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## FEATURE INTERVIEW

### “I became fascinated with conjugate gradients”

Anne Greenbaum Interviewed by Michael Overton<sup>1</sup>



Anne Greenbaum on the Summerland trail in Mount Rainier National Park, Washington, USA

**M.O. - When did you first get interested in math?**

**A.G. -** I think it was in high school. It was the easiest subject. History, you had to memorize too much. Literature, you had to read too much, but math, you could just reason your way through it. And I had an older brother who was studying math in college and later graduate school, and he would come home and tell me about interesting theorems and problems. He told me about Gödel’s Theorem and the Four Color Problem, which was unproved at that time. I spent a lot of time just drawing maze-like pictures, seeing if I could get something that couldn’t be colored with four colors. I never succeeded, but still enjoyed it.

**M.O. - Did you have any inspiring high school teachers?**

**A.G. -** My calculus teacher, Mrs. Taback. She had a master’s degree from the University of Chicago. She was very bright. I think she was inspiring.

**M.O. - Then you went to college at the University of Michigan. Can you tell us about that?**

**A.G. -** Okay, so I double-majored in math and philosophy at Michigan. Honestly, I got sick of math there. I could do the proofs and that sort of thing, but I kept asking *why*. I mean, the professors would say, well, this is a deep problem, this is a deep result, this is kind of trivial, and I never understood why. But I really enjoyed a numerical course at Michigan given by George

Fix. Charlie Van Loan was a TA for that course. I had worked at NASA over the summer and enjoyed that. So, although I considered graduate school in both math and philosophy, I decided to get a job for a while and I went to Lawrence Livermore Lab after college.

**M.O. - And what did you work on there?**

**A.G. -** Programming. I was a programmer.

**M.O. - In Fortran?**

**A.G. -** Yeah, that’s all we had. I enjoyed it, but what I didn’t like was being bossed around by Ph.D. physicists. So that’s when I decided to get a Ph.D. myself. I went to UC Berkeley. The lab let me continue working full-time, but gave me some time off to take classes, and they paid the tuition, because it was part of the University of California. So it was a good deal.

**M.O. - And who did you meet in Berkeley who had an influence on you?**

**A.G. -** All right, to be honest I didn’t go to graduate school at Berkeley to learn more. I figured I could learn more on my own. I went to get the letters “Ph.D.” after my name. So, I didn’t really interact that much with people at Berkeley. I mostly interacted with people at Livermore. Garry Rodrigue was somebody that I worked with there. And we had visitors, like Richard Varga and Gene Golub, who would come to Livermore. At Berkeley, my advisor was going to be Paul Concus, who was actually at LBL [Lawrence Berkeley Laboratory] but had an adjunct appointment at Berkeley. But it turned out that he didn’t have the right kind of appointment to advise students, and so, officially, my advisor became Beresford Parlett. I certainly enjoyed a numerical linear algebra class that I took from Vel Kahan. I mean, he scared me to death, but his class was really good. Every class, he would make some student take notes, and then they’d be Xeroxed and distributed to the class. I still have the notes from that class.

**M.O. - And did you have to take the notes on multiple occasions?**

**A.G. -** At least once. In fact, I took the notes the day he talked about showing that the numerical range, or the field of values, of a matrix is a convex set.

<sup>1</sup>New York University, USA; mo1@nyu.edu

**M.O. - So what was the title of your thesis?**

**A.G. -** Well, it was on conjugate gradient (CG) methods. The title was *Convergence Properties of the Conjugate Gradient Algorithm in Exact and Finite Precision Arithmetic*.

**M.O. - And did you do it on your own, or with Paul, or...?**

**A.G. -** Pretty much on my own. I became fascinated with conjugate gradients, because the lab was just starting to use that method in their codes. They had previously used ADI-type methods that sometimes had trouble converging and that sort of thing. And they were just beginning to incorporate conjugate gradients. So I was really interested in it. I went to a few conferences, and heard people give talks about it. And they would say things like, "Well, we have this bound on the convergence rate based on the square root of the condition number. But in fact, it converges much faster than that." And they would just wave their hands as to the reason why. So, the first problem that I tackled was why. It seemed like we should be able to say something about just why it converges as fast as it does. So I gave a sharp upper bound for the error in conjugate gradients. I guess the next thing that I became interested in was finite precision behavior. Parlett was interested in finite precision Lanczos, and I was interested in finite precision CG. And that was really a much harder problem to deal with. I came up with the idea very early, but the proof that I eventually came up with was ugly.

**M.O. - The proof of what, exactly?**

**A.G. -** When you do finite precision CG, or Lanczos, the polynomials that you construct are not orthogonal with respect to the measure that they're supposed to be orthogonal with respect to, namely a set of weights on the eigenvalues of the matrix. But I proved that they are orthogonal with respect to a smeared-out version of that measure, where the weights are on tiny intervals about the eigenvalues of the matrix.

**M.O. - So you were already doing this in your thesis, or was this after you got your Ph.D.?**

**A.G. -** Well, I was doing it while I was working on my thesis, but I couldn't really include it in my thesis, because I didn't have a proof of it.

**M.O. - So did getting the Ph.D. stop the Ph.D. physicists from bossing you around?**

**A.G. -** Yeah, I suppose it did. I had been working with various physics groups as a programmer, and then I moved to the math group and was allowed to do research. I started to get more and more interested in proving things, but you don't get rewarded at the lab for proving things, you get rewarded for making codes run faster and more accurately. So I started to think about going to academia.

**M.O. - That's when you came to NYU as a research professor.**

**A.G. -** Right. I believe my title there was: Senior Research Scientist. It was a soft money position, ostensibly to work on the "ultracomputer" project. I did some parallel computing there and some work on LAPACK, but this was not my main interest, so I worked on other things as well. It was in New York that I finally managed to eke out a proof of the finite precision CG thing. It was a long, ugly paper [2]. Nobody in the U.S. was reading it. But Zdeněk Strakoš in Prague read it very carefully. So we started communicating, by snail mail at first, and then by email when that became a thing. And so I invited him to come to New York. I had never met him before.

**M.O. - Was this in the 90s, after the Wall came down?**

**A.G. -** Yeah, when he came to New York, it was after the Wall came down. I think it was the first time he was allowed to leave the country. So he came out and we worked together writing a paper that illustrated the result without proofs, so it was much easier to read, much clearer to understand [5]. And then I visited him in Prague a number of times.

**M.O. - And eventually, you got the Bolzano Award given by the Czech Academy of Sciences.**

**A.G. -** I'm not actually sure why I got that, but I'm very grateful for it. I'm sure Strakoš had to pull some strings in order for me to get it. I think part of what did it, to be honest, was that I had been to the Czech Republic before. There was a meeting in Pilsen, when I was just a graduate student. A Russian gave a talk, and this was when it was behind the Iron Curtain, when it was Czechoslovakia. I asked a question about whether he had tried conjugate gradients or something. And this was considered kind of cheeky, coming from a young graduate student. And so he was a little bit upset about this. But the Czechs were not on good terms with the Russians, and so the Czechs were thrilled about it. Every time I came after that, they would bring up this conference in Pilsen.

**M.O. - Then, much later, you were honored by an invitation by the AWM to give the Sonia Kovalevsky Lecture. I was intrigued by the title of your talk there, *Two of my Favorite Problems*, so I thought I would ask you to tell us about those. I'm pretty sure I can guess what one of them is, but I'm not sure about the other one.**

**A.G. -** Well, the other one is CG and finite precision arithmetic, and trying to find a good proof of this analogy with constructing orthogonal polynomials on this smeared-out weight function.



*Michael Overton and Anne Greenbaum at Snow Lake*

**M.O. - So you still don't feel, even after the work with Strakoš, that you have really nailed that yet?**

**A.G. -** No, nobody that I know of has done that. I mean, the proof is there, it's just 30–40 pages long, and relies on a lot of Chris Paige's work, of course. But I still feel that there should be an elegant proof. And then the other problem, as you know, is Crouzeix's conjecture, which continues to fascinate me, though I've sort of run out of ideas on what to do about it. You and I worked on it a bunch [3].

**M.O. - Yes, I really enjoyed working with you on that.**

**A.G. -** It's a nice problem because, even if you don't solve the problem, there are lots of interesting things related to it that you have a much better chance of solving. So, I still think it's worthwhile working on it, even if you don't necessarily expect to prove the conjecture, or disprove it.

**M.O. - And another paper [4] of yours that is very well known is on how eigenvalues tell you nothing about GMRES. How did you get into that problem?**

**A.G. -** People often try to analyze the convergence of GMRES [the Generalized Minimal RESidual method, an iterative method for solving nonsymmetric linear systems] based on the eigenvalues of the matrix, much as they do in the symmetric case. But eigenvalues alone can't really tell you anything about its convergence – any nonincreasing convergence curve is possible for a matrix having any given eigenvalues. I began thinking about this when Nick Trefethen was beginning his work on pseudospectra – alternatives to eigenvalues that might give more information about the behavior of functions of a nonsymmetric, nonnormal matrix. Strakoš and I talked about it when we were both visiting the IMA [Institute for Mathematics and its Applications] in Minnesota. Then he got Vlastimil Pták involved as well and we wrote the joint paper showing how one could construct a matrix with any given eigenvalues for which the residual norm in GMRES would follow any nonincreasing convergence curve.

**M.O. - After your time in New York, you went to the University of Washington, and you've been there a long time now, I guess longer than anywhere else. So tell us about those years.**

**A.G. -** Okay, so when I came here, I came into the math department, and I really enjoyed being there. I learned a lot. I taught classes in sort of a different way, where you go to the board and do theorem, proof, theorem, proof. I really enjoyed teaching the graduate linear analysis course. I learned new things from that. But I never quite felt at home in the math department. Mathematicians went to different conferences from the ones that I went to. If I organized a numerical analysis seminar, nobody from the math department would come. And so I moved to the applied math department, where I feel much more at home. I've very much enjoyed being in applied math.

**M.O. - Who are some of the colleagues there that you've talked with the most?**

**A.G. -** Randy LeVeque and Loyce Adams, but they have both retired now. We've hired a new person, Heather Wilbur, who's really good. Tom Trogdon is our current chair, also very good. Tom and I co-advised a Ph.D. student, Tyler Chen, and the three of us recently wrote a joint paper [1].

**M.O. - And you like teaching there too, right?**

**A.G. -** I do, yeah. We have a numerical analysis sequence. The first quarter is numerical linear algebra, which I couldn't teach this year because I was at the Simons Workshop at Berkeley. And the next course, which I also enjoy teaching, and will teach next year, is everything numerical except linear algebra and PDEs. And then the third one is numerical PDEs, which I am teaching now.

**M.O. - And how was the Simons semester?**

**A.G. -** Oh, it was fun! I talked to a lot of people that I probably wouldn't have talked to otherwise. I'm still doing Zoom calls with a number of them. I finished a paper that I wrote with a couple of people who also visited Simons, though we had started it before.

**M.O. - Also, I know you really enjoy the Seattle area for the outdoors. You do a lot of hiking. Any particularly memorable ones?**

**A.G. -** Well, my favorite hike is Summerland at Mount Rainier. It's about 4 miles in each direction, but the first mile or two is not very steep. And then you get to the top, and there are these brooks and wildflowers, and big boulders where you can get on a boulder and eat your lunch, and enjoy the view of the glacier. So that's certainly one of my favorites.

**M.O. - I know you're a very good tennis player, because I played with you quite a lot. Maybe I can finish by telling a story, which is kind of fun. One time, I came and visited you, and you beat me only 6-1, unlike the usual 6-0. Then the next day, you beat me only 6-2, and in the lunch room later, I said, "So, Anne, maybe tomorrow morning I'll beat you at tennis." And you said, "Yeah, and in the afternoon, maybe we'll prove Crouzeix's conjecture!"**

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# LINEAR ALGEBRA EDUCATION

## The Linear Algebra Card Game: When Learning and Fun Become Linearly Dependent

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### 1. Introduction.

**Teacher:** How does this relate to eigenvalues?

**Student:** I . . . I'm not sure.

**Teacher:** Think back to how we described linear transformations . . .

**Student:** But . . . that was from week 2!

Moments like this are familiar to many Linear Algebra instructors. While students may successfully work through individual topics, they often struggle to recognize and explain the connections between them. Concepts introduced earlier in the course fade into isolated pieces of knowledge, rather than forming a connected structure. For instance, students may learn that a matrix is invertible if and only if its determinant is nonzero, yet later struggle to relate this to eigenvalues. This fragmentation limits both deep understanding and the ability to transfer knowledge to new contexts, such as machine learning, artificial intelligence, and economics, where linear algebra is fundamental. It is precisely this challenge that motivated the development of a linear algebra card game (Figure 1).

