The Bulletin of the International Linear Algebra Society

IMAGE

Serving the International Linear Algebra Community

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About *IMAGE*

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SPECIAL FEATURE

Three Observations on the International Linear Algebra Society: Steve Kirkland at the 2025 ILAS Conference

At the 2025 ILAS meeting, former ILAS President Steve Kirkland delivered some remarks at the start of the conference banquet. By popular request, and with Steve's assistance, his speech is reproduced here (with his footnotes).

Good evening, everyone. Let's show our appreciation to the organizers and volunteers for putting together this terrific conference, and especially this wonderful banquet. I'm sure that you're keen to catch up with your friends and colleagues here, so I will keep my remarks brief.

There's a pretty well-known paper from 1946 by the Harvard mathematician Garrett Birkhoff. The title is "Tres observaciones sobre el algebra lineal" [1]. Apologies to the Spanish-speakers in the room for my imperfect pronunciation! The title of Birkhoff's paper translates in English as "Three observations on linear algebra", and it includes one of my favourite results, namely that the set of the $n \times n$ doubly stochastic matrices is the convex hull of the $n \times n$ permutation matrices. Don't worry, I won't go into the mathematical details! Inspired by the title of that paper, and with apologies to Garrett Birkhoff, I have organized my remarks tonight under the heading "Tres observaciones sobre la Sociedade Internacional de Algebra Lineal" – "Three observations on the International Linear Algebra Society".

Before I go into them, I will give you fair warning: I do not claim that my observations will be deep!

Observation number 1: ILAS is International. (As I said, not deep!) I did a rough and highly unscientific survey, and it turns out that internationally-based mathematical societies are in the minority – about 75% of all mathematical societies are nationally or even locally based. Those math societies that take internationalism seriously are an even smaller minority, but fortunately ILAS is in that special category of societies that value internationalism. When the precursor to ILAS, the International Matrix Group, was founded in 1987, it was clear that internationalism was intended to be a cornerstone. The IMG had a board with members from 15 different countries. Two years later, when ILAS was founded, that international board widened its scope to include representation from 19 different countries. ILAS conferences have been held in 15 distinct nations in Asia, Europe, North America and South America. ILAS endorsed meetings have been held in 32 different countries, including Morocco and New Zealand. Taking both groups together – ILAS conferences and ILAS-endorsed meetings – those meetings give ILAS a presence on five and a half continents. I need to explain about the half. There's some fuzziness among geographers as to whether or not Oceania is considered a continent (some people do and some people don't), hence the half continent in my tally. But I'm feeling optimistic tonight, so I'll round it up to six continents for ILAS and ILAS-endorsed meetings. Not bad for a Society of some 500 members. We know that an election for the ILAS President will be held this fall, and the candidates will be identifying new directions in which to take our society. For those presidential candidates, I have a one-word suggestion: Antarctica. I say this for two reasons (i) Antarctica is the only continent without ILAS activity so far; and (ii) for me it's pleasant to think of an ILAS conference in which 30+ degree temperatures will not be an issue.[1] My point here is that through its conferences, its financial support, its endorsements, and its cooperation with other mathematical societies (such as AMS, IWOTA and SIAM), ILAS has proven itself to be in the business of advancing linear algebra across the globe.

Why is that important? It seems to me that ILAS operates on the understanding that if linear algebra is maintained as a big tent, a discipline that is open to wide participation, then better mathematics will result. By growing linear algebra around the world, we get more viewpoints, more collaborators, and a more richly connected research network. Or, to put it another way: From many peoples, strength.[2] In my view, there's value to upholding internationalism in mathematics, but you don't have to take my word for it. Quoting the mighty David Hilbert: "Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country."[3] I find that sentiment both inspirational and aspirational.

[1] The 2025 ILAS meeting in Kaohsiung, Taiwan was a terrific event, but boy it was hot!
[2] I did not come up with this phrasing myself – this is the motto of the Canadian province of Saskatchewan.
[3] Hilbert is reported as saying this in Eves' book *Mathematical Circles Squared* [2].

Observation number 2: ILAS is serious about promoting and enhancing Linear Algebra in all of its incarnations. (Again, no claim that my observation is deep.) The fact that ILAS supports the broad sweep of linear algebraic research is made obvious by the diversity of topics represented at this ILAS conference, and indeed at every ILAS conference.

Why is that important? Well, mathematical research has always profited from the healthy dialogue between theory and applications. For instance, problems in economics gave rise to the study of sign pattern matrices, now a thriving branch of combinatorial matrix theory; as an example in other direction, the rich structure of entry-wise nonnegative matrices provides the foundation for web search, epidemic models and image processing. In spite of an acknowledged symbiosis between theory and applications, many people (including many mathematicians) view mathematics as being neatly partitioned into the pure and the applied domains. The turnaround time for mathematical abstractions to become realizable in practice is diminishing, and consequently there is a confluence of the pure and the applied streams of mathematics. I predict that in the years ahead, the lines between the pure and the applied will continue to blur. The increasingly integrated nature of mathematical research will mean that the future will have neither pure mathematics nor applied mathematics, just mathematics. It will be our responsibility as practitioners to ensure that, whatever the mathematics of the future looks like, it is simultaneously rigorous, elegant, and relevant.

Now, ILAS has been ahead of the curve on this confluence of applied and pure for years, recognizing and facilitating that constructive dialogue between theory and applications. If you want an example, take a look at the list of Hans Schneider Prize winners, all 22 of them. Those winners are almost evenly divided between those who identify primarily as theorists, and those who are more focused on applications. And, if you dig a little further into the work of many of those prize winners, you'll see that both theory and applications are well represented in their bodies of work. Moreover, just as linear algebra is widely spread geographically, it's under the hood across all of the mathematical sciences, from scientific computing, to geometry, to quantum information theory. As ILAS founding president Hans Schneider was fond of saying: "Linear algebra is everywhere!" Of course, you don't have to take my word for it. You don't even have to take Hans Schneider's word for it, for that matter. According to William Stein, the founder of SageMath: "Mathematics is the art of reducing any problem to linear algebra." [4]

Observation number 3: ILAS is a Society. Yes, it's a professional society. But I'm also invoking the meaning of the word society as "the company of other people." Our ILAS meetings are not only a chance to stay abreast of the latest research developments in linear algebra, but also an opportunity to connect and reconnect with our friends, our colleagues and our collaborators.

Why is that important? Mathematics is undertaken by human beings, and human beings are social animals – yes, even the most introverted mathematicians are social animals! ILAS gives us a way to get together with the people that share our enthusiasm for our chosen subject. Many of us have travelled long distances just to get to this conference, just to connect with our fellow linear algebraists, maybe even just to attend this banquet! The society provided by ILAS enriches the discipline by bringing us together, in person, as a community. Again, you don't have to take my word for it. As Nobel laureate Desmond Tutu states [3]: "A person is a person through other persons. We need other human beings in order to learn how to be human."

So ends my three observations on the International Linear Algebra Society. The name says it all: ILAS is valuable because it's truly International, because it supports all of Linear Algebra, and because of the Society it provides.

Please, join me in raising a glass to our conference organizers, and to the many ILAS members who work hard to realize our Society's mission. Cheers and gān bēi![5] Enjoy your evening.

References.

[1] G. Birkhoff. Tres observaciones sobre el algebra lineal. *Univ. Nac. Tucumán. Revista A.*, 5:147–151, 1946.

[2] H. Eves. *Mathematical Circles Squared: A Third Collection of Mathematical Stories and Anecdotes.* Prindle, Weber & Schmidt, Boston, Mass.-London-Sydney, 1972.

[3] D. Tutu. Ubuntu: On the nature of human community. In *God Is Not a Christian: And Other Provocations.* HarperOne, New York, 2011.

[4] This quote is attributed to William Stein in several places online, but after much searching I was unable to pin down its source. I eventually emailed Dr. Stein to see if he could tell me. He replied very nicely that he's also unsure of the source, but confirmed that he has definitely said that sentence frequently and is happy to have it attributed to him.

[5] In Mandarin Chinese, "gān bēi" means "dry cup", a bit like the English phrase "bottoms up".

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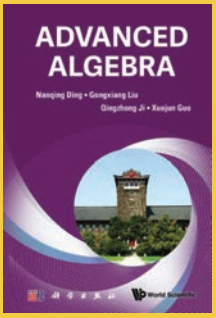

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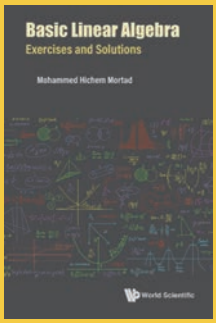

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This book is mainly intended for first-year university students who undertake a basic linear algebra course, as well as instructors. It contains the basic notions of linear algebra through solved exercises as well as a "True or False" section in each chapter. Each chapter also contains an essential background section, which makes the book easier to use.

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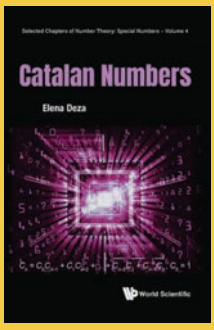

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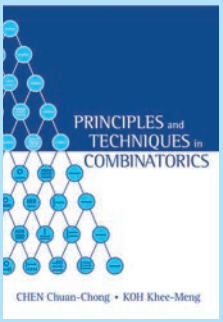

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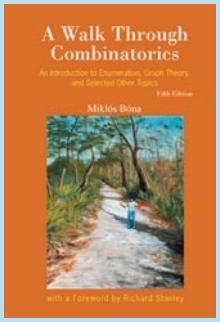

by **Miklós Bóna** *(University of Florida, USA)*

"This book covers all of the basic results one would expect and a selection of advanced topics as well. There are many very good problems in it. The problems in this book are, as the kids say, awesome. This is a comprehensive book on combinatorics. Exposition is clear, coverage is massive, and the problems are plentiful and excellent."
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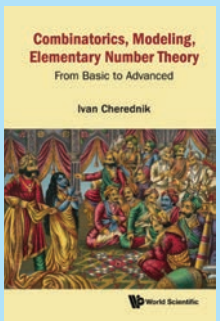

This combinatorics book is based on 25 years of teaching experience, requiring only high school algebra as a prerequisite. It covers a wide range of topics, from basic graph theory and enumerative combinatorics to advanced themes and applications. The book includes 135 problems and 275 exercises with varying difficulty levels, along with solutions, hints, and answers.

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LINEAR ALGEBRA EDUCATION

Workshop Series on Bridging Abstract and Numerical Linear Algebra

Sepideh Stewart, University of Oklahoma, Norman, OK, USA, `sepidehstewart@ou.edu`
Mike Michailidis, MathWorks, Natick, MA, USA, `mmichail@mathworks.com`
Rachel Quinlan, University of Galway, Galway, Ireland, `rachel.quinlan@universityofgalway.ie`

The traditional curriculum often treats abstract and numerical linear algebra as distinct domains. This separation may deprive students of the critical links between elegant theory, powerful practical application and computer implementation. Since many students worldwide may only take one linear algebra course during their degree, isn't it our responsibility to make that experience as beneficial as possible? Motivated by these considerations, the ILAS Education Committee has launched the workshop series *Bridging Abstract and Numerical Linear Algebra*. The goal of this initiative is to foster the thoughtful integration of these two fields through the sharing of expertise and experience, in order to provide students with a more comprehensive understanding of linear algebra. The first workshop of the series took place in Kaohsiung, Taiwan, on June 22nd, 2025, immediately before the 26th ILAS conference. This article provides a brief report on the workshop, which had 19 participants from eight countries.

