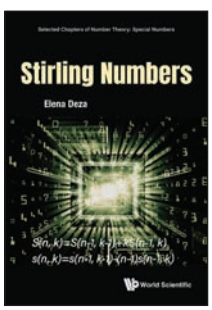

Stirling Numbers

by **Elena Deza** *(Moscow Pedagogical State University, Russia)*

This book collects together much of the scattered material on the two subclasses of Stirling numbers to provide a holistic overview of the topic. From the combinatorial point of view, Stirling numbers of the second kind $S(n, k)$ count the number of ways to partition a set of n different objects (i.e., a given n-set) into k non-empty subsets. Stirling numbers of the first kind $s(n, k)$ give the number of permutations of n elements with k disjoint cycles.

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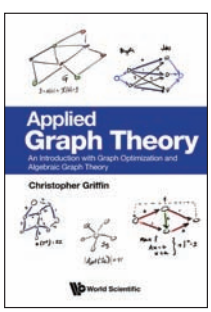

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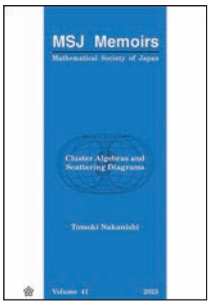

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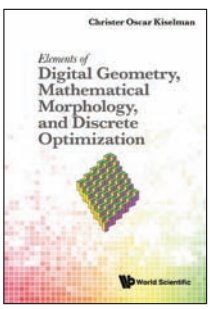

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Mathematical Reviews Clippings
American Mathematical Society

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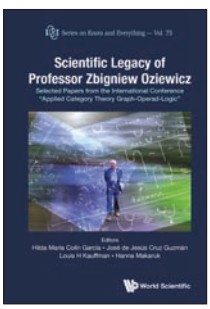

Scientific Legacy of Professor Zbigniew Oziewicz

Selected Papers from the International Conference "Applied Category Theory Graph-Operad-Logic"

edited by **Hilda María Colín García** *(National Autonomous University of Mexico, Mexico)*, **Joséde Jesús Cruz Guzmán** *(National Autonomous University of Mexico, Mexico)*, **Louis H Kauffman** *(University of Illinois at Chicago, USA)* & **Hanna Makaruk** *(Los Alamos National Laboratory, USA)*

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by **Simon Rubinstein-Salzedo** *(Euler Circle, USA)*

This textbook is aimed at transitioning high-school students who have already developed proficiency in mathematical problem solving from numerical-answer problems to proof-based mathematics. It serves to guide students on how to write and understand mathematical proofs techniques.

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JOURNAL ANNOUNCEMENTS

Special Issue of *Special Matrices* Dedicated to Frank J. Hall

Contributed announcement from Vilmar Trevison, *Special Matrices* Editor-in-Chief

The journal *Special Matrices* will publish a special issue for the Workshop on Spectral Graph Theory (WSGT) 2023 that took place October 24–27th in honor of Nair Abreu for her (70+3)rd birthday (`http://spectralgraphtheory.org`).

For the special issue, submissions are invited based on talks given at the Workshop, however other papers not presented at the conference but in connection with the Workshop are welcome as well. All manuscripts will undergo the standard peer review process (single-blind, with at least two independent reviewers) employed by the journal. The deadline for submissions is February 28th, 2024.

Special Matrices is indexed by SCOPUS, Web of Science, MathSciNet, zbMATH, and other major databases, and has earned the DOAJ Seal, awarded to journals that demonstrate best practices in open access publishing. The *Special Matrices* home page is `https://www.degruyter.com/journal/key/spma/html`; details on the special issue are at `https://www.degruyter.com/publication/journal_key/SPMA/downloadAsset/SPMA_CFP%20WSGT.pdf`.

Special Issues of *Advances in Operator Theory* Dedicated to Chi-Kwong Li and to Ilya Spitkovsky

Contributed announcement from M. S. Moslehian, *Advances in Operator Theory* Editor-in-Chief

The journal *Advances in Operator Theory* (AIOT), published by Springer/Birkhäuser, will publish two special issues.

The first special issue is dedicated to Professor Chi-Kwong Li on the occasion of his 65th birthday and for his significant contributions to several mathematics subjects including linear algebra, operator theory, quantum theory, and functional analysis. The guest editors for this special issue are Douglas Farenick (University of Regina, Canada), Nung-sing Raymond Sze (Hong Kong Polytechnic University, Hong Kong), Tin Yau Tam (University of Nevada, Reno, USA), and Man-Duen Choi (University of Toronto, Canada). The deadline for submissions is February 29th, 2024, and additional details may be found at `https://link.springer.com/collections/ejceaigbeg`.

The second special issue is dedicated to Professor Ilya M. Spitkovsky on the occasion of his 70th birthday and for his significant contributions to several mathematics subjects including operator theory, linear algebra, complex analysis, harmonic analysis, integral equations, and functional analysis. The guest editors for this special issue are Natália Bebiano (University of Coimbra, Portugal), Estelle Basor (American Institute of Mathematics, USA), Sergei Grudsky (National Polytechnic Institute, Mexico), and Maria Cristina Câmara (Universidade de Lisboa, Portugal). The deadline for submissions is July 31st, 2024, and additional details may be found at `https://link.springer.com/collections/ghaehaaadc`.

Manuscripts submitted for these special issues will undergo standard reviewing procedures and adhere to the established standards of the journal. All papers should be submitted via `https://www.editorialmanager.com/aiot`.

ILAS NEWS

Javad Mashreghi Selected as 2023 Fields Institute Fellow

Contributed announcement from Daniel Szyld, ILAS President

Congratulations to ILAS member Javad Mashreghi (of Université Laval) for his selection as a 2023 Fields Institute Fellow. Javad was cited for his "prolific organizational efforts for conferences, workshops and publications at the Fields Institute, including an exceptional scientific outcome during the 2021 Focus Program on Analytic Function Spaces and their Applications."

See `http://www.fields.utoronto.ca/news/Congratulations-to-2023-Fields-Institute-Fellows-0` for details.

Shanti Swarup Bhatnagar Prize Awarded to Apoorva Khare

Contributed announcement from Daniel Szyld, ILAS President

We wish to congratulate ILAS member (and member of the ILAS Board of Directors) Professor Apoorva Khare of the Indian Institute of Science, on his receiving the 2022 Shanti Swarup Bhatnagar Prize for Science and Technology.

This prize is awarded by the Council of Scientific & Industrial Research India (CSIR) each year for notable and outstanding research, applied or fundamental, in any of seven scientific disciplines. Apoorva was among two honorees to receive the 2023 prize in the Mathematical Sciences.

Previous recipients of this award include C. R. Rao (1959) and Rajendra Bhatia (1995).

For details, visit `https://ssbprize.gov.in`.

ILAS Members Selected as 2024 American Mathematical Society Fellows

The American Mathematical Society (AMS) has announced its 2024 class of AMS Fellows, recognizing individuals who have made out-standing contributions to the creation, exposition, advancement, communication, and utilization of mathematics. Congratulations to the following ILAS members for having been selected!

- Leslie Hogben (Iowa State University and the American Institute of Mathematics)
- Javad Mashreghi (Laval University)
- Cynthia Vinzant (University of Washington)

For more information and details on the AMS Fellows program, see `http://www.ams.org/profession/ams-fellows/ams-fellows`.