**2. The design of the game.** The design of the game was guided by ludo-didactical principles, in which game mechanics are intentionally aligned with learning processes [1]. Our goal was not simply to make linear algebra “fun.” This was not a gamification attempt, but an intentional design in which effective learning behaviors are embedded directly into the rules of the game. Players must retrieve concepts, connect them, and explain them to peers. These are core practices of meaningful learning that are naturally integrated into the game. Initial implementations suggested that this approach supports deeper engagement: students reported that explaining connections helped them better understand how topics fit together across the course, and instructors observed more frequent and precise use of mathematical language compared to traditional exercises.



Figure 1: *Playtest sessions with students (left) and colleagues in mathematics education (right). Student sessions provided feedback on clarity and engagement, while colleague sessions supported refinement of pedagogical alignment and usability.*

To realize these principles in practice, the development of the card game followed an iterative, design-based process. We started by identifying key concepts from the course and organizing them into a network of interconnected ideas, resulting in a set of 265 concept cards spanning 8 modules (fundamentals, systems of linear equations, linear transformations, matrix algebra and determinants, vector spaces and coordinate systems, eigenvalues and eigenvectors, orthogonality, symmetric matrices and quadratic forms). Early prototypes of the game were introduced in tutorial sessions, where we observed student interactions and collected informal feedback on clarity, engagement, and perceived learning. Based on these insights, we refined both the game mechanics and the selection of concepts, introducing additional elements (such as directional bonuses or different variants that make it easier or harder to connect concepts) to increase engagement and strategic depth. In a subsequent phase, the game was playtested with colleagues in mathematics education, whose

feedback helped us further improve its pedagogical alignment and usability (Figure 1). Finally, to transform the prototype into a polished and usable educational tool, we collaborated with a professional designer to develop a coherent visual identity and produce a high-quality card game (Figure 2).

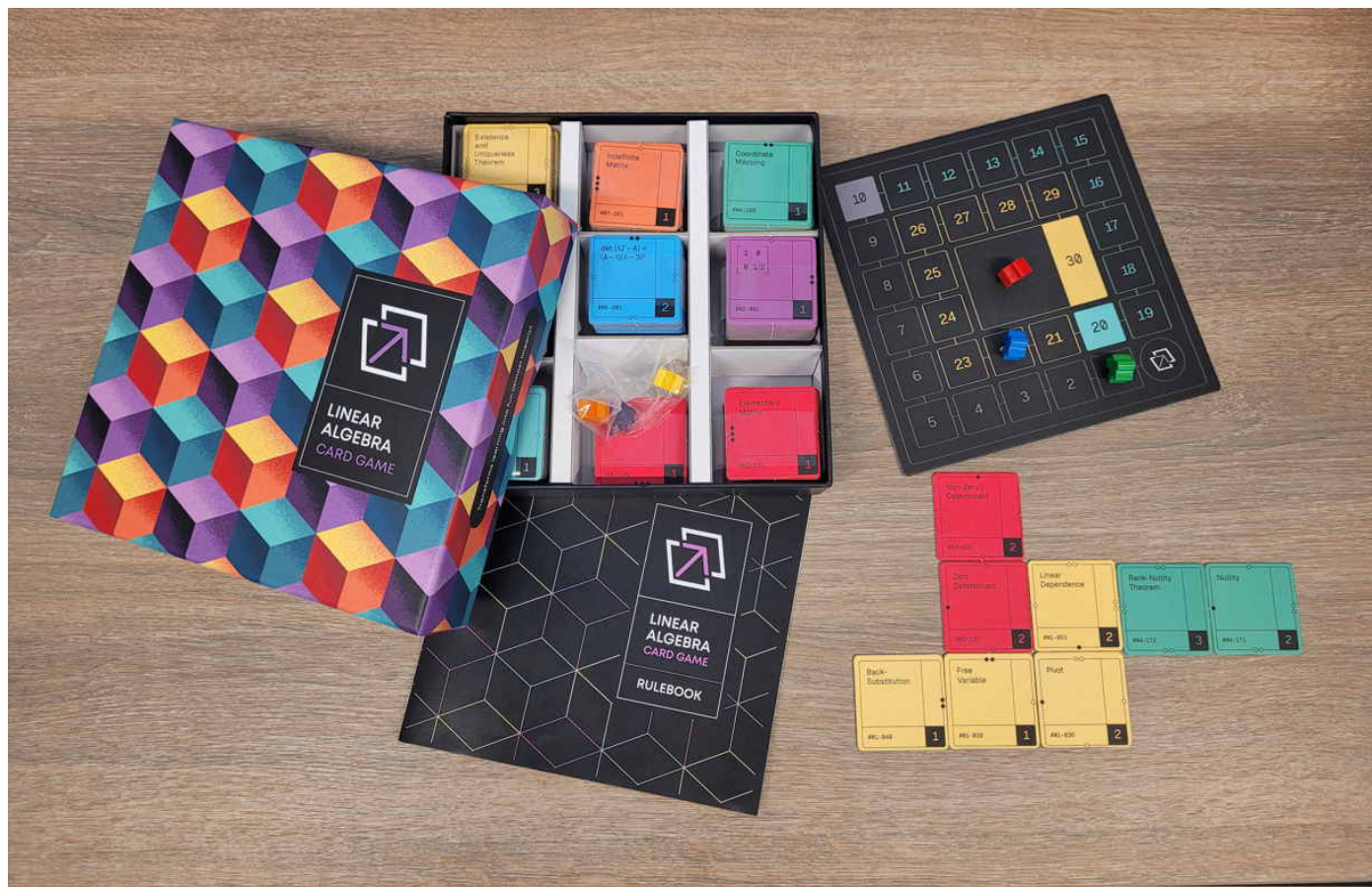


Figure 2: *The Linear Algebra Card Game, including the box design, the rulebook, the score tracker and concept cards used during gameplay.*

**3. How to play.** The game is played in groups of two to six players. Each player draws six cards from the shuffled deck, where each card represents a concept, theorem, or example in linear algebra (e.g., linear independence, the Invertible Matrix Theorem, an example of a triangular matrix with specific values) (Figure 3). On their turn, a player places a card adjacent to any other card already on the table, horizontally or vertically, gradually forming a grid. They must explain how their card relates to all cards in the corresponding row or column. For instance, a player placing a card on “invertible matrix” next to “determinant” and “linear independence” might argue that a matrix is invertible if and only if its determinant is nonzero, which in turn is equivalent to its columns being linearly independent. The other players evaluate the explanation. If they find it clear and mathematically sound, the player earns points equal to the total value of all cards in the row and column, plus any directional bonuses indicated by pips on the cards. The game typically ends when a player reaches at least 30 points.

In this way, the game makes relationships explicit that often are addressed only implicitly in typical classroom practice. As the course continues and new concepts are introduced, the starting deck can be expanded with additional modules. Moreover, instructors can choose which modules (or specific cards) to include or not to include, tailoring the deck to the experience they want to craft.



Figure 3: Arrangement of cards during a game.

**4. How to access the card game.** The game is available for pre-order via the project website:

<https://www.linearalgebragame.eu>

In addition, in line with our commitment to accessible and shareable educational innovation, we have made a print-and-play version of the game openly available, with support from Maastricht University Library [2]. This allows both instructors and students to freely download, print, and use the materials in their own teaching and learning contexts. Notably, some students in our courses expressed a desire to access the game independently to continue practicing outside the classroom. By publishing the game through open educational repositories, we aim to support broader adoption and adaptation across different courses and institutions. This open format not only facilitates dissemination but also invites further refinement and collaboration within the mathematics education community. The print-and-play version is available on the project website. By sharing the game as an open educational resource, we hope to contribute to more interactive and student-centered approaches to teaching linear algebra in line with Maastricht University's vision on teaching and learning, which emphasizes active and collaborative learning. In doing so, we aim to address the recurring challenge outlined at the beginning: helping students not only learn individual concepts, but also to recognize and articulate the connections between them. We invite colleagues to experiment with the game in their own teaching contexts and to share feedback or suggestions via [play@linearalgebragame.eu](mailto:play@linearalgebragame.eu).

#### References.

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- [2] <https://library.maastrichtuniversity.nl/news/from-classroom-to-the-world-the-linear-algebra-card-game-as-an-open-resource/>



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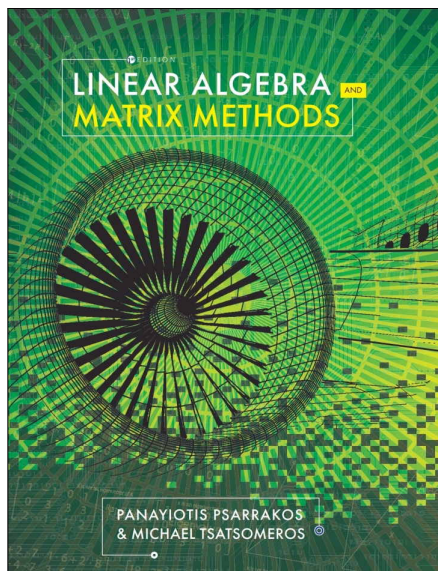


## BOOK REVIEW

### *Linear Algebra and Matrix Methods* by Michael Tsatsomeros and Panayiotis Psarrakos

Cognella, December 2024, ISBN 979-8-8233-7506-1, xix+423 pages

Reviewed by Mohsen Aliabadi, Clayton State University, maliabadi@clayton.edu



Here is a carefully organized textbook that presents linear algebra from both a structural and a matrix-theoretic point of view. It covers the standard core of undergraduate linear algebra while also including several topics that give the book a broader matrix-methods perspective, such as the minimal polynomial, Schur triangularization, singular value decomposition, Jordan canonical form, matrix norms, nonnegative matrix factorization, special classes of matrices, and applications. As a result, the book may serve not only as an introduction to vector spaces and linear transformations, but also as an introduction to some of the principal tools and viewpoints of matrix theory.

**1. Linear systems, rank, and vector spaces.** The book begins with systems of linear equations, Gaussian elimination, and the theory of homogeneous systems. This is a natural point of departure for an introductory linear algebra text. Row reduction gives students a concrete way to encounter several central notions, including pivot variables and free variables, existence and uniqueness of solutions, the rank of a coefficient matrix, the structure of the solution set of a homogeneous system, and the distinction between particular and general solutions. In this sense, the computational opening of the book prepares the reader for the later abstract language of subspaces, bases, and dimension.

The subsequent treatment of vector spaces develops the conceptual framework behind these computations. Subspaces, linear combinations, spanning sets, linear independence, bases, and dimension are indispensable notions in linear algebra, and their placement after systems of equations is pedagogically appropriate. The reader is led from the concrete geometry of solution sets to the more abstract structure of vector spaces. This transition is one of the central challenges in a first course, because it explains why row reduction is not only an algorithmic procedure, but also a tool for understanding dimension, coordinates, and linear dependence.

This part of the book also clarifies the relationship between algebraic and geometric thinking. A system of equations may be studied through its augmented matrix, through the column space and null space of its coefficient matrix, or through the geometry of affine subspaces. The organization of the material supports the reader in moving among these complementary viewpoints.

**2. Matrix algebra and linear transformations.** The chapters on matrix algebra and linear transformations form an important bridge between computation and theory. The book discusses matrix notation, matrix addition and scalar multiplication, matrix-vector products, matrix-matrix products, and entrywise descriptions of matrix operations. These topics are elementary in appearance, but they establish the algebra of matrices before matrices are interpreted as coordinate representations of linear transformations.

The treatment of linear transformations is a particularly important component of the text. The standard matrix of a linear map, the kernel and image of a transformation, invertibility, and the invertible matrix theorem connect matrix computations with the intrinsic structure of linear maps. This is the point at which students can begin to see that a matrix is not merely a rectangular array of scalars, but a representation of a linear operator or a linear transformation between finite-dimensional vector spaces.

The inclusion of inverse matrices and equivalent conditions for invertibility is also useful. Invertibility connects many of the basic themes of the subject: nonsingularity, full rank, trivial kernel, uniqueness of solutions, existence of inverse transformations, nonzero determinant, and linear independence of columns. Presenting the equivalence of these conditions gives students a unified view of linear algebra and prepares them for later material on determinants, eigenvalues, and diagonalization.

**3. Determinants and their algebraic structure.** The chapter on the determinant is broader than a purely computational treatment. It includes the Laplace expansion, general properties of determinants, consequences for invertibility, the Leibniz expansion, the permutation formula for the determinant, determinant formulas for inverses, and Cramer's

rule. This selection gives the reader several complementary perspectives on determinants.

From a computational perspective, Laplace expansion and row-operation properties provide methods for evaluating determinants. From a theoretical perspective, determinants measure invertibility, signed volume scaling, and orientation. From a combinatorial perspective, the Leibniz formula expresses the determinant as a signed sum indexed by permutations. This last viewpoint is useful in a matrix-theoretic text, since it exhibits the connection between determinants, permutation groups, alternating functions, and multilinearity. It also serves to connect determinants with ideas that arise in more advanced topics, such as exterior algebra, characteristic polynomials, compound matrices, permanents, and combinatorial matrix theory.