Participants at the workshop on Bridging Abstract and Numerical Linear Algebra, Kaohsiung, June 22nd, 2025

The workshop opened with informal presentations by Sepideh Stewart and Mike Michailidis. They gave an account of their recent collaboration at the University of Oklahoma, in which an abstract linear algebra course was augmented with a series of labs and projects to help students connect theory to practice. The course had 22 students, with diverse educational backgrounds in mathematics and computing. Students completed six lab sessions using MATLAB, and a computational final project on a choice of topics. Mike presented a description of some of the lab activities. Topics included an introduction to the use of MATLAB for matrix algebra, an investigation of why every spanning set contains a basis, the power method for computing eigenvalues, use of LU-factorization to solve linear systems, and image compression. Feedback from students on the course was very positive, with many commenting on how much the coding activities enhanced their understanding of the theoretical content. While the lab activities (with the exception of Lab 2) did not require students to construct mathematical proofs, students were motivated by their learning in the labs to return to the abstract context for explanations. It was noted that the ability of students to assume agency in mathematical investigations should not be underestimated, even at an early stage of their mathematical education. In this course, appropriately designed explorative challenges and the provision of suitable tools had an empowering effect that motivated engagement, stimulated curiosity and encouraged students to connect different forms of mathematical activity.

The workshop proceeded to an open discussion of opportunities, challenges and experiences. We posed the following topics for discussion:

1. Please share your classroom experiences of bridging abstract and numerical linear algebra.
2. What are the practical opportunities? What are the challenges/obstacles?
3. What are the theoretical challenges?
4. Does the rapid development of generative AI bring opportunities?

Participants mentioned the perennial constraints of time, availability of software and physical infrastructure, and inflexibility of curricula within institutions. Experiences such as the Oklahoma course suggest that the value of overcoming issues of this nature justifies the effort involved. While the workshop did not reach any definitive conclusions, there was a strong sense of the value of a forum of this nature and its potential to reinvigorate our practices. One key truth that was revealed: even with high-quality textbooks and resources available, the ultimate success of such an initiative depends on how it is implemented in the classroom. The strategies that instructors use to connect theory with practice and guide students through hands-on discovery are the most essential elements for creating a successful, meaningful learning experience.

The idea of integrating abstract theory with practical methods and applications is not new in mathematics education. At the 1967 International Commission on Mathematical Instruction (ICMI) colloquium (see [2] for details), Hans Freudenthal, then president of the ICMI, opened his introductory address [1] with a striking declaration: "I will not speak about how to teach mathematics so as to be useful but about why we should teach mathematics so as to be useful, or rather about why we should teach mathematics so as to be more useful" [1, p. 3]. He observed that while much research had focused on learning in controlled settings, very little was known about how individuals apply what they have learned. This gap explained why "most people never succeed in putting their theoretical knowledge to practical use" [1, p. 4]. Yet, he argued, mathematics had become indispensable for both the physical and social worlds, and was "needed not by a few people, but virtually by everybody" [1, p. 5]. This situation arguably persists today, despite the opportunities provided by the extent of our and our students' access to communications technology and computational resources.

Practical resources for integrating abstract and numerical linear algebra in the classroom remain scarce in the literature on linear algebra education. A recent paper [3] by Zhou, published in the journal *PRIMUS*, documents the use of numerical demonstrations to enhance learning and offers a clear model for what can be achieved. Zhou's approach promotes a "deeper understanding of... numerical linear algebra" that goes beyond surface-level knowledge to foster a vital sense of professional diligence, making students more "more mindful of certain pitfalls... that may lead to code inaccuracy or inefficiency" [3, p. 20]. A key argument of the paper is that teaching should emphasize the proper *implementation* of algorithms, not just the algorithms themselves.

Our hope is that the *Bridging* workshop series will support the creation of more literature of this nature, written by practitioners and informed by the learning experiences of their students. We envisage an international community of practice that maintains communication through a mailing list, through workshops aligned with major conferences, and through the Education section of the ILAS website.

The second workshop in the series will take place at the 2026 Joint Mathematics Meetings (JMM) in January 2026, and it is expected that the third will occur just before the 27th ILAS Conference in May 2026.

Here is the announcement for the next workshop:

JMM Workshop on Bridging Abstract and Numerical Linear Algebra
Organizers: Sepideh Stewart, Mike Michailidis, and Judi McDonald

January 7th, 2026, 8:30AM – 10:00AM, Room 207A (Level 2, Walter E. Washington Convention Center)

Come and explore methods for integrating abstract and numerical linear algebra in the classroom. Experience hands-on examples (MATLAB and Mathematica) for (a) Introducing programming to enhance understanding of matrix properties and abstract concepts, (b) Introducing modern applications, (c) Exploring using matrices and their properties and computational capabilities in understanding data to help students grow intuition and make conjectures, and (d) Pitfalls and Effective use of AI.

References.

[1] H. Freudenthal. Why to teach mathematics as to be useful. *Educational Studies in Mathematics*, 1(1): 3–8, 1968.

[2] F. Furinghetti and L. Giacardi. ICMI in the 1950s and 1960s: Reconstruction, Settlement, and "Revisiting Mathematics Education". In F. Furinghetti and L. Giacardi, editors, *The International Commission on Mathematical Instruction, 1908-2008: People, Events, and Challenges in Mathematics Education*, pages 43–94. Springer International Publishing, Cham, 2022.

[3] Y. Zhou. The power of randomly concrete examples and numerical demonstrations. *PRIMUS*, 2025. https://doi.org/10.1080/10511970.2024.2423344


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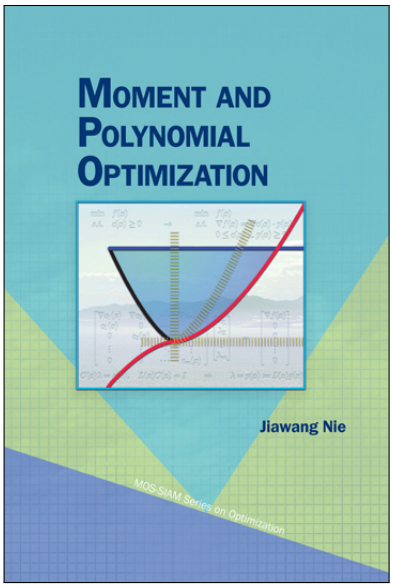
BOOK REVIEW

Moment and Polynomial Optimization
by Jiawang Nie

MOS-SIAM Series on Optimization, 2023, ISBN 978-1-61197-759-2, xvi+467 pages
Reviewed by Zaiwen Wen, Peking University, wenzw@pku.edu.cn



This monograph explores the dynamic and rapidly evolving field of moment and polynomial optimization, a domain that combines convex optimization, algebraic geometry, and moment theory to solve problems in mathematics, engineering, and applied sciences. The study focuses on the hierarchy of moment-SOS relaxations, a powerful framework that transforms nonconvex polynomial optimization problems into linear convex conic optimization problems that are solvable through semidefinite programming.

The monograph is structured into four comprehensive parts, each meticulously designed to build a cohesive understanding of the mathematics of the field. The first part establishes foundational mathematical tools and concepts crucial for moment and polynomial optimization. First, this part introduces basic knowledge of linear convex conic optimization, which provides powerful convex relaxations for solving moment and polynomial optimization problems. Then, it delves into the background of algebraic geometry, polynomials and moment problems, which are indispensable for understanding nonnegative polynomial representations. The author introduces sum-of-squares polynomials, Positivstellensätze, truncated moment problems, and other algebraic structures that underpin the Moment-SOS hierarchy. In particular, for a quick and easy introduction to the field, univariate polynomial optimization is explored with a focus on semidefinite programming representations and their unique mathematical properties. This part bridges the theoretical and computational aspects of moment and polynomial optimization, and sets the stage for the deeper exploration of the Moment-SOS hierarchy found in later chapters.

The second part focuses on the Moment-SOS hierarchy for solving both unconstrained and constrained polynomial optimization problems. The hierarchy's mathematical properties, such as asymptotic and finite convergence, extracting optimizers, and local and global optimality certificates are thoroughly discussed. The author also explores tight relaxations developed using optimality conditions and Lagrange multiplier expressions, offering practical solutions to some of the field's longstanding challenges.

The third part explores convex polynomial optimization and conic optimization over nonnegative polynomial cones and moment cones. It examines convex polynomials and sets, SOS-convexity, semidefinite programming representations, and proposed frameworks for solving non-convex optimization problems. By leveraging moment cones and nonnegative polynomial cones, many challenging optimization questions can be reformulated as linear convex conic optimization problems. In particular, the study highlights properties of copositive and completely positive cones, emphasizing their significance in solving optimization challenges.

The fourth part expands into advanced and specialized topics, highlighting the versatility and depth of moment and polynomial optimization. The topics include tensor optimization, positive semidefinite matrix polynomials, sparse optimization, and saddle point problems. Applications in distributionally robust optimization, multi-objective optimization, Nash equilibrium problems, and bilevel optimization are also discussed, demonstrating the broad applicability of the theory to contemporary mathematical and engineering problems.

One might initially assume that the focus on moments and polynomials is a niche area of mathematical optimization, but the scope and impact of this field are far-reaching. Moment and polynomial optimization has become central to research in areas ranging from algebraic geometry and numerical analysis to machine learning and quantum computing. The author provides a thorough and systematic presentation of the field, meticulously organizing results while connecting them to broader mathematical and practical contexts.

This monograph provides a clear and systematic exploration of moment and polynomial optimization, making complex topics accessible to a broad audience. It connects historical insights with modern developments across algebraic geometry, convex optimization, and moment theory. With exercises and examples, it will serve as an excellent resource for graduate students and researchers, offering both deep understanding and inspiration for future research in optimization and related fields.

ILAS NEWS

ILAS Members Selected as 2025 CMS Fellows

The Canadian Mathematical Society (CMS) recognizes fellows for "excellent contributions to mathematical research, teaching, or exposition; as well as having distinguished themselves in service to Canada's mathematical community." Among the seven mathematicians honored as 2025 CMS Fellows are ILAS member Douglas Farenick of the University of Regina, as well as ILAS member and former ILAS president Steve Kirkland, of the University of Manitoba.

ILAS Richard A. Brualdi Early Career Prize Awarded to John Urschel

The ILAS Richard A. Brualdi Early Career Prize is awarded every three years to an outstanding early career researcher in the field of linear algebra who is within seven years of receiving the Ph.D. or equivalent degree as of October 1st of the year before the award, for distinguished contributions to the field.