Lek-Heng Lim Selected as 2023 Vannevar Bush Faculty Fellow

Professor Lek-Heng Lim of the University of Chicago, ILAS member (and member of the ILAS Board of Directors), is among ten researchers selected by the Basic Research Office of the United States Department of Defense as its 2023 Vannevar Bush Faculty Fellows. This fellowship was granted to Lek-Heng in support of his proposal to study the "Geometry and Topology of Deep Neural Networks." This project aims to "build a complete picture of deep neural networks by piecing together current insights from algebraic, geometric, and topological studies" in order to "provide a theory of deep neural networks that could explain away the existing mysteries, allow for the design of more effective neural network architectures, and potentially unlock the door to explainable artificial intelligence and artificial general intelligence," which may ultimately "alleviate current fears that AI is an unpredictable and uncontrollable technology."

The fellowship includes funding of up to $3 million over five years.

For details, see `https://www.defense.gov/News/Releases/Release/Article/3463726`.

Board Approves New Funds for Students

Contributed announcement from Daniel Szyld, ILAS President

At its recent meeting, the ILAS Board of Directors approved two new initiatives with the goal of facilitating Ph.D. students with fewer funding opportunities to attend ILAS Conferences. There will be ten grants of US$700 each for travel expenses of Ph.D. students to attend an ILAS Conference. ILAS is setting aside funds to cover the registration of any deserving Ph.D. student who requests a registration fee waiver. This will be in place for the next two ILAS Conferences, and afterwards the Board will reassess the program.

OBITUARY NOTICES

Vladimir Nikiforov, 1954–2023

Submitted by Leonardo Silva de Lima

With deep sadness, we announce that Vladimir Nikiforov has passed away. He died on the first week of September 2023 in Memphis, Tennessee, USA, at the age of 69. Vlado, a professor of mathematics at the University of Memphis, collaborated with and influenced the spectral graph theory community with his nice ideas, fruitful talks, and elegant papers. Family, friends, and colleagues will miss him a lot. We honor his life and his many contributions.

C. R. Rao, 1920–2023

Submitted by Biswa Nath Datta

The legendary statistician Professor C. R. Rao has passed away at the age of 102.

Professor Rao made numerous outstanding and everlasting contributions to statistics and mathematical statistics. Among some of the most well-known of his results include the "Rao-Blackwell Theorem" and the "Theory of Generalized Inverse of a Matrix" (with Professor S. K. Mitra).

He received many highly prestigious awards, the most notable of which include the "2023 International Prize in Statistics", which is equivalent to Nobel Prize in the field, and the "US National Medal of Science Award" in 2002.

His work influenced many other areas of science, economics, genetics, and medicine. He is also well-known to the linear algebra community. His book *Matrix Algebra and its Applications to Statistics and Econometrics* (with Professor M. Bhaskar Rao) was published in 1998.

His death indeed is indeed a great loss to the world scientific community.

Send News for *IMAGE* Issue 72

IMAGE seeks to publish all news of interest to the linear algebra community. Issue 72 of *IMAGE* is due to appear online on June 1, 2024. Send your news for this issue to the appropriate editor by April 15, 2024. Photos are always welcome, as well as suggestions for improving the newsletter. Please send contributions directly to the appropriate editor:

- book reviews to Mohsen Aliabadi (`maliabadisr@ucsd.edu`)
- linear algebra education news and articles to Anthony Cronin (`anthony.cronin@ucd.ie`)
- interviews of senior linear algebraists to Adam Berliner (`berliner@stolaf.edu`)
- problems and solutions to Rajesh Pereira (`pereirar@uoguelph.ca`)
- advertisements to Amy Wehe (`awehe@fitchburgstate.edu`)
- announcements and reports of conferences/workshops/etc. to Jephian C.-H. Lin (`jephianlin@gmail.com`)
- other articles and proposals to the editor-in-chief, Louis Deaett (`louis.deaett@quinnipiac.edu`)

Send all other correspondence to the editor-in-chief, Louis Deaett (`louis.deaett@quinnipiac.edu`).

For past issues of *IMAGE*, please visit `https://www.ilasic.org/IMAGE`.

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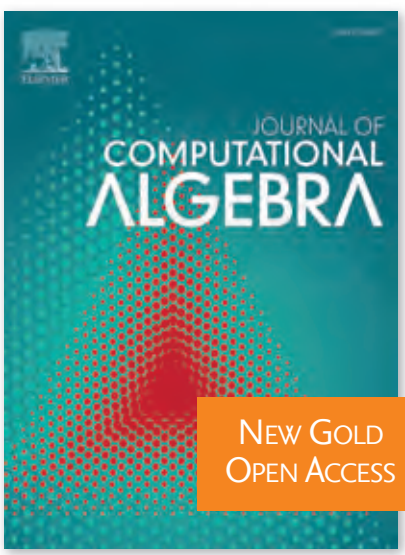

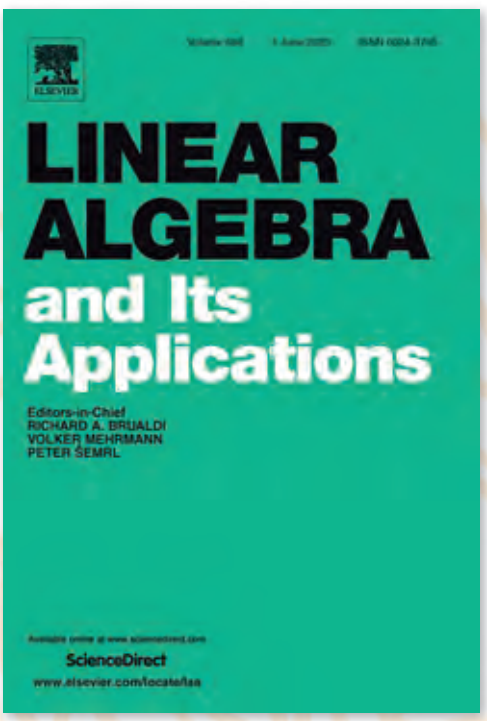

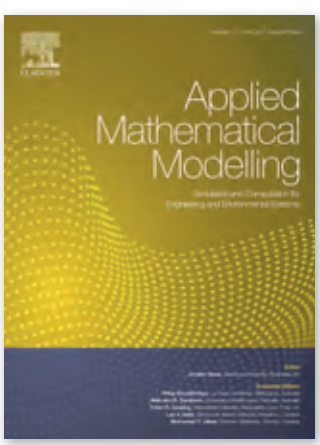

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CONFERENCE REPORTS

The 6th ALAMA Conference: Laplacian and M-matrices on Graphs Barcelona, Spain, May 11–12, 2023

Report by Á. Carmona

ALAMA participants in one of the modernista *buildings of the Hospital de Sant Pau*

The "Laplacian and M-Matrices on Graphs" conference held from May 11th to 12th, 2023, at the Facultat de Matemàtiques i Estadística (FME) of the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain, emerged as a significant gathering in the realm of linear algebra and matrix analysis. Organized by the ALAMA Network and UPC, this conference brought together experts, researchers, and scholars from both national and international sites. The scientific committee, comprising Ángeles Carmona, José J. Martínez, and Raquel Viaña, worked diligently to ensure the conference's academic rigor, while the organizing committee, led by Andrés M. Encinas, Margarida Mitjana, Ana Marco, and M. José Jiménez, oversaw the logistical aspects of the meeting. It is worth noting that this conference received significant support from various Spanish organizations and proudly bore the endorsement of ILAS.

This sixth ALAMA Conference was dedicated to the study of the Laplacian and M-matrices on graphs, covering some of the recent advances in this field, which is very broad. We had both international and national researchers; half of the speakers were active members of the ALAMA network. We dealt with problems related to M-matrices and their applications in random walks, inverse problems in networks, and to electrical impedance tomography.

Adding a delightful social dimension to the academic proceedings, Thursday evening witnessed a congenial banquet held at the restaurant El Cullerot de Sants. This gathering provided attendees with a splendid opportunity to engage in convivial conversations, following an enriching visit to the historic modernist building that housed the Hospital de Sant Pau. The banquet featured a delectable pica-pica with locally sourced products, followed by an exquisite main course.