The inclusion of Cramer's rule and formulas for the inverse of a matrix in terms of subdeterminants also serves a theoretical purpose. Although Cramer's rule is not usually the preferred numerical method for solving large systems, it gives explicit closed-form expressions for solutions, illustrates the role of the determinant in invertibility, and connects linear systems with algebraic formulas.

**4. Eigenvalues, diagonalization, and polynomial methods.** The spectral part of the book is one of its substantial components. The text treats eigenvalues, eigenvectors, eigenspaces, diagonalization, the Cayley–Hamilton theorem, and the minimal polynomial. These topics form the basis for understanding matrices through invariant subspaces and polynomial relations.

The discussion of diagonalization is important because it shows how a linear operator can sometimes be simplified by a change of basis. When a matrix is diagonalizable, many computations become more transparent: powers of matrices, systems of linear recurrences, certain differential equations, and spectral decompositions can be handled more efficiently. The placement of diagonalization after eigenvalues and eigenspaces follows the natural mathematical progression from spectral data to structural simplification.

The inclusion of the Cayley–Hamilton theorem and the minimal polynomial adds depth to the presentation. The Cayley–Hamilton theorem states that every square matrix satisfies its own characteristic polynomial, while the minimum polynomial records the lowest-degree monic polynomial relation satisfied by the matrix. This gives students a refined tool for understanding matrix identities, powers of matrices, diagonalizability, and canonical forms. In particular, the minimal polynomial is the appropriate object for formulating several structural criteria, including criteria related to diagonalizability and Jordan structure.

**5. Inner product spaces, orthogonality, and norms.** The chapter on inner product spaces introduces the geometric side of finite dimensional linear algebra. Inner products, vector norms, orthogonality, orthonormal bases, orthogonal complements, unitary matrices, and matrix norms are central topics. Their inclusion is important because linear algebra is not only an algebraic theory of vector spaces and linear maps; it is also a geometric theory of length, angle, projection, and approximation.

The material on orthogonality is especially relevant for applications. Orthogonal and unitary matrices preserve lengths and angles, and they play an important role in stable numerical algorithms. Orthogonal projections and orthonormal bases are fundamental for least-squares approximation, Fourier-type expansions, and numerical linear algebra. The inclusion of matrix norms is also useful. Matrix norms provide a language for measuring perturbations, errors, convergence of iterative methods, and stability of computations, which are central concerns in numerical and applied linear algebra.

**6. Matrix factorizations and canonical forms.** A notable feature of the book is its substantial treatment of matrix factorizations. The text includes LU factorization, Schur triangularization, QR factorization and least squares, singular value decomposition, Jordan canonical form, and nonnegative matrix factorization. This collection gives students a broad view of how matrix decompositions are used to reveal structure and perform computations.

LU factorization connects Gaussian elimination with matrix decompositions and is one of the basic tools for solving linear systems. QR factorization is central for least-squares problems and numerically stable orthogonalization. Schur triangularization provides a fundamental structural theorem for complex matrices and serves as a gateway to more advanced spectral theory. Singular value decomposition is one of the most important tools in applied linear algebra: it applies to arbitrary rectangular matrices, encodes rank and orthogonal approximation, and underlies low-rank approximation, image compression, principal component analysis, and related data analysis methods.

The inclusion of Jordan canonical form is also mathematically significant as it explains precisely how a matrix can fail to be diagonalizable: each Jordan block records both an eigenvalue and the nilpotent part of the operator on the corresponding generalized eigenspace. Together with the minimal polynomial, Jordan form gives students a deeper understanding of how polynomial relations reflect the internal structure of a matrix.

The discussion of nonnegative matrix factorization (NMF) gives the book a contemporary applied component. NMF

is less common in standard introductory linear algebra texts, but it is relevant in contexts where data are naturally nonnegative, such as image processing, text mining, signal processing, and source separation. Its inclusion broadens the range of matrix methods available to the reader.

**7. Special classes of matrices.** The chapter on special matrix classes gives the text a clear matrix-theory flavor. Hermitian matrices, positive definite matrices, normal matrices, and nonnegative matrices are among the most important families in matrix analysis. These classes are distinguished by concrete conditions, such as symmetry or self-adjointness, positivity of quadratic forms, unitary diagonalizability, and entrywise nonnegativity. Their inclusion helps students see how additional algebraic, geometric, or order-theoretic assumptions can lead to stronger conclusions.

Hermitian matrices are central because they have real eigenvalues and orthogonal eigenspaces, making them the finite-dimensional analogue of self-adjoint operators. Positive definite matrices are fundamental to the study of quadratic forms, optimization, numerical analysis, statistics, covariance matrices, and energy methods. Normal matrices provide a natural setting for unitary diagonalization and spectral analysis. Nonnegative matrices connect linear algebra to Perron–Frobenius theory, Markov chains, networks, population models, economics, and ranking algorithms.

This chapter is pedagogically useful because it shows students that special assumptions on a matrix, such as self-adjointness, positivity, normality, or entrywise nonnegativity, lead to stronger conclusions about eigenvalues, eigenvectors, decompositions, and applications.

**8. Applications.** The final chapter, on applications, gives the book an applied dimension that is well aligned with its emphasis on matrix methods. Topics in this chapter include differential equations and control systems, splittings and iterative methods for large linear systems, systems of linear differential equations, feedback controllability, Markov chains and stochastic matrices, the linear complementarity problem, image processing via SVD, and audio source separation via NMF.

The applications to differential equations and control systems show how eigenvalues, matrix exponentials, invariant subspaces, and controllability questions arise naturally in dynamical systems. The material on splittings and iterative methods connects matrix decompositions with the practical problem of solving large systems, where direct methods may be expensive or impractical. Markov chains and stochastic matrices provide a setting in which eigenvectors and nonnegative matrices acquire probabilistic interpretations. The linear complementarity problem introduces a class of problems lying at the intersection of matrix theory, optimization, and applied mathematics.

The applications to image processing through SVD and to audio source separation through NMF demonstrate how abstract matrix factorizations become concrete tools for compression, approximation, feature extraction, and separation of mixed signals. These examples help connect classical matrix methods with contemporary computational applications.

**9. Pedagogical value and audience.** One of the main strengths of the book is the way it balances computational practice, theoretical development, and applications. Students are introduced to concrete procedures such as row reduction, determinant computation, eigenvalue calculation, orthogonalization, and matrix factorization. At the same time, these procedures are placed within a coherent theoretical framework involving vector spaces, linear transformations, invariant subspaces, norms, canonical forms, and special matrix classes.

This balance makes the book suitable for several audiences. It can be used in a first course in linear algebra for mathematics, engineering, physics, computer science, economics, or data science students. It can also support a more advanced undergraduate course with emphasis on matrix methods, spectral theory, factorizations, and applications. Instructors may choose a more computational or a more theoretical path through the text, depending on the goals of the course.

The book also presents linear algebra as a connected subject. Row reduction leads to rank and dimension; linear transformations lead to matrices and invertibility; determinants lead to nonsingularity and characteristic polynomials; eigenvalues lead to diagonalization and canonical forms; inner products lead to orthogonality and QR factorization; matrix norms and decompositions lead to numerical and applied methods. This coherence is a valuable feature of the text.

**10. Conclusion.** Overall, *Linear Algebra and Matrix Methods* is a strong and useful textbook. It is clearly organized, mathematically rich, and broad in scope. The authors present linear algebra not merely as a collection of computational techniques, but as a coherent mathematical subject with connections to matrix analysis, geometry, numerical computation, optimization, dynamical systems, and data-driven applications. The inclusion of topics such as the minimum polynomial, Schur triangularization, singular value decomposition, Jordan canonical form, nonnegative matrix factorization, and special matrix classes gives the book considerable depth. It should be a valuable resource for students, instructors, and readers interested in a serious introduction to linear algebra and matrix methods.



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## OBITUARY NOTICES

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### Cleve Moler (1939–2026)

Submitted by Shaun Fallat and Michael Overton

Cleve Moler, a pioneer in our field and dear friend to many of us, died on May 20th, 2026.

Cleve changed all our lives through his development of MATLAB, an indispensable tool to so many of us. Despite his great success professionally and financially, he never lost touch with researchers in numerical linear algebra and continued to attend many scientific meetings even late in his life. He participated in nearly every Gatlinburg/Householder symposium since he first went to an early meeting as a graduate student from Stanford in 1964. He was awarded the Hans Schneider Prize in 2010. His passing is truly the end of an era, and he will be greatly missed.

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### Beresford Parlett (1932–2026)

Beresford Parlett passed away on February 7th, 2026. Among his many notable achievements, he was the author of *The Symmetric Eigenvalue Problem*, and was awarded the Hans Schneider Prize in 2010.

Memorial notes can be found at:

- <https://eecs.berkeley.edu/news/in-memoriam-beresford-parlett-1932-2026>
- <https://blogs.mathworks.com/cleve/2026/02/10/beresford-parlett>

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### Bob Plemmons (1938–2026)

Robert Plemmons passed away on February 6th, 2026. Among his many contributions, Bob co-wrote with Avi Berman the classic book *Nonnegative Matrices in the Mathematical Sciences*.

Memorial notes can be found at:

- <https://www.legacy.com/us/obituaries/winstonsalem/name/robert-plemmons-obituary?id=60741519>
- <https://inside.wfu.edu/2026/02/retired-faculty-member-robert-bob-plemmons-has-passed-away>

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### Ferran Puerta (1936–2026)

Submitted by Juan-Miguel Gracia Melero and Ion Zaballa

Ferran Puerta passed away on February 11th, 2026. An engineer, physicist and mathematician, he made a decisive contribution to the training of generations of researchers in geometry, algebra, linear algebra, and their applications, through his work in research and the writing of textbooks. He also played a key role in the early development of ILAS. In particular, he organized one of its first conferences, the eighth, in Barcelona in 1999.

Ferran was a vital and enthusiastic man who left a profound mark on many of us. May he rest in peace.

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## JOURNAL ANNOUNCEMENTS

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**Special Issue of *Numerical Algorithms* to honor the 65<sup>th</sup> birthday of Nicola Mastronardi**

**Contributed announcement from Marc Van Barel**

The journal *Numerical Algorithms* is pleased to announce a special issue in celebration of the 65th birthday of Nicola Mastronardi, with a focus on matrix computations, algorithms for structured rank matrices (like semiseparable and Toeplitz matrices), eigenvalue problems, and scientific computing applications.

Nicola is a prolific researcher in numerical linear algebra, having co-authored numerous highly cited scientific papers and collaborated with researchers from various international institutions. His work involves developing efficient algorithms for complex mathematical problems in fields such as medical diagnostics (e.g., magnetic resonance spectroscopy).

Editors for the special issue will include: Marc Van Barel (KU Leuven), Thomas Mach (University of Potsdam Institute for Mathematics), Beatrice Meini (University of Pisa), Valeria Simoncini (Università di Bologna), and Raf Vandebril (KU Leuven).

Topics of the special issue will include, but are not limited to, numerical linear algebra, numerical analysis, systems and control theory, scientific computing, and orthogonal polynomials.

Submissions should follow the guidelines of the journal *Numerical Algorithms* and can be sent in until the end of January 2027 using the submission system of the journal. The corresponding collection is called “NM65”.

More details can be found at:

<https://link.springer.com/collections/efabgiaici>

## ILAS NEWS

### Chi-Kwong Li Receives John von Neumann Fulbright Award

ILAS Member (and recent winner of the Hans Schneider Prize) Chi-Kwong Li of the College of William & Mary was selected to receive the John von Neumann Distinguished Award in STEM, a competitive grant offered through the Fulbright Program, the United States government’s flagship program of international educational and cultural exchange, and sponsored by Hungary’s Ministry of Culture and Innovation to support “cutting-edge STEM research preferably at one of Hungary’s high priority research facilities or universities” (see <https://fulbrightscholars.org/award/john-von-neumann-distinguished-award-stem-5>). The grant will support his visits to the Bolyai Institute at the University of Szeged in Spring 2027 and Spring 2028, where he will engage in cutting-edge research and inspiring educational activities in quantum information science and related areas.

### 2025 Béla Szőkefalvi-Nagy Medal Awarded to Chi-Kwong Li

The Bolyai Institute of the University of Szeged annually awards its Béla Szőkefalvi-Nagy Medal to honour a distinguished mathematician who has published significant, deep results in the Institute’s flagship journal, *Acta Scientiarum Mathematicarum*. The Institute recently bestowed the 2025 Medal on ILAS Member (and recent winner of the Hans Schneider Prize) Chi-Kwong Li of the College of William & Mary.

The Institute’s official citation for the medal can be read here:

<https://sci.u-szeged.hu/english/2026/bela-szokefalvi-nagy>

The award ceremony, including Chi-Kwong’s award lecture, *Generalized numerical ranges and The Smith Ward Problem*, can be viewed at:

[https://www.youtube.com/watch?v=nIVCpNF-\\_ew](https://www.youtube.com/watch?v=nIVCpNF-_ew)

### Stefan Güttel Selected as 2026 SIAM Fellow

The 2026 class of fellows of SIAM, the Society for Industrial and Applied Mathematics, was recently announced. Included among those honored was ILAS Member (and winner of the 2023 ILAS Olga Taussky and John Todd Prize) Stefan Güttel of the University of Manchester. Stefan was recognized for “research contributions to applied and numerical linear algebra with outstanding industrial impact.”