The 2026 prize has been awarded to John Urschel of the Massachusetts Institute of Technology, for "important contributions to numerical linear algebra including Gaussian elimination and the Lanczos method, and to spectral graph theory." Professor Urschel will present the prize lecture at the 2026 ILAS Conference in Blacksburg, Virginia, USA.

The prize committee consisted of Misha Kilmer, Stephen Kirkland (chair), Lek-Hem Lim, Alison Ramage, and Daniel B. Szyld.

ILAS Olga Tausky and John Todd Prize Awarded to Sirani Perera

The ILAS Olga Tausky and John Todd Prize is awarded to a person who has received the Ph.D. degree within about fifteen years of receiving the prize, and who has made significant contributions to linear algebra and matrix theory.

The 2026 prize has been awarded to Sirani Perera of Embry-Riddle Aeronautical University, for "substantial contributions to structured matrix computations, and for parlaying linear algebra insights into hardware implementation and signal processing." Professor Perera will present the ILAS Olga Taussky and John Todd Prize Lecture at the 2026 ILAS Conference in Blacksburg, Virginia, USA.

The prize committee consisted of Stephen Kirkland (chair), Lek-Hem Lim, Alison Ramage, and Daniel B. Szyld.

Send News for *IMAGE* Issue 76

IMAGE seeks to publish all news of interest to the linear algebra community. Issue 76 of *IMAGE* is due to appear online on June 1, 2026. Send your news for this issue to the appropriate editor by April 15, 2026. Photos are always welcome, as well as suggestions for improving the newsletter. Please send contributions directly to the appropriate editor:

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— Carol S. Woodward, SIAM President, Lawrence Livermore National Laboratory

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9/25

CONFERENCE REPORTS

VII ALAMA Conference — Tribute to Nick Higham
Castro Urdiales, Spain, May 14–16, 2025

Report by Ángeles Carmona

The seventh ALAMA Conference (Jornadas ALAMA) took place at the International Centre for Mathematical Meetings (CIEM) in Castro Urdiales, Spain, May 14–16, 2025. The meeting was explicitly dedicated to the memory and legacy of Professor Nicholas J. Higham (University of Manchester), who passed away in January 2024. The organizers designed the program to highlight Higham's influence on the fields of numerical linear algebra and matrix analysis, combining overview lectures by close collaborators with focused talks by Spanish colleagues.

CIEM's "La Residencia" (Pedro Velarde Building) hosted the workshop, with sessions held in an intimate setting that supported lively discussion. Approximately forty participants attended, representing universities and research centers from across Spain and abroad. A welcome reception on Wednesday evening fostered early interaction among attendees and set an engaging tone for the scientific program.

The welcome reception (left) and conference group photo (right) of the VII ALAMA conference

The program featured invited lectures and thematic sessions centered on Higham's contributions. Invited speakers included Françoise Tisseur (University of Manchester), Desmond J. Higham (University of Edinburgh), Theo Mary (Sorbonne Université), Julio Moro (Universidad Carlos III de Madrid), Fernando de Terán (Universidad Carlos III de Madrid), and Javier Pérez Álvaro (University of Montana). The talks surveyed topics such as matrix functions and matrix polynomials; probabilistic rounding error analysis, from early developments to modern perspectives; and structural and spectral questions in matrix theory and applications.

Two lectures that resonated strongly with the audience, both closely connected to the tribute theme, were:

- "Matrix structure: from Matrix Functions to Matrix Polynomials" (Françoise Tisseur) traced key threads in the modern theory of matrix functions and highlighted methodological advances aligned with Higham's foundational work.
- A commemorative talk by Julio Moro offered a tour through Nick Higham's life, illustrated with photographs and personal anecdotes, in which his human qualities stood out above all.

Beyond the technical program, the meeting emphasized Higham's role as a scholar, mentor, and communicator. A recurring theme across presentations was the way his work—particularly on the accuracy and stability of numerical algorithms, and on effective scientific communication—continues to shape best practices in numerical linear algebra. Personal recollections from collaborators and colleagues underscored the breadth of his impact on the ALAMA community and beyond.

The event was organized under the auspices of the ALAMA Network (Álgebra Lineal, Análisis Matricial y Aplicaciones), with local venue support from CIEM. The organizing and scientific committees included members of the Spanish linear algebra community, and the conference received the endorsement of the International Linear Algebra Society (ILAS). Institutional supporters included the Spanish Ministry of Science, Innovation and Universities (project RED2022-134176-T), the Universitat Politècnica de Catalunya (UPC) Department of Mathematics, Universidad Carlos III de Madrid, and the City Council of Castro Urdiales.

The conference offered a compact yet rich overview of current research threads influenced by Higham's work, provided a forum for young researchers to engage with core ideas in numerical linear algebra, and strengthened ties within the ALAMA network and the broader ILAS community. The atmosphere was both collegial and reflective, befitting a scientific tribute to a scholar whose ideas and guidance continue to inform the field.

The 8th Workshop on Design Theory, Hadamard Matrices and Applications (Hadamard 2025) Seville, Spain, May 26–30, 2025

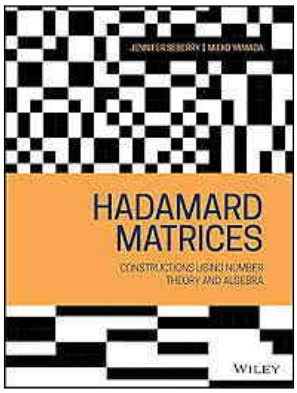
Report by José Andrés Armario

The purpose of the Hadamard series of conferencs is to bring together researchers and students interested in design theory, especially as it relates to Hadamard matrices and their applications, as well as in related areas in coding theory, association schemes, sequences, finite geometry, difference sets, quantum information theory, theoretical physics and computer security. Hadamard 2025 was the eighth conference in the series, the previous seven having been held in Wollongong, Australia (1993); Seville, Spain (2007); Galway, Ireland (2009); Melbourne, Australia (2011); Lethbridge, Canada (2014); Budapest, Hungary (2017); and Krakow, Poland (2022).

This edition took place May 26–30, 2025, at the Institute of Mathematics of the University of Seville (IMUS), and it was dedicated to honour Dane Flannery and Robert Craigen, two outstanding researchers in design theory.

Robert Craigen on his initial slide

Dane Flannery during his talk



Seberry and Yamada's book

Hadamard 2025 featured 21 invited talks, in addition to 36 contributed talks accepted by the Scientific Committee. In total, there were 75 participants from 24 different countries around the world. Attendees could learn about the latest developments, discuss the latest findings, take stock of what remains to be done on classical problems, and explore different visions for setting the direction for future work. Participants could also enjoy the social program, with flamenco, tapas and pottery. Both the Organizing and the Scientific Committees thank all the participants for their interest and contributions to making this conference a successful scientific event. Special thanks are due to Jennifer Seberry, whose generous donation of 15 copies of her book (with Mieko Yamada) were distributed among the students attending the conference.

The workshop was organized by Víctor Álvarez, José Andrés Armario, Raúl Falcón, María Dolores Frau, Manuel González-Regadera, Félix Gudiel and Belén Guemez, all from the University of Seville. For more information about the conference program, visit the conference webpage:

`https://gestioneventos.us.es/hadamard2025/programme`

Hadamard 2025 was partially supported by IMUS and the Department of Applied Math I (University of Seville). Papers presented during the conference may be published in a special issue of the *Journal of Algebraic Combinatorics* (*JACO*). For more information, see `https://link.springer.com/journal/10801/updates`.

Participants of Hadamard 2025

The 9th Linear Algebra Workshop (LAW'25) Portorož, Slovenia, June 2–6, 2025

Report by Matjaž Omladič

The LAW'xx meetings, held every three years, returned after a brief "pandemic delay," providing an opportunity to celebrate the milestone anniversary of their founder, Heydar Radjavi. The workshop covered a variety of topics in linear algebra and operator theory, and took place at the Faculty of Maritime Studies and Transport in Portorož, Slovenia, June 2–6, 2025. It was endorsed by ILAS, who generously provided funding for one invited speaker.

The conference attracted 58 registered participants. The scientific program featured 5 plenary lectures, 31 contributed talks, and 3 working groups. Among the plenary lectures was the ILAS Lecture, delivered by Helena Šmigoc (University College Dublin) on "Arbitrarily Finely Divisible Matrices." The talks covered a wide range of topics, including matrix theory, moment theory, spectral theory, subnormal operators, preserver problems, and trace inequalities.

The working groups, held in the afternoons, offered participants the opportunity to discuss open problems in three thematic areas: "Local to Global Properties of Collections of Matrices" (organized by Mitja Mastnak and Heydar Radjavi), "Preserver Problems" (organized by Chi-Kwong Li), and "Moment Problems, Positive Polynomials, and Applications" (organized by Konrad Schmüdgen and Aljaž Zalar).

The social program consisted of a welcome reception on Monday evening, a guided walking tour of Piran Wednesday afternoon, and a conference dinner on Wednesday evening.

LAW'25 was supported by the Institute of Mathematics, Physics, and Mechanics (IMFM), by the University of Ljubljana, and by the University of Primorska. For more details, please see the conference website at

`http://www.law05.si/law25`.

The LAW'25 conference photo

The 17th Workshop on Numerical Ranges and Numerical Radii (WONRA) Taichung, Taiwan, June 19–21, 2025

Report by Chi-Kwong Li, Ming-Cheng Tsai, Ya-Shu Wang and Ngai-Ching Wong

The 17th Workshop on Numerical Ranges and Numerical Radii (WONRA 2025) took place June 19–21, 2025, at National Chung Hsing University, Taichung, Taiwan. Organized by Ray-Kuang Lee, Chi-Kwong Li, Raymond Nung-Sing Sze, Ming-Cheng Tsai, Ya-Shu Wang and Ngai-Ching Wong, the workshop aimed to stimulate research and foster interaction among mathematicians in areas including operator theory, functional analysis, matrix norms, inequalities, numerical analysis, perturbation theory, matrix polynomials, quantum information science, and related topics. It also provided an opportunity for researchers to exchange ideas.

The workshop attracted approximately 60 participants from around the world. A total of 30 invited talks were delivered by distinguished speakers, including Natália Bebiano, Mao-Ting Chien, Man-Duen Choi, Kennett Dela Rosa, Osamu Hatori, Daisuke Hirota, Sejong Kim, Seung-Hyeok Kye, Rute Lemos, Chi-Kwong Li, Tejbir Lohan, Izuho Matsuzaki, Takeshi Miura, Lajos Molnar, Michiya Mori, Hiroshi Nakazato, Shiho Oi, Ryan O'Loughlin, Jyoti Rani, Sushil Singla, Tin-Yau Tam, Tamás Titkos, Frank Uhlig, Dániel Virosztek, Jani Virtanen, Kuo-Zhong Wang, Pei Yuan Wu, Takeaki Yamazaki, Fuzhen Zhang and Karol Życzkowski. A conference banquet was held on June 20th to celebrate the gathering and promote informal discussions among participants.

