The Bulletin of the International Linear Algebra Society

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Serving the International Linear Algebra Community
Issue Number 73, pp. 1–29, Fall 2024

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Ongoing Online Seminars

IMAGE Problem Corner: Old Problems with Solutions

IMAGE Problem Corner: New Problems

About *IMAGE*

ISSN 1553-8991. Produced by the International Linear Algebra Society (ILAS). Two issues are published each year, on June 1 and December 1. The editors reserve the right to select and edit material submitted. All issues of *IMAGE* may be viewed online at `http://www.ilasic.org/IMAGE`. *IMAGE* is typeset using LaTeX. Photographs for this issue were obtained from websites referenced in articles, university department websites, conference websites, or from contributors directly, unless indicated otherwise here:

With this issue, *IMAGE* welcomes Jeff Stuart as the new editor of the *IMAGE* Problem Corner!

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LINEAR ALGEBRA EDUCATION

Discussion on Teaching and Learning Linear Algebra International Congress of Mathematical Education (ICME-15)

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1. Introduction. In July 2024, as part of the 15th International Congress on Mathematical Education in Sydney, Australia (https://icme15.org), the authors organized two 90-minute sessions on teaching and learning linear algebra as a Discussion Group (DG) at the conference. The congress included 30 different DGs on a variety of topics. In the first session of the DG on Teaching and Learning Linear Algebra, 20 participants attended, and in the second session, there were 13 attendees. The delegates represented 12 countries: Australia, Canada, Chile, China, Germany, France, Hong Kong, Mexico, New Zealand, Poland, Slovakia, and the USA. The participants had broad backgrounds. Some were high school teachers teaching elementary linear algebra, some were in the process of creating courses equivalent to an AP Linear Algebra course, and some were university instructors teaching a variety of courses. A few had experience with research in linear algebra education, and some were research mathematicians. In this article, we present an overview of the topics and questions we posed during the DG and some of the participants' responses. In addition, we make some suggestions for ways forward. Our main intention was to discuss the overarching themes and guiding questions listed below during the DG sessions.

1. Theoretical approaches that can help students make sense of linear algebra concepts and establish connections within linear algebra
2. Applications of linear algebra in other disciplines and strategies for their inclusion in the curriculum. How does linear algebra interact with other disciplines?
3. Work with linear algebra instructors about their contributions, their difficulties, and their interpretations of student understanding
4. Contributions of research on linear algebra education
5. Transitional moments in linear algebra learning
6. Content of linear algebra courses and advances in teaching specific linear algebra concepts
7. What is the role of proof in understanding linear algebra concepts, and what are some challenges for teachers and students?
8. What are some teaching and learning resources (including technology) that can help students understand concepts?
9. As is customary in many universities worldwide, introducing linear algebra early in the degree program helps students encounter other mathematics topics besides calculus and encourages them to pursue different career paths. Studying linear algebra concepts gives students mathematical maturity earlier in their path.

2. Areas for Discussion. We organized the sessions into six different areas (three covered on each day): research, teaching, learning, resources, applications, and curriculum. Below, we describe each area, including its purpose, the questions posed, and some of the discussion that took place.

Research. The purpose of this section was to identify advances in linear algebra education research in both the first and the second courses. The session started with an introduction by the DG leaders to the state of research in linear algebra education, where different perspectives, such as APOS theory, modes of thinking, intuition, and representations were mentioned. In a survey paper, Stewart et al. [4] revealed the following promising directions for research in linear algebra teaching and learning. These provided an entry point to the discussion that followed.

1. Development of research-based curricula for teaching linear algebra, with implementation on a comprehensive scale
2. Careful analysis of textbooks and homework (online or handwritten)—with foci ranging from their content and exposition to how they are used by instructors and students to their effectiveness in supporting student learning
3. Examining topic areas such as systems of linear equations, properties of linear transformations, orthogonality, and least squares
4. Work on cross-cutting themes such as proof
5. Replication studies that carefully reference and build on previous work
6. Instructional recommendations that are based on empirical, replicable research studies

7. Documenting industry needs in ways that can inform research on the teaching and learning of undergraduate linear algebra (e.g., for curricular development in a first and/or second course)
8. Research on topics in a second course, which includes more abstract linear algebra

The leaders also presented some current research foci, including task design, connections between research and pedagogy, constructions of specific linear algebra concepts, transitional moments in linear algebra learning, and innovative ways of introducing linear algebra concepts. Among the advances and trends in linear algebra education research, the following directions were brought up: how students learn and construct linear algebra concepts; student difficulties; problem-solving; the influence of teaching methods and strategies on the learning of linear algebra concepts; socio-political approaches; proofs, justification, and argumentation; research on and with linear algebra instructors; online learning; applications of linear algebra; pedagogical approaches to bridge school learning and linear algebra; and modeling. After this brief introduction, the following questions were posed:

(a) What is the current focus of your research?
(b) What is the nature of the second course(s) at your institution?
(c) What is the latest from the literature on proof in linear algebra?
(d) What frameworks are you currently employing?
(e) What are the impacts of our existing research?
(f) What would be helpful in future research?

Teaching. We were interested in knowing about teaching approaches that led to better student understanding. The brief introduction to this area by the leaders included information about an abstract linear algebra course, including its nature, content, textbook used, participants, and laboratory sessions using MATLAB. A sample activity was presented, showing how students validated proofs with MATLAB. After this, we asked the group the following questions: What concepts do you teach in the first and second courses? What innovations in teaching are you implementing? Do you use programming languages (e.g., MATLAB) to teach linear algebra?

Learning. Knowing what might cause difficulties in the learning process can be a useful element for instructors. With this in mind, the questions posed for a discussion in this area were the following: In your experience, what are the significant obstacles to students' learning linear algebra concepts in the first and second courses? What are some ways to combat these obstacles? The leaders introduced the following general aspects as forming part of the possible obstacles: background influences from school or elsewhere; factors related to initial exposure in the first year; factors in ongoing study in the second year and beyond; and mismatch between expectations of students and teachers. They also mentioned some fundamental guiding principles for learning linear algebra, such as the following, and the importance of articulating them:

- The Linear Algebra Principle: "Move in straight lines whenever you can";
- The Conjugation Principle: $M = PDP^{-1}$ equivalent to $MP = PD$ (modified commutativity).

Resources. The purpose of this area was to facilitate an exchange of ideas between the participants about the resources used by instructors as well as by students and gather their impressions about different types of tools for linear algebra teaching and learning. In the introduction, the leaders stressed the importance of identifying the purpose of using a resource so that an adequate one could be chosen. Introducing a concept, strengthening a concept, designing or solving a problem, preparing or working on a project, and assessing or studying for an exam were mentioned among the possible reasons for using a resource. They also reminded the participants of the resources for improving the teaching of linear algebra, as mentioned by Day and Kalman [1]. In addition, some categories of resources, such as internal vs. external, material/digital, human/social, and cognitive resources [2], together with examples, were mentioned. The discussion was guided by the following questions: What resources (including technology) have you used and created that aid in the understanding of linear algebra concepts? What do you look for in a textbook?

Applications. The purpose of this area was for the participants to share their knowledge and experiences about the applications of linear algebra in other disciplines as well as resources and task design involving applications. Participants were presented with examples of projects on applications of linear algebra using MATLAB. We posed the following questions for discussion: How does linear algebra interact with other disciplines? How can we collaborate with experts from other fields to design and implement tasks involving applications?

Curriculum. We were interested in hearing about participants' experiences with strategies for including linear algebra applications in the curriculum, considering the needs of industry. We also wanted to discuss issues concerning content vs. depth and the development of a research-based curriculum. In the introduction, the evolution of the internet and searching techniques, computer graphics, simulations and virtual reality, automation and machine learning, and the evolution of artificial intelligence were mentioned as aspects to be considered. We posed the following questions: What are the pressing issues of the curriculum in this century? What are the needs of industry and disciplines that apply linear algebra? In what ways can we collaborate with experts from other fields?

3. Overall Outcomes. The participants' responses and conversations focused mostly on various linear algebra concepts and on students' meaningful understanding. Moreover, they also offered various helpful remedies as well as some new research questions.

In general, their responses indicated that linear algebra education research may not have been on their radars. Concerning teaching, some of the responses were:

- The main conflicting issue about the approach is whether to go from concrete to abstract, from abstract to concrete, or to blend the two methods.
  Remedies: Proof structure through interaction with objects, meaning before procedure, concepts should come first, more careful set-up of proofs, avoiding mimicry, and introducing "aha" moments.
- Students have a procedural orientation.
  Remedies: Pose questions with limited guidance, giving students the freedom to explore and experiment.
- Creativity can die if only following formulas.
  Remedies: MATLAB incorporated to varying degrees, sometimes as an ancillary and other times as a formal part of learning activities and assessment, computer algebra assisting students to learn concepts freely and not be burdened by technicalities.