The main speakers were: María Ángeles García Ferrero, Andrés M. Encinas, Álvaro Samperio, Ernesto Estrada, Minerva Catral, Juan Manuel Peña, Karel Devriendt and María José Jiménez. Those interested in the specific titles and content of their presentations can reach out to the organizing committee for further information, or visit the conference webpage:

`https://mapthe.upc.edu/en/congresses/jornadas`

The "Laplacian and M-Matrices on Graphs" conference in Barcelona was a resounding success, both academically and socially. It served as a platform for the exchange of cutting-edge ideas and fostered invaluable networking opportunities among the participants. This conference underscored the continued significance and relevance of linear algebra and matrix analysis in addressing complex real-world challenges.

16th Western Canadian Linear Algebra Meeting (WCLAM) Regina, Canada, May 27–28, 2023

Report by Shaun Fallat

The 16th iteration of the Western Canadian Linear Algebra Meeting (WCLAM) took place at the University of Regina, May 27–28, 2023. This 30-year-old conference series has been hosted across Western Canada and Washington State, providing an opportunity for researchers in linear algebra and related fields to present recent accounts, through lectures

or poster presentations, of their current work and to engage all participants in informal discussions. Traditionally, this regional conference attracts participants from Western Canada and the western United States. However, like previous editions of WCLAM, this conference included both national and international attendees. Unlike previous versions of this conference, WCLAM 2023 was delivered in a hybrid format, with both face-to-face and virtual presentations. Offering a remote modality option opened the meeting to a broader audience and afforded some presenters an opportunity to participate even though they were unable to travel to Regina. WCLAM has established a reputation for being a conference with a welcoming atmosphere and strong scientific program, and where researchers near the early stages of their careers are strongly encouraged to participate. In particular, of the fifteen presentations, five of them were delivered by graduate students and postdoctoral fellows. One participant commented: "I felt incredibly welcomed, and it warmed my heart to witness such a profound sense of admiration, goodwill, and gratitude among the members of WCLAM." WCLAM 2023 featured two invited speakers, Prof. Jane Breen (Ontario Tech University & ILAS Speaker) and Prof. Leslie Hogben (Iowa State University). For more information about the conference program, visit the conference webpage:

`https://uregina.ca/~sfallat/wclam`.

WCLAM 2023 was generously supported by the Pacific Institute for the Mathematical Sciences (PIMS), the International Linear Algebra Society (ILAS), and the Faculty of Science, Department of Mathematics & Statistics at the University of Regina.

Participants of WCLAM 2023

The 16th Workshop on Numerical Ranges and Numerical Radii (WONRA) Coimbra, Portugal, June 7–9, 2023

Report by Natália Bebiano, Rute Lemos, Ana Nata and Graça Soares Soares

The 16th Workshop on Numerical Ranges and Numerical Radii (WONRA 2023) was held at the Department of Mathematics of the University of Coimbra, Coimbra, Portugal, from June 7th to 9th, 2023. The purpose of this workshop was to stimulate research and foster interactions between researchers interested in the study of the numerical range, numerical radius, and their generalizations and connections to other branches of pure and applied science, including quantum information theory and quantum computing.

The informal workshop atmosphere facilitated the exchange of ideas and participants were informed on the latest developments on the subject. Eighteen talks, with contributions from both theoretical and applied perspectives, were presented, and there were twenty-five registered participants from thirteen countries. Chi-Kwong Li from the College of William and Mary, Williamsburg, Virginia, USA, presented a talk titled "Some recent results on quantum tomography."

The titles and abstracts of all the talks can be seen at:

`https://sites.google.com/view/wonra-2023/book-of-abstracts`

Participants had the opportunity to enjoy fruitful discussions and the friendly atmosphere. The social program included a welcome reception – *Porto d'Honra* at the panoramic restaurant of Machado de Castro National Museum, a workshop lunch, and a visit to the UNESCO World Cultural Heritage site *Paço das Escolas*, including the Royal Palace, Baroque Library, St. Michael's Chapel, and Great Hall of Acts.

The conference received support from the Center of Mathematics of the University of Coimbra (CMUC, `https://cmuc.mat.uc.pt`), the Portuguese Foundation for Science and Technology (FCT), and the Department of Mathematics of the University of Coimbra (DMUC). Moreover, it was also supported by the International Linear Algebra Society (ILAS).

The organizing committee consisted of Natália Bebiano, CMUC, University of Coimbra; Graça Soares, CMAT-UTAD, University of Trás-os-Montes e Alto Douro; Rute Lemos, CIDMA, University of Aveiro; and Ana Nata, CMUC, Polytechnic Institute of Tomar.

Participants of WONRA 2023

FMD60 Workshop on Matrix Analysis
Leganés, Spain, June 8–9, 2023

Report by Daniel B. Szyld and Fernando De Terán Vergara

The FMD60 Workshop on Matrix Analysis took place at the Universidad Carlos III de Madrid, Leganés, over two days, June 8–9, 2023, immediately before the 25$^{\text{th}}$ ILAS Conference in nearby Madrid.

About forty participants went to Leganés to celebrate Froilán M. Dopico's contributions to the field of linear algebra and to the community, on the occasion of his 60$^{\text{th}}$ birthday.

There were 19 talks given by speakers from 10 countries from Europe and North America, including Paul Van Dooren, who closed the meeting. The complete program can be found at:

`https://eventos.uc3m.es/92325/programme/fmd60-workshop-on-matrix-analysis.html`

The organizing committee, led by Fernando De Terán, also included Luis Miguel Anguas, María I. Bueno, Javier González Pizarro, Javier Pérez Álvaro, Kenet Pomás, and María C. Quintana.

The 25th ILAS Conference
Madrid, Spain, June 12–16, 2023

Report by Fernando De Terán Vergara

The 25th ILAS Conference took place in Madrid, Spain, from June 12th to June 16th, 2023, at the Escuela Técnica Superior de Ingenieros de Montes, Forestales y del Medio Natural of the Universidad Politécnica de Madrid.

The conference attracted 480 participants, mostly coming from Europe, but also from America and Asia. The program included 10 plenary lectures, 5 invited minisymposia, 26 contributed minisymposia, and 123 contributed talks, resulting in a total of 419 talks. The topics of the minisymposia covered a wide range of research areas in linear algebra, such as matrix equations, spectral theory, linear systems, and other topics in matrix analysis, as well as related areas, like combinatorics, group theory, Euclidean Jordan algebras, multilinear algebra (tensors), real algebraic geometry, graph theory, rational approximation, operator theory, and quantum information theory. Applications of linear algebra to other areas of science, including data science, computer science, and optimization, were covered as well. The program also included the usual minisymposium in ILAS conferences on linear algebra education (organized by Anthony Cronin and Judi McDonald), as well as a tribute to Steve Kirkland on the occasion of his 60th birthday (organized by Hermie Monterde, Sarah Plosker, Jane Breen, and Sooyeong Kim) and another one to Michael Overton for his 70th birthday (organized by Sara Grundel, Tim Mitchell, and Julio Moro).

The plenary speakers were

- Carlos Beltrán (Universidad de Cantabria, Spain),
- Erin C. Carson (Charles University, Czech Republic),
- Elias Jarlebring (KTH Stockholm, Sweden) as the SIAG-LA speaker,
- Shahla Nasserasr (Rochester Institute of Technology, USA),
- Vanni Noferini (Aalto University, Finland) as the *LAMA* speaker,
- Rachel Quinlan (University of Galway, Ireland),
- and Cynthia Vinzant (University of Washington, USA),

as well as

- Stefan Güttel (University of Manchester, England) as the recipient of the ILAS Taussky-Todd Prize,
- Nicholas J. Higham (University of Manchester, England) as the recipient of the Hans Schneider Prize, and
- Michael Tait (Villanova University, USA) as the recipient of the ILAS Richard A. Brualdi Early Career Prize, which was awarded for the first time.