Participants at WONRA 2025

The event was generously supported by the College of Science and the Department of Applied Mathematics of National Chung Hsing University, as well as by the Mathematics Research Promotion Center, Taiwan. The official conference web site, including the program of talks, can be found at:

`https://sites.google.com/email.nchu.edu.tw/wonra2025`

Photos from the conference can be found at `https://photos.app.goo.gl/FLDyzhHw89SDYH3w8`.

The previous 16 workshops have been held in different places since 1992. More information on the conference series can be found at `https://en.wikipedia.org/wiki/Workshop_on_Numerical_Ranges_and_Numerical_Radii`.

The next WONRA will be held in India in the summer of 2027. Further details will be announced soon.

The 26th ILAS Conference: Legacy and New Spectrum
Kaohsiung, Taiwan, June 23–27, 2025

Report by Jephian C.-H. Lin and Matthew M. Lin

The 26th Conference of the International Linear Algebra Society (ILAS2025) was successfully held in Kaohsiung, Taiwan, from June 23rd to 27th, 2025, bringing together researchers, educators, and enthusiasts in the field of linear algebra. Following ILAS2014 in Korea, ILAS2025 was the first ILAS conference to be hosted in the Asia-Pacific region after an 11-year interval. The event attracted more than 400 participants from 44 countries, reflecting the strong international engagement and collaboration within the linear algebra community.

The conference featured 9 plenary lectures, 35 minisymposia with a total of 290 talks, 43 contributed talks, and 9 poster presentations. The topics covered a broad range of areas, including numerical linear algebra, combinatorial matrix theory, matrix equations and inequalities, data science, quantum information, and operator theory. A special issue of *Linear Algebra and its Applications* will be published in connection with the conference.

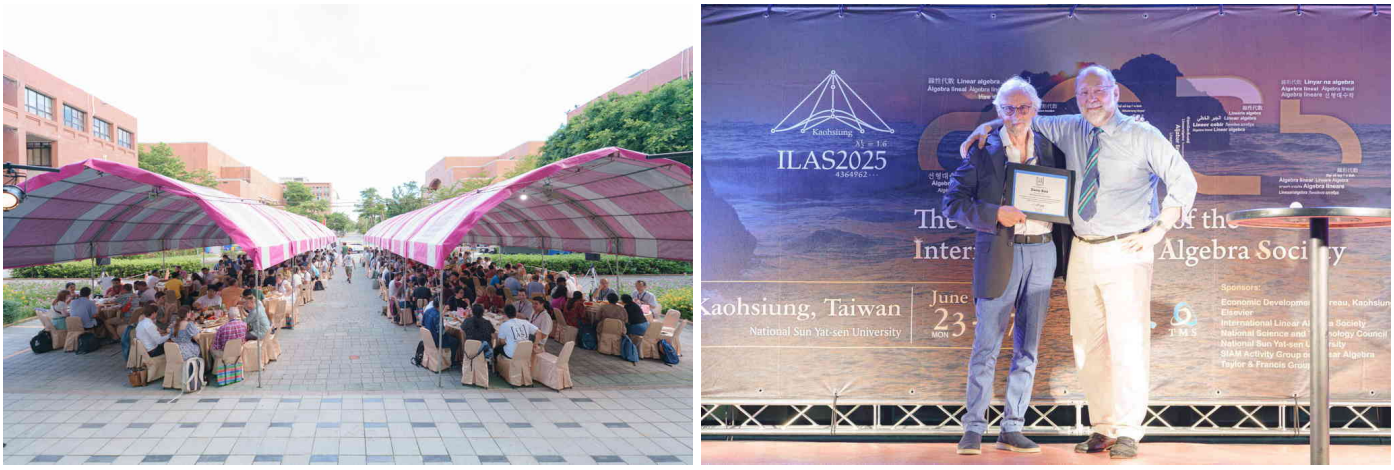
On Wednesday afternoon, ILAS held its business meeting, where organizational progress and future plans were discussed. Following the meeting, participants joined excursions to either Kaohsiung Harbor or the Lotus Pond, both offering rich scenic and cultural experiences. In the evening, the conference banquet was held in the pān-toh style—a traditional Taiwanese open-air feast celebrating special occasions.

During the banquet, the Hans Schneider Prize was awarded to Dario Bini in recognition of his outstanding contributions to matrix multiplication, polynomial and structured matrices, Markov chains, and other areas of computational linear algebra. After the award presentation, Steve Kirkland delivered a speech highlighting the success of ILAS and the spirit of the community. [*Editor's note:* Readers can find the text of the speech on page 3.]

Plenary speakers of ILAS 2025 (left); Karol Życzkowski delivers his plenary lecture (right)

While ILAS2025 celebrated the diversity of participants from around the world, it also strengthened connections and fostered new collaborations within the Asian mathematical community. The 17th Workshop on Numerical Ranges and Numerical Radii (WONRA) was held in Taichung, Taiwan, one week prior to ILAS2025, and the International Workshop on Matrix Analysis, Linear Algebra, and Applications (MALAA) took place in Quezon City, the Philippines, one week after. Together, these events created a vibrant period of academic exchange in the region.

The ILAS2025 Local Organizing Committee consisted of Chih-Wei Chen, Tsung-Ming Huang, Hao-Wei Huang, Yueh-Cheng Kuo, Jephian C.-H. Lin, Matthew M. Lin, Ching-Sung Liu, ShengLi Tzeng, Ngai-Ching Wong, and Suh-Yuh Yang. The Scientific Committee consisted of Melina Freitag, Tsung-Ming Huang, Sejong Kim, Chi-Kwong Li, Jephian C.-H. Lin, Shahla Nasserasr, Helena Šmigoc, Daniel B. Szyld, Raf Vandebril.



The ILAS 2025 conference banquet (left); Dario Bini receives the Hans Schneider Prize (right)

The success of ILAS2025 was made possible through the generous support and collaboration of the following organizations:

- Economic Development Bureau, Kaohsiung City Government
- Elsevier
- International Linear Algebra Society
- National Science and Technology Council
- National Sun Yat-sen University
- SIAM Activity Group on Linear Algebra
- Taylor & Francis Group

Their invaluable contributions and assistance are deeply appreciated. More details, including the conference program and photos can be found at `https://ilas2025.tw`.

The ILAS 2025 Conference Photo

MALAA 2025: Advancing Research and Collaboration in Matrix Analysis Quezon City, Philippines, July 1–2, 2025

Report by Kennett L. Dela Rosa

The International Workshop on Matrix Analysis, Linear Algebra, and Applications (MALAA 2025) was held July 1–2, 2025, at the Institute of Mathematics, University of the Philippines Diliman, Quezon City, Philippines. Organized by the Institute's Matrix Analysis and Linear Algebra (MALA) Group and its affiliates, MALAA 2025 featured a comprehensive program that included eight plenary sessions, twelve contributed talks, and four research discussion sessions. These talks and sessions covered a wide range of topics, showcasing the depth and diversity of current research in matrix analysis and linear algebra.

MALAA 2025 Day 1

MALAA 2025 Day 2

At the conference dinner, a surprise celebration honored Professor Agnes T. Paras on her retirement. Heartfelt speeches and warm tributes recognized her pioneering role in the study of matrix analysis at the Institute, her leadership of the MALA Group, and her significant influence in shaping the group's research direction.

Prof. Agnes T. Paras (holding flowers) with members and affiliates of the MALA Group at the conference dinner

The 36th International Workshop on Operator Theory and its Applications (IWOTA) Enschede, Netherlands, July 14–18, 2025

Report by Felix L. Schwenninger

The 36th International Workshop on Operator Theory and its Applications (IWOTA) took place at the University of Twente, Enschede, Netherlands, July 14th to 18th, 2025. Initiated by Israel Gohberg, the IWOTA conference series has flourished over the decades, establishing itself as a central annual meeting point for the community. While rooted in operator theory, IWOTA embraces a broad spectrum of topics across pure and applied mathematics and beyond, including complex analysis, harmonic analysis, linear algebra, random matrix theory, mathematical physics, and their applications in areas such as control theory, signal processing, and machine learning.

This year's IWOTA attracted 370 registered participants. The scientific programme featured 8 plenary lectures, 11 semi-plenary lectures, and 23 special sessions. The plenary and semi-plenary lectures addressed diverse themes such as operator-theoretic techniques in deep learning, modeling complex physical networks with port-Hamiltonian systems, manifestations of non-commutativity in geometric, complex, and Fourier-analytic settings, and developments in quantum information theory.

The special sessions again covered a wide range of areas within operator theory, providing opportunities for a greater focus within the talks and exchange of ideas, as well as for collaboration among participants with common interests. A notable feature of this year's IWOTA were the memorial special sessions dedicated to Harry Dym, Rien Kaashoek, Heinz Langer, and Nikolai Vasilevski. These towering figures were not only leaders in the field and inspiring teachers, but were also cherished colleagues and friends to many in the IWOTA community. Each helped shape IWOTA into the conference it is today.

The social programme on Wednesday afternoon offered participants a choice between a historic steam train ride to Haaksbergen, a guided city tour in Enschede, or a bus trip to the historic village of Ootmarsum. On Thursday evening, the conference dinner was held at the FC Twente football stadium, starting with welcoming drinks in the stands.

IWOTA was supported by funding from the Delft University of Technology, the University of Twente, the Dutch Research Council (NWO), the European Mathematical Society (EMS), the United States National Science Foundation (NSF), ILAS, the 4TU.Applied Mathematics Institute, the Dutch Institute of Systems and Control (DISC), and the Nonlinear Dynamics of Natural Systems (NDNS+) research cluster.

The meeting's proceedings will be published in the Birkhäuser series *Operator Theory: Advances and Applications*.

For more details, please see the conference website at `https://www.utwente.nl/en/iwota2025`.

Nordic Numerical Linear Algebra Meeting Uppsala, Sweden, August 19–20, 2025

Report by Roman Iakymchuk and Sven-Erik Ekström

The Nordic Numerical Linear Algebra Meeting (Nordic NLA 2025) was held at the Department of Information Technology, Ångström Laboratory, Uppsala University, Sweden, August 19–20, 2025. This event was part of a continuing series of Nordic meetings devoted to numerical linear algebra, algorithms, and applications, aiming to strengthen collaboration among Nordic and European researchers. The meeting was organized by Roman Iakymchuk (Uppsala University), Sven-Erik Ekström (Uppsala University), and Elias Jarlebring (KTH Royal Institute of Technology).

The program included 3 invited talks and 13 contributed talks. The invited speakers were Erna Begović Kovač (University of Zagreb, Croatia), Michele Benzi (Scuola Normale Superiore, Italy), and Jörn Zimmerling (Uppsala University, Sweden). In total, the meeting attracted 34 registered participants, with the majority from the Nordic countries.

The presentations covered a broad and up-to-date spectrum of topics in numerical linear algebra. In addition to classical themes—iterative solvers, preconditioning, and eigenvalue problems—several talks addressed randomized methods, data-driven matrix computations, tensor approaches, and mixed-precision algorithms. Contributions on tensor methods and structured matrix equations reflected the ongoing integration of multilinear techniques into linear algebra workflows. The discussion of mixed-precision and low-accuracy computations underscored the community's engagement with emerging high-performance computing paradigms and error-resilient algorithms.