Regarding learning, participants emphasized the following:

- Lack of prerequisite knowledge of matrices and vectors.
  Remedies: Use of online materials, using mastery models of learning.
- Differences in experience with the use of technology in teaching; some participants had never used it, others had;
  Pathological dislike of proofs, lack of earlier exposure and maturity;
  Lack of motivation and of attention to real-world examples.
  Remedies: Bridging courses, examples involving GPS and web searching on the internet, Pixar movies.
- The gap between high school and university;
  It is better to leave proofs for a second course and turn proofs into questions;
  Not everyone has a second course in linear algebra.
  Remedies: Have a series of investigations leading to understanding being revealed.

Regarding resources, the participants mentioned Gilbert Strang's videos from MIT OpenCourseWare, and visually attractive materials. Wolfram Alpha and Polypad virtual manipulatives, especially the algebra tiles, were among the resources used by the delegates. Participants referred to the big ideas in linear algebra that have applications in other fields. Physics, astronomy, computer science, and chemistry were mentioned as example fields.

Research concerning examples and non-examples, such as in the case of linear transformations, use of language, and notation in different fields where linear algebra notions are applied, were among possible research directions mentioned.

Overall, many good ideas and fruitful conversations emerged from both sessions. Our chosen topics also helped to focus the discussions. At the conclusion of the second session, the DG leaders shared some resources, such as recent books (e.g., the *ZDM* special issue and conference proceedings from ICME-13); some papers (e.g., the Linear Algebra Curriculum Study Group (LACSG 2.0) recommendations [3]); and the International Linear Algebra Society (ILAS) Education webpage, including a new page on linear algebra lesson plans. Participants were invited to consider involvement in future research projects. We believe the questions posed in this Discussion Group are important for the mathematics community to consider. The authors would like to continue the work on the nine overarching themes introduced above in the future. Mathematicians' input concerning these issues will undoubtedly lead to better outcomes for teaching and research.

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[1] J. M. Day and D. Kalman. Teaching linear algebra: Issues and resources. *College Math. J.*, 32(3):162–168, 2001.

[2] B. Pepin and J. Kock. Towards a better understanding of engineering students' use and orchestration of resources: Actual Student Study Paths. In U. T. Jankvist, M. van den Heuvel-Panhuizen, and M. Veldhuis, editors, *Eleventh Congress of the European Society for Research in Mathematics Education*, volume TWG14 of *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (CERME11)*, Utrecht, Netherlands, Feb. 2019. Utrecht University, Freudenthal Group.

[3] Stewart, S., Axler, S., Beezer, R., Boman, E., Catral, M., Harel, G., McDonald, J., Strong, D., and Wawro, M. The Linear Algebra Curriculum Study Group (LACSG 2.0) Recommendations. *Notices Amer. Math. Soc.*, 69(5):813–819, 2022.

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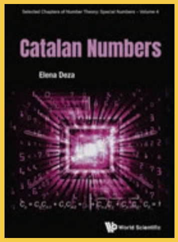
Catalan Numbers

by **Elena Deza** (Moscow Pedagogical State University, Russia)



Named after mathematician Eugène Charles Catalan, these numbers appear in various combinatorial problems. With rich history, intriguing properties, and diverse applications, Catalan numbers connect arithmetic, analysis, and combinatorics. This book explores their interesting properties, history, classical and modern applications, relations to other special numbers, and open problems.

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Contributions in Honor of Bruce C Berndt
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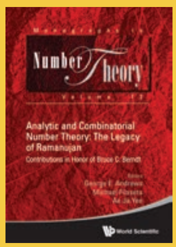

This volume reflects the contributions stemming from the conference Analytic and Combinatorial Number Theory: The Legacy of Ramanujan which took place at the University of Illinois at Urbana-Champaign on June 6 – 9, 2019. The conference included 26 plenary talks, 71 contributed talks, and 170 participants.

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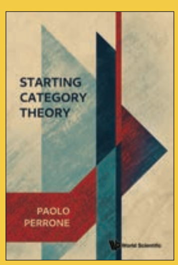
by **Paolo Perrone** (University of Oxford, UK)



Starting Category Theory serves as an accessible and comprehensive introduction to the fundamental concepts of category theory. Originally crafted as lecture notes for an undergraduate course, it has been developed to be equally well-suited for individuals pursuing self-study. Most crucially, it deliberately caters to those who are new to category theory, not requiring readers to have a background in pure mathematics, but only a basic understanding of linear algebra.

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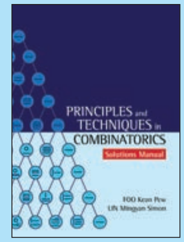

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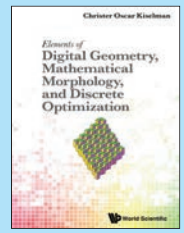

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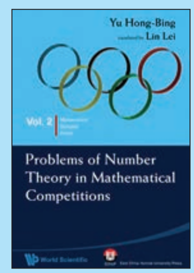

In this book, the author introduces some basic concepts and methods in elementary number theory via problems in mathematical competitions.

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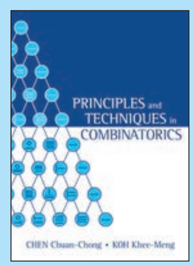

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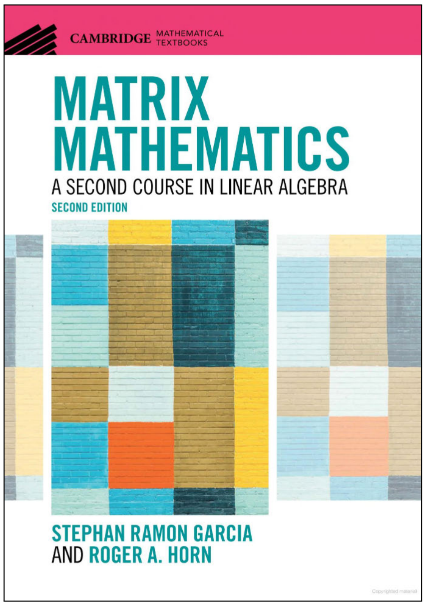
BOOK REVIEWS

Matrix Mathematics: A Second Course in Linear Algebra by Stephan Ramon Garcia and Roger A. Horn

Cambridge University Press, April 2023, ISBN 978-1-108-83710-1, xix+467 pages
Reviewed by Fuzhen Zhang, Nova Southeastern University, zhang@nova.edu



This is the second edition of *A Second Course in Linear Algebra* by Garcia and Horn. The new edition is entitled *Matrix Mathematics: A Second Course in Linear Algebra*, emphasizing “matrices” and reflecting a matrix-based approach to advanced linear algebra. The book can be used as the text for a second course in linear algebra and matrix theory for upper-level undergraduates in mathematics, statistics, computer science, data science, and the physical sciences.

I learned and benefited a great deal from the first (2017) edition of the book in my teaching and research in linear algebra and matrix theory, and I am now enjoying the second edition. The second edition is indeed much improved in several ways (from the mathematical content to the page layout to the color figures). Compared to the first edition, the new edition has added material on topics such as matrix norms, positive (nonnegative) matrices, polynomial interpolation, Gaussian quadrature, orthogonal polynomials, and the Google matrix.

The Preface (for the Second Edition) gives detailed information about what is new to this edition, as well as the readership, key features, coverage, and structure of the book, followed by a list of notation. Useful lists of notation and indices of terms are always handy and appreciated when one encounters an unfamiliar symbol or term in the book.

Topics are carefully selected (for the targeted readers), thoroughly discussed, and all results are rigorously derived and clearly shown in exposition. Take the Jordan canonical form (Chapter 12), which is one of my favorite topics, as an example. The authors start off with the ranks of the powers of a square matrix, then introduce Jordan blocks, then proceed to show the existence and uniqueness of the Jordan form. The topic is gradually introduced from basic theory (rank) to the fundamental result (the Jordan canonical form) in a concise way. But the authors do not stop there (just with the theory); they exhibit applications of Jordan canonical forms to differential equations and to stochastic matrices (Chapter 13). As a reader and an instructor, I would not and could not ask for more or less on the topic. It is clear how the material has been edited to reflect classroom experience.