Giovanni Barbarino (Aalto University, Finland), Andreas Bluhm (Université Grenoble Alpes, France), Teresa Conde (Universität Stuttgart, Germany), Massimiliano Fasi (Durham University, UK), and Luyining Gan (University of Nevada, USA) were the recipients of the "*LAA* early career speakers" grants, supported by Elsevier. This publisher also provided five grants for travel support to Ph.D. students, of US$500 each.

(Left to right) Nicholas J. Higham presented with the Hans Schneider Prize; Stefan Güttel presented with the Taussky-Todd Prize; Michael Tait presented with the Richard A. Brualdi Early Career Prize

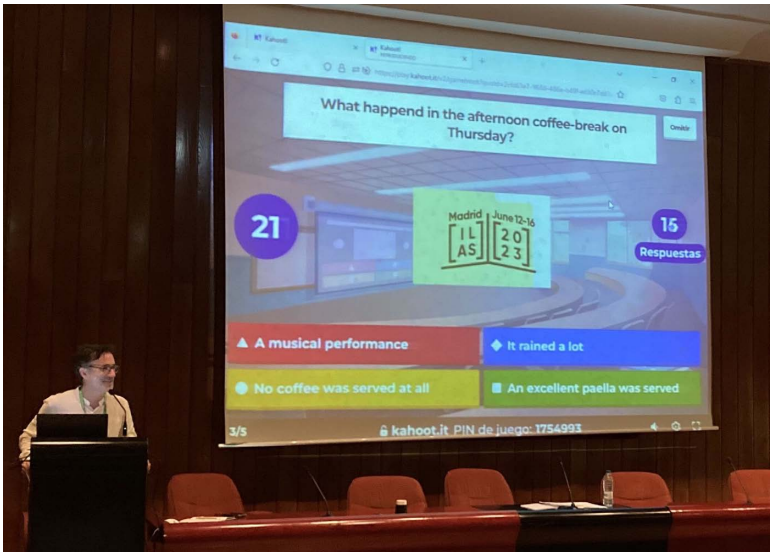
At the afternoon coffee break on Thursday, there was a musical performance.

The program also included social activities, including a cocktail reception on Monday evening, as well as a walking excursion in the Madrid city center and the conference banquet, both on Wednesday. The banquet included a short ceremony in which the ILAS President, Daniel B. Szyld, delivered certificates to the winners of the ILAS prizes (the Hans Schneider, Tausky-Todd, and Richard Brualdi prizes), and also featured an "after-banquet speech" delivered by former ILAS president Peter Šemrl.

The scientific committee of the conference consisted of Raymond H. Chan (City University of Hong Kong), Fernando De Terán (Universidad Carlos III de Madrid, Spain, chair), Gianna del Corso (Università di Pisa, Italy), Shaun Fallat (University of Regina, Canada), Heike Fassbender (Technische Universität Braunschweig, Germany), Elias Jarlebring (KTH Stockholm, Sweden), Linda Patton (CalPoly University, USA), Jeniffer Pestana (University of Strathclyde, UK), João Queiró (Universidade de Coimbra), Naomi Shaked-Monderer (Max Stern Yezreel Valley College, Israel), Zdenek Strakos (Charles University, Czech Republic), Daniel Szyld (Temple University, USA) as ILAS President, and Raf Vandebril (KU Leuven, Belgium) as ILAS Vice President for conferences.

The local organizing committee was comprised of Luis Miguel Anguas (Saint Louis University), Roberto Canogar (Universidad Nacional de Educación a Distancia), Fernando De Terán (Universidad Carlos III de Madrid, chair), Froilán M. Dopico (Universidad Carlos III de Madrid), Ana Luzón (Universidad Politécnica de Madrid), Ana Marco (Universidad de Alcalá), José Javier Martínez (Universidad de Alcalá), Manuel A. Morón (Universidad Complutense de Madrid), and Raquel Viaña (Universidad de Alcalá).

The conference was supported or sponsored by the International Linear Algebra Society (ILAS), Elsevier, Taylor and Francis, Universidad Complutense de Madrid (UCM), Ayuntamiento de Madrid (through the Madrid Convention Bureau), Comunidad Autónoma de Madrid (CAM), Universidad de Alcalá (UAH), Universidad Nacional de Educacióna Distancia (UNED), Universidad Politécnica de Madrid (UPM), Escuela de Montes, Forestales y del Medio Natural de la Universidad Politécnica de Madrid, Saint Louis University, Sociedad Española de Matemática Aplicada (SeMA), Instituto de Matemática Interdisciplinar (IMI), and the Society for Industrial and Applied Mathematics (SIAM).



The closing ceremony featured a challenging trivia contest.

The journal *Linear Algebra and its Applications* will publish a special issue featuring contributions to the conference. The editors of this special issue are Erin Carson, Fernando De Terán, Volker Mehrmann (responsible Editor-in-Chief), Vanni Noferini, and João Queiró.

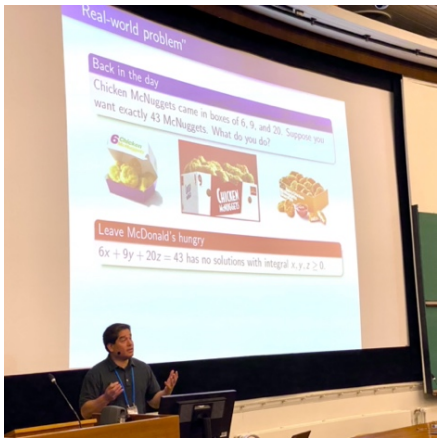
The group photo from ILAS 2023 in Madrid

The 34th International Workshop on Operator Theory and Its Applications (IWOTA) Helsinki, Finland, July 31–August 4, 2023

Report by Jani Virtanen

The 34th International Workshop on Operator Theory and its Applications (IWOTA) was held at the University of Helsinki from July 31st to August 4th, 2023, bringing together leading international experts for an exchange of results and methods, and for tracing the future development of operator theory, complex analysis, harmonic analysis, linear algebra, random matrix theory, mathematical physics, inverse problems, and their applications. There were 405 participants and the scientific program featured 8 plenary speakers (including Stephan Ramon Garcia as the Hans Schneider ILAS Lecturer), 15 semi-plenary speakers, and 19 special sessions.

IWOTA participants in the Porthania Building



Stephan Ramon Garcia delivering the Hans Schneider ILAS Lecture

Per Enflo performing at the conference dinner

The conference was supported by the University of Helsinki, the Federation of Finnish Learned Societies, the Magnus Ehrnrooth foundation, the Mathematics Fund of the Finnish Academy of Science and Letters, the International Linear Algebra Society, the National Science Foundation, and the City of Helsinki. The organizing committee consisted of Jani Virtanen (chair), Hans-Olav Tylli, and Kari Astala, working closely with J. William Helton, Igor Klep, Hugo J. Woerdeman, and other members of the IWOTA steering committee. The IWOTA 2023 conference proceedings will be published in the book series *Operator Theory: Advances and Applications* by Springer/Birkhäuser and edited by Joe Ball, Hans-Olav Tylli, and Jani Virtanen. Contributions for the proceedings should be received by February 15th, 2024 via `iwota2023@helsinki.fi`.

IWOTA 2023 featured over 400 participants

The IWOTA 2023 social program included the City Hall Reception on Monday the 31st, an excursion to Suomenlinna Sea Fortress on Wednesday the 2nd, and, on Thursday the 3rd, the conference dinner, including a concert by Per Enflo.

or further details, see `https://www.helsinki.fi/en/conferences/iwota2023`.