The meeting fostered lively discussions and exchanges of ideas in an informal and collaborative atmosphere. The compact format facilitated in-depth technical interactions, networking, and community building.

Participants of the 2025 Nordic Numerical Linear Algebra Meeting

Overall, the Nordic NLA Meeting in Uppsala continued the successful tradition of fostering a vibrant Nordic community in numerical linear algebra. The organizers gratefully acknowledge support from the eSSENCE (a national strategic research program in e-Science) via the eSSENCE MeetUps program, and from both the Division of Scientific Computing and the Section of Mathematics and Computer Science at Uppsala University.

2025 Workshop on Matrices and Operators (MAO)
Regina, Canada, August 19–21, 2025

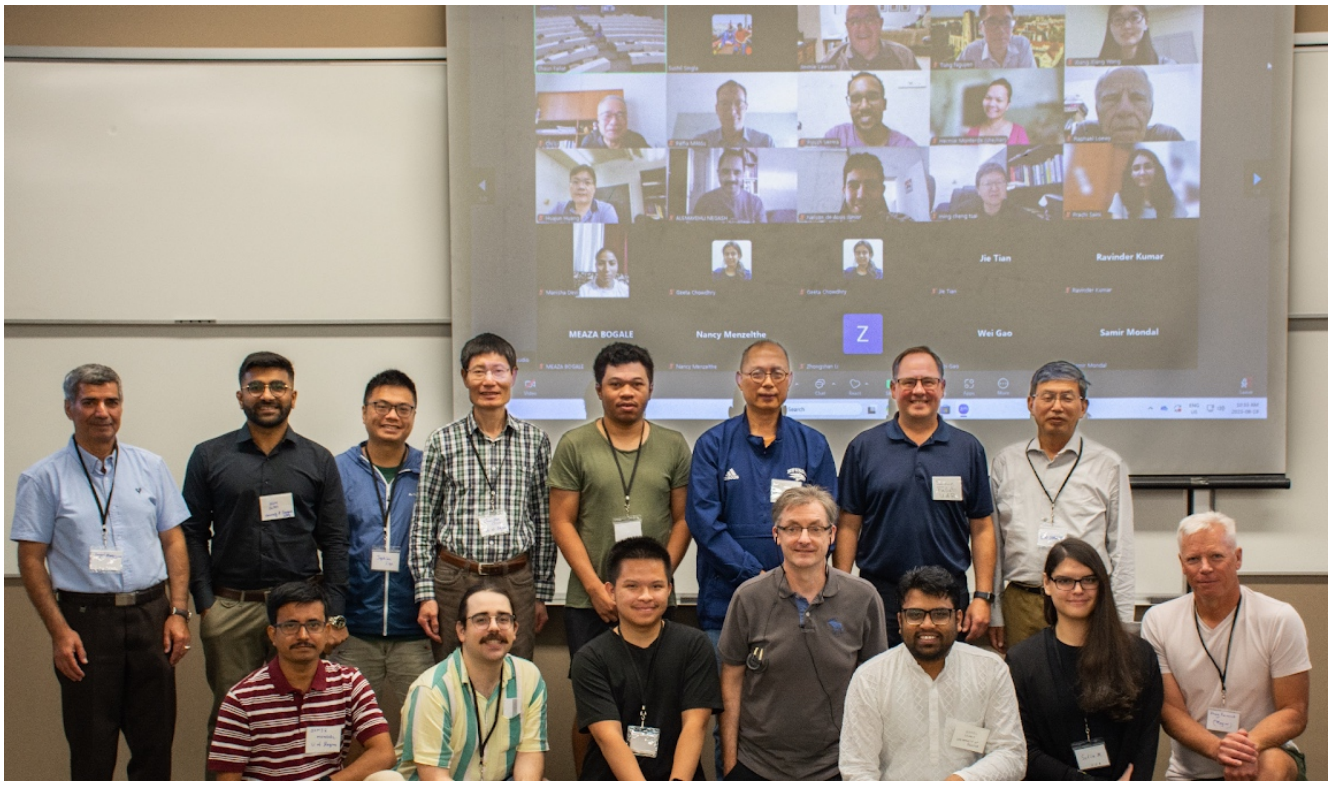
Report by Shaun Fallat

The 2025 Workshop on Matrices and Operators (MAO) took place at the University of Regina, Regina, Saskatchewan, Canada, August 19–21, 2025, part of a conference series that has been running for nearly 20 years, with most of its host venues in Asia. (Last year MAO took place in Reno, Nevada, USA, and it will take place in India in June 2026.)

The purpose of the workshop is to promote research and collaboration among researchers with an interest in matrix theory, operator theory, operator algebra, quantum information theory, and related topics. Participants are encouraged to present their latest findings and discoveries, and share their experiences and research problems, while fostering an atmosphere of collaboration and mutual learning. Through dialogue and the exchange of knowledge, the workshop aims to enhance the collective growth and development of the research community involved in matrix theory, operator theory, operator algebras, quantum information theory, and related fields.

MAO 2025 featured a hybrid program offering both in-person lectures and presentations via Zoom. Among the 45 participants, 26 participated online via Zoom. Furthermore, 20 of the 33 presentations were offered over Zoom. Running MAO 2025 as a hybrid event gave many graduate students and research scholars abroad the opportunity to participate, present their current research, and interact with all participants. Selected slide decks are available, along with the conference program and other relevant information, including the history of this workshop series, at the conference webpage: `https://sites.google.com/view/matrices-and-operators-2025`

MAO 2025 was generously supported by the Pacific Institute for the Mathematical Sciences (PIMS) and the University of Regina.



Participants (in-person and remote via Zoom) of MAO 2025

MAO 2026 will be held June 25–28, 2026 at Shiv Nadar University, Delhi-NCR, India. The registration deadline has passed, but more information can be found at `https://sites.google.com/snu.edu.in/mao2026`.

UPCOMING CONFERENCES AND WORKSHOPS

International Conference on Linear Algebra and its Applications (ICLAA 2025)
Manipal, India, December 17–20, 2025

The International Conference on Linear Algebra and its Applications (ICLAA 2025), organized by the Center for Advanced Research in Applied Mathematics & Statistics (CARAMS), Manipal Academy of Higher Education (MAHE), Manipal, India, will be held December 17–20, 2025. The Scientific Advisory Committee consists of Ravindra B. Bapat (chair), Manjunatha Prasad Karantha (convener), Steve Kirkland, and Simo Puntanen. The conference is endorsed by ILAS.

This conference is the sixth in its series and is in sequel to the conferences CMTGIM 2012, ICLAA 2014, ICLAA 2017, ICLAA 2020(21), and ICLAA 2023, held in Manipal during January 2012, December 2014, December 2017, December 2021, and December 2023, respectively. The present conference shall provide an avenue for leading mathematicians, statisticians, and applied scientists who are working around the globe in the theme area to get together in physical space, interact with each other, discuss research issues, and introduce new innovations. Besides arranging invited talks from eminent speakers, the organizers invite participants to present their research in the sessions of contributed talks.

The list of speakers includes eminent mathematicians such as Abraham Berman, T. E. S. Raghavan, Shaun M. Fallat, Stephen J. Haslett, Surender Kumar Jain, Stephen J. Kirkland, Andre Leroy, Lina Mallozzi, Simo Puntanen, Dietrich von Rosen, Apoorva Khare, Samir K. Neogy, T. S. S. R. K. Rao, Sukanta Pati, K. C. Sivakumar, Sivaramakrishnan Sivasubramanian, and Murali K. Srinivasan.

For an up-to-date list of speakers and other details, see the ICLAA 2025 webpage at `https://carams.in/events/iclaa2025`.

All original papers selected for presentation in the conference will be forwarded to the review process in the appropriate conference publications.

Preconference Workshop IWSMGA 2025: The ICLAA 2025 conference will take place following a preconference workshop, the "International Workshop on Special Matrices, Graphs, and Applications" that will be held December 10–16, 2025, and for which the resource persons are Abraham Berman, Stephen J. Kirkland, Simo Puntanen, T. E. S. Raghavan, S. K. Neogy, Sukanta Pati, Sivaramakrishnan Sivasubramanian, and other leading personalities in the subject. For more details, visit the IWSMGA 2025 webpage at `https://carams.in/events/iwsmga2025`.

For timelines, registration fees, and other details, visit `https://carams.in`. You may also write to the organizing secretary, K. Manjunatha Prasad, at `carams.mahe@gmail.com` or `kmprasad63@gmail.com`.

ILAS at the Joint Mathematics Meetings
Washington, D.C., USA, January 4–7, 2026

ILAS Invited Address

Dominique Guillot, University of Delaware
The Many Facets of Matrix Positivity: A Celebration of Linear Algebra
Monday, January 5th, 2026, 10:50AM–11:55AM

Abstract: Matrix positivity plays a fundamental role across mathematics and in applications, appearing in fields as diverse as analysis, combinatorics, data science, geometry, optimization, probability, statistics, and quantum physics. This talk will explore both classical and modern problems where various notions of matrix positivity are central. In particular, we will highlight recent breakthroughs, outstanding open questions, and unexpected applications in the area. By weaving together ideas from pure and applied mathematics, this talk aims to celebrate the richness of linear algebra and inspire further exploration of positivity in its many forms.

ILAS Special Sessions

ILAS Special Session on Combinatorial Matrix Theory
Sunday, January 4th, 2026, 8:00AM–12:00PM and 1:00PM–5:00PM
Organizers:

- Minerva Catral, Xavier University
- Bryan L Shader, University of Wyoming

ILAS Special Session on Matrix Analysis and Applications
Sunday, January 4th, 2026, 8:00AM–12:00PM and 1:00PM–5:00PM
Organizers:

- J. E. Pascoe, Drexel University
- Hugo Jan Woerdeman, Drexel University

ILAS Special Session on Matrix Analysis and Applications
Monday January 5th, 2026, 10:00AM–12:00PM and 1:00PM–5:00PM
Tuesday January 6th, 2026, 1:00PM–5:00PM
Organizers:

- Mohsen Aliabadi, University of California, San Diego
- Dominique Guillot, University of Delaware
- Tin-Yau Tam, University of Nevada, Reno
- Xiang Xiang Wang, University of Nevada, Reno

ILAS Special Session on Innovative and Effective Ways to Teach Linear Algebra
Tuesday January 6th, 2026, 8:00AM–11:30PM and 1:00PM–5:00PM
Organizers:

- Sepideh Stewart, University of Oklahoma
- Gil Strang, MIT
- David M. Strong, Pepperdine University
- Megan Wawro, Virginia Tech

ILAS Special Session on Algebraic Graph Theory: New Trends
Wednesday January 7th, 2026, 8:00AM–12:00PM and 1:00PM–5:00PM
Organizers:

- Milica Andelic, Kuwait University
- Renata Del-Vecchio, Universidade Federal Fluminense: Niterói
- Sudipta Mallik, Marshall University
- Zoran Stanic, University of Belgrade

ILAS Special Session on Recent Advances in Model Order Reduction and Data-Driven Modeling Theory and Computations
Wednesday January 7th, 2026, 8:00AM–11:30PM and 1:00PM–5:00PM
Organizers:

- Ionut Farcas, Virginia Tech
- Steffen W. R. Werner, Virginia Tech

At the JMM Awards Celebration on Monday, January 5th at 5:00PM, Dominique Guillot, as the ILAS speaker, will be presented with an award certificate.