The book contains 20 chapters; each chapter is focused on a single topic, as the title of the chapter reflects. The chapters, as listed in the Table of Contents, are:

1. Vector Spaces
2. Bases and Similarity
3. Block Matrices
4. Rank, Triangular Factorizations, and Row Equivalence
5. Inner Products and Norms
6. Orthonormal Vectors
7. Unitary Matrices
8. Orthogonal Complements and Orthogonal Projections
9. Eigenvalues, Eigenvectors, and Geometric Multiplicity
10. The Characteristic Polynomial and Algebraic Multiplicity
11. Unitary Triangularization and Block Diagonalization
12. The Jordan Form: Existence and Uniqueness
13. The Jordan Form: Applications
14. Normal Matrices and the Spectral Theorem
15. Positive Semidefinite Matrices
16. The Singular Value and Polar Decompositions
17. Singular Values and the Spectral Norm
18. Interlacing and Inertia
19. Norms and Matrix Norms
20. Positive and Nonnegative Matrices

Followed by:

Appendix A. Complex numbers
Appendix B. Polynomials
Appendix C. Basic Linear Algebra
Appendix D. Induction

The prerequisites for the book consist of a first course in linear algebra and a basic calculus sequence. The appendices are as essential as the prerequisites, because they are used frequently and freely. Some important preliminary facts include (a) the polar form of a complex number, (b) the Fundamental Theorem of Algebra, (c) elementary matrix algebra, and (d) mathematical induction.

Each chapter ends with a Notes section that provides more information about the chapter for any reader who wants to further explore the topic, followed by a bulleted list summarizing important concepts that helps students review and recap the basics of the chapter.

The book contains over 900 exercises (problems). Complete solutions to all problems are available in an instructor's solution manual. There are 350 conceptual and numerical examples and many color illustrations that help students to understand concepts and theorems, and to master ideas and skills.

Over the years (since my first time teaching a linear algebra course), I have had a few pedagogical issues of linear algebra on my mind. These issues might have been overlooked or dealt with carelessly by many authors (and perhaps readers too). Going through this book, I paid particular attention to my questions and checked with curiosity how the authors of this book chose to clarify these issues.

(a) **Vector addition associativity.**

One of the axioms for a vector space (say V) is the associativity of the vector addition, that is, $u+(v+w) = (u+v)+w$ for all $u, v, w \in V$.

After defining a vector space via the axioms for vector addition and scalar multiplication, some texts (including some popular ones) give no further introduction to the sum of multiple (say n) vectors. Here is a typical statement:

> "Let $v_1, \ldots, v_n$ be n vectors in V and let $v = v_1 + \cdots + v_n$."

Without proof or explanation, what does $v_1 + v_2 + \cdots + v_n$ really mean?

I have checked some textbooks on my desk. For example, in Lax's *Linear Algebra* [4], the axiomatic definition of a vector space is given on page 1, then the notion of a linear combination of multiple vectors is given on page 2, and there is no proof or explanation about the sum of multiple vectors. A student might ask, "What is $v_1+v_2+v_3+v_4$? Is $(v_1 + v_2) + (v_3 + v_4) = ((v_1 + v_2) + v_3) + v_4$?" Another such example can be found in Apostol's *Linear Algebra* [1], on pages 91 and 97.

(Note: That I single these books out does not imply that I dislike the books. In fact, I use the books often and I have great respect for their authors.)

In Garcia and Horn's book, on page 3, a brief explanation is given of the meaning of the sum $v_1 + v_2 + \cdots + v_n$ (a finite sum). A detailed treatment (usually involving a proof by induction) is found in an abstract algebra course. (See, for example, J. Rotman, *A First Course in Abstract Algebra with Applications* [7], page 130.) In a linear algebra course, such a detailed abstract proof would be superfluous (or may be left as an exercise for students), but a short explanation is still necessary. I think the authors have the right take on this issue.

Note: We face the same issue for the product of matrices, say $A_1A_2 \cdots A_n$. The associative law (of three matrices or vectors) permits us to omit parentheses when denoting such a product (or sum). A rigorous proof of this is a serious (mathematical) business.

(b) **List versus set.**

The authors use a *list* of vectors rather than a *set* of vectors. As they explain (on page 8), a *list of vectors* is a nonempty, finite, ordered sequence of vectors. A subtle, but important, point is that a given vector can appear more than once in a list. In contrast, a *set* is defined in Appendix C, page 432, to be *a collection of distinct elements.* This is the rigorous, formal definition of the mathematical term *set.*

For example, v_1, v_2, v_2, v_3 is a list of four vectors in V. However, the set of vectors in the list is $\{v_1, v_2, v_3\}$, a set of three distinct elements. Note that a list is different from a multiset, as the elements in a multiset (or set) are unordered.

When bases and coordinates are involved in a discussion, *list* is superior to *set* (or multiset). So, a *list* (of vectors) is more precise than a *set* (of vectors) that we (or at least I) often say. In the latter, we usually allow repetition of elements in a "set" (that is, we use the notion of *set* loosely).

(c) **Span of the empty set.**

When we teach linear algebra, it is inevitable to consider a subspace spanned by a set. What if the set is the empty set? Do we define a subspace spanned by the empty set? The question is about the statement *every subset spans a subspace.* Some authors of (elementary) linear algebra texts do not include the empty set.

In this book (as in many other texts), the span of the empty set is defined to be $\{0\}$, that is, $\operatorname{span}\emptyset = \{0\} = \operatorname{span}\{0\}$ (page 9).

In the context of vector spaces, since the empty set contains no vectors at all, it cannot span any non-trivial vector space. In view of this, the span of the empty set is defined to be the vector space consisting of just the zero vector. This definition is sometimes needed for technical reasons to simplify exposition and proofs. This would be my explanation (as a reviewer).

(d) **The degree of the zero polynomial is $-\infty$.**

Regarding the degree of the zero polynomial, I checked several (popular) references. Listed below are some exemplary ones; they show how the degree of the zero polynomial has been treated (defined):

(i) "symbol $-\infty$": N. Jacobson, *Basic Algebra I* [3], page 128, with the conventions $-\infty < n$ and $-\infty + n = -\infty$ (for $n = 0, 1, 2 \dots$) and $-\infty + (-\infty) = -\infty$.
(ii) not mentioned: R. Horn and C. Johnson, *Matrix Analysis* [2].
(iii) "has no degree": T. Apostol, *Linear Algebra* [1], page 93.
(iv) "for technical reasons, not defined": D. Lay et al. [5], page 194.

In this book, the degree of the zero polynomial is defined to be $-\infty$ (page 428, Appendix B), and without further explanation, evidently (to the reviewer) because this is dealt with in an elementary calculus course. I personally prefer (i).

(e) **The Gershgorin disk theorem.**

It is my personal belief that any linear algebra text that studies eigenvalues should contain the Gershgorin disk theorem, at least the first part, because it is beautiful, simple, and useful.

Part I of the Gershgorin disk theorem: The eigenvalues of a square matrix are all contained in (or on) the Gershgorin disks.

Part II of the Gershgorin disk theorem: If k of the Gershgorin disks form a region that is disjoint from all the other Gershgorin disks, then the region contains exactly k eigenvalues of the matrix.

By drawing disks (circles) in a plane, one can estimate the eigenvalues of the given matrix. There is no other result (in linear algebra) that is as intuitive as the Gershgorin disk theorem.

This book nicely presents Part I of the Gershgorin disk theorem, illustrated with several examples. As an application of the theorem, it is shown that a (strictly) diagonally dominant matrix is invertible.

The proof of Part II of the Gershgorin disk theorem requires advanced machinery (such as the argument principle) of complex analysis (while a proof by eigenvalue continuity is considered invalid by some [6]) that is not assumed as a prerequisite, and thus such a proof is omitted from the book. However, Part II of the theorem is addressed in the Notes for Chapter 9 (page 201).

I think this is the best way to expose the theorem to undergraduate students who are taking a second course of linear algebra.

To conclude, as a group of mathematicians who pre-read the book manuscript put it (as quoted on the book's back cover): "It is an excellent book." I like this book very much and highly recommend it.

This review reflects the reviewer's personal opinions and interests. The reviewer is grateful to Prof. R. Horn for his valuable comments.