The 10th International Conference on Matrix Analysis and Applications (ICMAA) Kunming, China, August 15–18, 2023

Report by Fuzhen Zhang

The 10th International Conference on Matrix Analysis and Applications (ICMAA) took place August 15th–18th, 2023 at Yunnan University, Kunming, China. Organized by Chaoqian Li, Yaotang Li, Tin-Yau Tam (Chair), Nian-Sheng Tang, Qing-Wen Wang and Fuzhen Zhang, this meeting aims to foster research and promote collaboration among mathematicians working in diverse areas of linear and multilinear algebra, matrix analysis, graph theory, and their applications. 155 participants from 6 countries attended the meeting either in person or online. The meeting featured 31 presentations, including the keynote talk by Zhaojun Bai of the University of California, Davis, USA. The meeting was generously supported by the School of Mathematics and Statistics of Yunnan University, and the K. C. Wong Education Foundation.

Participants of the 10th ICMAA in Kunming

The previous ICMAA conferences were held in China (Beijing; Hangzhou), the United States (twice at Nova Southeastern University; University of Nevada, Reno), Turkey (Selçuk University, Konya), Vietnam (Duy Tan University, Da Nang), Japan (Shinshu University, Nagano Prefecture), and Portugal (University of Aveiro, Aveiro).

For the scientific program and book of abstracts for ICMAA 2023 and other information about the conference, visit:

`https://icmaa2023.scimeeting.cn`

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UPCOMING CONFERENCES AND WORKSHOPS

ILAS at the Joint Mathematics Meetings
San Francisco, USA, January 3–6, 2024

https://www.jointmathematicsmeetings.org/jmm

ILAS Invited Address

Stephan Ramon Garcia, Pomona College
Fast food for thought: what can chicken nuggets tell us about linear algebra?
Thursday, January 4th, 2024, 9:45AM–10:50AM

Abstract: A simple question about chicken nuggets connects everything from analysis and combinatorics to probability theory and computer-aided design. Linear algebra is a recurring theme: determinantal formulas, piecewise-polynomial interpolation, random norms", trace polynomials, and operator algebras all make an appearance. This talk is aimed at a general mathematical audience: students are invited to attend!

ILAS Special Sessions

ILAS Special Session on Generalized Numerical Ranges and Related Topics
Wednesday, January 3rd, 2024, 8:00AM–11:30AM and 1:00PM–4:00PM

- Tin-Yau Tam, University of Nevada, Reno
- Pan-Shun Lau, University of Nevada, Reno

ILAS Special Session on Innovative and Effective Ways to Teach Linear Algebra
Wednesday, January 3rd, 2024, 9:00AM–12:00PM and 2:30PM–4:30PM

- David M. Strong, Pepperdine University
- Sepideh Stewart, University of Oklahoma
- Gil Strang, MIT
- Megan Wawro, Virginia Tech

ILAS Special Session on Linear Algebra, Matrix theory, and its Applications
Thursday, January 4th, 2024, 1:00PM–5:00PM and Friday, January 5th, 2024, 8:00AM–12:00PM

- Stephan Ramon Garcia, Pomona College
- Konrad Aguilar, Pomona College

ILAS Special Session on Sign-pattern Matrices and Their Applications
Friday, January 5th, 2024, 8:00AM–12:00PM and 1:00PM–5:00PM

- Bryan L. Shader, University of Wyoming
- Minerva Catral, Xavier University

ILAS Special Session on Spectral and combinatorial problems for nonnegative matrices and their generalizations
Saturday, January 6th, 2024, 8:00AM–12:00PM and 1:00PM–4:00PM

- Pietro Paparella, University of Washington Bothell
- Michael J. Tsatsomeros, Washington State University

ILAS Special Session on Graphs and Matrices
Saturday, January 6th, 2024, 8:00AM–12:00PM and 1:00PM–5:00PM
Organizers:

- Jane Breen, Ontario Tech University
- Stephen Kirkland, University of Manitoba

At the JMM Awards Celebration (i.e., the prize ceremony) on Wednesday, January 4th at 4:45PM, Stephan Ramon Garcia, as the ILAS speaker, will be presented with an award certificate.

Workshop on Matrices and Operators
Reno, USA, June 15–16, 2024

The University of Reno, Reno, Nevada, USA, will host the Workshop on Matrices and Operators on Saturday, June 15th and Sunday, June 16th, 2024. The purpose of the workshop is to promote research and collaboration among researchers with an interest in matrix theory, operator theory, and related topics. Participants are encouraged to present their latest findings and discoveries, and share their experiences and research problems, fostering an atmosphere of collaboration and mutual learning. Through dialogue and the exchange of knowledge, the workshop aims to enhance the collective growth and development of the research community involved in matrix theory, operator theory, and related fields.

Organizers:

- Valentin Deaconu, University of Nevada, Reno, USA, `vdeaconu@unr.edu`
- Alex Kumjian, University of Nevada, Reno, USA, `alex@unr.edu`
- Pan Shun Lau, University of Nevada, Reno, USA, `plau@unr.edu`
- Chi-Kwong Li, College of William and Mary, USA, `ckli@math.wm.edu`
- Tin-Yau Tam (Chair), University of Nevada, Reno, USA, `ttam@unr.edu`

The conference will be hosted at the Davidson Math and Science Center, University of Nevada, Reno, USA. The conference banquet will be held at 6PM on Saturday, June 15th, 2024.

The 35th International Workshop on Operator Theory and its Applications (IWOTA)
Canterbury, UK, August 12–16, 2024

The IWOTA conference series is the largest and most important annual event in operator theory and its applications, bringing together leading international experts from pure mathematics and application areas to trace the future development of operator theory and related areas such as complex analysis, harmonic analysis, linear algebra, random matrix theory, and mathematical physics, as well as their applications, including control theory, signal processing, and AI.

IWOTA 2024 will take place August 12th – 16th, 2024 at the University of Kent in Canterbury, UK. It will provide a medium for an intense exchange of new results, information and opinions, and for international collaboration in operator theory and its applications worldwide. It will further set directions for future research through the conference activities and proceedings, and also serve as the setting for the planning of future IWOTA conferences.

Deadlines:

- December 31st, 2023 – Special session proposals
- Spring 2024 – Submission of abstracts and registration

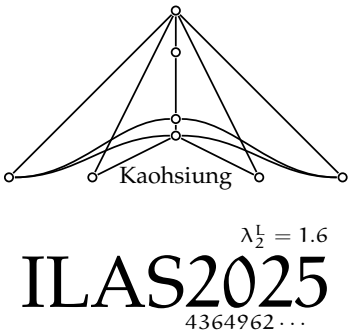
Invited Plenary Speakers:

- Mark Embree, Virginia Tech (Israel Gohberg ILAS-IWOTA Lecturer)
- Rupert Frank, LMU Munich
- Melina Freitag, Potsdam University
- Stéphane Gaubert, École Polytechnique, Paris
- Mariana Haragus, University of Franche-Comté
- Svetlana Jitomirskaya, University of California, Irvine and Georgia Tech
- Barbara Kaltenbacher, University of Klagenfurt
- Gitta Kutyniok, LMU Munich
- Malabika Pramanik, University of British Columbia

Organizing committee: Bas Lemmens, Ana Loureiro and Ian Wood (University of Kent, UK); Marina Iliopoulou (National and Kapodistrian University of Athens, Greece); and Marco Marletta (Cardiff University, UK) in collaboration with the IWOTA executive steering committee members: J. William Helton (chair), University of California, San Diego, USA; Sanne ter Horst, North-West University, South Africa; Igor Klep, University of Ljubljana, Slovenia; Irene Sabadini, Polytechnico di Milano, Italy; Hugo J. Woerdeman, Drexel University, USA.