For details, and to view the complete 2026 class of SIAM Fellows, visit:

<https://www.siam.org/publications/siam-news/articles/siam-announces-2026-class-of-fellows>

### ILAS Election Results

Shaun Fallat (University of Regina) has been elected to a three-year term as ILAS President, beginning March 1st, 2026. Aida Abiad Monge (Eindhoven University) and Christian Mehl (Technische Universität-Berlin) were elected to three-year terms as members of the ILAS Board of Directors. They began their terms on March 1st, 2026.

## ILAS President/Vice President Annual Report: May 1, 2026

Respectfully submitted by Shaun Fallat, ILAS President, [shaun.fallat@uregina.ca](mailto:shaun.fallat@uregina.ca)  
and Froilán M. Dopico, ILAS Vice President, [dopico@math.uc3m.es](mailto:dopico@math.uc3m.es)

Last year was another landmark year for ILAS. Highlights include the ILAS Olga Taussky and John Todd Prize awarded to Sirani Perera, the ILAS Richard A. Brualdi Early Career Prize awarded to John Urschel, a new agreement between ILAS and Elsevier that doubles the funding Elsevier allocates to ILAS conferences, increased funding for Ph.D. students and early-career researchers attending ILAS conferences, the 26th Conference of ILAS held in Kaohsiung, Taiwan (June 23–27, 2025), and the first ILAS elections held using a ranked choice instant-runoff voting system.

### 1. Board-of-Directors-approved actions since the last report include:

- The Board amended the ILAS Bylaws on June 24, 2025 as follows: (1) In Article 6, Section 2, Paragraph B, the sentence “Additional nominations for officers may be made by electronic or regular mail by any three members of the Society.” was replaced by “Additional nominations for officers may be made by electronic or regular mail by any number of members of the Society larger than or equal to the 5% of the total number of members of the Society.”; (2) In Article 10, Section 2, the sentence “At least two weeks shall be allowed for the additional nominations.” was replaced by “At least four weeks shall be allowed for the additional nominations.”
- The Board moved on October 17, 2025 to increase the maximum amount of each childcare grant from \$500 to \$750.
- The Board moved on October 17, 2025 to fund with \$1,000 one application to the “ILAS Grant Program in support of Mathematicians working in Linear Algebra affected by conflicts.”
- The Board moved on October 17, 2025 to upgrade the funding and the number of the grants provided by ILAS and by Elsevier, the publisher of *Linear Algebra and its Applications*, to cover partially the expenses of Ph.D. students and early-career researchers participating in ILAS Conferences. More precisely, the Board moved to fund in the 2026 ILAS Conference: 12 ILAS-LAA PhD grants of \$750 each + 6 ILAS-LAA Early career grants of \$1,000 each.
- The Board approved on October 17, 2025 a decision that ILAS membership fees will not increase in 2026.
- The Board approved on November 4, 2025 a new Agreement between ILAS and Elsevier Inc., publisher of the journal *Linear Algebra and its Applications*. The new Agreement began on January 1, 2026, and will conclude on December 31, 2028, but will be automatically renewed for one-year periods unless either party gives written notice to the other party at least twelve months prior to the expiration of the current term. Under the new agreement, Elsevier’s total funding for each ILAS conference has increased from \$5,000 to \$10,000.

### 2. Other news:

- The 26th Conference of the International Linear Algebra Society (ILAS 2025) “Legacy and New Spectrum” was held in Kaohsiung, Taiwan, from June 23 to 27, 2025. Following ILAS 2014 in Korea, ILAS 2025 was the first ILAS conference to be hosted in the Asia-Pacific region after an 11-year interval. The event attracted more than 400 participants from 44 countries, reflecting the strong international engagement within the linear algebra community. The conference featured 9 plenary lectures, 35 minisymposia with a total of 290 talks, 43 contributed talks, and 9 poster presentations. During the banquet, the Hans Schneider Prize was awarded to Dario Bini in recognition of his outstanding contributions to matrix multiplication, polynomial and structured matrices, Markov chains, and algebraic Riccati equations. After the award presentation, Steve Kirkland delivered a speech highlighting the success of ILAS and the spirit of the community. Jephian C.-H. Lin and Matthew M. Lin were the co-chairs of the local organizing committee.
- The 2026 Joint Mathematics Meetings (JMM 2026, an ILAS partner conference) was held in Washington, D.C., USA, January 4–7, 2026. The ILAS Invited Address was presented by the ILAS member Dominique Guillot (University of Delaware, USA) with the title “The Many Facets of Matrix Positivity: A Celebration of Linear Algebra”. There were six ILAS Special Sessions: (1) Combinatorial Matrix Theory (organized by Minerva Catral and Bryan L. Shader); (2) Matrix Analysis and Applications (organized by James E. Pascoe and Hugo J. Woerdeman); (3) Innovative and Effective Ways to Teach Linear Algebra (organized by David M. Strong, Sepideh Stewart, Gilbert Strang and Megan Wawro); (4) Matrix Analysis and Applications (organized

by Mohsen Aliabadi, Dominique Guillot, Tin-Yau Tam and Xiang Xiang Wang); (5) Algebraic Graph Theory: New Trends (organized by Milica Andelic, Renata Del-Vecchio, Sudipta Mallik and Zoran Stanic); and (6) Recent Advances in Model Order Reduction and Data-Driven Modeling: Theory and Computations (organized by Ionut Farcas and Steffen W. R. Werner).

- The Executive Board awarded on November 7, 2025 Sirani Perera the 2026 ILAS Olga Taussky and John Todd Prize, accepting the recommendation of the prize committee consisting of Steve Kirkland (chair), Lek-Hem Lim, Alison Ramage, and Daniel B. Szyld (ILAS President, ex officio). Sirani Perera was recognized for substantial contributions to structured matrix computations, and for parlaying linear algebra insights into hardware implementation and signal processing. Sirani Perera will deliver the prize lecture at the 2026 ILAS Conference.
- The Executive Board awarded on November 7, 2025 John Urschel the 2026 ILAS Richard A. Brualdi Early Career Prize, accepting the recommendation of the prize committee consisting of Misha Kilmer, Steve Kirkland (chair), Lek-Hem Lim, Alison Ramage, and Daniel B. Szyld (ILAS President, ex officio). John Urschel was recognized for important contributions to numerical linear algebra including Gaussian elimination and the Lanczos method, and to spectral graph theory. John Urschel will deliver the prize lecture at the 2026 ILAS Conference.
- The Board of Directors approved on October 17, 2025 the request of the organizers of the Western Canada Linear Algebra Meeting (WCLAM) 2026, to be held at Capilano University, Lonsdale, Canada, May 30–31, 2026, for an ILAS Lectureship for the ILAS member Beatrice Meini as an invited featured speaker.
- The Board of Directors approved on October 17, 2025 the request of the organizers of the ALAMA Meeting 2026 (Meeting of the Spanish Thematic Network of Linear Algebra, Matrix Analysis, and Applications), to be held at the Universidad de Huelva, Huelva, Spain, June 12–14, 2026, for an ILAS General support of conferences grant to be used for travel grants, partial support for plenary speakers, and registration waivers for students, low-income participants, or retired professors.
- The Board of Directors approved on October 17, 2025 the request of the organizers of the Saint-Girons V Conference, to be held at Saint-Girons, Ariège, France, June 29 – July 3, 2026, for an ILAS Lectureship for Per-Gunnar Martinsson as keynote speaker.
- The Board of Directors approved on October 17, 2025 the request of the organizers of the International Workshop on Operator Theory and its Applications (IWOTA) 2026, to be held at Université Laval, Quebec City, Canada, August 3–7, 2026, for an ILAS Lectureship for Felix Schwenninger as plenary lecturer.
- The Executive Board accepted on February 23, 2026 the recommendation of the JMM Committee that Misha Kilmer from Tufts University (USA) will give the ILAS Invited Address at the 2027 Joint Mathematics Meetings, to be held in Chicago, Illinois, USA, January 12–15, 2027. The JMM Committee consisted of Orly Alter, Ángeles Carmona Mejía, Mark Embree, Apoorva Khare (chair), and Daniel Szyld (ILAS President, ex officio).
- ILAS funded four applications for the program of childcare grants to participants in the 27th ILAS Conference (Virginia Tech, Blacksburg, Virginia, USA, May 18–22, 2026) with \$750 each.

**3. ILAS elections ran November 17 – December 17, 2025, and proceeded via electronic voting.** The following were elected to offices with three-year terms that began on March 1, 2026:

- President: Shaun Fallat
- Board of Directors: Aida Abiad Monge and Christian Mehl

The following continue in the ILAS offices which they currently hold:

- Vice President: Froilán Dopico (term ends February 29, 2028)
- Secretary/Treasurer: Minerva Catral (term ends February 28, 2027)
- Board of Directors: Enide Andrade (term ends February 29, 2028), Stefan Güttel (term ends February 28, 2027), Jephian C.-H. Lin (term ends February 29, 2028), and Naomi Shaked-Monderer (term ends February 28, 2027).

On February 28, 2026, Daniel B. Szyld completed two consecutive terms as President of ILAS. Daniel will continue to serve ILAS for one more year as a member of the Executive Board and of the Board of Directors in his capacity as Past President, i.e., until February 28, 2027. We would like to express our sincere gratitude to Daniel for his dedicated leadership at the helm of the Society over the past six years.

Fernando De Terán and Chi-Kwong Li completed their terms on the ILAS Board of Directors on February 28, 2026. We thank them for their valuable contributions as Board members; their service to ILAS is most appreciated.

We also thank the members of the Nominating Committee – Federico Poloni, Rachel Quinlan, Bryan Shader (chair), Pauline van den Driessche, and Paul Van Dooren – for their efforts on behalf of ILAS, and all the nominees for their participation in the elections.

#### 4. Appointments and re-appointments since the last report

- Second Vice-President (for ILAS Conferences):

Raf Vandebril

- Assistant Secretary/Treasurer:

Michael Tait

- ILAS-NET:

Leonardo Robol, Manager

- ILAS Website:

Dominique Guillot, Manager

- ILAS Resident Registered Agent in the State of Florida:

Fuzhen Zhang

- Advisory Committee:

Apoorva Khare

Steve Kirkland (chair)

Andre Ran

Françoise Tisseur

- Education Committee:

Anthony Cronin

Fernando De Terán

Jephian C.-H. Lin

Rachel Quinlan

Helena Šmigoc

Sepideh Stewart (chair)

David Strong

- Investment Committee:

Minerva Catral

Shaun Fallat (chair)

Daniel Szyld

Michael Tait

Michael Tsatsomeros

- Journals Committee:

Zlatko Drmac (chair)

Polona Oblak

Bryan Shader

- Outreach and Membership Committee:

Minerva Catral  
 Andrii Dmytryshyn  
 Shahla Nasserar (chair)  
 Federico Poloni

- Prize Canvassing Committee:

Richard Brualdi  
 Shaun Fallat (ex officio)  
 Chen Greif (chair)  
 Leslie Hogben  
 Ilse Ipsen

**5. ILAS endorsed the following conferences of interest to ILAS members that have taken place since the last President/Vice President annual report:**

- VII Thematic Conference of the Spanish ALAMA Network of Linear Algebra, Matrix Analysis, and Applications (ALAMA), “A tribute to Nick Higham”, International Center of Mathematical Meetings (CIEM) of the Universidad de Cantabria, Castro Urdiales, Spain, May 14–16, 2025.
- 9th Linear Algebra Workshop (LAW’25), Portorož, Slovenia, June 2–6, 2025. The ILAS member Helena Šmigoc was an ILAS-supported plenary lecturer. <http://www.law05.si/law25>
- XXII Householder Symposium, Cornell University, Ithaca, New York, USA, June 8–13, 2025. The ILAS member Ilse Ipsen was an ILAS-supported plenary lecturer. <https://householder-symposium.github.io>
- 36th International Workshop on Operator Theory and its Applications (IWOTA), University of Twente, Enschede, The Netherlands, July 14–18, 2025. The ILAS member Volker Mehrmann was an ILAS-supported plenary lecturer. <https://www.utwente.nl/en/iwota2025>
- International Conference on Linear Algebra its Applications (ICLAA 2025) CARAMS, MAHE, Manipal, India, December 17–20, 2025. <https://carams.in/events/iclaa2025>