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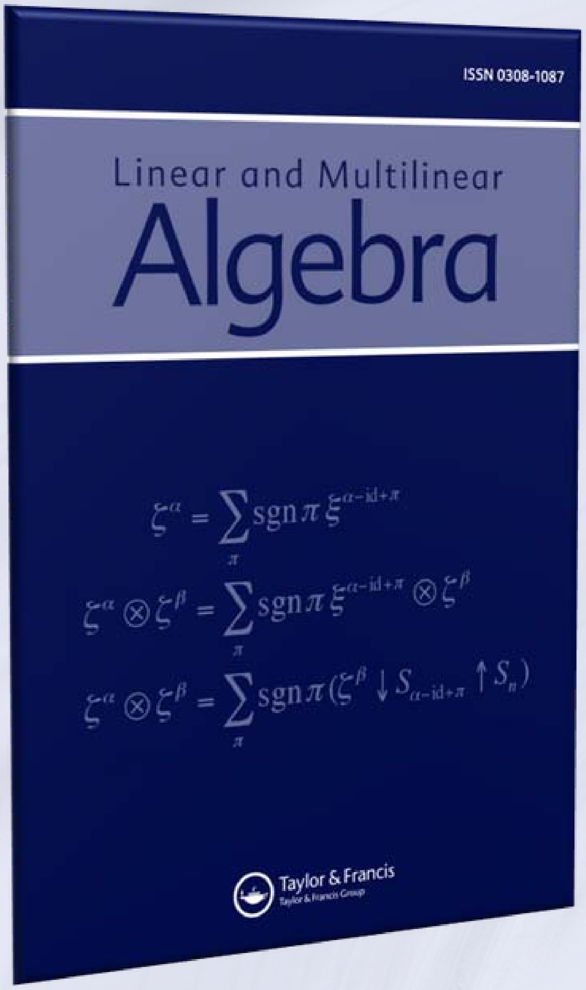
LINEAR AND MULTILINEAR ALGEBRA



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ILAS NEWS

2025 Hans Schneider Prize awarded to Dario Bini and Chi-Kwong Li

There are two recipients of the 2025 Hans Schneider Prize:

Dario Bini (Università di Pisa) was chosen in recognition of his substantial contributions to several areas of computational linear algebra, including matrix multiplication, polynomials and structured matrices, Markov chains, and algebraic Riccati equations. He will present the Hans Schneider Prize Lecture at the 26th ILAS Conference in Kaohsiung, Taiwan (`https://ilas2025.tw`), June 23–27, 2025.

Chi-Kwong Li (College of William and Mary) is cited for substantial contributions to several areas of linear algebra, including matrix analysis, operator theory, quantum information theory, and combinatorial matrix theory. He will present his Prize Lecture at ILAS 2026 in Blacksburg, Virginia (`https://ilas2026.math.vt.edu`), May 18–22, 2026.

Peter Šemrl Selected as 2025 AMS Fellow

The American Mathematical Society (AMS) has recently announced its 2025 class of AMS Fellows, recognizing individuals who have made outstanding contributions to the creation, exposition, advancement, communication, and utilization of mathematics.

ILAS member (and former ILAS president) Peter Šemrl, currently Director of the Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia, was among those cited.

More information and details on the AMS Fellows program, including a list of all of the 2025 fellows, can be found at `http://www.ams.org/profession/ams-fellows/ams-fellows`.

OBITUARY NOTICES

Ron Smith, 1936–2024

Submitted by Charlie Johnson

Ron Smith passed away on October 16th, 2024. He had been battling Parkinson's disease for several years.

Ron received his Ph.D. in 1976 from Auburn University, after working with Emilie Haynesworth (who also advised Tom Markham, 1939–2021). His thesis title was "Generalized Circulant and Block Centrosymmetric Matrices." This sort of structured matrix theory became more popular later. After a short stint as Assistant Professor at Northern Arizona University, he moved to the University of Tennessee at Chattanooga (UTC) in 1980, as Associate Professor, and was promoted to Professor in 1987. He spent nearly 40 years at UTC, was a leading academic in the Mathematics Department, and was a highly respected teacher. While at UTC, he received many internal honors, including becoming a UTC Distinguished Professor in 1991. He was also an organizer of several meetings in matrix analysis, including those at UTC. He was well known by most of us and well liked.

Ron was steadily productive in matrix analysis, working primarily, but not exclusively, in M-matrices, nonnegative matrices and related topics. Among his 40 publications was a long book chapter on Schur complements (with Charlie Johnson), stressing inheritance properties (and related topics), and the recent book *Matrix Positivity* from Cambridge University Press (with Charlie Johnson and Michael Tsatsomeros).

Despite his important work, it is not clear that mathematics was Ron's main claim to fame! Ron was a world-famous tournament bridge player, high in the top tier in North America. With 30,400 master points, he rose to be the 24th highest ranked player of all time in North America before he had to stop playing. Many mathematicians enjoy bridge, but Ron was likely the most successful among them. He had hundreds of regional and sectional titles to his name, as well as several national titles.

Ron is survived by his wife Linda, who was his long-time (most frequent and favorite) bridge partner, and by their four children. Linda was his devoted caregiver in his final years. Ron was well-liked by everyone and contributed to all phases of the profession. He organized meetings, was a frequent and careful referee, and helped with all kinds of administration. In a research project, he could be counted upon to contribute his share of ideas and do his share of the work. He will be missed, not only as a collaborator, and occasional bridge partner, but as a true friend.

Rien Kaashoek, 1937–2024

Submitted by Harm Bart, Gilbert Groenewald, Sanne ter Horst, Andé Ran and Hugo Woerdeman

Rien Kaashoek passed away peacefully in his sleep on November 21st, 2024, at the age of 87. Rien was a distinguished mathematician, renowned for his pioneering work in the fields of analysis and operator theory, with connections to matrix analysis and mathematical systems theory. Rien played an instrumental role in the development of the International Workshop on Operator Theory and its Applications (IWOTA), serving on the workshop's steering committee since its inception in 1981. He was the organizer of one of the early IWOTA meetings in Amsterdam in 1985 and crucial in initiating the collaboration between ILAS and IWOTA. A dedicated researcher and educator, Rien leaves behind a remarkable legacy of seventeen Ph.D. students, well over 200 research publications, and ten influential research monographs, many of which continue to be vital references in their respective fields. Beyond his professional accomplishments, Rien was known for his warmth, kindness, and dedication to his students and colleagues. His generosity as a mentor helped shape the early careers of many in the mathematical community, and he will be deeply missed by all who knew him.

JOURNAL ANNOUNCEMENTS

Special Issue of *Linear Algebra and its Applications* Dedicated to Daniel B. Szyld

Contributed announcement from
Volker Mehrmann, *Linear Algebra and its Applications* co-Editor-in-Chief

Linear Algebra and its Applications (*LAA*) is pleased to announce a special issue in honor of Professor Daniel B. Szyld (Temple University, Philadelphia, USA) on the occasion of his 70th birthday (which will occur in 2025) and in recognition of his many important contributions to applied and numerical linear algebra.

LAA solicits papers for the special issue within the entire scope of *LAA*, with a special emphasis on research topics related to the work of Daniel Szyld. The deadline for submissions is February 28th, 2025. All submissions will be subjected to normal refereeing procedures and the usual standards of *LAA* will be applied.

Manuscripts should be submitted via the Editorial Manager System for *LAA* (`https://www.editorialmanager.com/laa`), by choosing the special issue called "In Honor of Daniel Szyld" and selecting Volker Mehrmann as the responsible Editor-in-Chief.

Authors will have the opportunity to suggest one of the following special editors to handle their submission: Edmond Chow, Froilán Dopico, Marcel Schweitzer, Valeria Simoncini, Kirk Soodhalter.

Send News for *IMAGE* Issue 74

IMAGE seeks to publish all news of interest to the linear algebra community. Issue 74 of *IMAGE* is due to appear online on June 1, 2025. Send your news for this issue to the appropriate editor by April 15, 2025. Photos are always welcome, as well as suggestions for improving the newsletter. Please send contributions directly to the appropriate editor:

- book reviews to Mohsen Aliabadi (`maliabadisr@ucsd.edu`)
- linear algebra education news and articles to Anthony Cronin (`anthony.cronin@ucd.ie`)
- interviews of senior linear algebraists to the editor-in-chief, Louis Deaett (`louis.deaett@quinnipiac.edu`)
- problems and solutions to Jeffrey Stuart (`jeffrey.stuart@plu.edu`)
- advertisements to Amy Wehe (`awehe@fitchburgstate.edu`)
- announcements and reports of conferences/workshops/etc. to Jephian C.-H. Lin (`jephianlin@gmail.com`)
- other articles and proposals to the editor-in-chief, Louis Deaett (`louis.deaett@quinnipiac.edu`)

Send all other correspondence to the editor-in-chief, Louis Deaett (`louis.deaett@quinnipiac.edu`).