For further information, visit `https://blogs.kent.ac.uk/iwota2024`.

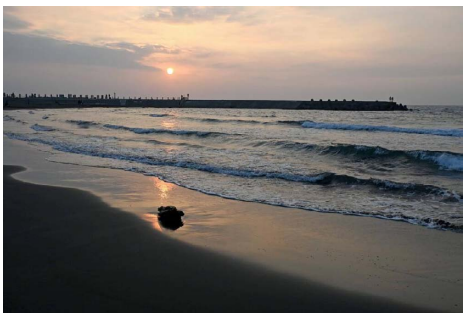
The 26th ILAS Conference
Kaohsiung, Taiwan, June 23–27, 2025



The 26th Conference of the International Linear Algebra Society will be held in the vibrant city of Kaohsiung, Taiwan, June 23–27, 2025. This prestigious event will bring together leading experts, researchers, and enthusiasts from around the world to share their knowledge and explore the latest advancements in the field of linear algebra. For further details, including registration information and program updates, please visit the conference website:

`https://ilas2025.tw`

We look forward to welcoming you to this exciting event in 2025!



ONGOING ONLINE SEMINARS

Algebraic Graph Theory Seminar

Host: University of Waterloo

Schedule: weekly on Mondays

Time: 11:30AM, Waterloo (Ontario, Canada) time (UTC+??hours)

URL: `https://math.uwaterloo.ca/~agtheory`

Matrix Seminar

Host: University of Nevada, Reno

Schedule: biweekly on Fridays

Time: 4:15PM, Reno (Nevada, USA) time (UTC+??hours)

`https://docs.google.com/document/d/1MswSd16JqsZE294kYCXujLio4cnAiuYv6QKRc6BxvI0/edit`

05C50 Online

Host: University of Manitoba

Schedule: biweekly on Fridays

Time: 10:00AM, Winnipeg (Manitoba, Canada) time (UTC+??hours)

`https://sites.google.com/view/05c50online/home`

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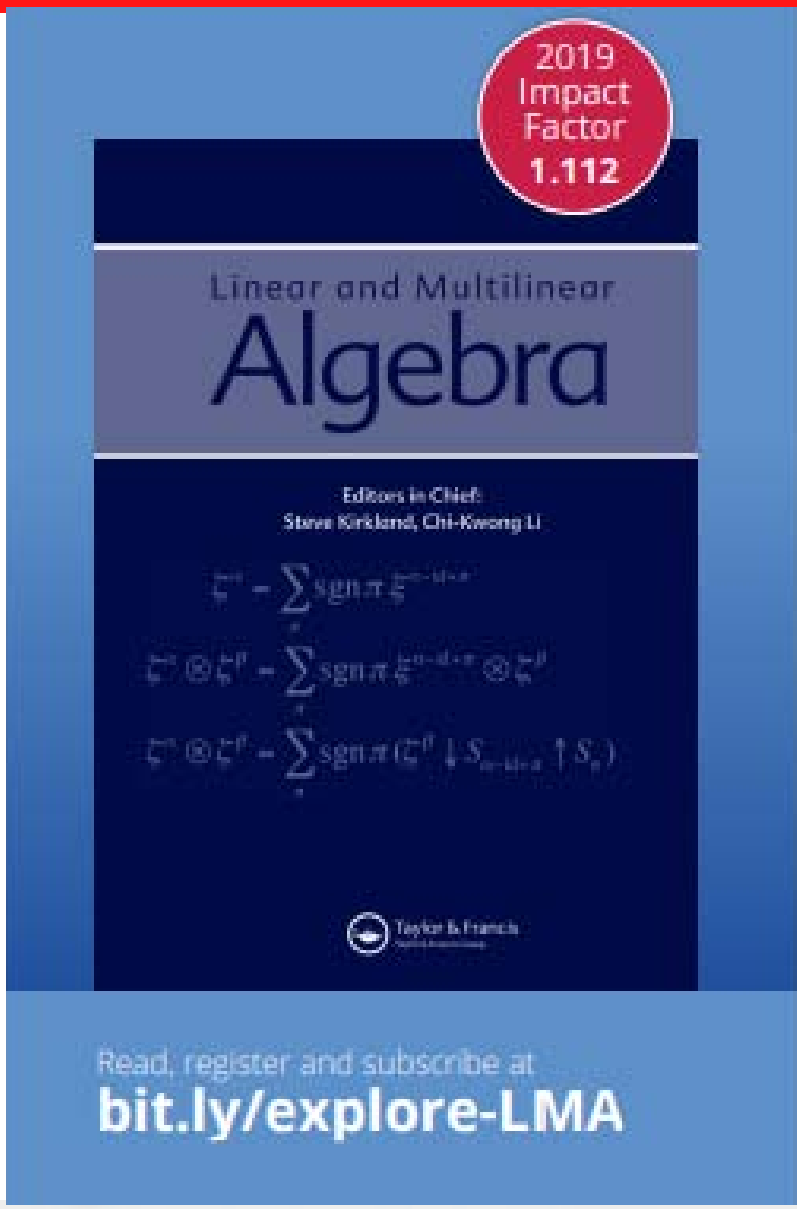

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IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to parts (a) and (b) of Problem 66-2, as well as to Problems 66-4, 70-1, and 70-3. Solutions are invited to Problems 68-2, 68-4, 69-2, 69-3, 70-2 and to all of the new problems from the present issue 71.

Problem 66-2: The Cardinality of Hadamard Matrices
Proposed by Richard William FAREBROTHER, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@hotmail.com

Let n be an even number. An $n \times n$ matrix H is called a Hadamard matrix if its entries lie in the set $\{-1, 1\}$ and it satisfies $HH^T = nI_n$.

(a) Show that the number of $n \times n$ Hadamard matrices is divisible by $2^{2n-1}(n-1)!$.

(b) Show that the number of $n \times n$ Hadamard matrices is exactly $2^{2n-1}(n-1)!$ if and only if $n = 2$ or 4.

(c) Show that the number of 4×4 symmetric Hadamard matrices is exactly 64.

Editor's Note: Issue 67 of *IMAGE* featured a solution to part (c) of this problem, given by Subhasish BEHERA, *Indian Institute of Technology, Bhubaneswar, India*, sb52@iitbbs.ac.in.

Solution 66-2a,b by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Call a Hadamard matrix *normalized* if every entry in its first row and in its first column is 1. Call two normalized Hadamard matrices H_1 and H_2 *row-permutation equivalent* if there exists a permutation matrix P keeping the first row unchanged such that $PH_1 = H_2$. Since the rows of any given Hadamard matrix H are distinct, $PH \neq QH$ for any two distinct permutation matrices P and Q, and we have exactly $(n-1)!$ normalized Hadamard matrices in each row-permutation equivalence class. Hence, the number of normalized Hadamard matrices is $(n-1)!k$, where k is the number of row-permutation equivalence classes.

Now call two Hadamard matrices H and M *diagonal equivalent* if there exist two diagonal matrices D_1 and D_2 with each main diagonal entry equal to ± 1 such that $D_1HD_2 = M$. Since there are 2^n choices each for D_1 and D_2 and $D_1HD_2 = (-D_1)H(-D_2)$, there are at most 2^{2n-1} Hadamard matrices in each equivalence class. Meanwhile, for any fixed Hadamard matrix H, by choosing D_1 and D_2 carefully, we can give the entries of the first row and first column of D_1HD_2 any sign pattern we want. Since there are 2^{2n-1} distinct possibilities here, every diagonal equivalence class contains exactly 2^{2n-1} Hadamard matrices, including exactly one normalized Hadamard matrix. Hence, the number of $n \times n$ Hadamard matrices is exactly $2^{2n-1}(n-1)!k$. Part (a) follows immediately from this.