**6. ILAS endorsed the following conferences of interest to ILAS members that will take place in the following months:**

- Western Canada Linear Algebra Meeting (WCLAM 2026). Capilano University, Lonsdale, Canada, May 30–31, 2026. The ILAS member Beatrice Meini will be an ILAS-supported plenary lecturer. <https://www.capilano.ca/about-capu/get-to-know-us/events/items/western-canadian-linear-algebra-meeting-wclam-2026.php>
- 9th Meeting of the Spanish Thematic Network of Linear Algebra, Matrix Analysis, and Applications (ALAMA 2026). Universidad de Huelva, Huelva, Spain, June 12–14, 2026. <https://congresosalcala.fgua.es/alam2026/?idioma=en>
- Saint-Girons V Conference. Generating ideas for a numerical world. Saint Girons, Ariège, France, June 29 – July 3, 2026. Per-Gunnar Martinsson will be an ILAS-supported plenary lecturer. <https://saintgironsconference.eu>
- 37th International Workshop on Operator Theory and its Applications (IWOTA). Université Laval, Quebec City, Canada, August 3–7, 2026. Felix Schwenninger will be an ILAS-supported plenary lecturer and Pamela Gorkin will be the Israel Gohberg ILAS-IWOTA Lecturer. <https://iwota-2026.fsg.ulaval.ca>

**The following ILAS conference is scheduled:**

- The 27th ILAS Conference (ILAS 2026) will be held at Virginia Tech in Blacksburg, Virginia, USA, May 18–22, 2026, with Mark Embree and Serkan Gugercin (Virginia Tech) as co-chairs of the Organizing Committee and Agnieszka Miedlar and Eric de Sturler (Virginia Tech) as co-chairs of the Scientific Committee. The plenary speakers will be Aida Abiad (Eindhoven University of Technology, The Netherlands), Haim Avron

(Tel Aviv University, Israel, SIAG/LA 2025 Lecture), Julianne Chung (Emory University, USA, SIAG/LA 2026 Lecture), Chi-Kwong Li (College of William and Mary, USA, Hans Schneider Prize Lecture), Sherry Li (Lawrence Berkeley National Laboratory, USA), Jephian C.-H. Lin (National Yang Ming Chiao Tung University, Taiwan, *LAMA* Lecture), Sirani Perera (Embry-Riddle Aeronautical University, USA, ILAS Olga Taussky and John Todd Prize Lecture), Arvind K. Saibaba (North Carolina State University, USA), John Urschel (MIT, USA, ILAS Richard A. Brualdi Early Career Prize Lecture), and Hugo Woerdeman (Drexel University, USA). Thirty-nine minisymposia covering a wide variety of topics related to linear algebra and its applications are scheduled. <https://ilas2026.math.vt.edu>

## 7. The following ILAS partner conference is scheduled:

- The 2027 Joint Mathematics Meetings (JMM 2027) will be held in Chicago, Illinois, USA, January 12–15, 2027. The ILAS member Misha Kilmer from Tufts University (USA) will present the ILAS Invited Address. Several ILAS Special Sessions will be scheduled.

## 8. The *Electronic Journal of Linear Algebra (ELA)* is now in its 42nd volume. *ELA*'s URL is <https://journals.uwo.edu/index.php/ela>. Volume 41 was published in 2025 and contains 49 papers. *ELA* received 331 new submissions in 2025, which is a record in terms of number of submissions (+49 papers and +19% with respect to the old record in 2022). The current acceptance rate is less than 16%. In 2025, 263493 downloads and 209204 abstract views of *ELA* papers occurred.

The current Editor-in-Chief is Vanni Noferini (Aalto University, Finland), who began his first three-year term on August 1, 2025, immediately after Froilán Dopico completed two consecutive three-year terms as Editor-in-Chief of *ELA*.

ILAS members are strongly encouraged to submit their work to *ELA*, the flagship research journal of our society.

## 9. *IMAGE* is the semi-annual bulletin for ILAS, available online at <https://ilasic.org/image>. The Editor-in-Chief is Louis Deaett (Quinnipiac University, USA). In 2025, the website of *IMAGE* received 1053 visits.

## 10. ILAS-NET is a moderated electronic newsletter for mathematicians worldwide, with a focus on linear algebra. It is currently managed by Leonardo Robol (Università di Pisa, Italy). An archive of ILAS-NET messages is available at <https://ilasic.org/ilas-net>. To send a message to ILAS-NET, please send the message (preferably in text format) in an email to [leonardo.robol@unipi.it](mailto:leonardo.robol@unipi.it) indicating that you would like it to be posted on ILAS-NET. If the message is approved, it will be posted soon afterwards. To subscribe to ILAS-NET, please go to <https://ilasic.org/ilas-net>. On April 28, 2026, there were 1471 active subscribers to the ILAS-NET newsletter (+177 new subscribers with respect to April 29, 2025).

## 11. ILAS's website is located at <https://ilasic.org> and highlights the main activities of ILAS (e.g., the *Electronic Journal of Linear Algebra (ELA)*, conferences, *IMAGE*, ILAS-NET and other activities). In addition, the website provides general information about ILAS (e.g., ILAS officers, bylaws, special lecturers, ILAS prizes, grant programs) as well as links to pages of interest to the ILAS community. Currently it is managed by Dominique Guillot (University of Delaware, USA). In 2025, the website of ILAS received 26914 pageviews from users from 109 countries. The front page received 14040 of these pageviews, the conference page 3799, and the *IMAGE* page 1053.

## 12. Number of current ILAS members. As of March 17, 2026, the number of ILAS members was 565.

Finally, we want to express our great gratitude to all the officers of ILAS who all show wonderful dedication to the society, as well as to all the individual members of ILAS and our corporate sponsors. Without any of them ILAS would not be what it is today.

Respectfully submitted,

Shaun Fallat, ILAS President ([shaun.fallat@uregina.ca](mailto:shaun.fallat@uregina.ca)); and

Froilán M. Dopico, ILAS Vice President ([dopico@math.uc3m.es](mailto:dopico@math.uc3m.es))

**ILAS 2025–2026 Treasurer’s Report**  
**April 1, 2025 – March 31, 2026**  
**by Minerva Catral, ILAS Secretary/Treasurer**

**Net Account Balance on March 31, 2025**

Checking Account – First Interstate Bank	\$ 24,093.70	
Certificate of Deposit 1	\$ 15,806.16	
PayPal	\$ 35,000.00	
Vanguard	\$ 240,721.46	
		\$ 315,621.32

General Fund	\$ 163,338.72	
Israel Gohberg ILAS-IWOTA Lecture Fund	\$ 7,357.62	
Conference Fund	\$ 11,267.20	
ILAS Olga Taussky and John Todd Prize Fund	\$ 11,678.57	
Hans Schneider Lecture Fund	\$ 10,223.52	
Frank Uhlig Education Fund	\$ 6,451.39	
Hans Schneider Prize Fund	\$ 27,164.43	
ILAS Richard A. Brualdi Early Career Prize Fund	\$ 5,763.47	
ELA Fund	\$ 1,656.87	
LAMA Fund	\$ 70,719.53	
		\$ 315,621.32

**INCOME:**

Dues	\$ 10,480.00	
Elsevier	\$ 15,000.00	
Israel Gohberg ILAS-IWOTA Lecture Fund Donations	\$ 0.00	
General Fund Donations	\$ 100.00	
Conference Fund Donations	\$ 0.00	
ILAS Olga Taussky and John Todd Prize Fund Donations	\$ 0.00	
Hans Schneider Lecture Fund Donations	\$ 0.00	
Uhlig Education Fund Donations	\$ 0.00	
Hans Schneider Prize Fund Donations	\$ 110.00	
ELA Fund Donations	\$ 100.00	
ILAS Richard A. Brualdi Early Career Prize Fund Donations	\$ 10.00	
Corporate Dues Income	\$ 0.00	
Interest - First Interstate Bank	\$ 8.42	
Interest on FIB Certificate of Deposit	\$ 1,427.91	
Vanguard CD Interest	\$ 45.26	
Vanguard Unrealized Capital Gains	\$ 34,463.69	
Accounts Receivable	\$ 300.00	
Miscellaneous	\$ 383.50	
Total Income		\$ 62,428.78

**EXPENSES:**

ILAS Conference Expenses	\$ 9,773.05	
ELA	\$ 5,418.78	
IMAGE	\$ 1,500.00	
JMM Expenses	\$ 4,020.08	
Hans Schneider Prize	\$ 93.40	
Taussky-Todd Prize	\$ 45.34	
Richard Brualdi Early Career Prize	\$ 45.34	
PayPal/Credit Card Processing & Bank Fees	\$ 1,037.78	
Non-ILAS Conferences	\$ 4,353.18	
Business License	\$ 61.25	
Election Costs	\$ 89.00	
Web Hosting, MailChimp & Online Membership Forms	\$ 3,036.97	
LAA Early Career Grants	\$ 3,700.00	
ILAS/LAA Ph.D. Grants	\$ 6,300.00	
Child Care Grants	\$ 2,020.00	
Grant for Mathematicians in Conflict	\$ 1,073.50	
Total Expenses		\$ 42,567.67

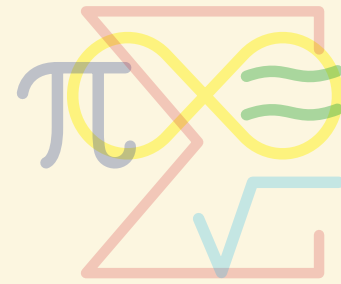
**Net Account Balance on March 31, 2026**

Checking Account - First Interstate Bank	\$ 22,798.56	
Certificate of Deposit - First Interstate Bank	\$ 25,000.00	
Certificate of Deposit - Vanguard	\$ 35,045.46	
Vanguard	\$ 240,184.85	
PayPal	\$ 12,153.47	
Accounts Receivable	\$ 300.00	
		\$ 335,482.34

General Fund	\$ 164,553.03	
Israel Gohberg ILAS-IWOTA Lecture Fund	\$ 8,004.63	
Conference Fund	\$ 12,359.93	
ILAS Olga Taussky and John Todd Prize Fund	\$ 12,876.94	
Hans Schneider Lecture Fund	\$ 11,515.91	
Frank Uhlig Education Fund	\$ 7,080.43	
Hans Schneider Prize Fund	\$ 30,092.59	
ILAS Richard A. Brualdi Early Career Prize Fund	\$ 6,576.44	
ELA Fund	\$ 1,933.00	
LAMA Fund	\$ 80,489.43	
		\$ 335,482.33

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Selected Chapters of Number Theory: Special Numbers - Vol 4

## Catalan Numbers

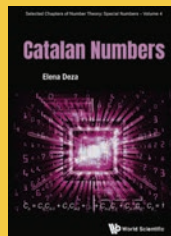
by **Elena Deza** (Moscow Pedagogical State University, Russia)

Named after mathematician Eugène Charles Catalan, these numbers appear in various combinatorial problems. With rich history, intriguing properties, and diverse applications, Catalan numbers connect arithmetic, analysis, and combinatorics. This book explores their interesting properties, history, classical and modern applications, relations to other special numbers, and open problems.

290pp | Pub date: Dec 2024

**Hardcover** 978-981-129-322-1 | **US\$98 / £90**

**eBook for Individual** 978-981-129-324-5 | **US\$78 / £70**



Monographs in Number Theory - Vol 12

## Analytic and Combinatorial Number Theory: The Legacy of Ramanujan

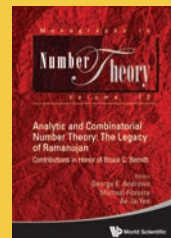
Contributions in Honor of Bruce C Berndt edited by **George E Andrews** (The Pennsylvania State University, USA), **Michael Filaseta** (University of South Carolina, USA) & **Ae Ja Yee** (The Pennsylvania State University, USA)

This volume reflects the contributions stemming from the conference Analytic and Combinatorial Number Theory: The Legacy of Ramanujan which took place at the University of Illinois at Urbana-Champaign on June 6 – 9, 2019. The conference included 26 plenary talks, 71 contributed talks, and 170 participants.

704pp | Pub. date: Sep 2024

**Hardcover** 978-981-127-736-8 | **US\$188 / £175**

**eBook for Individual** 978-981-127-738-2 | **US\$150 / £140**



## Starting Category Theory

by **Paolo Perrone** (University of Oxford, UK)

Starting Category Theory serves as an accessible and comprehensive introduction to the fundamental concepts of category theory. Originally crafted as lecture notes for an undergraduate course, it has been developed to be equally well-suited for individuals pursuing self-study. Most crucially, it deliberately caters to those who are new to category theory, not requiring readers to have a background in pure mathematics, but only a basic understanding of linear algebra.

464pp | Pub date: May 2024

**Hardcover** 978-981-128-600-1 | **US\$148 / £135**

**eBook for Individual** 978-981-128-602-5 | **US\$118 / £110**



## Principles and Techniques in Combinatorics

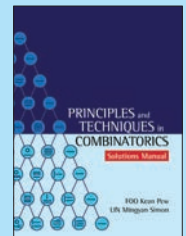
Solutions Manual

by **Kean Pew Foo, Mingyan, Simon Lin** (University of Illinois at Urbana-Champaign, USA)

Solutions are written in a relatively self-contained manner, with little undergraduate mathematics assumed. This caters to a wide range of readers, and makes it easy for the reader to understand the thought process behind the solutions to each problem.