For past issues of *IMAGE*, please visit `https://www.ilasic.org/IMAGE`.

CONFERENCE REPORTS

GODSIL75
Waterloo, Canada, March 15–17, 2024

Report by Ada Chan

The virtual workshop Godsil75 was held on March 15–17, 2024 and was supported and hosted by the Department of Combinatorics and Optimization, University of Waterloo. The workshop was organized to celebrate the life, work and birthday of Chris Godsil.

Chris Godsil's imprint on mathematics goes beyond algebraic combinatorics to combine theory and application, emerging research areas and well-developed mathematical tools. He is a research leader in discrete mathematics and continues to be influential in a wide range of mathematical subdisciplines and communities around the world. As a leading member of the Canadian combinatorial community, working first at Simon Fraser University and then at the University of Waterloo, he has shaped multiple generations of mathematicians in Canada and abroad. This workshop not only honoured Chris Godsil's contributions and his impact on mathematics, but also brought together researchers in discrete mathematics whose work has been influenced by him.

Chris Godsil on his final slide

The list of speakers and talks included:

- Gabriel Coutinho (Universidade Federal de Minas Gerais, Brazil), "Some results on the interplay of graph spectra and quantum walks"
- Chris Godsil (University of Waterloo, Canada), "Christopher, what have you done?" (`https://youtu.be/ZxFCjpDvXdI`)
- Wilfried Imrich (Montanuniversität Leoben, Austria), "Hierarchical graph products"
- Aleksandar Jurišić (University of Ljubljana, Slovenia), "Tight distance-regular graphs"
- William J. Martin (Worcester Polytechnic Institute, USA), "Navigating eigenspaces of association schemes"
- Brendan McKay (Australian National University, Australia), "Some exercises in combinatorial generation"
- Karen Meagher (University of Regina, Canada), "Algebraic approaches to Erdös–Ko–Rado theorem: an Afterword"
- Bojan Mohar (Simon Fraser University, Canada), "On extremal eigenvalues of trees and tree-like graphs"
- Gordon Royle (University of Western Australia, Australia), "Hamilton cycles in cubic and other graphs"
- Hanmeng Zhan (Worcester Polytechnic Institute, USA), "From continuous to discrete"
- Xiaohong Zhang (Université de Montréal, Canada), "Continuous quantum walks"

There were around 170 participants registered for the workshop. A few of the participants and speakers (who were locally situated, including the guest of honor) attended the workshop in person at the University of Waterloo.

Participants at the GODSIL75 workshop

The workshop was organized by Soffía Árnadóttir (Danmarks Tekniske Universitet), Ada Chan (York University), Qiuting Chen (Universität Paderborn), Sabrina Lato (Umeå Universitet) Mariia Sobchuk (University of Waterloo), and Christino Tamon (Clarkson University).

A Workshop on the Occasion of the 75th Birthday of David S. Watkins ([DW75]) Leuven, Belgium, May 9–10, 2024

Report by Raf Vandebril and Thomas Mach

The [DW75] Workshop took place at the Arenberg Castle of KU Leuven, over two days, May 9–10, 2024, immediately before the SIAM Conference on Applied Linear Algebra 2024 in Paris. About forty participants came together in Leuven to celebrate David S. Watkins's outstanding contributions to numerical linear algebra and in particular to the computation of eigenvalues and eigenvectors.

There were 22 talks—some light and some enlightening—on various subjects linked to David's research interests. The complete program is available at `https://sites.google.com/view/dw75atkuleuven/program`.

Participants at the DW75 workshop

The 17^{th} Western Canadian Linear Algebra Meeting (WCLAM) Calgary, Canada, May 25–26, 2024

Report by Shaun Fallat

The 17th iteration of the Western Canadian Linear Algebra Meeting (WCLAM) took place at the University of Calgary, on May 25–26, 2024. This 31-year-old conference series has been hosted across Western Canada and Washington State. It provides an opportunity for researchers in linear algebra and related fields to present accounts of their current work through lectures or poster presentations, and to engage all participants in informal discussions.

Traditionally, this regional conference attracts participants from Western Canada and the Western United States. However, WCLAM '24 represented a special iteration of this workshop series centered on celebrating the 95th birthday of Professor Peter Lancaster. WCLAM '24 had an additional focus of highlighting Peter Lancaster's work and lasting contributions by bringing together some of his colleagues and former highly qualified personnel (students, postdoctoral fellows, and trainees).

A unique feature of this special conference celebration was the hybrid format. Among the 45 participants, 17 participated online via Zoom. Furthermore, 7 of the 19 presentations were offered over Zoom, including that of one featured speaker. At the conclusion of the conference, there was time for closing remarks and tributes by friends and colleagues of Professor Peter Lancaster. A highlight of the meeting was the dinner at the restaurant "Notable," where participants were joined by some of Peter's family, and we enjoyed a heartfelt presentation by one of Peter's daughters discussing aspects of Peter's professional and personal life. WCLAM '24 featured three invited speakers: Prof. Amir Amiraslani (Capilano University), Prof. Panayiotis Psarrakos (National Technical University of Athens) and Prof. Françoise Tisseur (University of Manchester). For more information about the conference program, visit the conference webpage:

`https://uregina.ca/~sfallat/wclam24`

WCLAM '24 was generously supported by the Pacific Institute for the Mathematical Sciences and the Department of Mathematics and Statistics at the University of Calgary. Special thanks go out to Dr. Matthew Greenberg, Head of the Department of Mathematics and Statistics at the University of Calgary, for his time and help organizing this very special event!

Group photo from WCLAM '24

The 15$^{\text{th}}$ Workshop on Matrices and Operators (MAO) Reno, USA, June 14–17, 2024

Report by Valentin Deaconu, Alex Kumjian, Pan Shun Lau, Chi-Kwong Li, and Tin-Yau Tam

The 15th Workshop on Matrices and Operators (MAO 2024) took place June 14–17, 2024, at the University of Nevada, Reno (UNR), Nevada, USA. Organized by Valentin Deaconu, Alex Kumjian, Pan Shun Lau, Chi-Kwong Li, and Tin-Yau Tam, the workshop aimed to stimulate research and foster interaction among mathematicians in all aspects of matrix theory, operator theory, operator algebras, and related topics. It also provided an opportunity for researchers to exchange ideas. Approximately thirty participants from different parts of the world attended the workshop. Eighteen invited talks were delivered, with speakers including Bibhas Adhikari, Man Duen Choi, Dominique Guillot, Huajun Huang, Alex Kumjian, Jimmie Lawson, Pan Shun Lau, Avleen Kaur, Zehua Lai, Chi-Kwong Li, Jephian C.-H. Lin, Raymond Sze, Mohammad Shavandi, Ming-Cheng Tsai, Qing-Wen Wang, Xiang Xiang Wang, Kevin Wu and Run Zheng. A conference dinner was held on June 15th, followed by an excursion to Lake Tahoe and a party at Tin-Yau Tam's house on June 16th. The workshop concluded with an informal research discussion on June 17th.

Participants at MAO 2024

The event was generously supported by the Department of Mathematics and Statistics at UNR. The program, presentation slides, and conference photos can be found at:

`https://sites.google.com/view/tinyautam/homepage/mao-2024`

The previous fourteen workshops were held in different places since 2007. More information can be found at:

`https://cklixx.people.wm.edu/mao-history.html`

AESIM School on Linear Preserver Problems
Bhopal, India, July 8–13, 2024

Report by Rohit Dilip Holkar and Sushil Singla

An Asian and European Schools in Mathematics (AESIM) School on Linear Preserver Problems was held at the Indian Institute of Science Education and Research Bhopal (IISER Bhopal) from July 8–13, 2024. The chief guest, the Director of IISER Bhopal, Prof. Gobardhan Das, inaugurated the event. Prof. Peter Šemrl from Slovenia and Prof. C.-K. Li from the USA are the experts who guided around fifty participants from around the globe. The participants included faculty members, postdocs, and Ph.D. and M.Sc. students from various research institutes and universities. This programme was organized by Dr. Rohit Holkar and Dr. Atryee Bhattacharya (both of IISER Bhopal), and Dr. Sushil Singla (Slovenia). The programme was funded by the Centre International de Mathématiques Pures et Appliquées (CIMPA) and the National Board for Higher Mathematics India.

Group photo from the AESIM School on Linear Preserver Problems

Prof. Li used the Moore Method of teaching, while Prof. Šemrl had interactive sessions. Group discussions were conducted, and students remarked that they were useful and encouraging. Attendance was recorded; and made mandatory. All sessions were interactive. Some of Prof. Li's tutorials asked students to formulate research problems in their areas of research using his talks. A few of the tutorial problems were open research questions. Some students have taken these problems up for their research work, and they are possibly working on them at present.