The 11th Workshop on Matrix Equations and Tensor Techniques (METT XI) Leuven, Belgium, January 7–9, 2026

We are pleased to announce the 11th Workshop on Matrix Equations and Tensor Techniques (METT XI), which will take place January 7–9, 2026, at KU Leuven in Leuven, Belgium.

As in previous editions, the workshop will focus on recent advances in the theory, computation, and applications of linear and nonlinear matrix and tensor equations. Participants are invited to present their work either as a talk or a poster. There is no conference fee, but registration is mandatory.

The deadline for submission of abstracts is December 1st, 2025, while the registration deadline is December 5th.

Invited speakers:

- Markus Bachmayr (RWTH Aachen University, Germany)
- Gianluca Ceruti (Universität Innsbruck, Austria)
- Mariya Ishteva (KU Leuven, Belgium)
- Jens Saak (Max Planck Institute for Dynamics of Complex Technical Systems, Germany)

Further information is available at `https://homes.esat.kuleuven.be/~mettxi/index.html`

Special session on Applied and Numerical Linear Algebra *at the* 96th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM) Stuttgart, Germany, March 16–20, 2026

The 96th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM) will be hosted at the University of Stuttgart March 16–20, 2026 in Stuttgart, Germany.

On behalf of the organizing committee, we would like to invite you, your colleagues, postdocs, and graduate students to join **Section S17: Applied and Numerical Linear Algebra**. The topical speakers of the session are

- Maike Meier (University of Groningen) and
- Patrick Kürschner (HTWK Leipzig).

Contributed talks in this session will have a length of 15 minutes plus 5 minutes for discussion. Once the abstracts have been received, depending on the number of submissions and allowed time slots, we may have to select an appropriate number of abstracts for presentations.

For more detailed information concerning the submission of abstracts as well as registration and accommodation, please visit the conference website at

`https://jahrestagung.gamm.org/annual-meeting-2026/96th-annual-meeting-2/`

- The deadline for abstract submission is December 8th, 2025.
- The deadline for early online registration is January 22nd, 2026.
- Online registration will close on March 1st, 2026.

Please note that the organizers cannot provide financial support or exceptions from the registration fee.

Questions regarding the special session can be directed to the organizers:

Jemima Tabeart, TU Eindhoven, the Netherlands (`j.m.tabeart@tue.nl`)
Marcel Schweitzer, Bergische Universität Wuppertal, Germany (`marcel@uni-wuppertal.de`)

NSF-CBMS Regional Research Conference on Strong Matrix Properties and the Inverse Eigenvalue Problem Ypsilanti, Michigan, USA, May 11–15, 2026

Strong properties of matrices, modeled on the Strong Arnold Property, have led to significant recent advances on the Inverse Eigenvalue Problem for Graphs. (See `https://www.ams.org/notices/202002/rnoti-p257.pdf` for an introductory overview.) These new strong properties have also energized significant progress in the study of related invariants, such as the maximum multiplicity of an eigenvalue and the minimum number of distinct eigenvalues of a graph, and they have led to new connections with graph minors and graph propagation procedures. These connections have also

given rise to new matrix theory questions and results, new forbidden minor characterizations, new minor-monotone graph parameters, and new graph theoretic questions and results.

The conference will provide early-career researchers and graduate students, as well as established researchers, with access to cutting edge tools and ideas needed to explore this fruitful, new, and evolving area of mathematics. The conference is one of five Conference Board of Mathematical Sciences (CBMS) Regional Research Conferences in the 2025–2026 series, each of which is partially funded by the National Science Foundation (NSF) and features a distinguished team of lecturers delivering a sequence of lectures on a topic of important current research in one sharply focused area of the mathematical sciences.

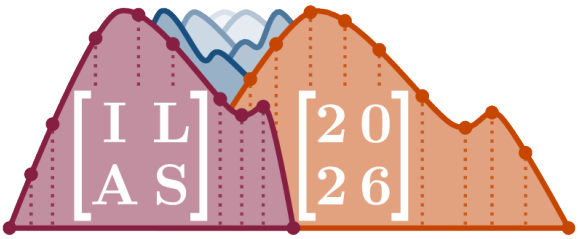
The primary speakers for this conference are:

Bryan Shader (University of Wyoming)
Helena Šmigoc (University College Dublin)
Kevin N. Vander Meulen (Redeemer University)

Participation from all countries is encouraged. Support of up to US$700 will be available for up to 30 participants affiliated with U.S. institutions with preference given to graduate students and junior faculty. The online application can be found at `https://sites.google.com/emich.edu/cbms` and the deadline to apply is January 23rd, 2026, with decisions available by February 1st.

Questions can be directed to the organizer, Lon Mitchell (`lon.mitchell@emich.edu`).

The 27th ILAS Conference: Linear Algebra on the Blue Ridge: Panoramas of Theory and Application
Blacksburg, Virginia, USA, May 18–22, 2026



The 27th Conference of the International Linear Algebra Society will be held in Blacksburg, Virginia from May 18–22, 2026, on the campus of Virginia Tech. The conference's theme, "Linear Algebra on the Blue Ridge: Panoramas of Theory and Application" highlights the geographical setting for the conference in the mountains of southwest Virginia, but also serves as an invitation to researchers from across linear algebra, ranging from core areas through to numerical analysis, applications, and linear algebra education.

Local Organizing Committee: Christopher Beattie, Paul Cazeaux, Eric de Sturler, Mark Embree, Serkan Gugercin, Agnieszka Miedlar, Mirjeta Pasha, Megan Wawro, Steffen Werner.

Scientific Committee: Shreemayee Bora, Geir Dahl, Eric de Sturler, Ioana Dumitriu, Stefan Güttel, Misha Kilmer, Agnieszka Miedlar, André Ran, Helena Šmigoc, David Strong, Daniel Szyld, Raf Vandebril, Heather Wilber.

Submission for minisymposium proposals is now open and applications will be accepted through December 15th, 2025. For more information, including proposal guidelines, visit the following link to the official Call for Minisymposia:

`https://ilas2026.math.vt.edu/submissions.html#section-mscall`

To submit a minisymposium proposal, you may use the following link:

`https://indico.math.vt.edu/event/2/abstracts/`

Contributed talk submissions will open on December 16th, 2025.

Further details, such as registration information, lodging details, and program updates, will be available in due course from the conference website `https://ilas2026.math.vt.edu`. For immediate questions, email the local organizing committee at `ilas2026@math.vt.edu`.

We look forward to a vibrant ILAS meeting in May 2026, and hope to see you in Blacksburg!

The 18th Western Canadian Linear Algebra Meeting (WCLAM) North Vancouver, Canada, May 30–31, 2026

The 18th Western Canadian Linear Algebra Meeting (WCLAM) will be held at Capilano University in North Vancouver, British Columbia, Canada, May 30–31, 2026. This long-running regional conference rotates among institutions in Western Canada and focuses on linear algebra and closely related areas.

WCLAM 2026 will continue the tradition of bringing together researchers, students, and practitioners working in linear algebra and its applications. The meeting will include invited plenary lectures, contributed talks, and poster presentations. A special emphasis will be placed on supporting and encouraging early-career researchers.

Confirmed Plenary Speakers:

- Minerva Catral (Xavier University, USA) — Combinatorial matrix theory; analytic number theory.
  Dr. Catral's research explores how combinatorial matrix properties (such as sign or zero-nonzero patterns) influence algebraic characteristics like power positivity and inertia.
- Beatrice Meini (University of Pisa, Italy) — Numerical linear algebra (ILAS Lecturer).
  Dr. Meini specializes in numerical methods for applied matrix problems, including algebraic Riccati equations and Markov chains. Her plenary lecture is supported by the International Linear Algebra Society.
- Pietro Paparella (University of Washington Bothell, USA) — Matrix analysis.
  Dr. Paparella's work focuses on entrywise nonnegative matrices and their spectral properties.

Participants from Western Canada, the rest of Canada, the United States, and abroad are warmly invited to attend. Contributions in all areas of linear algebra and matrix theory are welcome, including applications to computer science, data science, engineering, and related fields.

Further information about the meeting, including registration details, travel information, and local arrangements, will be posted on the conference webpage:

`https://capilanou.ca/wclam-2026`

A designated abstract submission link will also be provided on the conference webpage. Those interested in presenting a contributed talk or poster will be able to submit their abstracts there when the submission window opens (deadline expected around mid-April 2026).

WCLAM 2026 is organized by Hadi Kharaghani, Shaun Fallat, Steve Kirkland, Mike Tsatsomeros, Sarah Plosker, and Pauline van den Driessche, together with a local organizing team at Capilano University led by Amir Amiraslani. The meeting is supported by the Pacific Institute for the Mathematical Sciences, the International Linear Algebra Society, and Capilano University.

ALAMA 2026 Huelva, Spain, June 17–19, 2026

The Thematic Network on Linear Algebra, Matrix Analysis, and Applications (ALAMA) will hold its ninth biennial meeting at the University of Huelva, Huelva, Spain, June 17–19, 2026, continuing the trajectory started in Vitoria-Gasteiz (2008) and followed by Valencia, Leganés, Barcelona, León, Alicante, Alcalá de Henares and Gijón. ALAMA 2026 will be a forum for researchers to come together around linear algebra, matrix analysis, and their applications, fostering the exchange of ideas in an inclusive and diverse environment.

The program will include plenary lectures, parallel sessions, and mini-symposia, along with opportunities for interaction and strengthening the academic community. Technical organization is being handled by the General Foundation of the University of Alcalá with the support of various institutions.

Saint-Girons V Conference
Saint-Girons, France, June 29 – July 3, 2026

Science is based on observations, experiments, development of theories, and often development of models for simulating observed phenomena and/or predicting future events. Advances in computing capabilities have had a far-reaching impact not only on simulations, but also on data analysis and artificial intelligence (AI), leading to a digital society era. Numerical tools have played a fundamental role in these advances and in high-performance computing (HPC) in general. From the first meeting in Saint-Girons, this conference series has assessed the progress made in the full use of computing capabilities and identified potential challenges for discussion in subsequent meetings. One characteristic of the meetings is that in addition to keynote speakers and established researchers, participation of students is encouraged and the programme and budget are designed to support student attendance. The fifth meeting in the series – Saint-Girons V – will seek to cover a broad range of topics, as we move towards new frontiers of computing in applications and hardware.

Considering an evolving and increasingly complex HPC ecosystem, Saint-Girons V will give participants an opportunity to discuss pressing HPC-related topics, as well as the rise of AI and related massive data processing, the emerging area of quantum computing, and the potential implications of a post-Moore era of computing. The uniqueness of the meetings in Saint-Girons lies in the combination of a set of selected presentations, social events, and time for sharing and exchanging ideas at the pace of a picturesque mountain village. New research topics and collaborations have emerged from all these moments.