To prove part (b), we show that there is only one row-permutation equivalence class of $n \times n$ normalized Hadamard matrices if and only if $n = 2$ or 4. Let V_n be the set of n-vectors having $n/2$ entries equal to 1 and $n/2$ entries equal to -1 and with first entry 1. In a normalized Hadamard matrix, all rows except the first are members of V_n. Since $|V_n| = \binom{n-1}{n/2}$ and every row-permutation equivalence class corresponds to a distinct choice of $n-1$ row vectors from V_n, there will be only one row-permutation equivalence class of $n \times n$ normalized Hadamard matrices if and only if there is an $n \times n$ Hadamard matrix and $\binom{n-1}{n/2} = n-1$. We can simplify this last equation to $(n-2)! = (n/2)!(n/2-1)!$. For $n = 2$ or 4, this equation is satisfied, but an easy induction shows that $(n-2)! > (n/2)!(n/2-1)!$ and hence $\binom{n-1}{n/2} > n-1$ for all larger even n.

Problem 66-4: A Random Walk on an Abelian Group
Proposed by Rajesh PEREIRA, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let G under the operation $+$ be a finite abelian group of odd order. Let P be an arbitrary probability distribution on G. Consider the discrete random walk generated by this probability distribution (i.e., for all $g, h \in G$, if you are at g, then you have a probability $P(\{h\})$ of being at $g+h$ at the next time step). Show that the determinant of the transition matrix of this random walk lies in the interval $[0, 1]$.

Solution 66-4 by the proposer

Let T be the transition matrix. Index the rows and columns of T by the elements of G, so that $t_{gh} = P(\{g-h\})$. For each irreducible character ω of the (finite abelian) group G, let v_ω be the vector whose gth entry is $\omega(-g)$. Then

the gth entry of Tv_ω is $\sum_{h\in G} P(\{g-h\})\omega(-h) = \sum_{h\in G} P(\{g-h\})\omega(g-h)\omega(-g)$, and hence $Tv_\omega = \lambda_\omega v_\omega$, where $\lambda_\omega = \sum_{h\in G} P(\{g-h\})\omega(g-h) = \sum_{x\in G} P(\{x\})\omega(x)$. Moreover, the fact that distinct irreducible characters give rise to orthogonal vectors (see, e.g., [1, Chapter 5]) implies that the corresponding eigenvectors are linearly independent. Since the number of irreducible characters of an abelian group is always equal to the order of the group, it follows that every eigenvalue of T is of the form λ_ω for some character ω. Since $|\omega(g)| = 1$, we always have $|\lambda_\omega| \le 1$. The trivial character gives us an eigenvalue of 1. All other irreducible characters of an abelian group of odd order have nonreal elements and come in conjugate pairs. Thus, all other eigenvalues come in conjugate pairs, with the product of each pair lying in $[0, 1]$. The determinant thus also lies in $[0, 1]$.

Reference

[1] N. Jacobson. *Basic Algebra II.* W. H. Freeman and Company, New York, second edition, 1989.

Problem 70-1: A Decomposition for Nonzero Integer Matrices

Proposed by Gérald Bourgeois, *Université de la Polynésie française, FAA'A, Tahiti, Polynésie française*, bourgeois.gerald@gmail.com

Let $A \in M_n(\mathbb{Z})$ be a nonzero matrix. Show that there exist $G, N \in M_n(\mathbb{Z})$ satisfying $A = G + N$ where N is nilpotent of index n (i.e., $N^{n-1} \neq 0$ but $N^n = 0$) and all the eigenvalues of G (as a matrix over $\mathbb{C}$) are nonzero integers.

Solution 70-1 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

If $n = 1$, then $A = A + O$ satisfies the conditions, and so we may assume $n \ge 2$. If $A = aI_n$ for some integer a, then $a \neq 0$ since A is not the zero matrix. Write $A = G + N$, where N is the strictly upper-triangular matrix whose entries in the upper triangle all equal 1. Then N is nilpotent of index n, and $G = A - N$ is an upper-triangular matrix whose diagonal entries all equal a. The eigenvalues of G are therefore all equal to the nonzero integer a, and so we may assume that A is not a multiple of I_n.

Suppose now that the diagonal entries and superdiagonal entries of A are all nonzero. Then write $A = G + N$, where G is the lower-triangular matrix whose entries (including the main diagonal) in the lower triangle are exactly those of A. The eigenvalues of G are the n diagonal entries of A and hence are nonzero integers. Also $N = A - G$ is a strictly upper-triangular matrix whose superdiagonal entries are the superdiagonal entries of A and hence are nonzero; thus N is nilpotent of index n.

We will show that in all remaining cases A is similar to a matrix whose diagonal and superdiagonal entries are all nonzero and where the change-of-basis matrix and its inverse both belong to $M_n(\mathbb{Z})$. It follows that A itself satisfies the conditions given in the problem. Specifically, we prove, by induction on i, for $i = 1, \ldots, n$, that A is similar to a matrix in which the first i diagonal entries and first i superdiagonal entries are all nonzero.

Base case: Suppose there exist i and j with $i \neq j$ such that $a_{ij} \neq 0$. Let ρ be any permutation of $\{1, \ldots, n\}$ such that $\rho(i) = 1$ and $\rho(j) = 2$. Then the permutation similarity induced by ρ yields $a_{12} \neq 0$ in the transformed matrix, with the change-of-basis matrix and its inverse both belonging to $M_n(\mathbb{Z})$, since every permutation matrix belongs to $M_n(\mathbb{Z})$. If $a_{11} \neq 0$, we are done, so assume $a_{11} = 0$. Let L be the elementary matrix in which p times row 1 of I_n is added to row 2 of I_n. Then L^{-1} is the elementary matrix in which $-p$ time column 2 of I_n is added to column 1 of I_n. Choose any $p \neq 0$. Then, in the transformed matrix LAL^{-1}, we have $-pa_{12} \neq 0$ in the $(1, 1)$-position and a_{12} unchanged in the $(1, 2)$-position. By the way, all elementary matrices in which an integer times one row (or column) of I_n is added to another row (or column) of I_n belong to $M_n(\mathbb{Z})$. On the other hand, suppose $a_{ij} = 0$ for all $i \neq j$. Since A is not a multiple of the identity, there exist $i \neq j$ such that $a_{ii} \neq 0$ and $a_{jj} \neq a_{ii}$. Let ρ be any permutation such that $\rho(i) = 1$ and $\rho(j) = 2$. Then the permutation transformation induced by ρ yields $a_{11} \neq 0$ and $a_{22} \neq a_{11}$. Of course, all the off-diagonal entries are still zero. Let L be the elementary matrix that adds p times row 2 to row 1 and so L^{-1} is the elementary matrix that adds $-p$ times column 1 to column 2. For any $p \neq 0$, we have, in the transformed matrix LAL^{-1}, a_{11} unchanged in the $(1, 1)$-position and $p(a_{22} - a_{11}) \neq 0$ in the $(1, 2)$-position.