An informal gathering at the AESIM School on Linear Preserver Problems

Instruction was led by Peter Šemrl and Chi-Kwong Li

The 35th International Workshop on Operator Theory and its Applications (IWOTA) Kent, UK, August 12–16, 2024

Report by Ian Wood

The International Workshop on Operator Theory and its Applications (IWOTA) 2024 took place at the University of Kent, Canterbury, UK, August 12–16, 2024. IWOTA is a major series of annual workshops in mathematical analysis which covers operator-theoretic aspects of topics such as complex analysis, harmonic analysis, linear algebra, random matrix theory, mathematical physics, and their applications, such as control theory, signal processing and AI. As such, IWOTA has a strong overlap with the ILAS community; the conference was endorsed by ILAS, who generously provided funding for two of the invited speakers.

The conference attracted 384 registered participants. The busy programme consisted of 9 plenary talks, 12 semi-plenary talks, and 24 special sessions.

The plenary talks explored themes ranging from operator theory and AI to geometric measure theory and tropical geometry, as well as random matrices, nonlinear Schrödinger equations, fractional PDEs, graph Laplacians, and numerical methods for eigenvalue computations. They included the Israel Gohberg ILAS-IWOTA Lecture by Mark Embree (Virginia Tech) on "Contour Integral Eigensolvers through the Lens of System Identification" and the ILAS lecture "Solving Parameter Dependent Eigenvalue Problems using Taylor Series and Chebyshev Expansions" by Melina Freitag (University of Potsdam).

In addition, there were semi-plenary talks which covered topics including quantum network correlations, polynomial optimisation via computation of spectra, non-self-adjoint Schrödinger operators and Toeplitz operators, Fredholm determinants, dilation theory, invariant subspaces, and closed ideals of operator algebras. The talk by Froilán Dopico (Universidad Carlos III de Madrid) on "Polynomial and rational matrices with prescribed data" would have been of particular interest to ILAS members.

The special sessions again addressed a wide range of areas in operator theory. They provided the setting for more focused talks and gave opportunities for participants to present their results and interact with other researchers with similar interests. Among them were sessions on "Linear Algebra and Control Theory" organised by André Ran (Amsterdam) and Volker Mehrmann (TU Berlin), "Numerical Ranges" run by Ilya Spitkovsky (NYU, Abu Dhabi) and Tin-Yau Tam (University of Nevada, Reno), and "Special Matrices" coordinated by Natalia Bebiano (University of Coimbra) and Mikhail Tyaglov (Shanghai Jiao Tong University).

The social programme consisted of a walking tour of Canterbury on Wednesday afternoon and a conference dinner on Thursday evening.

IWOTA was supported by funding from the London Mathematical Society, the Heilbronn Institute for Mathematical Research, the National Science Foundation, the European Mathematical Society, ILAS, and the School of Mathematics, Statistics and Actuarial Science at the University of Kent.

Proceedings of the meeting will be published in the Birkhäuser series *Operator Theory: Advances and Applications*.

For more details, please see the conference website at `https://blogs.kent.ac.uk/iwota2024`.

IWOTA Conference Photo

Presentation of the certificate for the Israel Gohberg ILAS-IWOTA lecture to Mark Embree

UPCOMING CONFERENCES AND WORKSHOPS

ILAS at the Joint Mathematics Meetings
Seattle, USA, January 8–11, 2025

ILAS Invited Address

Anne Greenbaum, University of Washington
Are Iterative Linear System Solvers Backward Stable?
Thursday, January 9th, 2025, 9:40AM–10:45AM

Abstract: This is a question that I often discuss with colleagues but we seldom come to a definitive conclusion. One reason is that "backward stability" is not precisely defined – it is defined in a number of different ways throughout the literature. Roughly, a backward stable algorithm for solving $Ax = b$, when implemented on a machine with unit roundoff u, produces a result $\hat{x}$ that satisfies $(A+\Delta A)\hat{x} = b+\Delta b$, where $\|\Delta A\| \leq \epsilon\|A\|$ and $\|\Delta b\| \leq \epsilon\|b\|$, where $\epsilon = p(n)u+O(u^2)$ and $p(n)$ is a polynomial in the problem size n. However, one must be careful to specify the domain of allowable matrices A, which may depend on u, as well as the details of the implementation. Additionally, for some iterative methods, there is a dependence on k, the number of steps for which the algorithm is run or, perhaps, a bound on the number of steps needed for a corresponding problem assuming exact arithmetic; thus we may need to replace $p(n)$ by $p(n, k)$. In this talk, I will survey the literature on the attainable accuracy of iterative methods such as simple iteration (Jacobi, Gauss-Seidel, SOR, iterative refinement), steepest descent, and the conjugate gradient and Lanczos algorithms.

ILAS Special Sessions

ILAS Special Session on Strong Properties of Matrix Classes
Wednesday, January 8th, 2025, 8:00AM–12:00PM and 1:00PM–5:00PM

- Bryan L. Shader, University of Wyoming
- Minerva Catral, Xavier University

ILAS Special Session on Matrix Analysis and Applications
Wednesday, January 8th, 2025, 8:00AM–12:00PM and 1:00PM–5:00PM

- Tin-Yau Tam, University of Nevada, Reno
- Mohsen Aliabadi, University of California, San Diego
- Luyining Gan, Beijing University of Posts and Telecommunications

ILAS Special Session on Innovative and Effective Ways to Teach Linear Algebra
Thursday, January 9th, 2025, 1:00PM–5:00PM

- David M. Strong, Pepperdine University
- Sepideh Stewart, University of Oklahoma
- Gil Strang, MIT
- Megan Wawro, Virginia Tech

ILAS Special Session on Randomness in Numerical Linear Algebra
Saturday, January 11th, 2025, 9:00AM–12:00PM and 1:30PM–4:30PM

- Anne Greenbaum, University of Washington
- Heather Denise Wilber, University of Washington

ILAS Special Session on 05C50 Offline
Friday, January 10th, 2025, 8:00AM–12:00PM and 1:00PM–5:00PM

- Hermie Monterde, University of Manitoba
- Stephen Kirkland, University of Manitoba

ILAS Special Session on Preserver Problems
Saturday, January 11th, 2025, 8:30AM–11:00AM and 1:00PM–4:30PM

- Edward Poon, Embry-Riddle Aeronautical University
- Chi-Kwong Li, College of William and Mary
- Sushil Singla, PIMS Postdoctoral Fellow, University of Regina
- Bojan Kuzma, University of Primorska

ILAS Special Session on Inverse Spectral Problems for Nonnegative Matrices
Thursday, January 9th, 2025, 1:00PM–3:30PM

- Pietro Paparella, University of Washington Bothell

At the JMM Awards Celebration (i.e., the prize ceremony) on Wednesday, January 8th at 4:45PM, Anne Greenbaum, as the ILAS speaker, will be presented with an award certificate.

Special session on **Applied and Numerical Linear Algebra** ***at the*** **95th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM) Poznań, Poland, April 7–11, 2025**

The 95th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM) will be hosted at the Poznań University of Technology, April 7–11th, 2025 in Poznań, Poland.

On behalf of the organizing committee, we would like to invite you, your colleagues, postdocs and graduate students to join **Section S17: APPLIED AND NUMERICAL LINEAR ALGEBRA**. The Section organizers are

- Agnieszka Międlar, Virginia Tech, Blacksburg, VA, USA (amiedlar@vt.edu) and
- Michał Wojtylak, Jagiellonian University in Kraków, Poland (michal.wojtylak@uj.edu.pl).

The topical speakers of the session are

- Froilán M. Dopico (Universidad Carlos III de Madrid) and
- André Uschmajew (Augsburg University).

Contributed talks in this session will have a length of 15 minutes plus 5 minutes for discussion. Once the abstracts have been received, depending on the number of submissions and allowed time slots, we may have to select an appropriate number of abstracts for presentations.

For more detailed information concerning the submission of abstracts as well as registration and accommodation, please visit the conference website at

https://jahrestagung.gamm.org/annual-meeting-2025/95th-annual-meeting-2

The deadline for abstract submissions is December 15th, 2024. The deadline for early registration is January 21st, 2025. Online registration will close on March 21st, 2025.

Please note that we cannot provide any financial support or exception from the registration fee for participants.