440pp | Pub. date: Oct 2018

**Softcover** 978-981-3238-84-8 | **US\$45 / £40**



## Elements of Digital Geometry, Mathematical Morphology, and Discrete Optimization

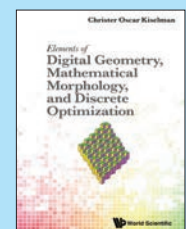
by **Christer Oscar Kiselman** (Uppsala University, Sweden)

*"The book offers an attractive option for a course dealing with set theory, general topology, analysis, and group theory, and their application to digital setting, mathematical morphology and discrete optimization."*

**Mathematical Reviews Clippings**  
**American Mathematical Society**

488pp | Pub. date: Feb 2022

**Hardcover** 978-981-124-829-0 | **US\$158 / £145**



Mathematical Olympiad Series - Vol 2

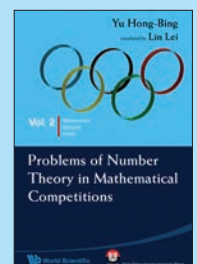
## Problems of Number Theory in Mathematical Competitions

by **Hong-Bing Yu** (Suzhou University, China) Translated by: **Lei Lin** (East China Normal University, China)

In this book, the author introduces some basic concepts and methods in elementary number theory via problems in mathematical competitions.

116pp | Pub. date: Sep 2009

**Softcover** 978-981-4271-14-1 | **US\$34 / £28**



## Principles and Techniques in Combinatorics

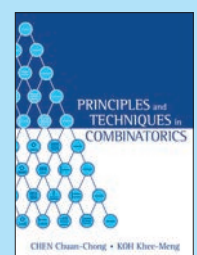
by **Chen Chuan-Chong** (NUS, Singapore), **Koh Khee-Meng** (NUS, Singapore)

*"This book should be a must for all mathematicians who are involved in the training of Mathematical Olympiad teams, but it will also be a valuable source of problems for university courses."*

**Mathematical Reviews**

312pp | Pub. date: Jul 1992

**Softcover** 978-981-02-1139-4 | **US\$39 / £32**



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## CONFERENCE REPORTS

### ILAS at the Joint Mathematics Meetings Washington, D.C., USA, January 4–7, 2026

Report by Minerva Catral

The 2026 Joint Mathematics Meetings (JMM) was held in Washington, D.C., USA, January 4–7, 2026. ILAS has been a JMM partner since 2022 and again had a strong presence at the meeting with six ILAS Special Sessions that spanned all time slots for the four days of the conference.

The ILAS Invited Address was delivered by Dominique Guillot, who gave a very engaging talk titled “The Many Facets of Matrix Positivity: A Celebration of Linear Algebra.” The talk highlighted the fundamental role that matrix positivity plays across mathematics and applications, and covered both classical and modern problems where various notions of positivity appear. The talk was very well-attended and drew great interest from mathematicians across disciplines.

The ILAS special sessions were also well-attended and provided a very good venue to promote ILAS and its various initiatives to the greater mathematics community. The titles and organizers of the ILAS special sessions were:

- Algebraic Graph Theory: New Trends (Sudipta Mallik, Milica Andelic, Zoran Stanic, Renata Del-Vecchio)
- Combinatorial Matrix Theory (Minerva Catral, Bryan Shader)
- Innovative and Effective Ways to Teach Linear Algebra (Sepideh Stewart, David M. Strong, Gilbert Strang, Megan Wawro)
- Matrix Analysis and Applications (Hugo Woerdeman, J.E. Pascoe)
- Matrix Analysis and Applications (Tin-Yau Tam, Mohsen Aliabadi, Dominique Guillot, Xiang Xiang Wang)
- Recent Advances in Model Order Reduction and Data-Driven Modeling: Theory and Computations (Steffen Werner, Ionut Farcas)

The next JMM will take place in Chicago, Illinois, USA, January 12–15, 2027. The ILAS Invited Address will be delivered by Professor Misha Kilmer of Tufts University.



*Dominique Guillot delivering the ILAS Invited Address (left) and receiving the corresponding certificate at the JMM awards ceremony (center); the ILAS education group at the ILAS Session on Innovative and Effective Ways to Teach Linear Algebra*

### The 11<sup>th</sup> Workshop on Matrix Equations and Tensor Techniques (METT XI) Leuven, Belgium, January 7–9, 2026

Report by Charlotte Vermeylen and Lieven De Lathauwer

The 11th edition of the Workshop on Matrix Equations and Tensor Techniques, organized biennially, took place for the first time in Leuven, Belgium, from January 7th to January 9th, 2026, where 63 participants gathered at the Department of Electrical Engineering (ESAT) at KU Leuven to learn about the latest developments in the field. The workshop’s steering committee consisted of Peter Benner, Heike Faßbender, Lars Grasedyck, Daniel Kressner, Beatrice Meini, Valeria Simoncini, and Lieven De Lathauwer, while the local organizing committee comprised Lieven De Lathauwer, Andreas De Vogel, and Charlotte Vermeylen.

Despite harsh weather conditions that made traveling to Leuven difficult for some participants, the effort was rewarded with a memorable group picture in the snow (see below). The workshop was also highly productive, featuring 21 talks, of which 3 were invited, alongside 13 poster presentations. The topics covered ranged from application-specific algorithms to more general methods for matrix and tensor decompositions. In particular, the program included contributions on randomized algorithms, multiparameter eigenvalue problems, Krylov methods, partial differential equations, Riemannian algorithms, numerical optimization, extrapolation techniques, tensor completion, and polynomial equations. The invited speakers were Markus Bachmayr, Jens Saak, and Mariya Ishteva. A more detailed version of the program can be found at <https://homes.esat.kuleuven.be/~mettxi/index.html>. The book of abstracts for the talks and posters can be downloaded from <https://homes.esat.kuleuven.be/~mettxi/download/booklet.pdf>.

Lunches were provided at ESAT, offering participants further opportunities to discuss the presentations and explore the posters in greater depth. The social program was extended on Wednesday evening with a reception in the Arenberg castle and on Thursday evening with a conference dinner.



*The METT XI group photo, taken in the snow in front of Arenberg Castle*

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## UPCOMING CONFERENCES AND WORKSHOPS

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### **9<sup>th</sup> Biennial Meeting of the Thematic Network on Linear Algebra, Matrix Analysis and Applications (ALAMA 2026) Heulva, Spain, June 17–19, 2026**

The Thematic Network on Linear Algebra, Matrix Analysis, and Applications (ALAMA) will hold its ninth biennial meeting at the University of Huelva, Huelva, Spain, June 17–19, 2026. The meeting will be hosted at the Faculty of Experimental Sciences of the University of Huelva, and will continue the trajectory started in Vitoria-Gasteiz (2008) and followed by Valencia, Leganés, Barcelona, León, Alicante, Alcalá de Henares and Gijón. ALAMA 2026 will be a forum for researchers to come together around linear algebra, matrix analysis, and their applications, fostering the exchange of ideas in an inclusive and diverse environment.

The plenary speakers at ALAMA 2026 will be Silvia Gazzola (Università di Pisa), Vanni Noferini (Aalto University) and Alicia Roca Martínez (Universitat Politècnica de València). In addition to the plenary lectures, the program will include parallel sessions of additional talks and minisymposia, along with opportunities for the interaction of participants and the strengthening of the academic community. Technical organization is being handled by the General Foundation of the University of Alcalá with the support of various institutions.

Registration is open until June 12th, 2026 and can be completed via:

<https://congresosalcala.fgua.es/alama2026/inscripcion/6481/online-registration>

All additional information regarding deadlines, the program, and practical details can be found on the ALAMA 2026 website at <https://www.alama2026.com>.

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## Saint-Girons V Conference

### Saint-Girons, France, June 29 – July 3, 2026



**SAINT-GIRONS CONFERENCE**  
GENERATING IDEAS FOR A NUMERICAL WORLD

Science is based on observations, experiments, development of theories, and often development of models for simulating observed phenomena and/or predicting future events. Advances in computing capabilities have had a far-reaching impact not only on simulations, but also on data analysis and artificial intelligence (AI), leading to a digital society era. Numerical tools have played a fundamental role in these advances and in high-performance computing (HPC) in general. From the first meeting in Saint-Girons, this conference series has assessed the progress made in the full use of computing capabilities and identified potential challenges for discussion in subsequent meetings. One characteristic of the meetings is that in addition to keynote speakers and established researchers, participation of students is encouraged

and the programme and budget are designed to support student attendance. The fifth meeting in the series, Saint-Girons V, will seek to cover a broad range of topics, as we move towards new frontiers of computing in applications and hardware.

Considering an evolving and increasingly complex HPC ecosystem, Saint-Girons V will give participants an opportunity to discuss pressing HPC-related topics, as well as the rise of AI and related massive data processing, the emerging area of quantum computing, and the potential implications of a post-Moore era of computing. The uniqueness of the meetings in Saint-Girons lies in the combination of a set of selected presentations, social events, and time for sharing and exchanging ideas at the pace of a picturesque mountain village. New research topics and collaborations have emerged from all these moments.

Topics of interest:

- numerical linear and multi-linear algebra
- numerical optimization
- numerical solution of PDEs
- AI for scientific computing and HPC for AI
- quantum computing
- approximate computing involving mixed precision
- low precision computation
- randomized algorithms
- low rank approximation

Keynote speakers:

- Jack Dongarra (Innovative Computing Laboratory)
- Sven Leyffer (Argonne National Laboratory)
- Per-Gunnar Martinsson (The University of Texas at Austin)
- Satoshi Matsuoka (Riken Center for Computational Science)
- Cleve Moler (MathWorks)
- Rick Stevens (Argonne National Laboratory)

Scientific Committee: Iain Duff (chair), Jean-Yves Bertou, Jack Dongarra, Laura Grigori, Torsten Hoefer, Sherry Li, Satoshi Matsuoka, Ulrich Rüde

Organization Committee: Pierre-Henri Cros (chair), Alfredo Buttari, Michel Daydé, Anaïs Douziech, Iain Duff, Serge Gratton, Osni Marques, Daniel Ruiz

For registration and further information, please visit <https://saintgironsconference.eu>.

**The 37<sup>th</sup> International Workshop on Operator Theory and its Applications (IWOTA)  
An ICM Satellite Conference  
Québec City, Canada, August 3–7, 2026**



The IWOTA conference series is the largest event in operator theory and its applications, bringing together leading international experts from pure mathematics and application areas to trace the future development of operator theory and related areas such as complex analysis, harmonic analysis, linear algebra, random matrix theory, and mathematical physics, as well as their applications, including control theory, signal processing, and machine learning.

IWOTA 2026 will take place August 3–7, 2026, at the Université Laval in Québec City, Canada. IWOTA 2026 is an International Congress of Mathematicians (ICM) Satellite Conference. The conference will provide

a medium for an intense exchange of new results, information and opinions, and for international collaboration in operator theory and its applications worldwide. It will further set directions for future research through its conference activities and proceedings. A substantial part of IWOTA consists of special sessions whose organizers have been selected to ensure a coherent, diverse and attractive agenda of research activity and talks. Special sessions provide opportunities for all participants to present their results and interact with other researchers with similar interests.

Plenary speakers of IWOTA 2026 include:

- Akram Aldroubi (Vanderbilt University, USA)
- Ilia Binder (University of Toronto, Canada)
- Emmanuel Fricain (University of Lille, France)
- Pam Gorkin (Bucknell University, USA)
- Matthew Kennedy (University of Waterloo, Canada)
- Lajos Molnár (University of Szeged, Hungary)
- Nilima Nigam (Simon Fraser University, Canada)
- Marek Ptak (University of Krakow, Poland)
- Thomas Ransford (Université Laval, Canada)
- Felix Schwenninger (University of Twente, Netherlands)
- William Slofstra (University of Waterloo, Canada)
- Joel Tropp (California Institute of Technology, USA)

The IWOTA 2026 main organizing committee consists of Javad Mashreghi and Frédéric Morneau-Guérin (Université Laval) in collaboration with the IWOTA steering committee chaired by J. William Helton (University of California, San Diego). They are further assisted by Ludovick Bouthat (Université Laval), George R. Exner (Bucknell University), Mostafa Nasri (University of Winnipeg), Marcu-Antone Orsoni (Université Laval), and William T. Ross (University of Richmond). Registration for the conference will open in Spring 2026. For updates and further information, including a list of special sessions and plenary speakers, visit <https://iwota-2026.fsg.ulaval.ca/en>.