Registration and more information about participation will be available in early 2026. For further information, please visit `https://saintgironsconference.eu`.

The 37th International Workshop on Operator Theory and its Applications (IWOTA)
An ICM Satellite Conference
Québec City, Canada, August 3–7, 2026

IWOTA International Workshop on Operator Theory and its Applications

The IWOTA conference series is the largest event in operator theory and its applications, bringing together leading international experts from pure mathematics and application areas to trace the future development of operator theory and related areas such as complex analysis, harmonic analysis, linear algebra, random matrix theory, and mathematical physics, as well as their applications, including control theory, signal processing, and machine learning.

IWOTA 2026 will take place August 3–7, 2026, at the Université Laval in Québec City, Canada. IWOTA 2026 is an International Congress of Mathematicians (ICM) Satellite Conference. The conference will provide a medium for an intense exchange of new results, information and opinions, and for international collaboration in operator theory and its applications worldwide. It will further set directions for future research through its conference activities and proceedings. A substantial part of IWOTA consists of special sessions whose organizers have been selected to ensure a coherent, diverse and attractive agenda of research activity and talks. Special sessions provide opportunities for all participants to present their results and interact with other researchers with similar interests.

The IWOTA 2026 main organizing committee consists of Javad Mashreghi and Frédéric Morneau-Guérin (Université Laval) in collaboration with the IWOTA steering committee chaired by J. William Helton (University of California, San Diego). They are further assisted by Ludovick Bouthat (Université Laval), George R. Exner (Bucknell University), Mostafa Nasri (University of Winnipeg), Marcu-Antone Orsoni (Université Laval), and William T. Ross (University of Richmond). Registration for the conference will open in Spring 2026. For updates and further information, including a list of special sessions and plenary speakers, visit `https://iwota-2026.fsg.ulaval.ca/en`.

ONGOING ONLINE SEMINARS

Algebraic Graph Theory Seminar

https://math.uwaterloo.ca/~agtheory

Host: University of Waterloo

Schedule: weekly on Mondays

Time: 11:30AM, Waterloo (Ontario, Canada) time

Most recent talk:

Eigenpolytopes

Chris Godsil (University of Waterloo, Canada)

Next talk: TBA

Contact: Sabrina Lato (smlato@uwaterloo.ca)

Matrix Seminar

https://docs.google.com/document/d/1MswSd16JqsZE294kYCXujLio4cnAiuYv6QKRc6BxvI0/edit

Host: University of Nevada, Reno

Schedule: biweekly on Fridays

Time: 4:15PM, Reno (Nevada, USA) time

Most recent talk:

On rank k and symplectic eigenvalue linear preservers

Aedan Jarrod A. Potot (University of Philippines, Diliman, Philippines)

Next talk:

December 5, 2025

Iana Fajardo (University of Philippines, Diliman, Philippines)

Contact: Pan Shun Lau (plau@unr.edu)

05C50 Online

https://sites.google.com/view/05c50online/home

Host: University of Manitoba

Schedule: biweekly on Fridays

Time: 10:00AM, Winnipeg (Manitoba, Canada) time

Most recent talk:

Algebraic bounds for sum-rank-metric codes

Antonina P. Khramova (Eindhoven University of Technology, Netherlands)

Next talk: January 2026

Contact: Hermie Monterde (monterdh@myumanitoba.ca)

Matrix Analysis and Linear Algebra Group Seminar

https://sites.google.com/up.edu.ph/mala/mala-seminar

Host: University of the Philippines Diliman

Schedule: biweekly on Wednesdays

Time: 3:00PM, Philippine Standard Time

Most recent talk:

On stable matrices exhibiting Turing instability

Nicole Joy P. Datu (University of the Philippines Diliman, Philippines)

Next talk: TBA

Contact: (mala@math.upd.edu.ph)

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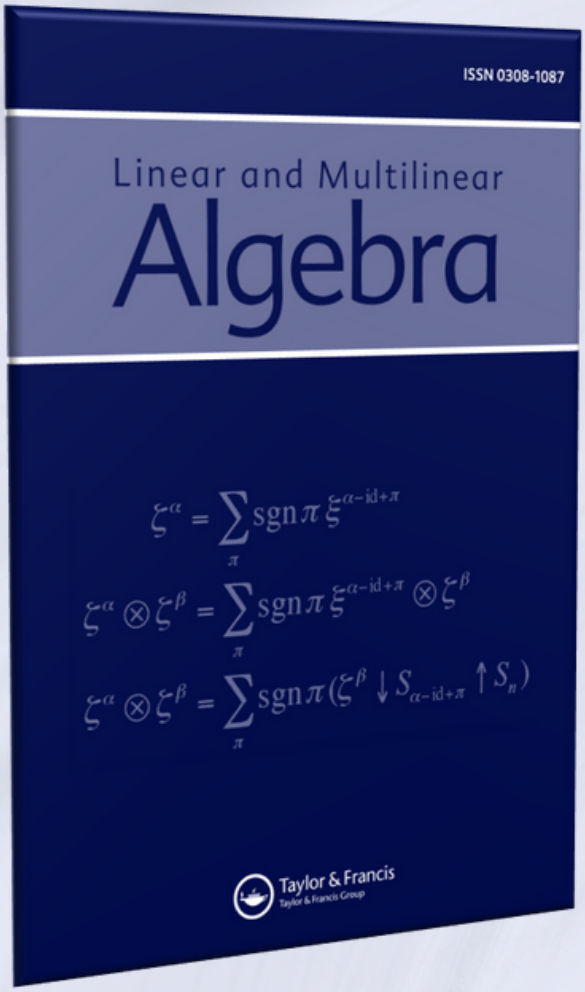

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IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to Problems 74-1 and 74-2. Solutions are still sought for Problems 68-2, 68-4, 69-2, 69-3, 70-2, 71-3, 72-1, 72-2, and 73-2, and we also invite solutions the one new problem in the present issue 75.

Problem 74-1: Determinant of a Matrix Difference
Proposed by Roger A. Horn, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu

Let $n \geq 2$. Let A and B be $n \times n$ complex matrices such that $\det A = \det B$ and $\operatorname{rank}(A+B) = 1$. Show: (a) If $\det A \neq 0$, then $\det(A - B) = 0$ if and only if n is odd. (b) If $\det A = 0$, then $\det(A - B) = 0$.

Solution 74-1.1 by Piyush Verma, *Indian Institute of Technology, Bhubaneswar, Odisha, India*, s21ma09010@iitbbs.ac.in

Since $\operatorname{rank}(A + B) = 1$, we have $A + B = \mathbf{x}\mathbf{y}^T$ for some nonzero column vectors $\mathbf{x}, \mathbf{y} \in \mathbb{C}^n$.

(a) Since $\det A \neq 0$, the matrix A is invertible. We have

$$A - B = 2A - (A + B) = 2A - \mathbf{x}\mathbf{y}^T = 2A\left(I - \frac{1}{2}A^{-1}\mathbf{x}\mathbf{y}^T\right) = 2A\left(I - \frac{1}{2}\mathbf{u}\mathbf{y}^T\right),$$

where $\mathbf{u} = A^{-1}\mathbf{x}$. From Sylvester's determinant identity, $\det(I + \mathbf{u}\mathbf{y}^T) = 1 + \mathbf{y}^T\mathbf{u}$. From this, we obtain

$$\begin{aligned}
\det(A - B) &= \det\left(2A\left(I - \frac{1}{2}\mathbf{u}\mathbf{y}^T\right)\right) \\
&= 2^n (\det A) \det\left(I - \frac{1}{2}\mathbf{u}\mathbf{y}^T\right) \\
&= 2^n (\det A)\left(1 - \frac{1}{2}\mathbf{y}^T\mathbf{u}\right) \\
&= 2^n (\det A)\left(1 - \frac{1}{2}\mathbf{y}^T A^{-1}\mathbf{x}\right).
\end{aligned}$$

Since $\det A \neq 0$, the above shows that $\det(A - B) = 0$ if and only if $\mathbf{y}^T A^{-1}\mathbf{x} = 2$. We have $A + B = \mathbf{x}\mathbf{y}^T$, so we can write $B = \mathbf{x}\mathbf{y}^T - A$. Thus, we get

$$\begin{aligned}
\det B &= \det(-A + \mathbf{x}\mathbf{y}^T) \\
&= \det(-A) \det(I - A^{-1}\mathbf{x}\mathbf{y}^T) \\
&= (-1)^n (\det A)(1 - \mathbf{y}^T A^{-1}\mathbf{x}).
\end{aligned}$$

Since $\det A = \det B$, we have

$$\det A = (-1)^n (\det A)(1 - \mathbf{y}^T A^{-1}\mathbf{x}).$$

Dividing both sides by $\det A$ gives

$$1 = (-1)^n (1 - \mathbf{y}^T A^{-1}\mathbf{x}).$$

If n is odd, then $\mathbf{y}^T A^{-1}\mathbf{x} = 2$; if n is even, then $\mathbf{y}^T A^{-1}\mathbf{x} = 0$. Therefore, $\det(A - B) = 0$ if and only if n is odd.

(b) Assume for the sake of contradiction that $\det A = 0$ while $\det(A - B) \neq 0$. Let $P = A + B = \mathbf{x}\mathbf{y}^T$ and let $Q = A - B$. Then $A = \frac{P+Q}{2}$ and $B = \frac{P-Q}{2}$. Since $\det A = \det B = 0$, we have $\det(P + Q) = \det(P - Q) = 0$. And Q is invertible, since $\det Q = \det(A - B) \neq 0$. Now,

$$0 = \det(P + Q) = \det(\mathbf{x}\mathbf{y}^T + Q) = (\det Q) \det(I + Q^{-1}\mathbf{x}\mathbf{y}^T)$$

and

$$0 = \det(P - Q) = \det(\mathbf{x}\mathbf{y}^T - Q) = \det(-Q) \det(I - Q^{-1}\mathbf{x}\mathbf{y}^T).$$

Since $\det Q \neq 0$, this yields $\det(I + Q^{-1}\mathbf{x}\mathbf{y}^T) = 0$ and $\det(I - Q^{-1}\mathbf{x}\mathbf{y}^T) = 0$. By Sylvester's determinant identity,

$$\det(I + Q^{-1}\mathbf{x}\mathbf{y}^T) = 1 + \mathbf{y}^T Q^{-1}\mathbf{x} = 0$$

and

$$\det(I - Q^{-1}\mathbf{x}\mathbf{y}^T) = 1 - \mathbf{y}^T Q^{-1}\mathbf{x} = 0.$$

Together, these yield

$$\mathbf{y}^T Q^{-1}\mathbf{x} = -1 \qquad \text{and} \qquad \mathbf{y}^T Q^{-1}\mathbf{x} = 1,$$

which is a contradiction. Therefore, our assumption that $\det(A - B) \neq 0$ must be false. Hence, if $\det A = 0$, then $\det(A - B) = 0$.