The 9th Linear Algebra Workshop (LAW'25) Portorož, Slovenia, June 2–6, 2025

http://www.law05.si/law25

The LAW'xx meetings are back after a short "pandemic delay," with an opportunity to mark the round anniversary of the founder of these gatherings, Heydar Radjavi. The event is co-organized by (in alphabetical order) the Institute of Mathematics, Physics, and Mechanics (IMFM); the University of Ljubljana; and the University of Primorska.

Confirmed Invited Speakers:

- Jane Breen (Ontario Tech University, Canada)
- Doug Farenick (University of Regina, Canada)
- João Gouveia (University of Coimbra, Portugal)
- Laurent Marcoux (University of Waterloo, Canada)
- Lajos Molnár (University of Szeged and University of Budapest, Hungary)
- Clément de Seguins Pazzis (Université de Versailles Saint-Quentin-en-Yvelines, France)
- Ryotaro Tanaka (Tokyo University of Science, Japan)

ILAS Lecturer at a non-ILAS conference:

- Helena Šmigoc (University College Dublin, Ireland)

Young researcher sections. Short presentations by Ph.D. students or postdocs are welcome, as usual at LAW'xx meetings.

Working groups. Much of math research is done through discussing open problems. We always do that, but not at conferences, at least not officially. At LAW'xx gatherings, we do. Thus, the number of published papers written or at least started at these meetings is greater than the number of meetings itself. Here are the prearranged working groups (in alphabetical order of the first leader).

- Chi-Kwong Li: Preserver problems
- Mitja Mastnak and Heydar Radjavi: Local to global properties of collections of matrices
- Konrad Schmüdgen and Aljaž Zalar: Moment problems, positive polynomials and applications

Geography: As past LAW'xx meetings have visited changing locations in Slovenia, a seaside area was chosen this time, a former Venetian salt-harvesting "colony", where people are still Slovene-Italian bilingual. It includes saltpans at Sečovlje (Sicciole), the touristy place Portorož (Portorose) in the middle, and an ancient salt exporting port-fortress, Piran (Pirano).

Scientific Committee: Dijana Ilišević (University of Zagreb, Croatia); Chi-Kwong Li (William & Mary, USA) Raphael Loewy (Technion - Israel Institute of Technology, Israel); Mitja Mastnak (Chair, Saint Mary's University, Canada); Martin Mathieu (Queen's University Belfast, United Kingdom); João Filipe Queiró (University of Coimbra, Portugal); and Konrad Schmüdgen (University of Leipzig, Germany).

Organizing Committee: Ljiljana Arambašić (University of Zagreb, Croatia); Bojan Kuzma (Local Organizer, University of Primorska, Slovenia); Matjaž Omladič (Chair, University of Ljubljana, Slovenia); Nik Stopar (University of Ljubljana, Slovenia); and Aljaž Zalar (University of Ljubljana, Slovenia).

Deadlines: Abstracts of contributions and early bird registrations are welcome before January 31st, 2025.

Welcoming Statement: The LAW'25 organizing team is committed to promoting an environment where everyone can be themselves and achieve their full potential regardless of protected characteristic or socio-economic background. LAW'25 will foster an atmosphere where exchange of ideas is highly encouraged and all participants feel included, valued and safe. We believe that in such atmosphere people can be more engaged and achieve more.

The 17$^{\text{th}}$ Workshop on Numerical Ranges and Numerical Radii (WONRA) Taichung, Taiwan, June 19–21, 2025

The 17$^{\text{th}}$ Workshop on Numerical Ranges and Numerical Radii (WONRA) will be held at the National Chung Hsing University, Taichung, Taiwan, June 19 (Thu) – 21 (Sat), 2025. The purpose of the workshop is to stimulate research and foster interactions between researchers interested in the study of numerical ranges and numerical radii. This subject has a long and distinguished history, with connections and applications to different branches of pure and applied science such as operator theory, functional analysis, matrix norms, inequalities, numerical analysis, perturbation theory, matrix polynomials, and quantum information science. In fact, the 2025 WONRA will have a special session on Quantum Information Theory, in connection to the International Quantum Year 2025.

The 2025 WONRA organizing committee consists of:

- Ray-Kuang Lee, National Tsing Hua University, Taiwan (`rklee@ee.nthu.edu.tw`)
- Chi-Kwong Li, College of William & Mary, USA (`ckli@math.wm.edu`)
- Raymond Nung-Sing Sze, Hong Kong Polytechnic University, Hong Kong (`raymond.sze@polyu.edu.hk`)
- Ming-Cheng Tsai, National Taipei University of Technology, Taiwan (`mctsai2@ntut.edu.tw`)
- Ya-Shu Wang, National Chung Hsin University, Taiwan (`yashu@dragon.nchu.edu.tw`)
- Ngai-Ching Wong, National Sun Yat-sen University, Taiwan (`wong@math.nsysu.edu.tw`)

For additional information, please visit the workshop website:

`https://sites.google.com/email.nchu.edu.tw/wonra2025`

The 26th ILAS Conference
Kaohsiung, Taiwan, June 23–27, 2025

The 26th Conference of the International Linear Algebra Society will be held in the vibrant city of Kaohsiung, Taiwan, June 23–27, 2025. This prestigious event will bring together leading experts, researchers, and enthusiasts from around the world to share their knowledge and explore the latest advancements in the field of linear algebra.

Plenary speakers:

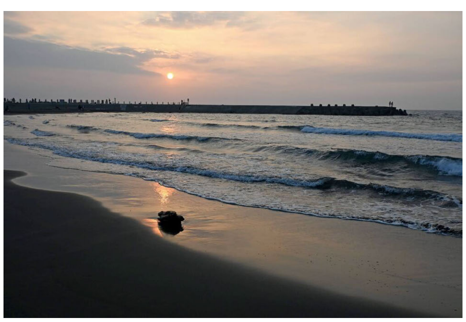
- Haim Avron (Tel Aviv University) **SIAG/LA Lecture**
- Dario Bini (Università di Pisa) **Hans Schneider Prize Lecture**
- Fan Chung Graham (University of California, San Diego)
- Fumio Hiai (Tohoku University)
- Daniel Kressner (EPFL) ***LAA* Lecture**
- Ren-Cang Li (University of Texas at Arlington)
- Karen Meagher (University of Regina)
- Polona Oblak (University of Ljubljana)
- Fernando De Terán Vergara (Universidad Carlos III de Madrid)
- Karol Życzkowski (Jagiellonian University)

ILAS2025 welcomes proposals for mini-symposia related to all topics within linear algebra. Each mini-symposium should feature between 4 and 12 speakers, with each talk having 20 minutes for the presentation and 5 minutes for questions. The deadline for submitting a mini-symposium proposal is December 31st.

For further details, including registration information and program updates, please visit the conference website:

`https://ilas2025.tw`

We look forward to welcoming you to this exciting event in 2025!



The 36th International Workshop on Operator Theory and its Applications (IWOTA)
Enschede, Netherlands, July 14–18, 2025

The IWOTA conference series is the largest and most important annual event in operator theory and its applications, bringing together leading international experts from pure mathematics and application areas to trace the future development of operator theory and related areas such as complex analysis, harmonic analysis, linear algebra, random matrix theory, and mathematical physics, as well as their applications, including control theory, signal processing, and AI.

IWOTA 2025 will take place July 14–18, 2025, at the University of Twente in Enschede, The Netherlands. It will provide a medium for an intense exchange of new results, information and opinions, and for international collaboration in operator theory and its applications worldwide. It will further set directions for future research through the conference activities and proceedings. A substantial part of IWOTA consists of special sessions whose organizers have been selected to ensure a coherent, diverse and attractive agenda of research activity and talks. Special sessions provide opportunities for all participants to present their results and interact with other researchers with similar interests.

The invited plenary speakers of IWOTA 2025 are:

- José Manuel Conde Alonso (Universidad Autònoma de Madrid)
- Hélène Frankowska (Sorbonne University)
- Philipp Grohs (University of Vienna)
- Volker Mehrmann (TU Berlin, ILAS lecture)
- Luz Roncal (Basque Center for Applied Mathematics)
- Roland Speicher (Saarland University)
- Walter van Suijlekom (Radboud University Nijmegen)
- Quanhua Xu (University of Franche-Comté, Harbin Institute of Technology)

The IWOTA 2025 organizing committee consists of Emiel Lorist (TU Delft), Felix Schwenninger (University of Twente), and Hans Zwart (University of Twente) in collaboration with the IWOTA executive steering committee members: J. William Helton (University of California, San Diego, chair), Sanne ter Horst (North-West University), Igor Klep (University of Ljubljana), Irene Sabadini (Polytecnico di Milano), Jani Virtanen (University of Helsinki and University of Reading), and Hugo J. Woerdeman (Drexel University).