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**Send News for *IMAGE* Issue 77**

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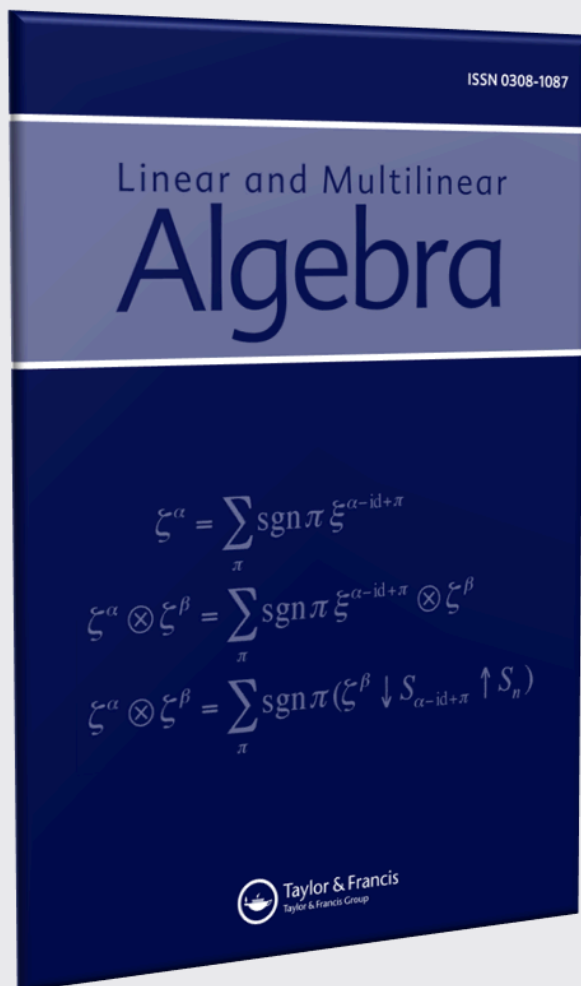
*IMAGE* seeks to publish all news of interest to the linear algebra community. Issue 77 of *IMAGE* is due to appear online on December 1, 2026. Send your news for this issue to the appropriate editor by October 15, 2026. Photos are always welcome, as well as suggestions for improving the newsletter. Please send contributions directly to the appropriate editor:

- book reviews to Mohsen Aliabadi ([maliabadisr@ucsd.edu](mailto:maliabadisr@ucsd.edu))
- linear algebra education news and articles to Sepideh Stewart ([sepidehstewart@ou.edu](mailto:sepidehstewart@ou.edu))
- interviews of senior linear algebraists to the editor-in-chief, Louis Deaett ([louis.deaett@quinnipiac.edu](mailto:louis.deaett@quinnipiac.edu))
- problems and solutions to Jeffrey Stuart ([jeffrey.stuart@plu.edu](mailto:jeffrey.stuart@plu.edu))
- advertisements to Amy Wehe ([awehe@fitchburgstate.edu](mailto:awehe@fitchburgstate.edu))
- announcements and reports of conferences/workshops/etc. to Jephian C.-H. Lin ([jephianlin@gmail.com](mailto:jephianlin@gmail.com))
- other articles and proposals to the editor-in-chief, Louis Deaett ([louis.deaett@quinnipiac.edu](mailto:louis.deaett@quinnipiac.edu))

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For past issues of *IMAGE*, please visit <https://www.ilasic.org/IMAGE>.

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## ONGOING ONLINE SEMINARS

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### Algebraic Graph Theory Seminar

<https://math.uwaterloo.ca/~agtheory>

**Host:** University of Waterloo

**Schedule:** weekly on Mondays

**Time:** 11:30AM, Waterloo (Ontario, Canada) time

**Most recent talk:**

*Applications of association schemes via Delsarte theory*

Jesse Lansdown (University of Galway, Ireland)

**Next talk:** TBA

**Contact:** Sabrina Lato ([smlato@uwaterloo.ca](mailto:smlato@uwaterloo.ca))

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### Matrix Seminar

<https://docs.google.com/document/d/1MswSd16JqsZE294kYCXujLio4cnAiuYv6QKRc6BxvI0/edit>

**Host:** University of Nevada, Reno

**Schedule:** biweekly on Fridays

**Time:** 4:15PM, Reno (Nevada, USA) time

**Most recent talk:**

*A Mostow Decomposition of Symmetric Cones*

Khalid Koufany (University of Lorraine, France)

**Next talk:** TBA

**Contact:** Pan Shun Lau ([plau@unr.edu](mailto:plau@unr.edu))

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### 05C50 Online

<https://sites.google.com/view/05c50online/home>

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**Schedule:** biweekly on Fridays

**Time:** 10:00AM, Winnipeg (Manitoba, Canada) time

**Most recent talk:**

*Graphs whose least eigenvalue does not change after deleting the neighbourhood of any vertex*

Domingos Cardoso (University of Aveiro, Portugal)

**Next talk:** September 2026

**Contact:** Hermie Monterde ([monterdh@myumanitoba.ca](mailto:monterdh@myumanitoba.ca))

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### Matrix Analysis and Linear Algebra Group Seminar

<https://sites.google.com/up.edu.ph/mala/seminar>

**Host:** University of the Philippines Diliman

**Schedule:** biweekly on Wednesdays

**Time:** 3:00PM, Philippine Standard Time

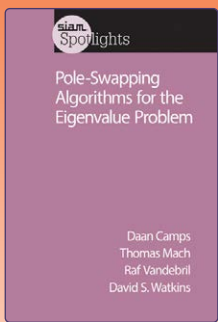
**Most recent talk:**

*On the  $m$ th Roots of a Complex Matrix*

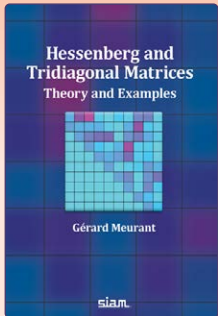
Jesus Paolo E. Joven (University of the Philippines Diliman, Philippines)

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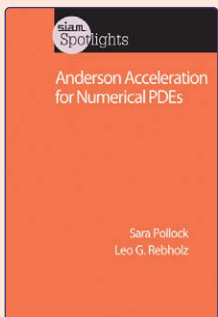
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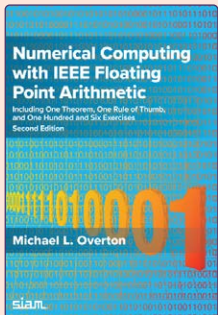
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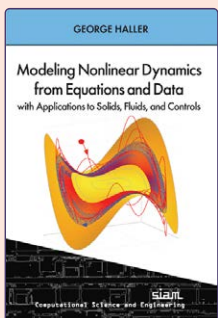
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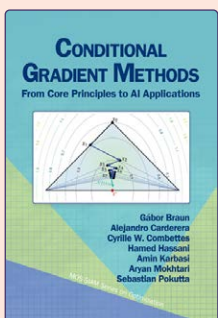
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## IMAGE PROBLEM CORNER: OLD PROBLEMS WITH SOLUTIONS

We present solutions to Problem 75-1. Solutions are still sought for Problems 68-2, 68-4, 69-2, 69-3, 70-2, 71-3, 72-1, 72-2, and 73-2, and we also invite solutions to the one new problem in the present issue 76.

### Problem 75-1: A Very Revealing Factorization

Proposed by Eugene A. HERMAN, *Grinnell College, Grinnell, Iowa, USA*, eaherman@gmail.com

Let  $n$  be a positive integer, and let  $M_n(\mathbb{C})$  denote the set of all  $n \times n$  complex matrices. Suppose that  $A \in M_n(\mathbb{C})$  satisfies  $A^2 = 0$ . Prove that there exist  $B, C \in M_n(\mathbb{C})$  such that  $A = BC$  and  $CB = 0$ .

**Solution 75-1.1** by Roger A. HORN, *University of Utah, Salt Lake City, Utah, USA*, rhorn@math.utah.edu

If  $A = 0$ , we may take  $B = C = 0$ , so assume that  $A \neq 0$ . Let

$$J_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$

and notice that  $J_2 = E_2 J_2$  and  $J_2 E_2 = 0$ . Since  $A^2 = 0$ , there is a nonsingular  $S \in M_n(\mathbb{C})$  and a positive integer  $k$  such that  $1 \leq k \leq n/2$  and  $A = S \left( \left( \bigoplus_{i=1}^k J_2 \right) \oplus 0_{n-2k} \right) S^{-1}$ . If we take  $B = S \left( \left( \bigoplus_{i=1}^k E_2 \right) \oplus 0_{n-2k} \right) S^{-1}$  and  $C = A$ , then  $BC = A$  and  $CB = 0$ .

A similar solution was given by each of Subhasish BEHERA, *Binayak Acharya College, Berhampur, India*, sb52@iitbbs.ac.in, Debarota MONDAL, *Indian Institute of Technology Guwahati, Guwahati, India*, dm27@iitbbs.ac.in, M.J. DE LA PUENTE, *Universidad Complutense de Madrid, Madrid, Spain*, mpuente@ucm.es, and Piyush VERMA, *Indian Institute of Technology Bhubaneswar, Odisha, India*, s21ma09010@iitbbs.ac.in.

**Solution 75-1.2** by Carl MEYER, *North Carolina State University, Raleigh, North Carolina, USA*, meyer@ncsu.edu

Assume  $A \neq 0$  and let  $r = \text{rank}(A)$ . Then  $A^2 = 0$  implies  $0 < r < n$ . Let  $A = XY$ , where  $X$  is  $n \times r$  and  $Y$  is  $r \times n$  with  $\text{rank}(X) = r = \text{rank}(Y)$ . This is generally called a *full-rank factorization* [2, p.190]. Since  $X$  has full column rank and  $Y$  has full row rank, we have [1, Section 1.3, Lemma 2] that  $X^\dagger X = I = Y Y^\dagger$ , where  $\star^\dagger$  denotes the Moore–Penrose pseudoinverse. From  $0 = A^2 = X(YX)Y$ , then, multiplication on the left by  $X^\dagger$  and on the right by  $Y^\dagger$  yields  $0 = YX$ . Therefore, the matrices

$$B = \left( X \mid 0_{n \times (n-r)} \right) \quad \text{and} \quad C = \begin{pmatrix} Y \\ \hline 0_{(n-r) \times n} \end{pmatrix}$$

produce the desired conclusion.

### References

- [1] A. Ben-Israel and T. N. E. Greville. *Generalized Inverses: Theory and Applications*, volume 15 of *CMS Books in Mathematics*. Springer-Verlag, New York, second edition, 2003.
- [2] C. D. Meyer. *Matrix Analysis and Applied Linear Algebra*. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, second edition, 2023.

**Solution 75-1.3** by Giovanni BARBARINO, *University of Mons, Mons, Belgium*, giovanni.barbarino@gmail.com

By properties of the Moore–Penrose inverse  $A^\dagger$ , we have

$$A = AA^\dagger A = (AA^\dagger)A \quad \text{and} \quad A(AA^\dagger) = A^2 A^\dagger = 0,$$

so it is enough to take  $B = AA^\dagger$  and  $C = A$ .

**Solution 75-1.4** by Oskar Maria BAKSALARY, *Adam Mickiewicz University, Poznań, Poland*, [obaksalary@gmail.com](mailto:obaksalary@gmail.com) and Götz TRENKLER, *Dortmund University of Technology, Dortmund, Germany*, [trenkler@statistik.tu-dortmund.de](mailto:trenkler@statistik.tu-dortmund.de)

This solution is based on [2, Corollary 6]. According to the decomposition established therein, every  $n \times n$  complex matrix  $A$  of rank  $r$  can be represented as

$$A = U \begin{pmatrix} \Sigma K & \Sigma L \\ 0 & 0 \end{pmatrix} U^*, \quad (1)$$

where  $U \in M_n(\mathbb{C})$  is unitary,  $\Sigma = \text{diag}(\sigma_1 I_{r_1}, \dots, \sigma_t I_{r_t})$  is the diagonal matrix of singular values  $\sigma_1 > \sigma_2 > \dots > \sigma_t > 0$  of  $A$ , and  $r_1 + r_2 + \dots + r_t = r$ , with  $K$  an  $r \times r$  complex matrix and  $L$  an  $r \times n - r$  complex matrix satisfying

$$KK^* + LL^* = I_r. \quad (2)$$

From (1) it follows that  $A^2 = 0$  if and only if  $\Sigma K \Sigma K = 0$  and  $\Sigma K \Sigma L = 0$ , or, in view of the nonsingularity of  $\Sigma$ ,  $K \Sigma K = 0$  and  $K \Sigma L = 0$ . Combining modified versions of these equalities, obtained by postmultiplying the former one by  $K^*$  and the latter one by  $L^*$ , gives  $K \Sigma K K^* + K \Sigma L L^* = 0$ , which, by (2), yields  $K \Sigma = 0$ . By referring once again to the fact that  $\Sigma$  is nonsingular, we arrive at  $K = 0$ . Since sufficiency is clearly visible, we conclude that  $A^2 = 0 \iff K = 0$ . In consequence,

$$A = U \begin{pmatrix} 0 & \Sigma L \\ 0 & 0 \end{pmatrix} U^*,$$

where  $LL^* = I_r$ . Picking now

$$B = U \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 0 & L \\ 0 & 0 \end{pmatrix} U^*$$

visibly yields  $A = BC$  and  $CB = 0$ .

A related fact is that when  $A^2 = 0$ , there exist idempotent matrices  $B$  and  $C$  such that  $A = BC - CB$ , the result asserted in [1, Proposition 6].