Solution 74-1.2 by the proposer

(a) Suppose that $\det A = \det B \neq 0$. Let $C = A^{-1}B$, so $\det C = \frac{\det B}{\det A} = \frac{\det A}{\det A} = 1$. We have $1 = \operatorname{rank}(A + B) = \operatorname{rank}(A(I + C))$, which, since A is nonsingular, implies $\operatorname{rank}(I + C) = 1$. Hence, the geometric multiplicity of -1 as an eigenvalue of C is $n - 1$. Therefore, the algebraic multiplicity of -1 is at least $n - 1$, and hence $1 = \det C = (-1)^{n-1}\lambda$ for $\lambda \in \{-1, 1\}$ with $\lambda = 1$ if and only if n is odd. In particular, 1 is an eigenvalue of C if and only if n is odd. From this we see that n is odd if and only if $0 = \det(I - C)$, which holds if and only if $0 = (\det A)\det(I - C)$. To complete the proof, then, note that $(\det A)\det(I - C) = \det(A(I - C)) = \det(A - B)$.

(b) Suppose that $\det A = \det B = 0$, and let $\mathbf{x}$ and $\mathbf{y}$ be nonzero vectors such that $A + B = \mathbf{x}\mathbf{y}^T$. Then there are nonzero vectors $\mathbf{w}$ and $\mathbf{z}$ such that $A\mathbf{w} = B\mathbf{z} = \mathbf{0}$. If $A\mathbf{z} = \mathbf{0}$, then $A\mathbf{z} - B\mathbf{z} = (A - B)\mathbf{z} = \mathbf{0}$, and hence $\det(A - B) = 0$. Suppose that $A\mathbf{z} \neq \mathbf{0}$, which ensures that $\mathbf{w}$ is not a scalar multiple of $\mathbf{z}$. Then $\mathbf{0} \neq A\mathbf{z} = (A + B)\mathbf{z} = \mathbf{x}\mathbf{y}^T\mathbf{z} = (\mathbf{y}^T\mathbf{z})\mathbf{x}$, so $\mathbf{y}^T\mathbf{z} \neq 0$, while $B\mathbf{w} = (A + B)\mathbf{w} = \mathbf{x}\mathbf{y}^T\mathbf{w} = (\mathbf{y}^T\mathbf{w})\mathbf{x}$. Let $\mu = (\mathbf{y}^T\mathbf{w})/(\mathbf{y}^T\mathbf{z})$ and observe that $\mu A\mathbf{z} = \mu(\mathbf{y}^T\mathbf{z})\mathbf{x} = (\mathbf{y}^T\mathbf{w})\mathbf{x} = B\mathbf{w}$. Then $\mu\mathbf{z} + \mathbf{w} \neq \mathbf{0}$ and $(A - B)(\mu\mathbf{z} + \mathbf{w}) = \mu A\mathbf{z} - B\mathbf{w} = \mathbf{0}$, so $\det(A - B) = 0$.

Similar, albeit longer, solutions were given (separately) by Ranjita Behera, *Binayak Acharya College, Berhampur, Odisha, India*, ranjitabehera7809@gmail.com, by Kadali Kranthi Behera, *Indian Institute of Technology, Madras, Tamil Nadu, India*, ma21d010@smail.iitm.ac.in, by Kuldeep Sarma, *Indian Institute of Science, Bengaluru, Karnataka, India*, kuldeep.sarma65@gmail.com, and by Nèstor Thome, *Universitat Politècnica de Valèencia, Spain*, njthome@mat.upv.es.

Also solved by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Problem 74-2: Determinant of a Matrix of Factorials

Proposed by Jeffrey Stuart, *Pacific Lutheran University, Tacoma, Washington, USA*, jeffrey.stuart@plu.edu

Let n and p be positive integers. For the $n \times n$ matrix $A = (a_{ij})$ with $a_{ij} = (i + j - 1)!$ for $1 \leq i, j \leq n$,

$$\det A = \prod_{k=0}^{n-1} [k!(n - k)!] .$$

For the $n \times n$ matrix $B = (b_{ij})$ with $b_{ij} = (p + i + j - 1)!$ for $1 \leq i, j \leq n$,

$$\det B = \prod_{k=0}^{n-1} [k!(p + k)!] .$$

Solution 74-2.1 by Piyush Verma, *Indian Institute of Technology, Bhubaneswar, Odisha, India*, s21ma09010@iitbbs.ac.in

Note that for all $i, j \geq 1$,

$$a_{ij} = (i + j - 1)! = (i - 1)! \cdot \binom{i + j - 1}{j} \cdot j!.$$

Let D_1 and D_2 be the $n \times n$ diagonal matrices $D_1 = \operatorname{diag}(0!, 1!, \ldots, (n-1)!)$ and $D_2 = \operatorname{diag}(1!, 2!, \ldots, n!)$. Let $S = (s_{ij})$ be the $n \times n$ matrix with

$$s_{ij} = \binom{i+j-1}{j} \text{ for } 1 \le i, j \le n.$$

Then $A = D_1 S D_2$. Observe that

$$\det D_1 = \prod_{k=0}^{n-1} k! \qquad \text{and} \qquad \det D_2 = \prod_{j=1}^{n} j! = \prod_{k=0}^{n-1} (n-k)!,$$

where the last product is achieved by reindexing with $j = n - k$. From [1], the matrix S admits an LU decomposition, where L is a lower triangular matrix with all diagonal entries 1, and where U is an upper triangular matrix with all diagonal entries 1. Thus, $\det S = 1$. Therefore,

$$\begin{aligned} \det A &= (\det D_1)(\det S)(\det D_2) = \prod_{k=0}^{n-1} k! \cdot 1 \cdot \prod_{k=0}^{n-1} (n-k)! \\ &= \prod_{k=0}^{n-1} [k!(n-k)!]. \end{aligned}$$

The claim that

$$\det B = \prod_{k=0}^{n-1} [k!(p+k)!]$$

for all $p > 0$ is not valid in general. It suffices to provide a counterexample. Let $p = n = 2$. Then

$$\det \begin{bmatrix} 3! & 4! \\ 4! & 5! \end{bmatrix} = 144$$

but

$$\prod_{k=0}^{1} [k!(2+k)!] = 0!2! \cdot 1!3! = 12.$$

References

[1] A. Edelman and G. Strang. Pascal matrices. *Amer. Math. Monthly*, 111(3):189–197, 2004.

Solution 74-2.2 by Ranjita Behera, *Binayak Acharya College, Berhampur, Odisha, India*, ranjitabehera7809@gmail.com

Editor's note: This solution was submitted with a 2×2 counterexample to part (b) and a clarification that the flaw lay in a typographical error in the definition of the matrix B. In particular, with $b_{ij} = (p+i+j-2)!$ rather than $b_{ij} = (p+i+j-1)!$, the stated formula for $\det B$ is correct, as the following solution shows.

Let the $n \times n$ matrix $B = (b_{ij})$ be given by $b_{ij} = (p+i+j-2)!$ for $1 \le i, j \le n$. Then the matrix A given in part (a) is the special case of B where $p = 1$. As a result, it suffices to prove the conclusion of part (b) for this corrected matrix B.

Toward this end, observe that, for $1 \le i, j \le n$,

$$b_{ij} = (p+i+j-2)! = (p+(i-1)+(j-1))! = (i-1)! \cdot \binom{p+(i-1)+(j-1)}{i-1} \cdot (p+(j-1))!.$$

Let $S = (s_{ij})$ be the $n \times n$ matrix with

$$s_{ij} = \binom{p+(i-1)+(j-1)}{i-1} \text{ for } 1 \le i, j \le n.$$

Let the $n \times n$ diagonal matrices D_{row} and D_{col} be defined by

$$D_{\text{row}} = \operatorname{diag}(0!, 1!, \ldots, (n-1)!)$$

and

$$D_{\text{col}} = \operatorname{diag}(p!, (p+1)!, \ldots, (p+n-1)!)\,.$$

Then we have

$$B = D_{\text{row}} S D_{\text{col}}.$$

Consequently,

$$\begin{aligned}\det B &= (\det D_{\text{row}})(\det S)(\det D_{\text{col}}) \\ &= \left(\prod_{k=0}^{n-1} k!\right)(\det S)\left(\prod_{k=0}^{n-1}(p+k)!\right) \\ &= (\det S)\prod_{k=0}^{n-1}[k!\,(p+k)!]\end{aligned}$$

and it suffices to show that $\det S = 1$ for all positive integers p. First, reindex the rows and columns of S so that the indices are $0, 1, \ldots, n-1$, i.e., so that

$$s_{ij} = \binom{p+i+j}{i} \quad \text{for } 0 \le i, j \le n-1.$$

Then repeatedly perform differencing of adjacent columns $C_k \to C_k - C_{k-1}$ as follows. First, reduce the first row of S to $\begin{bmatrix}1 & 0 & 0 & \cdots & 0 & 0\end{bmatrix}$ via the sequence of column operations $C_n - C_{n-1}, C_{n-1} - C_{n-2}, \ldots, C_2 - C_1$. Call the resulting matrix $S^{(1)}$. Note that the effect of the above operations on the second row is given by Pascal's identity for binomial coefficients,

$$\binom{m}{r} = \binom{m-1}{r} + \binom{m-1}{r-1}$$

or, equivalently,

$$\binom{m}{r} - \binom{m-1}{r} = \binom{m-1}{r-1}.$$

In particular, in the matrix $S^{(1)}$, all entries in the second row save for the initial entry are 1. So it is possible then to reduce the second row of $S^{(1)}$, via column operations, to the form $\begin{bmatrix}? & 1 & 0 & \cdots & 0\end{bmatrix}$ using the sequence $C_n - C_{n-1}, C_{n-1} - C_{n-2}, \ldots, C_3 - C_2$. Call the resulting matrix $S^{(2)}$. In this matrix, each entry in row 2 beyond the first two entries is equal to 1. Thus, we can reduce the third row of $S^{(2)}$ by differencing adjacent columns until the diagonal entry in the row becomes 1, with subsequent entries 0. Continuing in this fashion, eventually the resulting matrix obtained is lower triangular with all diagonal entries 1. Since column differencing is an elementary column operation that preserves the determinant, this shows that $\det S = 1^n = 1$.

IMAGE PROBLEM CORNER: NEW PROBLEMS

Problems: We offer one new problem in this issue and invite readers to submit solutions for publication in *IMAGE*.
Submissions: Please submit proposed problems and solutions in macro-free LaTeX along with the PDF file by e-mail to *IMAGE* Problem Corner editor Jeff Stuart (`jeffrey.stuart@plu.edu`).

New Problems:

Problem 75-1: A Very Revealing Factorization

Proposed by Eugene A. Herman, *Grinnell College, Grinnell, Iowa, USA*, `eaherman@gmail.com`

Let n be a positive integer, and let $M_n(\mathbb{C})$ denote the set of all $n \times n$ complex matrices. Suppose that $A \in M_n(\mathbb{C})$ satisfies $A^2 = 0$. Prove that there exist $B, C \in M_n(\mathbb{C})$ such that $A = BC$ and $CB = 0$.