Registration for the conference will open in Spring 2025. For further information, including a list of special sessions, see:

`https://www.utwente.nl/en/iwota2025`

The 3rd Workshop on Low-Rank Models and Applications (LRMA)
Mons, Belgium, September 11–12, 2025

The third workshop on Low-Rank Models and Applications (LRMA) will take place on the 11th and 12th of September 2025 at the University of Mons, Belgium. The LRMA workshop will offer a vibrant and intimate venue for interactions between researchers from fields such as numerical analysis, computer science, information theory, mathematics, and signal processing. The scientific program will include invited plenary lectures, as well as regular contributed talks and posters. The plenary speakers are:

- Stanislav Budzinskiy (University of Vienna)
- Luca Calatroni (CNRS, i3S laboratory of Sophia Antipolis)
- Alice Cortinovis (University of Pisa)
- Mariya Ishteva (KU Leuven)
- Paul Magron (LORIA, Centre INRIA de l'Université de Lorraine)
- Margherita Porcelli (University of Florence)
- Bertrand Rivet (Grenoble-INP)
- Lawrence Saul (Flatiron Institute)

The call for papers is available at `https://bit.ly/LRMA25`. The deadline for submitting an abstract is July 4th, 2025.

All relevant information can be found at: `https://sites.google.com/view/lrma25`

The 26th ILAS Conference
Blacksburg, USA, May 18–22, 2026

The 27th Conference of the International Linear Algebra Society will be held in Blacksburg, Virginia, May 18th–22nd, 2026, on the campus of Virginia Tech. The conference's theme, "Linear Algebra on the Blue Ridge: Panoramas of Theory and Application," highlights the geographical setting for the conference in the mountains of southwest Virginia, but also serves as an invitation to researchers from across linear algebra, ranging from core areas through to numerical analysis, applications, and linear algebra education.

Further details, such as registration information, lodging details, and program updates, will be available in due course from the conference website: `https://ilas2026.math.vt.edu`. For immediate questions, email the local organizing committee at `ilas2026@math.vt.edu`.

We look forward to a vibrant ILAS meeting in May 2026, and hope to see you in Blacksburg!

ONGOING ONLINE SEMINARS

Algebraic Graph Theory Seminar

https://math.uwaterloo.ca/~agtheory

Host: University of Waterloo

Schedule: weekly on Mondays

Time: 11:30AM, Waterloo (Ontario, Canada) time

Most recent talk:

On the Q-polynomial property of bipartite graphs with a uniform structure

Giusy Monzillo (University of Primorska, Slovenia)

Next talk:

December 2, 2024

Minimal spectral radius in a given class of graphs

Vishal Gupta (University of Delaware, USA)

Contact: Sabrina Lato (smlato@uwaterloo.ca)

Matrix Seminar

https://docs.google.com/document/d/1MswSd16JqsZE294kYCXujLio4cnAiuYv6QKRc6BxvI0/edit

Host: University of Nevada, Reno

Schedule: biweekly on Fridays

Time: 4:15PM, Reno (Nevada, USA) time

Most recent talk:

Lie all-derivable points of triangular algebras

Lei Liu (Xidian University, China)

Next talk:

December 6, 2024

Linear maps preserving disjoint idempotents

Ya-Shu Wang (National Chung-Hsing University, Taiwan)

Contact: Pan Shun Lau (plau@unr.edu)

05C50 Online

https://sites.google.com/view/05c50online/home

Host: University of Manitoba

Schedule: biweekly on Fridays

Time: 10:00AM, Winnipeg (Manitoba, Canada) time

Most recent talk:

Some results involving the Aα-matrix

Carla Oliviera (National School of Statistical Sciences - ENCE, Brazil)

Next talk:

December 6, 2024

From graph decomposition to matrix apportionment and back

Edinah Gnang (Johns Hopkins University, USA)

Contact: Hermie Monterde (monterdh@myumanitoba.ca)

IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present a solution to Problem 72-3. Solutions are invited to Problems 68-2, 68-4, 69-2, 69-3, 70-2, 71-3, 72-1, 72-2, and to the two new problems from the present issue 73.

Problem 72-3: Numerical Range of a 2×2 matrix

Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, pereirar@uoguelph.ca

It is well known that the numerical range of a 2×2 matrix is an ellipse. Suppose M is a 2×2 matrix whose numerical range $W(M)$ is a non-degenerate ellipse. Show that M is unitarily equivalent to a matrix A where $2\sqrt{|a_{12}a_{21}|}$ is the distance between the two foci of $W(M)$.

Solution 72-3 by Rute Lemos, *University of Aveiro, Aveiro, Portugal*, rute@ua.pt and
Graça Soares, *CMAT-UTAD and Universidade de Trás-os-Montes e Alto Douro, Villa Real, Portugal*, gsoares@utad.pt

The *numerical range* of an $n \times n$ complex matrix A is the subset of the complex plane denoted and defined by

$$W(A) = \{x^* A\, x : \, x \in \mathbb{C}^n, \, x^* x = 1\}\, .$$

It is a compact, convex set containing the spectrum of A. By the Elliptical Range Theorem (see, e.g., [2]), the numerical range of a 2×2 matrix is an elliptical disc (eventually degenerated into a line segment, or a point) with its foci at the eigenvalues of the matrix.

It is well known that for any $n \times n$ matrix X, there exists a unitary matrix U such that the diagonal entries of U^*XU are equal to $\frac{1}{n}\operatorname{Tr}(X)$ (see, e.g., [1]). In particular, for a 2×2 matrix M whose numerical range $W(M)$ is a non-degenerate elliptical disc, there exists a unitary matrix U such that

$$U^* M\, U = \begin{bmatrix} \frac{1}{2}\operatorname{Tr}(M) & a_{12} \\ a_{21} & \frac{1}{2}\operatorname{Tr}(M) \end{bmatrix}$$

for some complex numbers a_{12} and a_{21}. We claim that the matrix $A = U^*MU$ solves the problem. In fact, since the numerical range is unitarily invariant, we have $W(A) = W(M)$ and, by the Elliptical Range Theorem, $W(A)$ is an elliptical disc whose foci are the eigenvalues of A, namely

$$\tfrac{1}{2}\operatorname{Tr}(M) \pm \sqrt{a_{12}a_{21}} = \tfrac{1}{2}\operatorname{Tr}(M) \pm \mathrm{e}^{i\alpha}\sqrt{|a_{12}a_{21}|},$$

with $\alpha = \frac{1}{2}(\arg a_{12} + \arg a_{21})$. In particular, the distance between the foci of the ellipse is equal to $2\sqrt{|a_{12}a_{21}|}$.

References

[1] R. A. Horn and C. R. Johnson. *Topics in matrix analysis.* Cambridge University Press, Cambridge, 1991.

[2] F. D. Murnaghan. On the field of values of a square matrix. *Proc. Natl. Acad. Sci. USA*, 18(3):246–248, 1932.

Also solved by Pan Shun Lau, *University of Nevada, Reno, Nevada, USA*, plau@unr.edu

IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We introduce two new problems in this issue and invite readers to submit solutions for publication in *IMAGE*.
Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Jeff Stuart (`jeffrey.stuart@plu.edu`).

New Problems:

Problem 73-1: Generalised Quadratic Equations of Higher Order

Proposed by Richard William Farebrother, *Bayston Hill, Shrewsbury, England*, `R.W.Farebrother@hotmail.com`

Let n be a positive integer. Suppose that A is a nonzero $n \times n$ complex matrix satisfying

$$A^{r+2} = \alpha A^{r+1} + \beta A^r$$

for some positive integer $r \geq 1$ and some nonzero complex numbers α and β.

Show that, provided A is not nilpotent, A also satisfies the generalised quadratic equation

$$A^2 = \alpha A + \beta P,$$

for some $n \times n$ complex matrix P satisfying $P^2 = P$ and $AP = PA = A$.

Problem 73-2: A Characteristic Conundrum

Proposed by Rajesh Pereira, *University of Guelph, Guelph, Canada*, `pereirar@uoguelph.ca`

Let S be an $n \times n$ positive semidefinite doubly stochastic matrix and let $p(x)$ be its characteristic polynomial. Let $M = [p(ns_{ij})]$, i.e., M is the $n \times n$ matrix whose (i, j)-entry is $p(ns_{ij})$. Show that M is positive semidefinite.

A solution to Problem 72-3 is given on page 27.