Induction step: Assume that, for some i, the matrix A is similar to a matrix in which the diagonal entries $a_{11}, \ldots, a_{ii}$ and the superdiagonal entries $a_{12}, \ldots, a_{i,i+1}$ are all nonzero. In the transformed matrix LAL^{-1}, where L is the elementary matrix in which p times row i is added to row $i + 1$, all the diagonal and superdiagonal entries preceding row i and following row $i + 1$ are unchanged, as is the $(i, i + 1)$-entry. However, the (i, i)-entry is now $a_{ii} - pa_{i,i+1}$ and the $(i + 1, i + 1)$-entry is now $a_{i+1,i+1} + pa_{i,i+1}$. So we choose p such that $p \neq 0$, $a_{ii} - pa_{i,i+1} \neq 0$, $a_{i+1,i+1} + pa_{i,i+1} \neq 0$ and

$a_{i+1,i+1}+pa_{i,i+1} \neq a_{i+2,i+2}$. Thus, in the transformed matrix, we have that a_{ii}, $a_{i,i+1}$ and $a_{i+1,i+1}$ are nonzero, and that $a_{i+2,i+2} \neq a_{i+1,i+1}$. This last condition, which applies only when $i \leq n-3$, is needed in the next step. In the transformed matrix LAL^{-1}, where L^{-1} is the elementary matrix in which $-p$ times column $i+1$ is added to column $i+2$, all the diagonal and superdiagonal entries preceding row $i+1$ are unchanged, as are the (i,i)- and $(i,i+1)$-entries. However, the $(i+1,i+1)$-entry is now $a_{i+1,i+1}+pa_{i+2,i+1}$ and the $(i+1,i+2)$-entry is $p(a_{i+2,i+2}-a_{i+1,i+1}-pa_{i+2,i+1})$. So we must choose p such that these two entries are not zero. This completes the proof by induction, but it is still possible that $a_{nn}=0$. In that case, we make another transformation of the form LAL^{-1} just like the first of the two transformations in the induction step.

Problem 70-3: The Diagonal Entries of l^1-Normaloid Matrices

Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

The l^1-norm on $\mathbb{R}^n$ is given by $\|x\|_1 = \sum_{i=1}^n |x_i|$. The l^1 operator norm of an $n \times n$ matrix A is $\|A\|_1 = \max_{x\neq 0} \frac{\|Ax\|_1}{\|x\|_1}$ and its spectral radius is defined by $r(A) = \max_{1\leq i\leq n} |\lambda_i|$, where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A. We say that A is l^1-*normaloid* if $r(A) = \|A\|_1$.

Let A be an l^1-normaloid real matrix with all entries nonzero. Show that the main diagonal entries of A are either all positive or all negative.

Solution 70-3 by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

This solution is a modification of the usual proof of Gershgorin's circle theorem, where we make use of the fact that $\|A\|_1 = \max\{\|C_1\|_1, \ldots, \|C_n\|_1\}$, where C_i is the ith column of A. Choose λ such that $r(A) = |\lambda|$, and choose x to be an eigenvector associated with λ. Choose $k \in \{1, \ldots, n\}$ such that $|x_k| \geq |x_j|$ for all $j \in \{1, \ldots, n\}$. Let $\{a_{ij}\}_{i,j=1}^n = A$. Since $Ax = \lambda x$, we have, in particular, $\sum_{j=1}^n a_{kj}x_j = \lambda x_k$, and so $\sum_{j=1,j\neq k}^n a_{kj}x_j = (\lambda - a_{kk})x_k$. Furthermore,

$$|\lambda| = r(A) = \|A\|_1 = \max\{\|C_1\|_1, \ldots, \|C_n\|_1\} \geq \|C_k\|_1.$$

Therefore,

$$\begin{aligned}
\|C_k\|_1 - |a_{kk}| &\leq |\lambda| - |a_{kk}| \\
&\leq |\lambda - a_{kk}| \\
&= \left| \sum_{j=1,j\neq k}^n a_{kj}\frac{x_j}{x_k} \right| \\
&\leq \sum_{j=1,j\neq k}^n |a_{kj}|\frac{|x_j|}{|x_k|} \\
&\leq \sum_{j=1,j\neq k}^n |a_{kj}| \qquad (1) \\
&= \|C_k\|_1 - |a_{kk}|,
\end{aligned}$$

and so all of the above inequalities are equalities. In particular, $(|\lambda| - |a_{kk}|)^2 = |\lambda - a_{kk}|^2$. Meanwhile, we have $(|\lambda| - |a_{kk}|)^2 = |\lambda|^2 - 2|\lambda a_{kk}| + a_{kk}^2$ and, since a_{kk} is real,

$$\begin{aligned}
|\lambda - a_{kk}|^2 &= \operatorname{Re}(\lambda - a_{kk})^2 + \operatorname{Im}(\lambda - a_{kk})^2 \\
&= (\operatorname{Re}(\lambda) - a_{kk})^2 + \operatorname{Im}(\lambda)^2 \\
&= \operatorname{Re}(\lambda)^2 - 2\operatorname{Re}(\lambda)a_{kk} + a_{kk}^2 + \operatorname{Im}(\lambda)^2 \\
&= \operatorname{Re}(\lambda)^2 + \operatorname{Im}(\lambda)^2 - 2\operatorname{Re}(\lambda)a_{kk} + a_{kk}^2 \\
&= |\lambda|^2 - 2\operatorname{Re}(\lambda)a_{kk} + a_{kk}^2,
\end{aligned}$$

from which it follows that $|\lambda a_{kk}| = \operatorname{Re}(\lambda)a_{kk}$. Hence, λ is real and has the same sign as a_{kk}. (Note that $\lambda \neq 0$, since $\lambda = \|C_k\|_1$ and, since all entries of A are nonzero, $\|C_k\|_1 > 0$.) From the fact that the inequality (1) is an equality and that $|a_{kj}| > 0$ for all k and j, we also have that $|x_j| = |x_k|$ for all j. Thus, by repeating the above string of inequalities for each j, we conclude that λ has the same sign as a_{jj} for all j. So all the diagonal elements of A have the same sign.

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IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce three new problems in this issue and invite readers to submit solutions for publication in *IMAGE*.
Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Rajesh Pereira (pereirar@uoguelph.ca).

New Problems:

Problem 71-1: A Property of 4×4 Hadamard Matrices
Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England*, R.W.Farebrother@hotmail.com

Let n be an even number. An $n \times n$ matrix H is called a Hadamard matrix if its entries lie in the set $\{-1, 1\}$ and it satisfies $HH^T = nI_n$. A *monomial matrix* is a square matrix that has at most one nonzero entry in each row and in each column.

(a) Show that, for any 4×4 Hadamard matrix H, there exists a unitary monomial matrix U and a rank-one matrix R such that $H = 2U + R$.

(b) Show that no $n \times n$ Hadamard matrix is the sum of a monomial matrix and a rank-one matrix when $n > 4$.

Problem 71-2: Cauchy-Schwarz and the Characteristic Polynomial
Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let $\mathbb{F}$ be either the field of real numbers or the field of complex numbers. Suppose N is a nonzero nilpotent $n \times n$ matrix with entries in $\mathbb{F}$, that $k \in (0, 1)$, and that $p(x)$ is a monic polynomial of degree n with all of its coefficients in $\mathbb{F}$. Show that there exists an $n \times n$ matrix M with entries in $\mathbb{F}$ having characteristic polynomial $p(x)$ and satisfying $|\mathrm{Tr}(MN)|^2 > k\,\mathrm{Tr}(M^*M)\,\mathrm{Tr}(N^*N)$.

Problem 71-3: An $(n^2 - n)$th Derivative Inequality
Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

Let $\|\cdot\|$ denote the usual operator norm on the space of $n \times n$ complex matrices. For any $n \times n$ complex matrix A, define ϕ_A to be the linear operator on the space of $n \times n$ matrices which maps any such matrix B to the commutator $AB - BA$. Let $p_A(z)$ be the characteristic polynomial of ϕ_A. Find the smallest positive number C_n such that $|p_A^{(n^2-n)}(0)| \leq C_n \|A\|^{n^2-n}$ for all $n \times n$ complex matrices A.

Editor's Note: This is a correction and replacement for Problem 65-4.

Solutions to Problems 66-2 (a) and (b), 66-4, 70-1 and 70-2 are on pages 29–31.