#### References

- [1] R. Drnovšek, H. Radjavi, and P. Rosenthal. A characterization of commutators of idempotents. *Linear Algebra Appl.*, 347:91–99, 2002.
- [2] R. E. Hartwig and K. Spindelböck. Matrices for which  $A^*$  and  $A^\dagger$  commute. *Linear and Multilinear Algebra*, 14(3):241–256, 1983.

A similar solution was given by Éric PITÉ, *Paris, France*, [eric.pite@telecom-paristech.org](mailto:eric.pite@telecom-paristech.org).

**Solution 75-1.5** by Teng ZHANG, *Xi'an Jiaotong University, Xi'an, China*, [teng.zhang@stu.xjtu.edu.cn](mailto:teng.zhang@stu.xjtu.edu.cn)

Let  $V = \mathbb{C}^n$  and let  $T : V \rightarrow V$  be the linear operator represented by  $A$  with respect to the standard basis. Since  $A^2 = 0$ , we have  $T^2 = Z$ , the zero operator on  $V$ . Thus,  $T(Tv) = 0$  for all  $v \in V$ , giving  $\text{Im}(T) \subseteq \ker(T)$  where  $\text{Im}(T)$  and  $\ker(T)$  are the image and the kernel of  $T$ , respectively. Since  $V$  is finite-dimensional, there is a subspace  $W$  of  $V$  such that  $V = \ker(T) \oplus W$ . That is, for each  $v \in V$ , there exist unique  $k \in \ker(T)$  and  $w \in W$  such that  $v = k + w$ . Define  $P$  to be the orthogonal projection of  $V$  onto  $W$ . That is,  $P(v) = P(k + w) = w$  for all  $v \in V$ , making  $P$  a linear operator on  $V$ . Clearly,  $\text{Im}(P) = W$  and  $\ker(P) = \ker(T)$ . For each  $v \in V$ ,

$$(T \circ P)(v) = T(P(v)) = T(w) = T(w) + T(k) = T(v).$$

That is,  $T \circ P = T$ . Letting  $C$  be the matrix representing  $P$  with respect to the standard basis, this is equivalent to  $AC = A$ . In addition, from the observations above, we have  $T(V) = \text{Im}(T) \subseteq \ker(T) = \ker(P)$ , giving

$$(P \circ T)(V) = P(T(V)) \subseteq P(\ker(P)) = 0.$$

That is,  $P \circ T = Z$  as operators on  $V$ . Expressed in terms of matrices,  $CA = 0$ . Set  $B = A$  to get the stated conclusion.

**Solution 75-1.6** by K.C. SIVAKUMAR, *Indian Institute of Technology Madras, Chennai, India*, [kcskumar@iitm.ac.in](mailto:kcskumar@iitm.ac.in)

We show that the statement of the problem applies to a larger class of matrices. Recall that  $A \in M_n(\mathbb{C})$  is said to be *group invertible* if there exists  $X \in M_n(\mathbb{C})$  such that  $AXA = A$ ,  $XAX = X$  and  $AX = XA$ . While these equations may fail to have a solution for a given matrix  $A$ , when such a solution does exist, it is unique. In that case, we denote the unique  $X$  by  $A^\#$ . It is well known [1, Section 4.4, Theorem 1] that  $A^\#$  exists if and only if  $\text{rank}(A) = \text{rank}(A^2)$ . From this, it follows that the group inverse does not exist for a nilpotent matrix.

**Theorem.** *Let  $A \in M_n(\mathbb{C})$  be such that  $A^\#$  does not exist. Then there exist  $B, C \in M_n(\mathbb{C})$  such that  $A = BC$  and  $CB$  is not invertible. In particular, if  $A$  is nilpotent of index  $\ell$ , then the matrices  $B$  and  $C$  may be chosen such that  $(CB)^{\ell-1} = 0$ . Specifically, if  $A^2 = 0$ , then there exist  $B, C \in M_n(\mathbb{C})$  such that  $A = BC$  and  $CB = 0$ .*

*Proof.* Assume  $A \neq 0$ . Let  $r = \text{rank}(A)$ . Since  $A^\#$  does not exist, we have by [1, Theorem 4.4.1] that  $\text{rank}(A) \neq \text{rank}(A^2)$ , and so  $0 < r < n$ . Let  $A = FG$  be a full-rank factorization of  $A$ . That is,  $F$  is  $n \times r$  and  $G$  is  $r \times n$  with  $\text{rank}(F) = \text{rank}(G) = r$ .

Since  $\text{rank}(A) < \text{rank}(A^2)$ , some vector  $w$  has  $Aw \neq 0$  while  $A^2w = 0$ . In particular,  $0 \neq Aw = FGw = F(Gw)$ , so that  $Gw \neq 0$ . At the same time,  $0 = A^2w = (FG)(FG)w = F(GF(Gw))$ . Since  $F$  has full column rank, this gives  $GF(Gw) = 0$ . Hence, letting  $v = Gw$  gives a nonzero vector  $v$  with  $(GF)v = 0$  and  $Fv \neq 0$ .

Since  $F$  has full column rank, it has a left inverse  $H$  giving  $HF = I_r$ . Let  $B = FH$  and let  $C = A$ . Then

$$BC = (FH)A = (FH)(FG) = F(HF)G = FG = A,$$

and  $CB = A(FH) = (FG)(FH) = F(GF)H$ . Moreover,

$$(CB)(Fv) = (F(GF)H)(Fv) = F(GF)(HF)v = F(GF)v = F(0) = 0,$$

showing that  $CB$  is singular, and hence not invertible, as claimed.

Finally, suppose  $A$  is nilpotent of index  $\ell$ , with  $\ell \geq 2$ . Then we obtain  $0 = A^\ell = (FG)^\ell = F(GF)^{\ell-1}G$ . Since  $F$  has independent columns,  $(GF)^{\ell-1}G = 0$ . Since  $G$  has independent rows,  $(GF)^{\ell-1} = 0$ . For  $\ell = 2$ , this gives  $CB = F(GF)H = F(0)H = 0$ , while for  $\ell > 2$ , we have (using  $HF = I_r$ ) that

$$\begin{aligned} (CB)^{\ell-1} &= (F(GF)H)^{\ell-1} \\ &= \underbrace{(F(GF)H) \cdots (F(GF)H)}_{\ell-2 \text{ factors of } F(GF)H} (F(GF)H) \\ &= F \underbrace{(GF)(HF)(GF)(HF) \cdots (GF)(HF)(GF)H}_{\ell-2 \text{ factors of } (GF)(HF)} \\ &= F \underbrace{(GF)(GF) \cdots (GF)(GF)H}_{\ell-2 \text{ factors of } GF} \\ &= F(GF)^{\ell-1}H = F(0)H = 0. \end{aligned}$$

□

## References

- [1] A. Ben-Israel and T. N. E. Greville. *Generalized Inverses: Theory and Applications*, volume 15 of *CMS Books in Mathematics*. Springer-Verlag, New York, second edition, 2003.

*Editor's Note:* It is also possible to extend some of the solutions above to obtain the following more general result that contains the original result as a special case.

**Theorem.** *Let  $m$  and  $n$  be positive integers with  $m \geq 2$ . Suppose  $A \in M_n(\mathbb{C})$  such that  $A^m = 0$ . Then there exist  $B, C \in M_n(\mathbb{C})$  such that  $A = BC$ ,  $(CB)^{m-1} = 0$ , and  $\text{rank}(A) = \text{rank}(B) = \text{rank}(C)$ .*

A proof of this theorem can be obtained as an extension of Solution 75-1.1 and also as an extension of Solution 75-1.3.

Extending Solution 75-1.1: Since  $A^m = 0$ , there is an invertible  $S \in M_n(\mathbb{C})$  such that  $SAS^{-1} = J$ , where  $J$  is a strictly upper triangular Jordan form of  $A$ . Further,  $J$  can be chosen so that there exist a positive integer  $k \leq n$  and a sequence of positive integers  $n_1, n_2, \dots, n_k$  satisfying  $m \geq n_1 \geq n_2 \geq \dots \geq n_k$  and  $n = n_1 + n_2 + \dots + n_k$  such that

$$J = J_{n_1} \oplus J_{n_2} \oplus \dots \oplus J_{n_k},$$

where  $J_{n_h}$  for  $1 \leq h \leq k$  is an  $n_h \times n_h$  upper triangular Jordan block for 0. We may now observe the equivalences below.

$$\begin{aligned} A^m = 0 &\iff J^m = 0 \\ A = BC &\iff J = (SBS^{-1})(SCS^{-1}) \\ (CB)^{m-1} = 0 &\iff ((SCS^{-1})(SBS^{-1}))^{m-1} = 0 \end{aligned}$$

Consequently, we will assume  $A = J$  and  $S = I_n$ .

For each  $h$ , if  $n_h > 1$ , then let  $B_h$  be the  $n_h \times n_h$  diagonal matrix  $B_h = \text{diag}(1, 1, \dots, 1, 0)$ , while if  $n_h = 1$ , then let  $B_h = [0]$ . Thus,  $\text{rank}(B_h) = \text{rank}(J_{n_h})$  and  $B_h J_{n_h} = J_{n_h}$  for each  $h$ . When  $n_h = 1$ , we have  $J_{n_h} B_h = [0]$ . When  $n_h > 1$ , we obtain  $J_{n_h} B_h$  from  $J_{n_h}$  by replacing the 1 in the last column of  $J_{n_h}$  with a column of zeros. Thus, when  $n_h > 1$ ,

$$J_{n_h} B_h = \begin{bmatrix} J_{n_h-1} & \hat{0} \\ \hat{0}^T & 0 \end{bmatrix},$$

where  $\hat{0}$  is the zero column vector in  $\mathbb{C}^{n_h-1}$ . Since  $n_h - 1 < n_h \leq m$ , we have  $(J_{n_h} B_h)^{m-1} = 0_{n_h \times n_h}$ . Letting  $C = J$  and  $B = B_1 \oplus B_2 \oplus \dots \oplus B_k$ , it follows that  $BC = A$  and  $(CB)^{m-1} = 0$ , and it is clear that  $\text{rank}(B) = \text{rank}(C) = \text{rank}(J)$ .

Extending Solution 75-1.3: Let  $B = AA^\dagger$ , where  $A^\dagger$  denotes the Moore–Penrose inverse of  $A$ , and let  $C = A$ . Then  $BC = AA^\dagger A = A$  by the definition of the Moore–Penrose inverse. Obviously,  $\text{rank}(C) = \text{rank}(A)$ . Since the rank of a matrix product is bounded above by the rank of each factor,  $\text{rank}(B) = \text{rank}(AA^\dagger) \leq \text{rank}(A)$  while at the same time  $\text{rank}(B) \geq \text{rank}(BC) = \text{rank}(A)$ , so that  $\text{rank}(B) = \text{rank}(A)$ . Finally,

$$\begin{aligned} (CB)^{m-1} &= \underbrace{(AAA^\dagger)(AAA^\dagger)(AAA^\dagger) \cdots (AAA^\dagger)(AAA^\dagger)}_{m-1 \text{ factors of } AAA^\dagger} \\ &= A \underbrace{(AA^\dagger)A(AA^\dagger)A(AA^\dagger) \cdots A(AA^\dagger)A(AA^\dagger)}_{\text{with the singleton } A \text{ occurring as a factor } m-1 \text{ times}} \\ &= A \underbrace{(AA^\dagger A)(AA^\dagger A)(AA^\dagger A) \cdots (AA^\dagger A)(AA^\dagger A)}_{m-2 \text{ factors of } AA^\dagger A} \\ &= AA^{m-2}(AA^\dagger) = A^m A^\dagger = 0A^\dagger = 0. \end{aligned}$$

## IMAGE PROBLEM CORNER: NEW PROBLEMS

**Problems:** We offer one new problem in this issue and invite readers to submit solutions for publication in *IMAGE*.

**Submissions:** Please submit proposed problems and solutions in macro-free L<sup>A</sup>T<sub>E</sub>X along with the PDF file by e-mail to *IMAGE* Problem Corner editor Jeff Stuart ([jeffrey.stuart@plu.edu](mailto:jeffrey.stuart@plu.edu)).

### NEW PROBLEMS:

#### Problem 76-1: Cholesky Decomposition and 1-Condition Number of a Ubiquitous Matrix

Proposed by M.J. DE LA PUENTE, *Universidad Complutense de Madrid, Madrid, Spain*, [mpuente@ucm.es](mailto:mpuente@ucm.es)

Let  $d$  be a positive integer, and let  $a$  and  $\ell$  be real numbers with  $0 < a < \ell$ . Define

$$A_d = aJ_d + (\ell - a)I_d,$$

where  $I_d$  is the  $d \times d$  identity matrix and  $J_d$  is the  $d \times d$  all-1s matrix. Show that  $A_d$  is positive definite, find its Cholesky decomposition, and compute its 1-condition number (i.e., its condition number with respect to the  $L_1$ -norm).